Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.4-Improper/1.1.4.2-c-x-^m-a-x^j+b-x^n-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [454]. This is test number [30].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (454)	% 0.00 (0)
Mathematica	% 100.00 (454)	% 0.00 (0)
Maple	% 84.80 (385)	% 15.20 (69)
Maxima	% 33.70 (153)	% 66.30 (301)
Fricas	% 56.39 (256)	% 43.61 (198)
Sympy	% 25.11 (114)	% 74.89 (340)
Giac	% 52.42 (238)	% 47.58 (216)
Mupad	% 42.51 (193)	% 57.49 (261)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

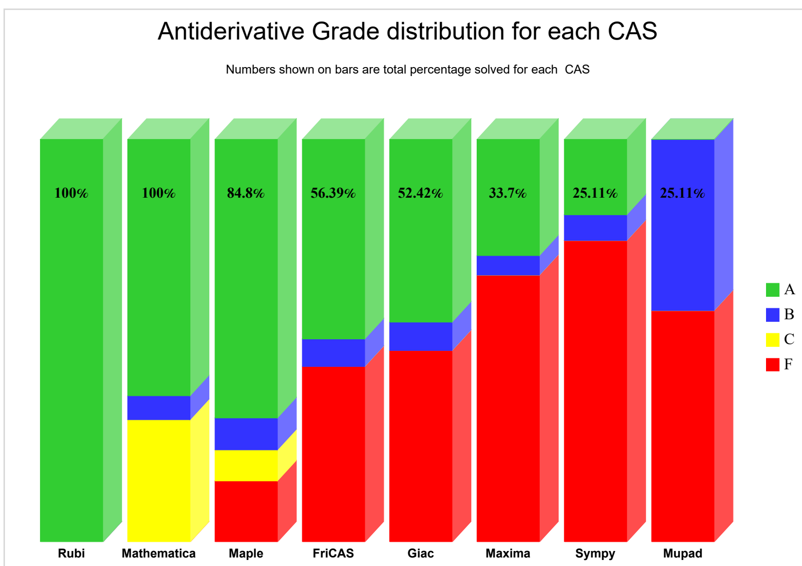
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

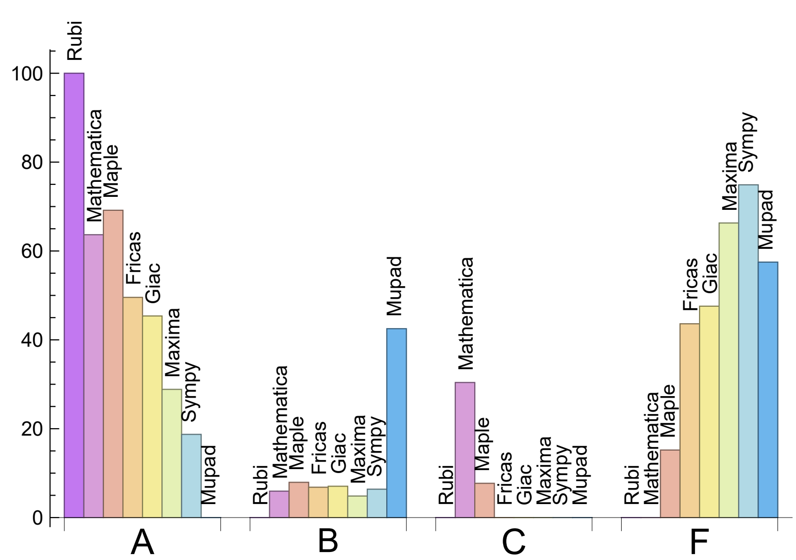
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	63.66	5.95	30.40	0.00
Maple	69.16	7.93	7.71	15.20
Maxima	28.85	4.85	0.00	66.30
Fricas	49.56	6.83	0.00	43.61
Sympy	18.72	6.39	0.00	74.89
Giac	45.37	7.05	0.00	47.58
Mupad	0.00	42.51	0.00	57.49

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	69	100.00 %	0.00 %	0.00 %
Maxima	301	100.00 %	0.00 %	0.00 %
Fricas	198	51.52 %	25.76 %	22.73 %
Sympy	340	86.76 %	13.24 %	0.00 %
Giac	216	88.43 %	0.00 %	11.57 %
Mupad	261	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

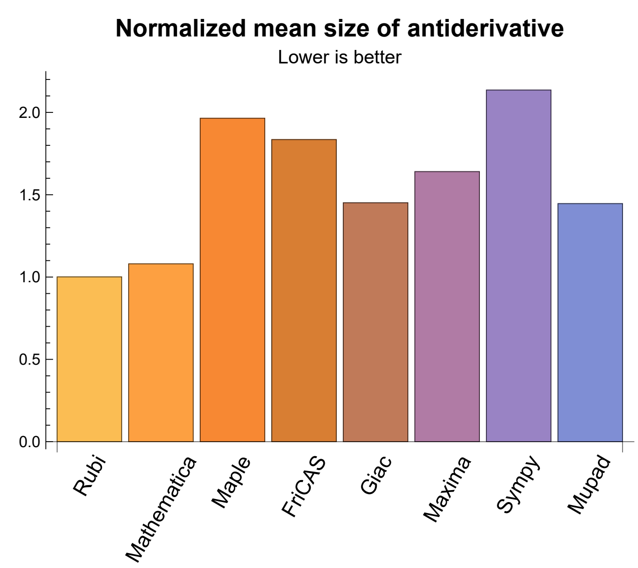
1.3 Performance

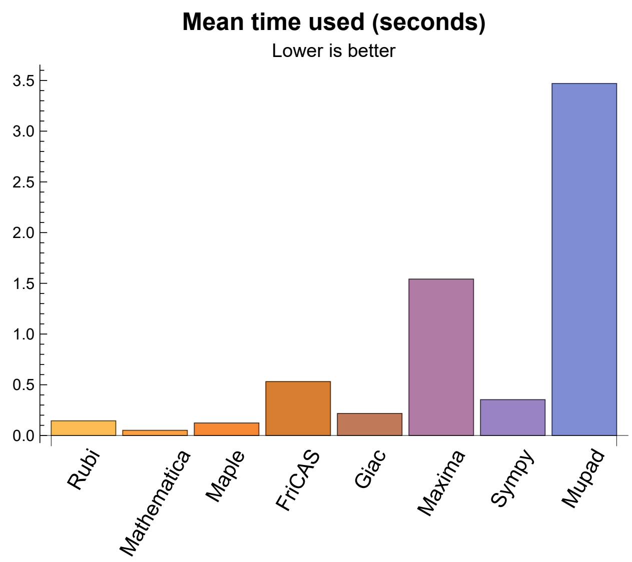
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	119.28	1.00	74.00	1.00
Mathematica	0.05	62.66	1.08	55.00	0.89
Maple	0.12	192.66	1.96	79.00	0.94
Maxima	1.54	42.06	1.64	27.00	0.89
Fricas	0.53	78.83	1.83	57.00	1.18
Sympy	0.35	45.86	2.14	26.00	0.88
Giac	0.22	79.02	1.45	47.00	0.97
Mupad	3.47	46.89	1.45	37.00	0.87

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {347}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

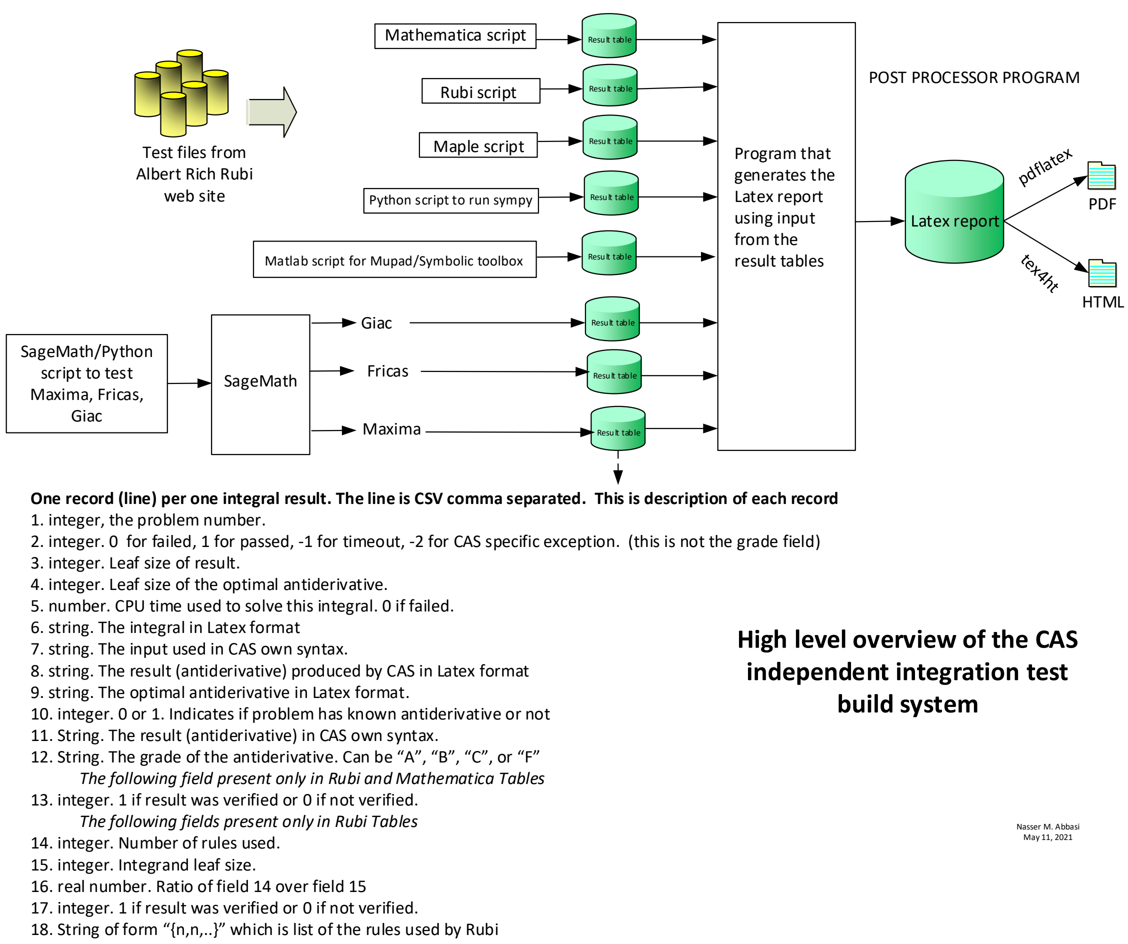
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 89, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 167, 168, 169, 170, 171, 176, 177, 178, 179, 180, 185, 186, 187, 188, 189, 190, 194, 195, 196, 197, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 240, 241, 242, 243, 244, 245, 246, 248, 252, 253, 254, 255, 256, 257, 260, 261, 262, 263, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 331, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 407, 408, 409, 411, 412, 413, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { 26, 28, 30, 32, 34, 328, 329, 330, 332, 333, 334, 349, 350, 351, 352, 353, 354, 392, 410, 414, 415, 416, 417, 418, 419, 420, 421 }

C grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 86, 88, 90, 97, 98, 99, 100, 101, 102, 110, 111, 112, 124, 125, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 172, 173, 174, 175, 181, 182, 183, 184, 191, 192, 193, 198, 199, 200, 201, 202, 238, 239, 247, 249, 250, 251, 258, 259, 264, 265, 266, 267, 268, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 402, 403, 404, 405, 406, 437, 438 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 94, 95, 96, 103, 105, 117, 118, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 276, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 303, 306, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 355, 356, 358, 360, 362, 363, 369, 377, 382, 383, 384, 385, 386, 388, 389, 394, 402, 408, 431, 432, 433, 434, 435, 436, 448, 449, 450 }

B grade: { 28, 32, 86, 104, 110, 111, 112, 119, 120, 124, 125, 126, 127, 272, 313, 328, 329, 330, 331, 332, 333, 334, 335, 336, 348, 349, 350, 351, 352, 353, 354, 357, 359, 361, 387, 412 }

C grade: { 92, 93, 97, 98, 99, 100, 101, 102, 106, 107, 108, 109, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 307, 407, 411 }

F grade: { 277, 278, 279, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 409, 410, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 34, 35, 36, 37, 94, 95, 96, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 260, 261, 262, 263, 284, 285, 286, 303, 306, 308, 309, 319, 320, 321, 322, 323, 324, 325, 326, 338, 339, 340, 344, 345, 346, 347, 349, 352, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 383, 387, 394, 402, 433, 434, 435, 436, 448, 449, 450 }

B grade: { 28, 30, 32, 283, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 348, 350, 351, 353 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 256, 257, 258, 259, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 384, 385, 386, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 451, 452, 453, 454 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 108, 109, 113, 114, 115, 116, 121, 122, 123, 128, 129, 130, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 280, 281, 282, 284, 285, 286, 287, 288, 297, 300, 303, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 338, 339, 340, 344, 345, 346, 355, 356, 357, 358, 359, 360, 361, 362, 363, 369, 377, 382, 383, 384, 385, 386, 387, 388, 389, 394, 402, 407, 408, 409, 410, 411, 412, 413, 414, 416, 419, 431, 432, 433, 434, 435, 436, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

B grade: { 28, 30, 32, 34, 79, 84, 127, 283, 323, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 341, 342, 343, 347, 348, 349, 350, 351, 352, 353, 354 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 98, 99, 100, 101, 102, 103, 104, 105, 110, 111, 112, 117, 118, 119, 120, 124, 125, 126, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 277, 278, 279, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 415, 417, 418, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 16, 18, 20, 22, 23, 24, 25, 27, 29, 31, 33, 34, 35, 36, 37, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 308, 309, 319, 320, 321, 322, 324, 325, 326, 327, 338, 339, 340, 344, 345, 346, 347, 355, 356, 357, 358, 359, 360, 362, 363, 369, 377, 394, 433 }

B grade: { 9, 13, 15, 17, 19, 21, 26, 28, 30, 32, 283, 323, 328, 329, 330, 332, 333, 334, 341, 342, 343, 349, 350, 351, 352, 353, 354, 361, 402 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 314, 315, 316, 317, 318, 331, 335, 336, 337, 348, 364, 365, 366, 367, 368, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 36, 37, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 103, 104, 105, 106, 107, 108, 109, 112, 113, 117, 118, 119, 120, 121, 122, 123, 127, 167, 168, 169, 170, 171, 172, 173, 174, 175, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 255, 256, 262, 263, 269, 270, 271, 272, 273, 274, 275, 276, 283, 287, 288, 297, 300, 303, 306, 308, 309, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 352, 354, 360, 361, 382, 384, 386, 387, 388, 408, 412, 432, 433, 434, 435, 436 }

B grade: { 26, 28, 30, 32, 34, 176, 177, 178, 179, 235, 240, 241, 242, 243, 244, 245, 328, 329, 330, 331, 332, 333, 334, 335, 336, 348, 350, 351, 353, 362, 383, 431 }

C grade: { }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 97, 98, 99, 100, 101, 102, 110, 111, 114, 115, 116, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 252, 253, 254, 257, 258, 259, 260, 261, 264, 265, 266, 267, 268, 277, 278, 279, 280, 281, 282, 284, 285, 286, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 301, 302, 304, 305, 307, 310, 311, 325, 326, 327, 337, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 409, 410, 411, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 41, 48, 61, 72, 94, 95, 96, 98, 105, 113, 134, 142, 153, 162, 170, 178, 189, 198, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 258, 260, 261, 262, 263, 266, 268, 280, 281, 282, 283, 284, 285, 286, 291, 308, 309, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 371, 382, 383, 384, 386, 387, 388, 392, 415, 416, 417, 418, 419, 420, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 444 }

C grade: { }

F grade: { 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 97, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 171, 172, 173, 174, 175, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 199, 200, 201, 202, 235, 237, 238, 239, 246, 247, 248, 249, 250, 251, 256, 257, 259, 264, 265, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 310, 311, 312, 313, 364, 365, 366, 367, 368, 369, 370, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 385, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 421, 422, 423, 424, 425, 439, 440, 441, 442, 443, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.005	0.002	0.039	1.297	0.485	0.098	0.162	0.022

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.004	0.002	0.041	1.303	0.484	0.085	0.147	0.022

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.000	0.043	1.349	0.637	0.059	0.146	0.019

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.003	0.000	0.045	1.338	0.730	0.064	0.146	0.017

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	11	11	10	14	11
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.77	1.08	0.85
time (sec)	N/A	0.005	0.001	0.052	1.304	0.573	0.110	0.150	0.024

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.015	0.001	0.040	1.307	0.589	0.071	0.148	0.041

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.020	0.001	0.038	1.341	0.796	0.071	0.157	0.033

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.011	0.002	0.043	1.318	0.723	0.073	0.150	0.030

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	25	24	24	24	24	24
normalized size	1	1.00	1.00	1.56	1.50	1.50	1.50	1.50	1.50
time (sec)	N/A	0.007	0.002	0.042	1.292	0.671	0.073	0.197	0.031

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	21	21	22	21	21
normalized size	1	1.00	1.00	0.88	0.84	0.84	0.88	0.84	0.84
time (sec)	N/A	0.012	0.001	0.051	1.296	0.888	0.078	0.152	0.030

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	36	36	42	36	36
normalized size	1	1.00	1.00	0.80	0.78	0.78	0.91	0.78	0.78
time (sec)	N/A	0.016	0.002	0.036	1.332	0.537	0.074	0.153	0.039

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	23	22	20	24	22
normalized size	1	1.00	1.00	0.89	0.85	0.81	0.74	0.89	0.81
time (sec)	N/A	0.023	0.005	0.043	1.322	0.783	0.152	0.155	0.040

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	26	82	56	26	23
normalized size	1	1.00	1.00	0.87	0.84	2.65	1.81	0.84	0.74
time (sec)	N/A	0.016	0.009	0.044	2.942	0.828	0.166	0.148	4.913

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.007	0.002	0.044	1.423	0.877	0.128	0.191	4.916

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	15	67	53	15	16
normalized size	1	1.00	1.00	0.67	0.62	2.79	2.21	0.62	0.67
time (sec)	N/A	0.009	0.004	0.044	2.947	0.589	0.147	0.153	0.045

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	18	15	24	18
normalized size	1	1.00	1.00	0.95	0.91	0.82	0.68	1.09	0.82
time (sec)	N/A	0.012	0.005	0.046	1.343	0.802	0.211	0.147	0.063

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	30	29	82	65	29	26
normalized size	1	1.00	1.00	0.88	0.85	2.41	1.91	0.85	0.76
time (sec)	N/A	0.017	0.013	0.049	2.962	0.718	0.222	0.153	4.956

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	31	33	31	43	31
normalized size	1	1.00	1.00	0.91	0.89	0.94	0.89	1.23	0.89
time (sec)	N/A	0.026	0.007	0.051	1.269	0.596	0.292	0.167	0.060

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	39	40	106	87	40	37
normalized size	1	1.00	1.00	0.91	0.93	2.47	2.02	0.93	0.86
time (sec)	N/A	0.026	0.023	0.047	2.967	0.641	0.252	0.153	4.937

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	44	44	45	42	57	46
normalized size	1	1.00	1.00	0.90	0.90	0.92	0.86	1.16	0.94
time (sec)	N/A	0.033	0.007	0.054	1.388	0.921	0.335	0.157	0.062

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	35	120	78	35	33
normalized size	1	1.00	1.00	0.80	0.78	2.67	1.73	0.78	0.73
time (sec)	N/A	0.015	0.027	0.049	2.984	0.677	0.236	0.169	4.955

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	33	35	34	47	34	47	34
normalized size	1	1.00	0.87	0.92	0.89	1.24	0.89	1.24	0.89
time (sec)	N/A	0.028	0.018	0.054	1.402	0.716	0.334	0.158	0.049

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	54	46	49	136	92	47	44
normalized size	1	1.00	0.95	0.81	0.86	2.39	1.61	0.82	0.77
time (sec)	N/A	0.018	0.038	0.055	2.961	0.771	0.329	0.152	4.975

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	46	50	73	51	51	51
normalized size	1	1.00	0.84	0.94	1.02	1.49	1.04	1.04	1.04
time (sec)	N/A	0.039	0.037	0.054	1.365	0.795	0.411	0.209	0.053

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	59	64	172	114	59	58
normalized size	1	1.00	0.99	0.87	0.94	2.53	1.68	0.87	0.85
time (sec)	N/A	0.029	0.040	0.061	2.918	1.029	0.386	0.168	5.026

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	29	22	21	21	19	23	11
normalized size	1	1.00	2.23	1.69	1.62	1.62	1.46	1.77	0.85
time (sec)	N/A	0.011	0.005	0.046	1.375	0.840	0.107	0.174	4.977

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	19	18	14	14	15	14
normalized size	1	1.00	0.90	0.95	0.90	0.70	0.70	0.75	0.70
time (sec)	N/A	0.015	0.003	0.041	1.322	0.878	0.084	0.151	0.036

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	22	17	16	16	14	18	6
normalized size	1	1.00	3.67	2.83	2.67	2.67	2.33	3.00	1.00
time (sec)	N/A	0.008	0.003	0.049	1.294	0.917	0.108	0.166	0.058

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	13	8	8	15	8
normalized size	1	1.00	1.00	1.17	1.08	0.67	0.67	1.25	0.67
time (sec)	N/A	0.006	0.002	0.046	1.295	0.984	0.099	0.148	0.029

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	2	2	19	3	13	13	12	15	2
normalized size	1	1.00	9.50	1.50	6.50	6.50	6.00	7.50	1.00
time (sec)	N/A	0.004	0.002	0.039	1.279	0.934	0.116	0.168	0.033

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	11	10	16	11
normalized size	1	1.00	1.00	1.07	1.00	0.73	0.67	1.07	0.73
time (sec)	N/A	0.008	0.003	0.057	1.317	0.941	0.106	0.156	4.960

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	24	19	18	20	15	20	8
normalized size	1	1.00	3.00	2.38	2.25	2.50	1.88	2.50	1.00
time (sec)	N/A	0.008	0.003	0.051	1.327	0.915	0.125	0.153	0.033

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	20	24	17	26	16
normalized size	1	1.00	1.00	0.95	0.91	1.09	0.77	1.18	0.73
time (sec)	N/A	0.015	0.004	0.049	1.292	0.679	0.113	0.151	0.033

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	31	24	25	30	24	27	13
normalized size	1	1.00	2.07	1.60	1.67	2.00	1.60	1.80	0.87
time (sec)	N/A	0.010	0.004	0.057	1.263	0.592	0.137	0.148	4.924

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	27	30	22	33	23
normalized size	1	1.00	1.00	0.90	0.93	1.03	0.76	1.14	0.79
time (sec)	N/A	0.016	0.004	0.046	1.346	0.556	0.131	0.160	0.033

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	12	18	14
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.80	1.20	0.93
time (sec)	N/A	0.009	0.004	0.045	1.289	0.549	0.150	0.149	4.950

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	15	15	12	18	16
normalized size	1	1.00	1.00	0.89	0.83	0.83	0.67	1.00	0.89
time (sec)	N/A	0.010	0.004	0.047	1.262	0.590	0.143	0.148	0.052

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	95	168	0	0	0	0	-1
normalized size	1	1.00	0.58	1.03	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.059	0.096	0.000	0.753	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	80	197	0	0	0	0	-1
normalized size	1	1.00	0.28	0.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.264	0.056	0.063	0.000	0.659	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	79	146	0	0	0	0	-1
normalized size	1	1.00	0.58	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.033	0.067	0.000	0.549	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	51	175	0	0	0	0	40
normalized size	1	1.00	0.20	0.69	0.00	0.00	0.00	0.00	0.16
time (sec)	N/A	0.173	0.012	0.068	0.000	0.613	0.000	0.000	5.051

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	48	124	0	0	0	0	-1
normalized size	1	1.00	0.42	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.010	0.066	0.000	0.574	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	51	177	0	0	0	0	-1
normalized size	1	1.00	0.21	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.201	0.013	0.083	0.000	0.540	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	53	123	0	0	0	0	-1
normalized size	1	1.00	0.46	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.014	0.041	0.000	0.710	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	53	201	0	0	0	0	-1
normalized size	1	1.00	0.19	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	0.015	0.044	0.000	0.731	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	94	188	0	0	0	0	-1
normalized size	1	1.00	0.51	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.089	0.073	0.000	0.590	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	84	217	0	0	0	0	-1
normalized size	1	1.00	0.28	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.046	0.075	0.000	0.616	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	83	166	0	0	0	0	40
normalized size	1	1.00	0.53	1.05	0.00	0.00	0.00	0.00	0.25
time (sec)	N/A	0.134	0.035	0.073	0.000	0.784	0.000	0.000	5.005

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	52	195	0	0	0	0	-1
normalized size	1	1.00	0.19	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.016	0.068	0.000	0.821	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	49	144	0	0	0	0	-1
normalized size	1	1.00	0.37	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	0.016	0.077	0.000	0.584	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	52	194	0	0	0	0	-1
normalized size	1	1.00	0.19	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.233	0.016	0.069	0.000	0.686	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	54	139	0	0	0	0	-1
normalized size	1	1.00	0.40	1.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.017	0.073	0.000	0.939	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	54	196	0	0	0	0	-1
normalized size	1	1.00	0.19	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	0.015	0.077	0.000	0.615	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	54	142	0	0	0	0	-1
normalized size	1	1.00	0.39	1.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	0.020	0.081	0.000	0.501	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	54	223	0	0	0	0	-1
normalized size	1	1.00	0.18	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.312	0.017	0.081	0.000	0.675	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	54	169	0	0	0	0	-1
normalized size	1	1.00	0.33	1.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.016	0.083	0.000	0.762	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	80	149	0	0	0	0	-1
normalized size	1	1.00	0.57	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.034	0.067	0.000	0.623	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	66	178	0	0	0	0	-1
normalized size	1	1.00	0.26	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.034	0.072	0.000	0.483	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	64	127	0	0	0	0	-1
normalized size	1	1.00	0.55	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.042	0.063	0.000	0.609	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	53	158	0	0	0	0	-1
normalized size	1	1.00	0.23	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	0.021	0.067	0.000	0.713	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	49	108	0	0	0	0	40
normalized size	1	1.00	0.53	1.17	0.00	0.00	0.00	0.00	0.43
time (sec)	N/A	0.049	0.017	0.066	0.000	0.595	0.000	0.000	5.035

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	48	182	0	0	0	0	-1
normalized size	1	1.00	0.19	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.013	0.072	0.000	0.540	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	53	129	0	0	0	0	-1
normalized size	1	1.00	0.45	1.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.025	0.071	0.000	0.466	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	53	204	0	0	0	0	-1
normalized size	1	1.00	0.19	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.254	0.015	0.072	0.000	0.599	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	80	172	0	0	0	0	-1
normalized size	1	1.00	0.50	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.039	0.095	0.000	0.582	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	68	200	0	0	0	0	-1
normalized size	1	1.00	0.24	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.269	0.029	0.073	0.000	0.496	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	67	147	0	0	0	0	-1
normalized size	1	1.00	0.49	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.034	0.073	0.000	0.591	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	57	182	0	0	0	0	-1
normalized size	1	1.00	0.23	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.025	0.073	0.000	0.557	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	54	130	0	0	0	0	-1
normalized size	1	1.00	0.47	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.029	0.071	0.000	0.579	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	56	184	0	0	0	0	-1
normalized size	1	1.00	0.22	0.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.209	0.016	0.074	0.000	0.699	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	54	132	0	0	0	0	-1
normalized size	1	1.00	0.47	1.16	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.032	0.068	0.000	0.617	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	51	206	0	0	0	0	40
normalized size	1	1.00	0.19	0.75	0.00	0.00	0.00	0.00	0.15
time (sec)	N/A	0.224	0.026	0.074	0.000	0.650	0.000	0.000	5.168

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	56	150	0	0	0	0	-1
normalized size	1	1.00	0.40	1.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.018	0.074	0.000	0.842	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	56	228	0	0	0	0	-1
normalized size	1	1.00	0.18	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.323	0.017	0.080	0.000	0.763	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	130	212	0	376	0	100	-1
normalized size	1	1.00	0.82	1.33	0.00	2.36	0.00	0.63	-0.01
time (sec)	N/A	0.248	0.213	0.092	0.000	0.698	0.000	0.325	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	77	70	0	108	0	80	-1
normalized size	1	1.00	0.61	0.56	0.00	0.86	0.00	0.63	-0.01
time (sec)	N/A	0.200	0.042	0.044	0.000	0.653	0.000	0.214	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	120	198	0	348	0	86	-1
normalized size	1	1.00	0.92	1.52	0.00	2.68	0.00	0.66	-0.01
time (sec)	N/A	0.206	0.286	0.078	0.000	0.816	0.000	0.305	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	66	59	0	97	0	64	-1
normalized size	1	1.00	0.65	0.58	0.00	0.96	0.00	0.63	-0.01
time (sec)	N/A	0.161	0.034	0.046	0.000	0.599	0.000	0.205	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	0	61	0	17	-1
normalized size	1	1.00	1.00	1.08	0.00	2.44	0.00	0.68	-0.04
time (sec)	N/A	0.037	0.019	0.048	0.000	0.488	0.000	0.293	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	55	48	0	86	0	50	-1
normalized size	1	1.00	0.72	0.63	0.00	1.13	0.00	0.66	-0.01
time (sec)	N/A	0.119	0.028	0.047	0.000	0.702	0.000	0.200	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	37	0	76	0	29	-1
normalized size	1	1.00	0.86	0.73	0.00	1.49	0.00	0.57	-0.02
time (sec)	N/A	0.075	0.024	0.063	0.000	0.668	0.000	0.291	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	37	0	75	0	33	-1
normalized size	1	1.00	0.86	0.73	0.00	1.47	0.00	0.65	-0.02
time (sec)	N/A	0.074	0.023	0.058	0.000	0.613	0.000	0.303	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	55	48	0	87	0	43	-1
normalized size	1	1.00	0.72	0.63	0.00	1.14	0.00	0.57	-0.01
time (sec)	N/A	0.113	0.023	0.048	0.000	0.650	0.000	0.284	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	0	63	0	23	-1
normalized size	1	1.00	1.00	1.08	0.00	2.52	0.00	0.92	-0.04
time (sec)	N/A	0.037	0.016	0.042	0.000	0.426	0.000	0.197	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	66	59	0	95	0	55	-1
normalized size	1	1.00	0.65	0.58	0.00	0.94	0.00	0.54	-0.01
time (sec)	N/A	0.157	0.025	0.043	0.000	0.527	0.000	0.248	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	43	217	0	360	0	114	-1
normalized size	1	1.00	0.33	1.67	0.00	2.77	0.00	0.88	-0.01
time (sec)	N/A	0.203	0.014	0.054	0.000	0.500	0.000	0.267	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	77	70	0	110	0	90	-1
normalized size	1	1.00	0.61	0.56	0.00	0.87	0.00	0.71	-0.01
time (sec)	N/A	0.192	0.041	0.049	0.000	0.904	0.000	0.269	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	44	234	0	396	0	104	-1
normalized size	1	1.00	0.28	1.47	0.00	2.49	0.00	0.65	-0.01
time (sec)	N/A	0.241	0.023	0.057	0.000	0.596	0.000	0.250	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	88	81	0	121	0	147	-1
normalized size	1	1.00	0.58	0.53	0.00	0.80	0.00	0.97	-0.01
time (sec)	N/A	0.233	0.045	0.045	0.000	0.901	0.000	0.324	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	46	247	0	422	0	138	-1
normalized size	1	1.00	0.24	1.31	0.00	2.23	0.00	0.73	-0.01
time (sec)	N/A	0.293	0.027	0.061	0.000	0.688	0.000	0.267	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	99	92	0	132	0	202	-1
normalized size	1	1.00	0.55	0.51	0.00	0.73	0.00	1.12	-0.01
time (sec)	N/A	0.285	0.043	0.046	0.000	1.108	0.000	0.412	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	81	997	0	133	0	45	-1
normalized size	1	1.00	1.47	18.13	0.00	2.42	0.00	0.82	-0.02
time (sec)	N/A	0.070	0.050	0.089	0.000	0.958	0.000	0.298	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	61	979	0	94	0	23	-1
normalized size	1	1.00	1.91	30.59	0.00	2.94	0.00	0.72	-0.03
time (sec)	N/A	0.032	0.032	0.087	0.000	0.886	0.000	0.210	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	26	19	0	14	19
normalized size	1	1.00	1.00	1.17	1.13	0.83	0.00	0.61	0.83
time (sec)	N/A	0.034	0.029	0.041	1.461	0.796	0.000	0.222	5.134

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	31	35	38	29	0	30	27
normalized size	1	1.00	0.65	0.73	0.79	0.60	0.00	0.62	0.56
time (sec)	N/A	0.066	0.028	0.040	1.512	1.018	0.000	0.192	5.129

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	44	48	50	40	0	47	40
normalized size	1	1.00	0.59	0.65	0.68	0.54	0.00	0.64	0.54
time (sec)	N/A	0.102	0.028	0.046	1.494	0.757	0.000	0.199	5.266

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	64	688	0	0	0	0	-1
normalized size	1	1.00	0.29	3.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.213	0.036	0.096	0.000	0.772	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	49	671	0	0	0	0	40
normalized size	1	1.00	0.25	3.41	0.00	0.00	0.00	0.00	0.20
time (sec)	N/A	0.130	0.011	0.086	0.000	0.648	0.000	0.000	5.157

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	53	696	0	0	0	0	-1
normalized size	1	1.00	0.24	3.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.187	0.014	0.093	0.000	0.914	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	503	66	1079	0	0	0	0	-1
normalized size	1	1.00	0.13	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.514	0.030	0.087	0.000	0.865	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	53	1054	0	0	0	0	-1
normalized size	1	1.00	0.11	2.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	0.013	0.095	0.000	0.748	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	497	497	48	1083	0	0	0	0	-1
normalized size	1	1.00	0.10	2.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.472	0.013	0.094	0.000	0.646	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	151	223	0	0	0	111	-1
normalized size	1	1.00	0.87	1.28	0.00	0.00	0.00	0.64	-0.01
time (sec)	N/A	0.151	0.187	0.093	0.000	0.000	0.000	0.288	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	129	181	0	0	0	83	-1
normalized size	1	1.00	1.11	1.56	0.00	0.00	0.00	0.72	-0.01
time (sec)	N/A	0.087	0.116	0.051	0.000	0.000	0.000	0.312	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	88	83	0	0	0	54	72
normalized size	1	1.00	1.57	1.48	0.00	0.00	0.00	0.96	1.29
time (sec)	N/A	0.060	0.053	0.049	0.000	0.000	0.000	0.285	5.243

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	159	0	19	0	25	-1
normalized size	1	1.00	1.00	6.36	0.00	0.76	0.00	1.00	-0.04
time (sec)	N/A	0.038	0.009	0.055	0.000	0.872	0.000	0.177	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	48	218	0	42	0	84	-1
normalized size	1	1.00	0.57	2.60	0.00	0.50	0.00	1.00	-0.01
time (sec)	N/A	0.117	0.052	0.066	0.000	1.027	0.000	0.234	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	72	262	0	64	0	146	-1
normalized size	1	1.00	0.51	1.85	0.00	0.45	0.00	1.03	-0.01
time (sec)	N/A	0.203	0.054	0.065	0.000	0.880	0.000	0.191	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	96	306	0	86	0	208	-1
normalized size	1	1.00	0.48	1.53	0.00	0.43	0.00	1.04	-0.00
time (sec)	N/A	0.297	0.062	0.068	0.000	0.661	0.000	0.198	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	64	549	0	0	0	0	-1
normalized size	1	1.00	0.32	2.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.175	0.065	0.062	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	64	503	0	0	0	0	-1
normalized size	1	1.00	0.46	3.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.075	0.064	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	64	236	0	0	0	94	-1
normalized size	1	1.00	0.83	3.06	0.00	0.00	0.00	1.22	-0.01
time (sec)	N/A	0.074	0.043	0.048	0.000	0.000	0.000	0.321	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	404	0	36	0	34	40
normalized size	1	1.00	1.00	16.16	0.00	1.44	0.00	1.36	1.60
time (sec)	N/A	0.005	0.019	0.059	0.000	0.884	0.000	0.190	5.426

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	48	524	0	63	0	0	-1
normalized size	1	1.00	0.61	6.63	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.082	0.066	0.000	0.867	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	72	570	0	87	0	0	-1
normalized size	1	1.00	0.53	4.16	0.00	0.64	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.078	0.064	0.000	0.754	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	96	614	0	109	0	0	-1
normalized size	1	1.00	0.49	3.15	0.00	0.56	0.00	0.00	-0.01
time (sec)	N/A	0.300	0.085	0.067	0.000	1.055	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	164	245	0	0	0	125	-1
normalized size	1	1.00	0.80	1.20	0.00	0.00	0.00	0.61	-0.00
time (sec)	N/A	0.172	0.264	0.055	0.000	0.000	0.000	0.283	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	142	203	0	0	0	97	-1
normalized size	1	1.00	0.97	1.39	0.00	0.00	0.00	0.66	-0.01
time (sec)	N/A	0.120	0.209	0.054	0.000	0.000	0.000	0.406	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	102	160	0	0	0	69	-1
normalized size	1	1.00	1.17	1.84	0.00	0.00	0.00	0.79	-0.01
time (sec)	N/A	0.079	0.117	0.051	0.000	0.000	0.000	0.269	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	65	136	0	0	0	37	-1
normalized size	1	1.00	1.91	4.00	0.00	0.00	0.00	1.09	-0.03
time (sec)	N/A	0.049	0.074	0.050	0.000	0.000	0.000	0.263	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	37	194	0	29	0	53	-1
normalized size	1	1.00	0.69	3.59	0.00	0.54	0.00	0.98	-0.02
time (sec)	N/A	0.074	0.061	0.063	0.000	1.374	0.000	0.189	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	59	240	0	50	0	115	-1
normalized size	1	1.00	0.53	2.14	0.00	0.45	0.00	1.03	-0.01
time (sec)	N/A	0.154	0.069	0.060	0.000	1.248	0.000	0.211	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	83	284	0	72	0	177	-1
normalized size	1	1.00	0.49	1.67	0.00	0.42	0.00	1.04	-0.01
time (sec)	N/A	0.245	0.086	0.065	0.000	1.413	0.000	0.236	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	62	527	0	0	0	0	-1
normalized size	1	1.00	0.36	3.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.083	0.064	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	62	440	0	0	0	0	-1
normalized size	1	1.00	0.55	3.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.068	0.060	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	79	240	0	0	0	0	-1
normalized size	1	1.00	1.32	4.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.067	0.113	0.049	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	45	111	0	54	0	26	-1
normalized size	1	1.00	1.50	3.70	0.00	1.80	0.00	0.87	-0.03
time (sec)	N/A	0.047	0.042	0.059	0.000	1.199	0.000	0.183	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	57	548	0	79	0	0	-1
normalized size	1	1.00	0.53	5.12	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.051	0.063	0.000	1.355	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	83	592	0	101	0	0	-1
normalized size	1	1.00	0.50	3.59	0.00	0.61	0.00	0.00	-0.01
time (sec)	N/A	0.257	0.069	0.067	0.000	1.565	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	107	636	0	123	0	0	-1
normalized size	1	1.00	0.48	2.85	0.00	0.55	0.00	0.00	-0.00
time (sec)	N/A	0.353	0.066	0.094	0.000	1.419	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	155	264	0	0	0	0	-1
normalized size	1	1.00	0.51	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	0.178	0.115	0.000	2.156	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	136	273	0	0	0	0	-1
normalized size	1	1.00	0.33	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.578	0.129	0.073	0.000	7.402	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	118	198	0	0	0	0	-1
normalized size	1	1.00	0.55	0.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	0.104	0.076	0.000	1.758	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	94	207	0	0	0	0	40
normalized size	1	1.00	0.29	0.64	0.00	0.00	0.00	0.00	0.12
time (sec)	N/A	0.326	0.059	0.071	0.000	8.097	0.000	0.000	5.180

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	54	132	0	0	0	0	-1
normalized size	1	1.00	0.44	1.07	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.048	0.065	0.000	1.583	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	59	213	0	0	0	0	-1
normalized size	1	1.00	0.18	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	0.072	0.096	0.000	7.388	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	59	179	0	0	0	0	-1
normalized size	1	1.00	0.31	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	0.052	0.077	0.000	1.573	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	59	281	0	0	0	0	-1
normalized size	1	1.00	0.14	0.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.544	0.051	0.082	0.000	8.289	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	59	245	0	0	0	0	-1
normalized size	1	1.00	0.21	0.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.413	0.060	0.085	0.000	1.296	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	142	196	0	0	0	0	-1
normalized size	1	1.00	0.48	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.501	0.154	0.144	0.000	1.515	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	123	261	0	0	0	0	-1
normalized size	1	1.00	0.30	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.554	0.109	0.079	0.000	7.472	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	106	164	0	0	0	0	40
normalized size	1	1.00	0.51	0.79	0.00	0.00	0.00	0.00	0.19
time (sec)	N/A	0.271	0.101	0.107	0.000	1.539	0.000	0.000	5.193

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	60	228	0	0	0	0	-1
normalized size	1	1.00	0.19	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.048	0.085	0.000	8.472	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	60	130	0	0	0	0	-1
normalized size	1	1.00	0.42	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.202	0.060	0.074	0.000	1.313	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	62	339	0	0	0	0	-1
normalized size	1	1.00	0.18	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.431	0.060	0.081	0.000	7.825	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	62	168	0	0	0	0	-1
normalized size	1	1.00	0.29	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	0.066	0.086	0.000	1.157	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	62	411	0	0	0	0	-1
normalized size	1	1.00	0.14	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.640	0.058	0.088	0.000	8.550	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	62	201	0	0	0	0	-1
normalized size	1	1.00	0.21	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.069	0.089	0.000	1.631	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	161	196	0	0	0	0	-1
normalized size	1	1.00	0.53	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.506	0.101	0.105	0.000	1.572	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	414	414	143	263	0	0	0	0	-1
normalized size	1	1.00	0.35	0.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.566	0.077	0.087	0.000	12.048	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	124	163	0	0	0	0	-1
normalized size	1	1.00	0.57	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	0.072	0.054	0.000	3.071	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	106	228	0	0	0	0	-1
normalized size	1	1.00	0.33	0.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.089	0.057	0.000	7.889	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	80	127	0	0	0	0	40
normalized size	1	1.00	0.63	1.01	0.00	0.00	0.00	0.00	0.32
time (sec)	N/A	0.118	0.039	0.050	0.000	1.440	0.000	0.000	5.268

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	54	253	0	0	0	0	-1
normalized size	1	1.00	0.18	0.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.294	0.071	0.077	0.000	6.561	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	59	142	0	0	0	0	-1
normalized size	1	1.00	0.36	0.87	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.062	0.092	0.000	1.476	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	59	365	0	0	0	0	-1
normalized size	1	1.00	0.15	0.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.476	0.057	0.087	0.000	7.389	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	59	179	0	0	0	0	-1
normalized size	1	1.00	0.24	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.345	0.091	0.101	0.000	1.698	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	131	384	0	0	0	0	-1
normalized size	1	1.00	0.30	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.674	0.109	0.115	0.000	8.994	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	124	260	0	0	0	0	-1
normalized size	1	1.00	0.52	1.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	0.132	0.088	0.000	1.659	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	94	312	0	0	0	0	-1
normalized size	1	1.00	0.27	0.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.426	0.093	0.067	0.000	8.226	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	82	184	0	0	0	0	-1
normalized size	1	1.00	0.55	1.23	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.089	0.063	0.000	1.620	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	62	242	0	0	0	0	40
normalized size	1	1.00	0.21	0.82	0.00	0.00	0.00	0.00	0.14
time (sec)	N/A	0.260	0.044	0.059	0.000	7.068	0.000	0.000	5.351

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	62	181	0	0	0	0	-1
normalized size	1	1.00	0.39	1.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.205	0.070	0.069	0.000	1.695	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	64	339	0	0	0	0	-1
normalized size	1	1.00	0.17	0.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.078	0.066	0.000	7.168	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	64	261	0	0	0	0	-1
normalized size	1	1.00	0.26	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.365	0.072	0.079	0.000	1.888	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	64	411	0	0	0	0	-1
normalized size	1	1.00	0.14	0.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.694	0.083	0.067	0.000	7.592	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	181	156	0	0	0	396	-1
normalized size	1	1.00	0.49	0.42	0.00	0.00	0.00	1.07	-0.00
time (sec)	N/A	0.627	0.184	0.070	0.000	0.000	0.000	0.223	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	144	123	0	0	0	312	-1
normalized size	1	1.00	0.51	0.43	0.00	0.00	0.00	1.10	-0.00
time (sec)	N/A	0.441	0.130	0.048	0.000	0.000	0.000	0.216	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	107	90	0	0	0	228	-1
normalized size	1	1.00	0.55	0.46	0.00	0.00	0.00	1.17	-0.01
time (sec)	N/A	0.273	0.090	0.049	0.000	0.000	0.000	0.194	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	70	57	0	0	0	143	40
normalized size	1	1.00	0.64	0.52	0.00	0.00	0.00	1.31	0.37
time (sec)	N/A	0.137	0.047	0.045	0.000	0.000	0.000	0.184	5.192

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	27	0	0	0	23	-1
normalized size	1	1.00	1.00	1.17	0.00	0.00	0.00	1.00	-0.04
time (sec)	N/A	0.040	0.012	0.039	0.000	0.000	0.000	0.208	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	57	80	0	0	0	72	-1
normalized size	1	1.00	0.63	0.89	0.00	0.00	0.00	0.80	-0.01
time (sec)	N/A	0.139	0.055	0.049	0.000	0.000	0.000	0.280	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	57	125	0	0	0	126	-1
normalized size	1	1.00	0.32	0.70	0.00	0.00	0.00	0.71	-0.01
time (sec)	N/A	0.296	0.075	0.061	0.000	0.000	0.000	0.278	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	57	167	0	0	0	177	-1
normalized size	1	1.00	0.21	0.63	0.00	0.00	0.00	0.67	-0.00
time (sec)	N/A	0.475	0.050	0.059	0.000	0.000	0.000	0.352	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	57	209	0	0	0	228	-1
normalized size	1	1.00	0.16	0.59	0.00	0.00	0.00	0.64	-0.00
time (sec)	N/A	0.660	0.052	0.061	0.000	0.000	0.000	0.432	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	172	145	0	0	0	770	-1
normalized size	1	1.00	0.50	0.42	0.00	0.00	0.00	2.24	-0.00
time (sec)	N/A	0.616	0.161	0.048	0.000	0.000	0.000	0.308	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	135	112	0	0	0	602	-1
normalized size	1	1.00	0.53	0.44	0.00	0.00	0.00	2.36	-0.00
time (sec)	N/A	0.422	0.112	0.051	0.000	0.000	0.000	0.303	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	98	79	0	0	0	434	40
normalized size	1	1.00	0.58	0.47	0.00	0.00	0.00	2.57	0.24
time (sec)	N/A	0.249	0.070	0.049	0.000	0.000	0.000	0.230	5.141

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	63	48	0	0	0	265	-1
normalized size	1	1.00	0.75	0.57	0.00	0.00	0.00	3.15	-0.01
time (sec)	N/A	0.139	0.080	0.051	0.000	0.000	0.000	0.211	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	88	69	0	0	0	83	-1
normalized size	1	1.00	1.13	0.88	0.00	0.00	0.00	1.06	-0.01
time (sec)	N/A	0.137	0.104	0.051	0.000	0.000	0.000	0.231	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	61	93	0	0	0	92	-1
normalized size	1	1.00	0.54	0.82	0.00	0.00	0.00	0.81	-0.01
time (sec)	N/A	0.184	0.074	0.061	0.000	0.000	0.000	0.261	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	61	139	0	0	0	143	-1
normalized size	1	1.00	0.30	0.68	0.00	0.00	0.00	0.70	-0.00
time (sec)	N/A	0.340	0.054	0.062	0.000	0.000	0.000	0.368	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	61	181	0	0	0	194	-1
normalized size	1	1.00	0.21	0.62	0.00	0.00	0.00	0.67	-0.00
time (sec)	N/A	0.522	0.060	0.059	0.000	0.000	0.000	0.383	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	61	223	0	0	0	245	-1
normalized size	1	1.00	0.16	0.59	0.00	0.00	0.00	0.65	-0.00
time (sec)	N/A	0.718	0.096	0.069	0.000	0.000	0.000	0.472	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	401	401	185	167	0	0	0	206	-1
normalized size	1	1.00	0.46	0.42	0.00	0.00	0.00	0.51	-0.00
time (sec)	N/A	0.728	0.229	0.051	0.000	0.000	0.000	0.299	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	148	134	0	0	0	164	-1
normalized size	1	1.00	0.47	0.43	0.00	0.00	0.00	0.52	-0.00
time (sec)	N/A	0.531	0.141	0.044	0.000	0.000	0.000	0.188	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	111	101	0	0	0	122	-1
normalized size	1	1.00	0.49	0.45	0.00	0.00	0.00	0.54	-0.00
time (sec)	N/A	0.346	0.128	0.046	0.000	0.000	0.000	0.269	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	74	68	0	0	0	80	-1
normalized size	1	1.00	0.54	0.50	0.00	0.00	0.00	0.58	-0.01
time (sec)	N/A	0.180	0.084	0.049	0.000	0.000	0.000	0.176	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	36	36	0	0	0	36	40
normalized size	1	1.00	0.77	0.77	0.00	0.00	0.00	0.77	0.85
time (sec)	N/A	0.050	0.038	0.044	0.000	0.000	0.000	0.488	5.219

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	90	61	0	0	0	51	-1
normalized size	1	1.00	1.48	1.00	0.00	0.00	0.00	0.84	-0.02
time (sec)	N/A	0.093	0.130	0.050	0.000	0.000	0.000	0.217	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	48	126	0	0	0	109	-1
normalized size	1	1.00	0.31	0.82	0.00	0.00	0.00	0.71	-0.01
time (sec)	N/A	0.239	0.063	0.048	0.000	0.000	0.000	0.260	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	48	188	0	0	0	160	-1
normalized size	1	1.00	0.20	0.78	0.00	0.00	0.00	0.66	-0.00
time (sec)	N/A	0.408	0.079	0.050	0.000	0.000	0.000	0.310	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	48	248	0	0	0	211	-1
normalized size	1	1.00	0.15	0.75	0.00	0.00	0.00	0.64	-0.00
time (sec)	N/A	0.577	0.071	0.053	0.000	0.000	0.000	0.359	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	161	143	0	0	0	214	-1
normalized size	1	1.00	0.48	0.43	0.00	0.00	0.00	0.64	-0.00
time (sec)	N/A	0.599	0.192	0.050	0.000	0.000	0.000	0.245	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	122	110	0	0	0	163	-1
normalized size	1	1.00	0.49	0.44	0.00	0.00	0.00	0.66	-0.00
time (sec)	N/A	0.414	0.127	0.052	0.000	0.000	0.000	0.215	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	85	77	0	0	0	112	-1
normalized size	1	1.00	0.53	0.48	0.00	0.00	0.00	0.70	-0.01
time (sec)	N/A	0.242	0.100	0.052	0.000	0.000	0.000	0.228	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	60	45	0	0	0	60	-1
normalized size	1	1.00	0.88	0.66	0.00	0.00	0.00	0.88	-0.01
time (sec)	N/A	0.084	0.063	0.047	0.000	0.000	0.000	0.219	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	45	56	0	0	0	71	40
normalized size	1	1.00	0.75	0.93	0.00	0.00	0.00	1.18	0.67
time (sec)	N/A	0.056	0.042	0.051	0.000	0.000	0.000	0.212	5.358

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	48	88	0	0	0	105	-1
normalized size	1	1.00	0.33	0.60	0.00	0.00	0.00	0.72	-0.01
time (sec)	N/A	0.241	0.088	0.057	0.000	0.000	0.000	0.259	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	48	126	0	0	0	156	-1
normalized size	1	1.00	0.20	0.53	0.00	0.00	0.00	0.66	-0.00
time (sec)	N/A	0.410	0.076	0.064	0.000	0.000	0.000	0.391	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	48	159	0	0	0	207	-1
normalized size	1	1.00	0.15	0.49	0.00	0.00	0.00	0.64	-0.00
time (sec)	N/A	0.597	0.075	0.069	0.000	0.000	0.000	0.376	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	48	192	0	0	0	258	-1
normalized size	1	1.00	0.12	0.47	0.00	0.00	0.00	0.63	-0.00
time (sec)	N/A	0.840	0.089	0.079	0.000	0.000	0.000	0.452	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.009	0.001	0.037	1.334	0.502	0.061	0.187	0.022

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.008	0.001	0.040	1.337	0.423	0.061	0.150	0.022

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.003	0.000	0.045	1.337	0.582	0.067	0.161	0.022

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13
normalized size	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76
time (sec)	N/A	0.006	0.001	0.055	1.363	0.514	0.062	0.147	0.020

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	10	8	10	10
normalized size	1	1.00	1.00	0.92	0.83	0.83	0.67	0.83	0.83
time (sec)	N/A	0.004	0.001	0.046	1.344	0.519	0.062	0.146	0.017

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.031	0.002	0.045	1.354	0.448	0.070	0.150	0.040

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.017	0.002	0.056	1.205	0.515	0.070	0.148	0.033

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.013	0.002	0.043	1.332	0.473	0.070	0.160	0.031

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	26	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.87	0.80	0.80
time (sec)	N/A	0.017	0.002	0.038	1.270	0.559	0.072	0.166	0.034

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24
normalized size	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.016	0.002	0.046	1.326	0.646	0.070	0.152	0.032

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	52	52	52	49	53	51
normalized size	1	1.00	1.00	0.91	0.91	0.91	0.86	0.93	0.89
time (sec)	N/A	0.036	0.028	0.043	1.274	0.539	0.143	0.151	5.093

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	41	42	41	37	43	40
normalized size	1	1.00	1.00	0.93	0.95	0.93	0.84	0.98	0.91
time (sec)	N/A	0.027	0.004	0.043	1.305	0.546	0.131	0.140	0.042

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	29	29	26	30	29
normalized size	1	1.00	1.00	0.97	0.94	0.94	0.84	0.97	0.94
time (sec)	N/A	0.021	0.004	0.045	1.366	0.474	0.120	0.191	0.041

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	17	14	19	18
normalized size	1	1.00	1.00	1.06	1.00	0.94	0.78	1.06	1.00
time (sec)	N/A	0.016	0.003	0.049	1.300	0.384	0.114	0.148	0.036

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	11	10
normalized size	1	1.00	1.00	1.10	1.00	1.00	0.70	1.10	1.00
time (sec)	N/A	0.007	0.001	0.042	1.323	0.380	0.071	0.146	0.022

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	16	10	20	15
normalized size	1	1.00	1.00	1.06	1.00	0.89	0.56	1.11	0.83
time (sec)	N/A	0.007	0.004	0.047	1.297	0.394	0.164	0.160	5.120

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
normalized size	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.015	0.005	0.050	1.357	0.395	0.198	0.153	0.052

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	38
normalized size	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90
time (sec)	N/A	0.021	0.032	0.053	1.366	0.379	0.214	0.153	0.059

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	51	54	44	56	48
normalized size	1	1.00	1.00	0.95	0.91	0.96	0.79	1.00	0.86
time (sec)	N/A	0.027	0.006	0.053	1.332	0.392	0.235	0.161	0.061

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	54	57	59	73	54	62	62
normalized size	1	1.00	0.93	0.98	1.02	1.26	0.93	1.07	1.07
time (sec)	N/A	0.040	0.023	0.052	1.307	0.387	0.210	0.172	0.038

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	45	47	62	44	48	50
normalized size	1	1.00	0.93	0.98	1.02	1.35	0.96	1.04	1.09
time (sec)	N/A	0.030	0.015	0.048	1.317	0.368	0.207	0.155	0.045

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	34	36	47	31	34	36
normalized size	1	1.00	0.88	1.03	1.09	1.42	0.94	1.03	1.09
time (sec)	N/A	0.024	0.014	0.050	1.277	0.372	0.183	0.150	0.040

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	24	26	28	20	24	23
normalized size	1	1.00	0.87	1.04	1.13	1.22	0.87	1.04	1.00
time (sec)	N/A	0.018	0.007	0.049	1.330	0.387	0.141	0.154	0.037

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	13	13	10	12	12
normalized size	1	1.00	1.00	1.08	1.08	1.08	0.83	1.00	1.00
time (sec)	N/A	0.007	0.003	0.049	1.301	0.379	0.141	0.146	5.169

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	30	28	39	22	31	26
normalized size	1	1.00	0.83	1.03	0.97	1.34	0.76	1.07	0.90
time (sec)	N/A	0.019	0.013	0.065	1.291	0.392	0.219	0.150	0.045

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	45	41
normalized size	1	1.00	0.83	1.02	1.07	1.50	0.88	1.07	0.98
time (sec)	N/A	0.024	0.051	0.051	1.306	0.389	0.272	0.155	5.343

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	53	57	64	86	54	64	57
normalized size	1	1.00	0.91	0.98	1.10	1.48	0.93	1.10	0.98
time (sec)	N/A	0.032	0.089	0.059	1.316	0.386	0.311	0.152	5.315

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	73	95	66	73	69
normalized size	1	1.00	0.96	0.99	1.06	1.38	0.96	1.06	1.00
time (sec)	N/A	0.039	0.070	0.051	1.343	0.397	0.330	0.147	0.071

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	79	86	108	80	86	79
normalized size	1	1.00	0.94	0.94	1.02	1.29	0.95	1.02	0.94
time (sec)	N/A	0.052	0.047	0.054	1.366	0.382	0.362	0.153	0.077

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	53	57	53	62	0	131	62
normalized size	1	1.00	0.50	0.54	0.50	0.59	0.00	1.25	0.59
time (sec)	N/A	0.120	0.042	0.043	1.459	0.400	0.000	0.163	5.487

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	42	46	42	51	0	108	51
normalized size	1	1.00	0.52	0.58	0.52	0.64	0.00	1.35	0.64
time (sec)	N/A	0.074	0.044	0.046	1.444	0.389	0.000	0.157	5.475

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	31	35	30	39	0	81	39
normalized size	1	1.00	0.60	0.67	0.58	0.75	0.00	1.56	0.75
time (sec)	N/A	0.043	0.024	0.049	1.417	0.382	0.000	0.180	5.302

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	12	26	0	50	-1
normalized size	1	1.00	0.92	1.08	0.48	1.04	0.00	2.00	-0.04
time (sec)	N/A	0.036	0.011	0.041	1.426	0.386	0.000	0.155	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	51	0	111	0	67	73
normalized size	1	1.00	1.04	1.00	0.00	2.18	0.00	1.31	1.43
time (sec)	N/A	0.049	0.033	0.043	0.000	0.405	0.000	0.165	5.357

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	56	0	127	0	45	-1
normalized size	1	1.00	0.92	1.08	0.00	2.44	0.00	0.87	-0.02
time (sec)	N/A	0.052	0.054	0.053	0.000	0.397	0.000	0.257	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	42	73	0	149	0	72	-1
normalized size	1	1.00	0.50	0.87	0.00	1.77	0.00	0.86	-0.01
time (sec)	N/A	0.092	0.013	0.055	0.000	0.409	0.000	0.209	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	42	89	0	175	0	92	-1
normalized size	1	1.00	0.38	0.79	0.00	1.56	0.00	0.82	-0.01
time (sec)	N/A	0.138	0.014	0.056	0.000	0.417	0.000	0.268	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	80	79	86	95	0	282	80
normalized size	1	1.00	0.50	0.49	0.53	0.59	0.00	1.75	0.50
time (sec)	N/A	0.231	0.048	0.048	1.520	0.397	0.000	0.179	5.238

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	69	68	75	84	0	246	69
normalized size	1	1.00	0.51	0.50	0.55	0.62	0.00	1.81	0.51
time (sec)	N/A	0.172	0.040	0.051	1.549	0.402	0.000	0.193	5.237

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	58	57	64	73	0	210	58
normalized size	1	1.00	0.54	0.53	0.59	0.68	0.00	1.94	0.54
time (sec)	N/A	0.141	0.029	0.043	1.439	0.393	0.000	0.234	5.187

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	47	46	53	62	0	173	47
normalized size	1	1.00	0.59	0.58	0.66	0.78	0.00	2.16	0.59
time (sec)	N/A	0.133	0.027	0.045	1.547	0.377	0.000	0.239	5.175

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	36	35	41	50	0	136	36
normalized size	1	1.00	0.69	0.67	0.79	0.96	0.00	2.62	0.69
time (sec)	N/A	0.083	0.021	0.039	1.468	0.387	0.000	0.180	5.174

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	28	37	0	89	28
normalized size	1	1.00	0.92	1.08	1.12	1.48	0.00	3.56	1.12
time (sec)	N/A	0.041	0.014	0.041	1.388	0.397	0.000	0.162	5.618

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	61	0	130	0	85	-1
normalized size	1	1.00	0.92	0.82	0.00	1.76	0.00	1.15	-0.01
time (sec)	N/A	0.096	0.047	0.048	0.000	0.415	0.000	0.174	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	40	72	0	136	0	62	-1
normalized size	1	1.00	0.55	0.99	0.00	1.86	0.00	0.85	-0.01
time (sec)	N/A	0.093	0.015	0.055	0.000	0.408	0.000	0.207	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	72	74	0	154	0	70	-1
normalized size	1	1.00	0.89	0.91	0.00	1.90	0.00	0.86	-0.01
time (sec)	N/A	0.092	0.051	0.059	0.000	0.417	0.000	0.265	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	42	87	0	175	0	92	-1
normalized size	1	1.00	0.39	0.80	0.00	1.61	0.00	0.84	-0.01
time (sec)	N/A	0.134	0.022	0.056	0.000	0.425	0.000	0.229	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	42	101	0	197	0	109	-1
normalized size	1	1.00	0.31	0.74	0.00	1.44	0.00	0.80	-0.01
time (sec)	N/A	0.184	0.017	0.053	0.000	0.408	0.000	0.241	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	42	113	0	219	0	126	-1
normalized size	1	1.00	0.25	0.68	0.00	1.33	0.00	0.76	-0.01
time (sec)	N/A	0.236	0.029	0.054	0.000	0.411	0.000	0.267	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	53	55	53	51	0	0	51
normalized size	1	1.00	0.51	0.53	0.51	0.50	0.00	0.00	0.50
time (sec)	N/A	0.148	0.041	0.048	1.462	0.380	0.000	0.000	5.188

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	42	44	42	40	0	0	40
normalized size	1	1.00	0.56	0.59	0.56	0.53	0.00	0.00	0.53
time (sec)	N/A	0.100	0.027	0.046	1.459	0.403	0.000	0.000	5.201

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	30	33	30	28	0	0	31
normalized size	1	1.00	0.61	0.67	0.61	0.57	0.00	0.00	0.63
time (sec)	N/A	0.055	0.021	0.043	1.433	0.386	0.000	0.000	5.164

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	12	21	0	26	17
normalized size	1	1.00	0.91	1.09	0.52	0.91	0.00	1.13	0.74
time (sec)	N/A	0.010	0.008	0.043	1.410	0.394	0.000	0.197	5.142

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	46	39	0	74	0	45	-1
normalized size	1	1.00	1.53	1.30	0.00	2.47	0.00	1.50	-0.03
time (sec)	N/A	0.011	0.010	0.044	0.000	0.405	0.000	0.159	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	66	55	0	127	0	0	-1
normalized size	1	1.00	1.22	1.02	0.00	2.35	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.064	0.048	0.000	0.411	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	40	77	0	153	0	0	44
normalized size	1	1.00	0.46	0.89	0.00	1.76	0.00	0.00	0.51
time (sec)	N/A	0.091	0.011	0.046	0.000	0.414	0.000	0.000	5.408

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	40	95	0	175	0	0	-1
normalized size	1	1.00	0.35	0.83	0.00	1.52	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.011	0.055	0.000	0.420	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	50	56	41	60	0	0	57
normalized size	1	1.00	0.51	0.57	0.42	0.61	0.00	0.00	0.58
time (sec)	N/A	0.151	0.035	0.046	1.481	0.393	0.000	0.000	5.282

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	39	46	30	49	0	0	47
normalized size	1	1.00	0.54	0.64	0.42	0.68	0.00	0.00	0.65
time (sec)	N/A	0.105	0.018	0.048	1.442	0.386	0.000	0.000	5.223

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	26	34	19	38	0	28	35
normalized size	1	1.00	0.55	0.72	0.40	0.81	0.00	0.60	0.74
time (sec)	N/A	0.057	0.014	0.047	1.536	0.387	0.000	0.280	5.175

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	19	27	12	29	0	37	28
normalized size	1	1.00	0.90	1.29	0.57	1.38	0.00	1.76	1.33
time (sec)	N/A	0.018	0.007	0.044	1.479	0.379	0.000	0.226	5.074

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	35	54	0	156	0	0	-1
normalized size	1	1.00	0.67	1.04	0.00	3.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.029	0.053	0.000	0.426	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	36	62	0	189	0	0	-1
normalized size	1	1.00	0.48	0.83	0.00	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.010	0.059	0.000	0.406	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	38	76	0	219	0	0	42
normalized size	1	1.00	0.35	0.69	0.00	1.99	0.00	0.00	0.38
time (sec)	N/A	0.105	0.019	0.057	0.000	0.427	0.000	0.000	5.432

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	38	86	0	241	0	0	-1
normalized size	1	1.00	0.28	0.62	0.00	1.75	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.010	0.061	0.000	0.401	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	38	100	0	263	0	0	44
normalized size	1	1.00	0.23	0.60	0.00	1.58	0.00	0.00	0.27
time (sec)	N/A	0.232	0.012	0.064	0.000	0.418	0.000	0.000	5.683

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	104	103	0	180	0	64	-1
normalized size	1	1.00	0.83	0.82	0.00	1.44	0.00	0.51	-0.01
time (sec)	N/A	0.169	0.135	0.069	0.000	0.412	0.000	0.204	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	90	92	0	159	0	52	-1
normalized size	1	1.00	0.95	0.97	0.00	1.67	0.00	0.55	-0.01
time (sec)	N/A	0.125	0.055	0.048	0.000	0.416	0.000	0.185	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	73	78	0	131	0	38	-1
normalized size	1	1.00	1.22	1.30	0.00	2.18	0.00	0.63	-0.02
time (sec)	N/A	0.083	0.043	0.049	0.000	0.424	0.000	0.178	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	55	58	0	77	0	23	-1
normalized size	1	1.00	1.62	1.71	0.00	2.26	0.00	0.68	-0.03
time (sec)	N/A	0.042	0.018	0.047	0.000	0.398	0.000	0.175	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	0	21	0	30	-1
normalized size	1	1.00	0.92	1.08	0.00	0.84	0.00	1.20	-0.04
time (sec)	N/A	0.038	0.010	0.037	0.000	0.400	0.000	0.236	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	31	33	0	29	0	55	-1
normalized size	1	1.00	0.55	0.59	0.00	0.52	0.00	0.98	-0.02
time (sec)	N/A	0.077	0.016	0.063	0.000	0.383	0.000	0.214	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	44	46	0	40	0	77	-1
normalized size	1	1.00	0.51	0.53	0.00	0.47	0.00	0.90	-0.01
time (sec)	N/A	0.118	0.016	0.045	0.000	0.382	0.000	0.202	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	55	57	0	51	0	103	-1
normalized size	1	1.00	0.47	0.49	0.00	0.44	0.00	0.89	-0.01
time (sec)	N/A	0.163	0.020	0.045	0.000	0.390	0.000	0.211	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	59	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.022	0.497	0.000	0.409	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	61	59	0	0	0	0	0	-1
normalized size	1	1.27	1.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	0.014	0.508	0.000	0.410	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.008	0.500	0.000	0.415	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	30	36	0	38	0	0	54
normalized size	1	1.00	0.94	1.12	0.00	1.19	0.00	0.00	1.69
time (sec)	N/A	0.025	0.016	0.046	0.000	0.417	0.000	0.000	5.281

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	44	50	0	70	0	0	98
normalized size	1	1.00	0.63	0.71	0.00	1.00	0.00	0.00	1.40
time (sec)	N/A	0.054	0.023	0.048	0.000	0.423	0.000	0.000	5.275

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	72	84	0	111	0	0	157
normalized size	1	1.00	0.62	0.72	0.00	0.96	0.00	0.00	1.35
time (sec)	N/A	0.091	0.032	0.049	0.000	0.411	0.000	0.000	5.365

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
normalized size	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.010	0.008	0.056	1.286	0.364	0.405	0.176	5.110

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	46	48	46	42	0	0	42
normalized size	1	1.00	0.58	0.60	0.58	0.52	0.00	0.00	0.52
time (sec)	N/A	0.115	0.034	0.050	1.433	0.388	0.000	0.000	5.184

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	34	37	34	30	0	0	33
normalized size	1	1.00	0.65	0.71	0.65	0.58	0.00	0.00	0.63
time (sec)	N/A	0.065	0.019	0.056	1.443	0.387	0.000	0.000	5.327

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	27	14	21	0	0	21
normalized size	1	1.00	1.00	1.08	0.56	0.84	0.00	0.00	0.84
time (sec)	N/A	0.017	0.008	0.048	1.405	0.373	0.000	0.000	5.190

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	54	43	0	75	0	47	-1
normalized size	1	1.00	1.69	1.34	0.00	2.34	0.00	1.47	-0.03
time (sec)	N/A	0.011	0.011	0.048	0.000	0.415	0.000	0.191	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	71	66	0	127	0	57	-1
normalized size	1	1.00	1.20	1.12	0.00	2.15	0.00	0.97	-0.02
time (sec)	N/A	0.055	0.071	0.051	0.000	0.417	0.000	0.210	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	68	248	0	0	0	0	-1
normalized size	1	1.00	0.29	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.142	0.028	1.292	0.000	0.405	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	52	231	0	0	0	0	-1
normalized size	1	1.00	0.25	1.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.067	0.010	0.049	0.000	0.417	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	55	248	0	0	0	0	44
normalized size	1	1.00	0.23	1.02	0.00	0.00	0.00	0.00	0.18
time (sec)	N/A	0.129	0.013	0.524	0.000	0.405	0.000	0.000	5.712

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	68	676	0	0	0	0	-1
normalized size	1	1.00	0.13	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.308	0.025	0.539	0.000	0.411	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	55	394	0	0	0	0	-1
normalized size	1	1.00	0.11	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.190	0.030	0.059	0.000	0.408	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	50	673	0	0	0	0	-1
normalized size	1	1.00	0.10	1.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	0.011	0.545	0.000	0.404	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	86	2017	0	0	0	0	-1
normalized size	1	1.00	0.32	7.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	0.053	0.846	0.000	0.418	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	70	2586	0	0	0	0	-1
normalized size	1	1.00	0.13	4.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.500	0.029	0.668	0.000	0.428	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	81	3347	0	148	0	44	-1
normalized size	1	1.00	1.25	51.49	0.00	2.28	0.00	0.68	-0.02
time (sec)	N/A	0.090	0.042	1.140	0.000	0.553	0.000	0.246	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	70	1793	0	0	0	0	-1
normalized size	1	1.00	0.30	7.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.035	1.135	0.000	0.404	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	57	2374	0	0	0	0	-1
normalized size	1	1.00	0.12	4.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.398	0.024	1.166	0.000	0.426	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	59	480	0	101	0	41	-1
normalized size	1	1.00	1.64	13.33	0.00	2.81	0.00	1.14	-0.03
time (sec)	N/A	0.048	0.023	1.237	0.000	0.544	0.000	0.201	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	55	437	0	0	0	0	-1
normalized size	1	1.00	0.27	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.166	0.014	1.225	0.000	0.411	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	55	2860	0	0	0	0	-1
normalized size	1	1.00	0.11	5.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.487	0.025	0.910	0.000	0.399	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	29	26	21	0	23	-1
normalized size	1	1.00	1.00	1.07	0.96	0.78	0.00	0.85	-0.04
time (sec)	N/A	0.040	0.012	0.045	1.421	0.385	0.000	0.204	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	57	1795	0	0	0	0	-1
normalized size	1	1.00	0.24	7.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.221	0.016	1.081	0.000	0.398	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	555	555	57	3048	0	0	0	0	-1
normalized size	1	1.00	0.10	5.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.573	0.026	1.086	0.000	0.407	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	35	37	38	31	0	38	-1
normalized size	1	1.00	0.62	0.66	0.68	0.55	0.00	0.68	-0.02
time (sec)	N/A	0.083	0.019	0.043	1.421	0.386	0.000	0.232	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	57	2009	0	0	0	0	-1
normalized size	1	1.00	0.22	7.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.284	0.017	1.235	0.000	0.403	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	28	26	19	30	25
normalized size	1	1.00	1.00	1.04	1.00	0.93	0.68	1.07	0.89
time (sec)	N/A	0.017	0.005	0.048	1.363	0.378	0.207	0.151	5.209

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	41	40	41	31	45	38
normalized size	1	1.00	1.00	0.98	0.95	0.98	0.74	1.07	0.90
time (sec)	N/A	0.019	0.005	0.048	1.358	0.387	0.216	0.144	5.724

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	94	120	0	171	0	0	-1
normalized size	1	1.00	0.84	1.07	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.196	0.047	0.000	0.398	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	92	98	0	150	0	0	-1
normalized size	1	1.00	1.07	1.14	0.00	1.74	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.073	0.053	0.000	0.419	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	78	0	122	0	48	-1
normalized size	1	1.00	1.34	1.39	0.00	2.18	0.00	0.86	-0.02
time (sec)	N/A	0.082	0.050	0.049	0.000	0.408	0.000	0.231	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	59	56	0	74	0	23	-1
normalized size	1	1.00	1.84	1.75	0.00	2.31	0.00	0.72	-0.03
time (sec)	N/A	0.034	0.018	0.051	0.000	0.391	0.000	0.281	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	25	0	21	0	27	21
normalized size	1	1.00	0.91	1.09	0.00	0.91	0.00	1.17	0.91
time (sec)	N/A	0.005	0.008	0.048	0.000	0.379	0.000	0.206	5.140

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	29	30	0	29	0	27	42
normalized size	1	1.00	0.56	0.58	0.00	0.56	0.00	0.52	0.81
time (sec)	N/A	0.045	0.015	0.054	0.000	0.378	0.000	0.231	5.062

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	42	46	0	40	0	43	40
normalized size	1	1.00	0.52	0.58	0.00	0.50	0.00	0.54	0.50
time (sec)	N/A	0.086	0.021	0.044	0.000	0.398	0.000	0.239	5.140

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	53	57	0	51	0	57	92
normalized size	1	1.00	0.49	0.53	0.00	0.47	0.00	0.53	0.85
time (sec)	N/A	0.134	0.024	0.046	0.000	0.393	0.000	0.268	5.137

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	64	68	0	62	0	71	116
normalized size	1	1.00	0.47	0.50	0.00	0.46	0.00	0.52	0.85
time (sec)	N/A	0.174	0.021	0.046	0.000	0.390	0.000	0.233	5.145

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	22	28	22	32	22
normalized size	1	1.00	1.00	0.88	0.85	1.08	0.85	1.23	0.85
time (sec)	N/A	0.018	0.014	0.045	1.337	0.387	0.221	0.147	0.050

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	22	28	22	32	22
normalized size	1	1.00	1.00	0.85	0.81	1.04	0.81	1.19	0.81
time (sec)	N/A	0.018	0.006	0.050	1.323	0.382	0.233	0.180	5.209

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	14	9	14	8	5	15	8
normalized size	1	1.00	1.75	1.12	1.75	1.00	0.62	1.88	1.00
time (sec)	N/A	0.003	0.005	0.043	1.333	0.378	0.072	0.148	5.274

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	16	18	15	16	10
normalized size	1	1.00	1.00	1.10	1.60	1.80	1.50	1.60	1.00
time (sec)	N/A	0.003	0.003	0.034	1.337	0.359	0.078	0.145	0.032

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	16	26	27	16	26
normalized size	1	1.00	1.00	0.92	1.33	2.17	2.25	1.33	2.17
time (sec)	N/A	0.003	0.004	0.046	1.310	0.369	0.088	0.147	0.037

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	7	10	10
normalized size	1	1.00	1.00	1.10	1.00	1.00	0.70	1.00	1.00
time (sec)	N/A	0.002	0.001	0.043	1.357	0.369	0.077	0.148	0.027

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	21	22	32	0	19
normalized size	1	1.00	1.00	0.95	1.05	1.10	1.60	0.00	0.95
time (sec)	N/A	0.006	0.004	0.045	1.345	0.409	0.635	0.000	5.178

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	40	36	82	0	21
normalized size	1	1.00	1.00	1.05	2.00	1.80	4.10	0.00	1.05
time (sec)	N/A	0.008	0.004	0.043	1.361	0.411	1.027	0.000	5.135

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	53	52	119	0	21
normalized size	1	1.00	1.00	1.05	2.65	2.60	5.95	0.00	1.05
time (sec)	N/A	0.009	0.004	0.041	1.402	0.394	1.355	0.000	5.122

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
normalized size	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.004	0.005	0.048	1.314	0.338	0.118	0.145	5.157

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
normalized size	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.011	0.020	0.039	1.369	0.344	0.130	0.151	5.156

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
normalized size	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.012	0.014	0.037	1.312	0.326	0.138	0.149	5.153

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	339	275	231	0	285	287
normalized size	1	1.00	0.89	12.56	10.19	8.56	0.00	10.56	10.63
time (sec)	N/A	0.015	0.031	0.108	1.428	0.412	0.000	0.515	5.875

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
normalized size	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.004	0.004	0.047	1.325	0.346	0.128	0.167	0.002

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
normalized size	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.005	0.006	0.043	1.340	0.338	0.127	0.148	5.188

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
normalized size	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.004	0.007	0.043	1.347	0.342	0.127	0.164	5.158

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	287	275	205	0	205	285
normalized size	1	1.00	0.89	10.63	10.19	7.59	0.00	7.59	10.56
time (sec)	N/A	0.008	0.005	0.085	1.481	0.417	0.000	0.271	5.950

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	126	121	93	0	93	124
normalized size	1	1.00	1.00	4.67	4.48	3.44	0.00	3.44	4.59
time (sec)	N/A	0.010	0.014	0.064	1.406	0.411	0.000	0.199	5.437

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	39	66	82	0	0	38
normalized size	1	1.00	1.00	1.44	2.44	3.04	0.00	0.00	1.41
time (sec)	N/A	0.014	0.022	0.068	1.401	0.408	0.000	0.000	5.211

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.005	0.003	0.039	1.288	0.367	0.122	0.147	0.046

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.006	0.004	0.041	1.273	0.381	0.148	0.145	5.124

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	10	14	13
normalized size	1	1.00	1.00	0.93	0.87	0.87	0.67	0.93	0.87
time (sec)	N/A	0.005	0.005	0.036	1.368	0.377	0.164	0.146	0.047

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
normalized size	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.005	0.010	0.049	1.334	0.373	0.282	0.191	0.042

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
normalized size	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.006	0.017	0.051	1.290	0.365	0.500	0.173	0.062

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	24	31	36	36	36	22	37
normalized size	1	1.00	1.26	1.63	1.89	1.89	1.89	1.16	1.95
time (sec)	N/A	0.006	0.013	0.055	1.306	0.373	0.745	0.150	5.184

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	22	25	19	22	22
normalized size	1	1.00	1.00	0.96	0.92	1.04	0.79	0.92	0.92
time (sec)	N/A	0.011	0.002	0.043	1.359	0.370	0.103	0.154	0.042

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	34	38	37	46	34
normalized size	1	1.00	1.00	0.88	0.85	0.95	0.92	1.15	0.85
time (sec)	N/A	0.023	0.008	0.044	1.318	0.379	0.151	0.169	0.037

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	45	44	48	49	45	47
normalized size	1	1.00	1.00	0.90	0.88	0.96	0.98	0.90	0.94
time (sec)	N/A	0.021	0.007	0.048	1.322	0.378	0.188	0.150	0.048

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	144	156	124	637	36	112	197
normalized size	1	1.00	0.78	0.84	0.67	3.44	0.19	0.61	1.06
time (sec)	N/A	0.346	0.143	0.108	2.902	1.253	1.542	0.181	5.914

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	363	325	297	0	269	363
normalized size	1	1.00	1.00	12.52	11.21	10.24	0.00	9.28	12.52
time (sec)	N/A	0.017	0.017	0.211	1.479	0.411	0.000	2.472	6.777

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	14	134	160	14	14
normalized size	1	1.00	10.00	8.44	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.003	0.006	0.043	1.329	0.327	0.115	0.150	5.325

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
normalized size	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.009	0.006	0.044	1.343	0.339	0.127	0.155	0.002

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
normalized size	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.012	0.008	0.039	1.355	0.340	0.140	0.153	5.222

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	14	134	160	14	14
normalized size	1	1.00	10.00	8.44	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.003	0.005	0.039	1.352	0.343	0.111	0.147	5.202

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	134	134	160	134	134
normalized size	1	1.00	10.00	8.44	8.38	8.38	10.00	8.38	8.38
time (sec)	N/A	0.008	0.007	0.042	1.311	0.349	0.131	0.214	0.002

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	160	135	14	134	160	14	14
normalized size	1	1.00	10.00	8.44	0.88	8.38	10.00	0.88	0.88
time (sec)	N/A	0.003	0.006	0.043	1.298	0.339	0.117	0.153	5.177

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	36	37	27	53	0	26
normalized size	1	1.00	1.00	1.57	1.61	1.17	2.30	0.00	1.13
time (sec)	N/A	0.011	0.009	0.051	1.424	0.404	0.675	0.000	5.260

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	22	39	27	28	41	0	31
normalized size	1	1.00	0.96	1.70	1.17	1.22	1.78	0.00	1.35
time (sec)	N/A	0.013	0.006	0.053	1.335	0.406	1.810	0.000	5.227

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	41	19	28	39	0	34
normalized size	1	1.00	1.00	2.73	1.27	1.87	2.60	0.00	2.27
time (sec)	N/A	0.008	0.005	0.056	1.369	0.416	2.142	0.000	5.225

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	32	16	26	20	0	26
normalized size	1	1.00	1.00	1.45	0.73	1.18	0.91	0.00	1.18
time (sec)	N/A	0.010	0.006	0.051	1.321	0.399	1.508	0.000	5.231

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	34	11	26	22	0	28
normalized size	1	1.00	1.00	2.27	0.73	1.73	1.47	0.00	1.87
time (sec)	N/A	0.006	0.004	0.050	1.321	0.407	1.702	0.000	5.203

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	8	8	8	9	8
normalized size	1	1.00	1.00	1.00	0.67	0.67	0.67	0.75	0.67
time (sec)	N/A	0.004	0.003	0.046	1.330	0.391	0.178	0.153	0.096

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	116	17	8	22	18	8
normalized size	1	1.00	1.00	8.29	1.21	0.57	1.57	1.29	0.57
time (sec)	N/A	0.005	0.003	0.436	1.300	0.389	0.374	0.198	5.296

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	8	8	10	32	8
normalized size	1	1.00	1.00	0.75	0.67	0.67	0.83	2.67	0.67
time (sec)	N/A	0.004	0.002	0.044	2.888	0.386	0.259	0.152	0.094

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	39	31	24	32	0	26
normalized size	1	1.00	1.00	1.62	1.29	1.00	1.33	0.00	1.08
time (sec)	N/A	0.016	0.024	0.162	2.929	0.422	0.445	0.000	5.249

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	104	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.114	0.226	0.866	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	109	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.067	0.950	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	103	0	0	0	0	0	-1
normalized size	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.270	0.761	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	99	0	0	0	0	0	-1
normalized size	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.160	0.723	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.286	0.971	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	39	58	97	78	0	-1
normalized size	1	1.00	0.90	0.76	1.14	1.90	1.53	0.00	-0.02
time (sec)	N/A	0.029	0.014	0.044	2.956	0.416	1.776	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	85	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	0.068	0.715	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	78	0	0	0	0	0	97
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	1.59
time (sec)	N/A	0.083	0.056	0.716	0.000	0.000	0.000	0.000	5.169

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.074	0.739	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	131	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	0.263	0.868	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	126	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.326	0.710	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	117	0	0	0	0	0	-1
normalized size	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.173	0.709	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	120	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.276	0.687	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	58	51	73	120	88	0	-1
normalized size	1	1.00	0.79	0.70	1.00	1.64	1.21	0.00	-0.01
time (sec)	N/A	0.041	0.028	0.049	3.001	0.414	3.076	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	97	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.078	0.668	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	94	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.072	0.696	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	100	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	0.090	0.708	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	96	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.079	0.727	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	58	47	0	93	0	67	67
normalized size	1	1.00	1.14	0.92	0.00	1.82	0.00	1.31	1.31
time (sec)	N/A	0.077	0.029	0.076	0.000	0.413	0.000	0.162	5.184

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	62	61	53	108	0	69	55
normalized size	1	1.00	1.48	1.45	1.26	2.57	0.00	1.64	1.31
time (sec)	N/A	0.023	0.032	0.076	2.935	0.409	0.000	0.169	5.574

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	66	55	0	104	0	71	63
normalized size	1	1.00	1.29	1.08	0.00	2.04	0.00	1.39	1.24
time (sec)	N/A	0.065	0.038	0.080	0.000	0.413	0.000	0.191	5.337

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	70	74	0	112	0	0	-1
normalized size	1	1.00	1.15	1.21	0.00	1.84	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.022	1.870	0.000	0.442	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	55	0	98	0	61	67
normalized size	1	1.00	1.25	1.04	0.00	1.85	0.00	1.15	1.26
time (sec)	N/A	0.079	0.031	0.085	0.000	0.408	0.000	0.159	5.182

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	68	81	34	118	0	63	54
normalized size	1	1.00	1.58	1.88	0.79	2.74	0.00	1.47	1.26
time (sec)	N/A	0.023	0.034	0.087	2.995	0.414	0.000	0.240	5.384

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	73	73	0	109	0	65	63
normalized size	1	1.00	1.38	1.38	0.00	2.06	0.00	1.23	1.19
time (sec)	N/A	0.068	0.042	0.110	0.000	0.407	0.000	0.189	5.354

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	78	105	0	118	0	0	-1
normalized size	1	1.00	1.24	1.67	0.00	1.87	0.00	0.00	-0.02
time (sec)	N/A	0.080	0.032	1.987	0.000	0.424	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	98	0	0	0	0	0	-1
normalized size	1	1.00	1.58	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.108	0.161	0.792	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	89	0	0	0	0	0	-1
normalized size	1	1.00	1.68	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.099	0.169	0.729	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
normalized size	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.015	0.074	0.709	0.000	0.000	0.000	0.000	5.387

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	87	0	0	0	0	0	-1
normalized size	1	1.00	1.71	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.085	0.135	0.715	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	42	76	27	0	-1
normalized size	1	1.00	1.00	0.84	1.35	2.45	0.87	0.00	-0.03
time (sec)	N/A	0.020	0.007	0.050	2.957	0.425	1.976	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.111	0.050	0.724	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	66	0	0	0	0	0	-1
normalized size	1	1.00	1.65	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.048	0.704	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	68	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.136	0.057	0.723	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	117	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.199	0.832	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	109	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.209	0.731	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	91	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.152	0.702	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	104	0	0	0	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.201	0.738	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	40	42	61	148	185	0	-1
normalized size	1	1.00	0.74	0.78	1.13	2.74	3.43	0.00	-0.02
time (sec)	N/A	0.033	0.014	0.051	3.065	0.418	3.721	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	55	0	0	0	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.049	0.681	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	51	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.043	0.720	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	55	0	0	0	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.217	0.055	0.694	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	51	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	0.065	0.730	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	63	477	0	102	0	0	-1
normalized size	1	1.00	1.97	14.91	0.00	3.19	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.025	0.570	0.000	0.543	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	59	49	0	80	0	40	-1
normalized size	1	1.00	1.84	1.53	0.00	2.50	0.00	1.25	-0.03
time (sec)	N/A	0.016	0.020	0.194	0.000	0.417	0.000	0.214	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	63	0	0	102	0	0	-1
normalized size	1	1.00	1.97	0.00	0.00	3.19	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.029	0.115	0.000	0.907	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	76	0	0	102	0	0	-1
normalized size	1	1.00	2.05	0.00	0.00	2.76	0.00	0.00	-0.03
time (sec)	N/A	0.024	0.050	0.732	0.000	0.414	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	66	471	0	111	0	0	-1
normalized size	1	1.00	2.00	14.27	0.00	3.36	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.033	6.219	0.000	0.542	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	62	53	0	88	0	47	-1
normalized size	1	1.00	1.88	1.61	0.00	2.67	0.00	1.42	-0.03
time (sec)	N/A	0.015	0.023	0.190	0.000	0.409	0.000	0.187	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	66	0	0	111	0	0	-1
normalized size	1	1.00	2.00	0.00	0.00	3.36	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.028	0.157	0.000	0.918	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	78	0	0	106	0	0	-1
normalized size	1	1.00	2.05	0.00	0.00	2.79	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.059	1.377	0.000	0.415	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
normalized size	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.023	0.103	0.760	0.000	0.000	0.000	0.000	5.209

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	109	0	0	67
normalized size	1	1.00	2.11	0.00	0.00	2.95	0.00	0.00	1.81
time (sec)	N/A	0.019	0.040	2.102	0.000	0.414	0.000	0.000	5.323

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	78	0	0	0	0	0	67
normalized size	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.81
time (sec)	N/A	0.018	0.029	0.720	0.000	0.000	0.000	0.000	5.127

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0	66
normalized size	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.74
time (sec)	N/A	0.021	0.106	0.696	0.000	0.000	0.000	0.000	5.166

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	109	0	0	66
normalized size	1	1.00	2.11	0.00	0.00	2.87	0.00	0.00	1.74
time (sec)	N/A	0.020	0.025	2.152	0.000	0.421	0.000	0.000	5.102

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	80	0	0	0	0	0	66
normalized size	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	1.74
time (sec)	N/A	0.019	0.028	0.694	0.000	0.000	0.000	0.000	5.117

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	218	0	0	0	0	0	-1
normalized size	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.412	0.884	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	156	0	0	0	0	0	-1
normalized size	1	1.00	1.56	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.210	0.839	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	106	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.091	0.820	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	116	0	0	0	0	0	-1
normalized size	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.145	0.793	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	166	0	0	0	0	0	-1
normalized size	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.551	0.807	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	177	0	0	0	0	0	82
normalized size	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.85
time (sec)	N/A	0.055	0.229	0.806	0.000	0.000	0.000	0.000	5.245

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	134	0	0	0	0	0	82
normalized size	1	1.00	1.54	0.00	0.00	0.00	0.00	0.00	0.94
time (sec)	N/A	0.053	0.166	0.847	0.000	0.000	0.000	0.000	5.225

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	88	0	0	0	0	0	83
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.051	0.058	0.783	0.000	0.000	0.000	0.000	5.273

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	104	0	0	0	0	0	83
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.057	0.123	0.773	0.000	0.000	0.000	0.000	5.484

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	185	0	0	0	0	0	83
normalized size	1	1.00	1.83	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.057	0.382	0.796	0.000	0.000	0.000	0.000	5.604

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	16	0	16	0	50	15
normalized size	1	1.00	1.06	0.89	0.00	0.89	0.00	2.78	0.83
time (sec)	N/A	0.007	0.006	0.047	0.000	0.387	0.000	0.185	5.262

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	18	0	19	0	11	27
normalized size	1	1.00	1.00	0.90	0.00	0.95	0.00	0.55	1.35
time (sec)	N/A	0.003	0.010	0.054	0.000	0.438	0.000	0.172	5.322

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
normalized size	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.004	0.002	0.044	3.144	0.381	0.213	0.155	5.244

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	22
normalized size	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	0.88
time (sec)	N/A	0.010	0.013	0.050	1.463	0.392	0.000	0.169	5.231

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	27	29
normalized size	1	1.00	1.00	1.16	0.56	1.12	0.00	1.08	1.16
time (sec)	N/A	0.011	0.003	0.044	1.427	0.387	0.000	0.163	5.258

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	14	28	0	14	22
normalized size	1	1.00	1.00	1.16	0.56	1.12	0.00	0.56	0.88
time (sec)	N/A	0.008	0.012	0.053	1.465	0.374	0.000	0.186	5.211

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	988	988	61	0	0	0	0	0	42
normalized size	1	1.00	0.06	0.00	0.00	0.00	0.00	0.00	0.04
time (sec)	N/A	2.245	0.021	0.542	0.000	0.000	0.000	0.000	5.410

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	61	0	0	0	0	0	42
normalized size	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	0.09
time (sec)	N/A	0.974	0.017	0.480	0.000	0.000	0.000	0.000	5.251

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	92	92	0	0	0	0	0	-1
normalized size	1	1.03	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.135	0.744	0.000	0.611	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	73	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.175	0.809	0.000	0.810	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	74	74	0	0	0	0	0	-1
normalized size	1	1.12	1.12	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.129	0.760	0.000	0.727	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	82	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.198	0.786	0.000	0.632	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	0.147	0.804	0.000	0.623	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	64	0	0	76
normalized size	1	1.00	0.98	0.00	0.00	1.45	0.00	0.00	1.73
time (sec)	N/A	0.015	0.039	0.856	0.000	0.602	0.000	0.000	5.303

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	0	0	61	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	1.39	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.003	0.848	0.000	0.561	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	76	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	1.65	0.00	0.00	-0.02
time (sec)	N/A	0.075	0.046	0.908	0.000	0.406	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	79	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	1.72	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.005	0.882	0.000	0.420	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	-1
normalized size	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.033	0.188	1.551	0.403	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	-1
normalized size	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.033	0.079	1.495	0.399	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	36	40	17	44	0	0	-1
normalized size	1	1.00	0.82	0.91	0.39	1.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.033	0.069	1.580	0.397	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	47	0	0	47	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.82	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.023	0.720	0.000	0.407	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	0	0	54	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.89	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.027	0.726	0.000	0.413	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	0	0	76	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	1.95	0.00	0.00	-0.03
time (sec)	N/A	0.050	0.026	1.269	0.000	0.417	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	38	0	0	64	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	1.60	0.00	0.00	-0.02
time (sec)	N/A	0.074	0.045	0.869	0.000	0.413	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [347] had the largest ratio of [.7778]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	13	0.077
2	A	2	1	1.00	11	0.091
3	A	1	0	1.00	9	0.000
4	A	2	1	1.00	13	0.077
5	A	2	1	1.00	13	0.077
6	A	3	2	1.00	15	0.133
7	A	4	3	1.00	13	0.231
8	A	3	2	1.00	11	0.182
9	A	2	2	1.00	15	0.133
10	A	3	2	1.00	15	0.133
11	A	3	2	1.00	11	0.182
12	A	4	3	1.00	15	0.200
13	A	3	3	1.00	15	0.200
14	A	2	2	1.00	15	0.133
15	A	2	2	1.00	13	0.154
16	A	5	5	1.00	11	0.454
17	A	3	3	1.00	15	0.200
18	A	4	3	1.00	15	0.200
19	A	4	3	1.00	15	0.200
20	A	4	3	1.00	15	0.200
21	A	3	3	1.00	15	0.200
22	A	4	3	1.00	13	0.231
23	A	4	4	1.00	11	0.364
24	A	4	3	1.00	15	0.200
25	A	5	4	1.00	15	0.267
26	A	4	3	1.00	13	0.231
27	A	4	3	1.00	13	0.231
28	A	3	3	1.00	13	0.231
29	A	2	2	1.00	13	0.154
30	A	2	2	1.00	11	0.182
31	A	5	5	1.00	9	0.556
32	A	3	3	1.00	13	0.231
33	A	4	3	1.00	13	0.231
34	A	4	3	1.00	13	0.231
35	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
36	A	5	5	1.00	9	0.556
37	A	5	5	1.00	11	0.454
38	A	6	5	1.00	17	0.294
39	A	7	7	1.00	17	0.412
40	A	5	5	1.00	15	0.333
41	A	6	6	1.00	13	0.462
42	A	4	4	1.00	17	0.235
43	A	6	6	1.00	17	0.353
44	A	4	4	1.00	17	0.235
45	A	7	7	1.00	17	0.412
46	A	7	5	1.00	17	0.294
47	A	8	7	1.00	15	0.467
48	A	6	6	1.00	13	0.462
49	A	7	7	1.00	17	0.412
50	A	5	4	1.00	17	0.235
51	A	7	7	1.00	17	0.412
52	A	5	5	1.00	17	0.294
53	A	7	6	1.00	17	0.353
54	A	5	4	1.00	17	0.235
55	A	8	7	1.00	17	0.412
56	A	6	5	1.00	17	0.294
57	A	5	4	1.00	17	0.235
58	A	6	6	1.00	17	0.353
59	A	4	4	1.00	17	0.235
60	A	5	5	1.00	15	0.333
61	A	3	3	1.00	13	0.231
62	A	6	6	1.00	17	0.353
63	A	4	4	1.00	17	0.235
64	A	7	6	1.00	17	0.353
65	A	6	5	1.00	17	0.294
66	A	7	7	1.00	17	0.412
67	A	5	5	1.00	17	0.294
68	A	6	6	1.00	17	0.353
69	A	4	4	1.00	17	0.235
70	A	6	6	1.00	17	0.353
71	A	4	4	1.00	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	7	7	1.00	13	0.538
73	A	5	5	1.00	17	0.294
74	A	8	7	1.00	17	0.412
75	A	7	4	1.00	19	0.210
76	A	5	2	1.00	19	0.105
77	A	6	3	1.00	19	0.158
78	A	4	2	1.00	19	0.105
79	A	1	1	1.00	19	0.053
80	A	3	2	1.00	19	0.105
81	A	2	2	1.00	19	0.105
82	A	2	2	1.00	19	0.105
83	A	3	2	1.00	19	0.105
84	A	1	1	1.00	19	0.053
85	A	4	2	1.00	19	0.105
86	A	6	3	1.00	19	0.158
87	A	5	2	1.00	19	0.105
88	A	7	4	1.00	19	0.210
89	A	6	3	1.00	19	0.158
90	A	8	4	1.00	19	0.210
91	A	7	3	1.00	19	0.158
92	A	3	3	1.00	17	0.176
93	A	2	2	1.00	15	0.133
94	A	1	1	1.00	17	0.059
95	A	2	2	1.00	17	0.118
96	A	3	2	1.00	17	0.118
97	A	4	4	1.00	17	0.235
98	A	3	3	1.00	13	0.231
99	A	4	4	1.00	17	0.235
100	A	6	6	1.00	17	0.353
101	A	5	5	1.00	17	0.294
102	A	6	6	1.00	17	0.353
103	A	8	5	1.00	19	0.263
104	A	6	5	1.00	17	0.294
105	A	4	4	1.00	15	0.267
106	A	1	1	1.00	19	0.053
107	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
108	A	5	2	1.00	19	0.105
109	A	7	2	1.00	19	0.105
110	A	9	6	1.00	19	0.316
111	A	7	6	1.00	19	0.316
112	A	5	5	1.00	17	0.294
113	A	1	1	1.00	15	0.067
114	A	3	3	1.00	19	0.158
115	A	5	3	1.00	19	0.158
116	A	7	3	1.00	19	0.158
117	A	9	5	1.00	21	0.238
118	A	7	5	1.00	21	0.238
119	A	5	5	1.00	21	0.238
120	A	3	3	1.00	21	0.143
121	A	2	2	1.00	21	0.095
122	A	4	2	1.00	21	0.095
123	A	6	2	1.00	21	0.095
124	A	8	6	1.00	21	0.286
125	A	6	6	1.00	21	0.286
126	A	4	4	1.00	21	0.190
127	A	2	2	1.00	21	0.095
128	A	4	3	1.00	21	0.143
129	A	6	3	1.00	21	0.143
130	A	8	3	1.00	21	0.143
131	A	11	6	1.00	19	0.316
132	A	11	8	1.00	19	0.421
133	A	8	6	1.00	17	0.353
134	A	8	8	1.00	15	0.533
135	A	5	5	1.00	19	0.263
136	A	8	8	1.00	19	0.421
137	A	7	6	1.00	19	0.316
138	A	11	8	1.00	19	0.421
139	A	10	6	1.00	19	0.316
140	A	11	6	1.00	19	0.316
141	A	11	8	1.00	17	0.471
142	A	8	7	1.00	15	0.467
143	A	8	8	1.00	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
144	A	6	6	1.00	19	0.316
145	A	9	8	1.00	19	0.421
146	A	8	6	1.00	19	0.316
147	A	12	8	1.00	19	0.421
148	A	11	6	1.00	19	0.316
149	A	11	5	1.00	19	0.263
150	A	11	7	1.00	19	0.368
151	A	8	5	1.00	19	0.263
152	A	8	7	1.00	17	0.412
153	A	5	5	1.00	15	0.333
154	A	7	7	1.00	19	0.368
155	A	6	5	1.00	19	0.263
156	A	10	7	1.00	19	0.368
157	A	9	5	1.00	19	0.263
158	A	12	8	1.00	19	0.421
159	A	9	6	1.00	19	0.316
160	A	9	8	1.00	19	0.421
161	A	6	6	1.00	17	0.353
162	A	7	7	1.00	15	0.467
163	A	6	6	1.00	19	0.316
164	A	10	8	1.00	19	0.421
165	A	9	6	1.00	19	0.316
166	A	13	8	1.00	19	0.421
167	A	13	3	1.00	19	0.158
168	A	10	3	1.00	19	0.158
169	A	7	3	1.00	17	0.176
170	A	4	3	1.00	15	0.200
171	A	1	1	1.00	19	0.053
172	A	4	4	1.00	19	0.210
173	A	7	4	1.00	19	0.210
174	A	10	4	1.00	19	0.210
175	A	13	4	1.00	19	0.210
176	A	12	3	1.00	19	0.158
177	A	9	3	1.00	17	0.176
178	A	6	3	1.00	15	0.200
179	A	3	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	4	3	1.00	19	0.158
181	A	5	4	1.00	19	0.210
182	A	8	4	1.00	19	0.210
183	A	11	4	1.00	19	0.210
184	A	14	4	1.00	19	0.210
185	A	14	3	1.00	19	0.158
186	A	11	3	1.00	19	0.158
187	A	8	3	1.00	19	0.158
188	A	5	3	1.00	17	0.176
189	A	2	2	1.00	15	0.133
190	A	3	3	1.00	19	0.158
191	A	6	3	1.00	19	0.158
192	A	9	3	1.00	19	0.158
193	A	12	3	1.00	19	0.158
194	A	12	4	1.00	19	0.210
195	A	9	4	1.00	19	0.210
196	A	6	4	1.00	19	0.210
197	A	3	3	1.00	17	0.176
198	A	3	3	1.00	15	0.200
199	A	6	4	1.00	19	0.210
200	A	9	4	1.00	19	0.210
201	A	12	4	1.00	19	0.210
202	A	15	4	1.00	19	0.210
203	A	2	1	1.00	15	0.067
204	A	2	1	1.00	13	0.077
205	A	1	0	1.00	11	0.000
206	A	2	1	1.00	15	0.067
207	A	2	1	1.00	15	0.067
208	A	3	2	1.00	17	0.118
209	A	3	2	1.00	15	0.133
210	A	3	2	1.00	13	0.154
211	A	3	2	1.00	17	0.118
212	A	3	2	1.00	17	0.118
213	A	3	2	1.00	17	0.118
214	A	3	2	1.00	17	0.118
215	A	3	2	1.00	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
216	A	3	2	1.00	17	0.118
217	A	2	2	1.00	17	0.118
218	A	4	4	1.00	15	0.267
219	A	3	2	1.00	13	0.154
220	A	3	2	1.00	17	0.118
221	A	3	2	1.00	17	0.118
222	A	3	2	1.00	17	0.118
223	A	3	2	1.00	17	0.118
224	A	3	2	1.00	17	0.118
225	A	3	2	1.00	17	0.118
226	A	2	2	1.00	17	0.118
227	A	3	2	1.00	17	0.118
228	A	3	2	1.00	17	0.118
229	A	3	2	1.00	15	0.133
230	A	3	2	1.00	13	0.154
231	A	3	2	1.00	17	0.118
232	A	4	3	1.00	19	0.158
233	A	3	3	1.00	17	0.176
234	A	2	2	1.00	15	0.133
235	A	1	1	1.00	19	0.053
236	A	3	3	1.00	19	0.158
237	A	3	3	1.00	19	0.158
238	A	4	4	1.00	19	0.210
239	A	5	4	1.00	19	0.210
240	A	6	3	1.00	19	0.158
241	A	5	3	1.00	17	0.176
242	A	4	3	1.00	15	0.200
243	A	3	2	1.00	19	0.105
244	A	2	2	1.00	19	0.105
245	A	1	1	1.00	19	0.053
246	A	4	3	1.00	19	0.158
247	A	4	4	1.00	19	0.210
248	A	4	3	1.00	19	0.158
249	A	5	4	1.00	19	0.210
250	A	6	4	1.00	19	0.210
251	A	7	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
252	A	4	2	1.00	19	0.105
253	A	3	2	1.00	19	0.105
254	A	2	2	1.00	19	0.105
255	A	1	1	1.00	17	0.059
256	A	2	2	1.00	15	0.133
257	A	3	3	1.00	19	0.158
258	A	4	3	1.00	19	0.158
259	A	5	3	1.00	19	0.158
260	A	4	3	1.00	19	0.158
261	A	3	3	1.00	19	0.158
262	A	2	2	1.00	19	0.105
263	A	1	1	1.00	19	0.053
264	A	3	3	1.00	19	0.158
265	A	4	4	1.00	17	0.235
266	A	5	4	1.00	15	0.267
267	A	6	4	1.00	19	0.210
268	A	7	4	1.00	19	0.210
269	A	5	3	1.00	21	0.143
270	A	4	3	1.00	21	0.143
271	A	3	3	1.00	21	0.143
272	A	2	2	1.00	21	0.095
273	A	1	1	1.00	21	0.048
274	A	2	2	1.00	21	0.095
275	A	3	2	1.00	21	0.095
276	A	4	2	1.00	21	0.095
277	A	3	3	1.00	21	0.143
278	A	3	3	1.27	19	0.158
279	A	3	3	1.00	21	0.143
280	A	1	1	1.00	21	0.048
281	A	2	2	1.00	21	0.095
282	A	3	2	1.00	21	0.095
283	A	2	2	1.00	17	0.118
284	A	3	2	1.00	19	0.105
285	A	2	2	1.00	19	0.105
286	A	1	1	1.00	19	0.053
287	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
288	A	3	3	1.00	19	0.158
289	A	3	3	1.00	19	0.158
290	A	2	2	1.00	17	0.118
291	A	3	3	1.00	19	0.158
292	A	5	5	1.00	19	0.263
293	A	4	4	1.00	19	0.210
294	A	5	5	1.00	19	0.263
295	A	5	4	1.00	21	0.190
296	A	6	6	1.00	21	0.286
297	A	3	3	1.00	21	0.143
298	A	4	4	1.00	21	0.190
299	A	5	5	1.00	21	0.238
300	A	2	2	1.00	21	0.095
301	A	3	3	1.00	21	0.143
302	A	6	6	1.00	21	0.286
303	A	1	1	1.00	21	0.048
304	A	4	4	1.00	21	0.190
305	A	7	6	1.00	21	0.286
306	A	2	2	1.00	21	0.095
307	A	5	4	1.00	21	0.190
308	A	3	2	1.00	15	0.133
309	A	3	2	1.00	13	0.154
310	A	5	3	1.00	19	0.158
311	A	4	3	1.00	19	0.158
312	A	3	3	1.00	19	0.158
313	A	2	2	1.00	17	0.118
314	A	1	1	1.00	15	0.067
315	A	2	2	1.00	19	0.105
316	A	3	2	1.00	19	0.105
317	A	4	2	1.00	19	0.105
318	A	5	2	1.00	19	0.105
319	A	4	3	1.00	11	0.273
320	A	4	3	1.00	13	0.231
321	A	3	3	1.00	9	0.333
322	A	3	3	1.00	9	0.333
323	A	3	3	1.00	9	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
324	A	3	3	1.00	13	0.231
325	A	3	3	1.00	13	0.231
326	A	3	3	1.00	13	0.231
327	A	3	3	1.00	13	0.231
328	A	2	2	1.00	11	0.182
329	A	2	2	1.00	15	0.133
330	A	2	2	1.00	15	0.133
331	A	2	2	1.00	23	0.087
332	A	2	2	1.00	11	0.182
333	A	2	2	1.00	13	0.154
334	A	2	2	1.00	13	0.154
335	A	2	2	1.00	17	0.118
336	A	2	2	1.00	17	0.118
337	A	2	2	1.00	17	0.118
338	A	2	2	1.00	11	0.182
339	A	2	2	1.00	11	0.182
340	A	2	2	1.00	11	0.182
341	A	2	2	1.00	11	0.182
342	A	2	2	1.00	13	0.154
343	A	2	2	1.00	13	0.154
344	A	3	2	1.00	11	0.182
345	A	4	3	1.00	11	0.273
346	A	3	2	1.00	11	0.182
347	A	7	7	1.00	9	0.778
348	A	2	2	1.00	22	0.091
349	A	1	1	1.00	13	0.077
350	A	2	2	1.00	15	0.133
351	A	2	2	1.00	17	0.118
352	A	1	1	1.00	13	0.077
353	A	2	2	1.00	15	0.133
354	A	1	1	1.00	13	0.077
355	A	2	2	1.00	11	0.182
356	A	5	5	1.00	13	0.385
357	A	2	2	1.00	15	0.133
358	A	5	5	1.00	13	0.385
359	A	2	2	1.00	15	0.133

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	2	2	1.00	11	0.182
361	A	2	2	1.00	11	0.182
362	A	2	2	1.00	9	0.222
363	A	5	5	1.00	11	0.454
364	A	3	3	1.00	25	0.120
365	A	4	4	1.00	27	0.148
366	A	4	4	1.00	23	0.174
367	A	4	4	1.00	22	0.182
368	A	4	4	1.00	21	0.190
369	A	5	5	1.00	18	0.278
370	A	4	4	1.00	23	0.174
371	A	3	3	1.00	15	0.200
372	A	4	4	1.00	23	0.174
373	A	5	4	1.00	27	0.148
374	A	5	4	1.00	23	0.174
375	A	5	4	1.00	22	0.182
376	A	5	4	1.00	21	0.190
377	A	6	5	1.00	18	0.278
378	A	5	4	1.00	23	0.174
379	A	5	5	1.00	22	0.227
380	A	5	4	1.00	23	0.174
381	A	5	4	1.00	22	0.182
382	A	5	5	1.00	13	0.385
383	A	5	5	1.00	15	0.333
384	A	4	4	1.00	15	0.267
385	A	4	4	1.00	15	0.267
386	A	5	5	1.00	15	0.333
387	A	5	5	1.00	17	0.294
388	A	4	4	1.00	17	0.235
389	A	4	4	1.00	17	0.235
390	A	3	3	1.00	27	0.111
391	A	3	3	1.00	23	0.130
392	A	2	2	1.00	15	0.133
393	A	3	3	1.00	21	0.143
394	A	4	4	1.00	18	0.222
395	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
396	A	3	3	1.00	22	0.136
397	A	3	3	1.00	23	0.130
398	A	4	4	1.00	27	0.148
399	A	4	4	1.00	23	0.174
400	A	4	4	1.00	22	0.182
401	A	4	4	1.00	21	0.190
402	A	5	5	1.00	18	0.278
403	A	4	4	1.00	23	0.174
404	A	4	4	1.00	22	0.182
405	A	4	4	1.00	23	0.174
406	A	4	4	1.00	22	0.182
407	A	3	3	1.00	15	0.200
408	A	3	3	1.00	15	0.200
409	A	3	3	1.00	15	0.200
410	A	3	3	1.00	19	0.158
411	A	3	3	1.00	16	0.188
412	A	3	3	1.00	16	0.188
413	A	3	3	1.00	16	0.188
414	A	3	3	1.00	20	0.150
415	A	3	3	1.00	19	0.158
416	A	3	3	1.00	17	0.176
417	A	3	3	1.00	17	0.176
418	A	3	3	1.00	20	0.150
419	A	3	3	1.00	19	0.158
420	A	3	3	1.00	18	0.167
421	A	3	3	1.00	21	0.143
422	A	3	3	1.00	21	0.143
423	A	3	3	1.00	21	0.143
424	A	3	3	1.00	21	0.143
425	A	3	3	1.00	21	0.143
426	A	3	3	1.00	15	0.200
427	A	3	3	1.00	15	0.200
428	A	3	3	1.00	15	0.200
429	A	3	3	1.00	15	0.200
430	A	3	3	1.00	15	0.200
431	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
432	A	1	1	1.00	11	0.091
433	A	3	3	1.00	13	0.231
434	A	1	1	1.00	17	0.059
435	A	1	1	1.00	17	0.059
436	A	2	2	1.00	15	0.133
437	A	11	10	1.00	19	0.526
438	A	9	8	1.00	19	0.421
439	A	3	3	1.03	17	0.176
440	A	3	3	1.00	22	0.136
441	A	3	3	1.12	22	0.136
442	A	3	3	1.00	27	0.111
443	A	3	3	1.00	27	0.111
444	A	1	1	1.00	18	0.056
445	A	2	2	1.00	17	0.118
446	A	2	2	1.00	22	0.091
447	A	1	1	1.00	23	0.043
448	A	2	2	1.00	19	0.105
449	A	2	2	1.00	19	0.105
450	A	2	2	1.00	19	0.105
451	A	2	2	1.00	19	0.105
452	A	2	2	1.00	19	0.105
453	A	1	1	1.00	25	0.040
454	A	2	2	1.00	28	0.071

Chapter 3

Listing of integrals

3.1 $\int x^2 \left(ax+bx^3\right) dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(ax+bx^3\right) dx &= \int \left(ax^3+bx^5\right) dx\\ &= \frac{ax^4}{4}+\frac{bx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3),x]
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

fricas [A] time = 0.49, size = 13, normalized size = 0.76

$$\frac{1}{6}x^6b + \frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b + 1/4*x^4*a
```

giac [A] time = 0.16, size = 13, normalized size = 0.76

$$\frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{6}b\,x^6 + \frac{1}{4}a\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x),x)
```

```
[Out] 1/4*a*x^4+1/6*b*x^6
```

maxima [A] time = 1.30, size = 13, normalized size = 0.76

$$\frac{1}{6}bx^6 + \frac{1}{4}ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/4*a*x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^6}{6} + \frac{a\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3),x)
```

```
[Out] (a*x^4)/4 + (b*x^6)/6
```

sympy [A] time = 0.10, size = 12, normalized size = 0.71

$$\frac{ax^4}{4} + \frac{bx^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x),x)
```

```
[Out] a*x**4/4 + b*x**6/6
```

3.2 $\int x\left(ax+bx^3\right)\,dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

```
[Out] 1/3*a*x^3+1/5*b*x^5
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(ax+bx^3\right)\,dx &= \int \left(ax^2+bx^4\right)\,dx\\ &= \frac{ax^3}{3}+\frac{bx^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3),x]
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

fricas [A] time = 0.48, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5b+\frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*b + 1/3*x^3*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{5}b\,x^{5}+\frac{1}{3}a\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x),x)
```

```
[Out] 1/3*a*x^3+1/5*b*x^5
```

maxima [A] time = 1.30, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^{5}+\frac{1}{3}\,ax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/3*a*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^{5}}{5}+\frac{a\,x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3),x)
```

```
[Out] (a*x^3)/3 + (b*x^5)/5
```

sympy [A] time = 0.08, size = 12, normalized size = 0.71

$$\frac{ax^{3}}{3}+\frac{bx^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x),x)
```

```
[Out] a*x**3/3 + b*x**5/5
```

3.3 $\int \left(ax+bx^3\right)\,dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

```
[Out] 1/2*a*x^2+1/4*b*x^4
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a*x + b*x^3,x]
[Out] (a*x^2)/2 + (b*x^4)/4
```

Rubi steps

$$\int \left(ax+bx^3\right)\,dx=\frac{ax^2}{2}+\frac{bx^4}{4}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x + b*x^3,x]
[Out] (a*x^2)/2 + (b*x^4)/4
```

fricas [A] time = 0.64, size = 13, normalized size = 0.76

$$\frac{1}{4}x^4b+\frac{1}{2}x^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="fricas")
[Out] 1/4*x^4*b + 1/2*x^2*a
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="giac")
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{4}b\,x^4+\frac{1}{2}a\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^3+a*x,x)
```

```
[Out] 1/2*a*x^2+1/4*b*x^4
```

maxima [A] time = 1.35, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4 + \frac{1}{2}\,ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/2*a*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{b\,x^4}{4} + \frac{a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x + b*x^3,x)
```

```
[Out] (a*x^2)/2 + (b*x^4)/4
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^2}{2} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a*x,x)
```

```
[Out] a*x**2/2 + b*x**4/4
```

3.4 $\int \frac{ax+bx^3}{x} \, dx$

Optimal. Leaf size=12

$$ax + \frac{bx^3}{3}$$

[Out] a*x+1/3*b*x^3

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^3)/x,x]

[Out] a*x + (b*x^3)/3

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax+bx^3}{x} \, dx &= \int \left(a+bx^2\right) \, dx \\ &= ax + \frac{bx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^3)/x,x]

[Out] a*x + (b*x^3)/3

fricas [A] time = 0.73, size = 10, normalized size = 0.83

$$\frac{1}{3} bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^3+a*x)/x,x, algorithm="fricas")

[Out] 1/3*b*x^3 + a*x

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$\frac{1}{3} bx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="giac")
```

```
[Out] 1/3*b*x^3 + a*x
```

maple [A] time = 0.04, size = 11, normalized size = 0.92

$$\frac{1}{3}b\,x^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)/x,x)
```

```
[Out] a*x+1/3*b*x^3
```

maxima [A] time = 1.34, size = 10, normalized size = 0.83

$$\frac{1}{3}\,bx^{3}+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x,x, algorithm="maxima")
```

```
[Out] 1/3*b*x^3 + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^3}{3}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)/x,x)
```

```
[Out] a*x + (b*x^3)/3
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$ax+\frac{bx^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)/x,x)
```

```
[Out] a*x + b*x**3/3
```

3.5 $\int \frac{ax+bx^3}{x^2}\,dx$

Optimal. Leaf size=13

$$a\log(x)+\frac{bx^2}{2}$$

```
[Out] 1/2*b*x^2+a*ln(x)
```

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$a\log(x)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)/x^2,x]
```

```
[Out] (b*x^2)/2 + a*Log[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax+bx^3}{x^2}\,dx &= \int \left(\frac{a}{x}+bx\right)dx\\ &= \frac{bx^2}{2}+a\log(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$a\log(x)+\frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)/x^2,x]
```

```
[Out] (b*x^2)/2 + a*Log[x]
```

fricas [A] time = 0.57, size = 11, normalized size = 0.85

$$\frac{1}{2}\,bx^2+a\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

giac [A] time = 0.15, size = 14, normalized size = 1.08

$$\frac{1}{2}\,bx^2+\frac{1}{2}\,a\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + 1/2*a*log(x^2)
```

maple [A] time = 0.05, size = 12, normalized size = 0.92

$$\frac{b\,x^{2}}{2}+a\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)/x^2,x)
```

```
[Out] 1/2*b*x^2+a*ln(x)
```

maxima [A] time = 1.30, size = 11, normalized size = 0.85

$$\frac{1}{2}\,bx^{2}+a\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*log(x)
```

mupad [B] time = 0.02, size = 11, normalized size = 0.85

$$\frac{b\,x^{2}}{2}+a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)/x^2,x)
```

```
[Out] (b*x^2)/2 + a*log(x)
```

sympy [A] time = 0.11, size = 10, normalized size = 0.77

$$a\log\left(x\right)+\frac{bx^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)/x**2,x)
```

```
[Out] a*log(x) + b*x**2/2
```

3.6 $\int x^2 \left(ax + bx^3\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

```
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 270}

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(ax + bx^3\right)^2 \, dx &= \int x^4 \left(a + bx^2\right)^2 \, dx\\ &= \int \left(a^2x^4 + 2abx^6 + b^2x^8\right) \, dx\\ &= \frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5} + \frac{2}{7}abx^7 + \frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (2*a*b*x^7)/7 + (b^2*x^9)/9
```

fricas [A] time = 0.59, size = 24, normalized size = 0.80

$$\frac{1}{9}x^9b^2 + \frac{2}{7}x^7ba + \frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="fricas")
[Out] 1/9*x^9*b^2 + 2/7*x^7*b*a + 1/5*x^5*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{9}b^2x^9+\frac{2}{7}ab\,x^7+\frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x)^2,x)
[Out] 1/5*a^2*x^5+2/7*a*b*x^7+1/9*b^2*x^9
```

maxima [A] time = 1.31, size = 24, normalized size = 0.80

$$\frac{1}{9}\,b^2x^9+\frac{2}{7}\,abx^7+\frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/9*b^2*x^9 + 2/7*a*b*x^7 + 1/5*a^2*x^5
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5}+\frac{2\,a\,b\,x^7}{7}+\frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3)^2,x)
[Out] (a^2*x^5)/5 + (b^2*x^9)/9 + (2*a*b*x^7)/7
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^5}{5}+\frac{2abx^7}{7}+\frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**2,x)
[Out] a**2*x**5/5 + 2*a*b*x**7/7 + b**2*x**9/9
```

3.7 $\int x\left(ax+bx^3\right)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 43}

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x + b*x^3)^2,x]

[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int x\left(ax+bx^3\right)^2\,dx &= \int x^3\left(a+bx^2\right)^2\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int x(a+bx)^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx^2+b^2x^3\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{1}{3}abx^6+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3)^2,x]
[Out] (a^2*x^4)/4 + (a*b*x^6)/3 + (b^2*x^8)/8
```

fricas [A] time = 0.80, size = 24, normalized size = 0.80

$$\frac{1}{8}x^8b^2 + \frac{1}{3}x^6ba + \frac{1}{4}x^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="fricas")
[Out] 1/8*x^8*b^2 + 1/3*x^6*b*a + 1/4*x^4*a^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.80

$$\frac{1}{8}b^2x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{8}b^2x^8 + \frac{1}{3}ab\,x^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x)^2,x)
[Out] 1/4*a^2*x^4+1/3*a*b*x^6+1/8*b^2*x^8
```

maxima [A] time = 1.34, size = 24, normalized size = 0.80

$$\frac{1}{8}b^2x^8 + \frac{1}{3}abx^6 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/8*b^2*x^8 + 1/3*a*b*x^6 + 1/4*a^2*x^4
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3)^2,x)
[Out] (a^2*x^4)/4 + (b^2*x^8)/8 + (a*b*x^6)/3
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^4}{4} + \frac{abx^6}{3} + \frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**2,x)
[Out] a**2*x**4/4 + a*b*x**6/3 + b**2*x**8/8
```

3.8 $\int \left(ax+bx^3\right)^2\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^3\right)^2\,dx &= \int x^2\left(a+bx^2\right)^2\,dx\\ &= \int \left(a^2x^2+2abx^4+b^2x^6\right)dx\\ &= \frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3}+\frac{2}{5}abx^5+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2,x]
```

```
[Out] (a^2*x^3)/3 + (2*a*b*x^5)/5 + (b^2*x^7)/7
```

fricas [A] time = 0.72, size = 24, normalized size = 0.80

$$\frac{1}{7}x^7b^2+\frac{2}{5}x^5ba+\frac{1}{3}x^3a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*b^2 + 2/5*x^5*b*a + 1/3*x^3*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{7}b^2x^7 + \frac{2}{5}ab\,x^5 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2,x)
```

```
[Out] 1/3*a^2*x^3+2/5*a*b*x^5+1/7*b^2*x^7
```

maxima [A] time = 1.32, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{2}{5}\,abx^5 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*x^7 + 2/5*a*b*x^5 + 1/3*a^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3}+\frac{2\,a\,b\,x^5}{5}+\frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^2,x)
```

```
[Out] (a^2*x^3)/3 + (b^2*x^7)/7 + (2*a*b*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^3}{3}+\frac{2abx^5}{5}+\frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2,x)
```

```
[Out] a**2*x**3/3 + 2*a*b*x**5/5 + b**2*x**7/7
```

3.9 $\int \frac{(ax+bx^3)^2}{x} dx$

Optimal. Leaf size=16

$$\frac{(a+bx^2)^3}{6b}$$

```
[Out] 1/6*(b*x^2+a)^3/b
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{(a+bx^2)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2/x,x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax+bx^3)^2}{x} dx &= \int x(a+bx^2)^2 dx\\ &= \frac{(a+bx^2)^3}{6b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{(a+bx^2)^3}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2/x,x]
```

```
[Out] (a + b*x^2)^3/(6*b)
```

fricas [A] time = 0.67, size = 24, normalized size = 1.50

$$\frac{1}{6}b^2x^6 + \frac{1}{2}abx^4 + \frac{1}{2}a^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

giac [A] time = 0.20, size = 24, normalized size = 1.50

$$\frac{1}{6} b^2 x^6 + \frac{1}{2} abx^4 + \frac{1}{2} a^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="giac")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

maple [A] time = 0.04, size = 25, normalized size = 1.56

$$\frac{1}{6}b^2 x^6 + \frac{1}{2}ab\,x^4 + \frac{1}{2}a^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2/x,x)
```

```
[Out] 1/6*b^2*x^6+1/2*a*b*x^4+1/2*a^2*x^2
```

maxima [A] time = 1.29, size = 24, normalized size = 1.50

$$\frac{1}{6} b^2 x^6 + \frac{1}{2} abx^4 + \frac{1}{2} a^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/6*b^2*x^6 + 1/2*a*b*x^4 + 1/2*a^2*x^2
```

mupad [B] time = 0.03, size = 24, normalized size = 1.50

$$\frac{a^2\,x^2}{2} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^2/x,x)
```

```
[Out] (a^2*x^2)/2 + (b^2*x^6)/6 + (a*b*x^4)/2
```

sympy [B] time = 0.07, size = 24, normalized size = 1.50

$$\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2/x,x)
```

```
[Out] a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6
```

3.10 $\int \frac{(ax+bx^3)^2}{x^2}\,dx$

Optimal. Leaf size=25

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 194}

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{(ax+bx^3)^2}{x^2}\,dx &= \int \left(a+bx^2\right)^2 dx\\ &= \int \left(a^2+2abx^2+b^2x^4\right) dx\\ &= a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$a^2x+\frac{2}{3}abx^3+\frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^2/x^2,x]
```

```
[Out] a^2*x + (2*a*b*x^3)/3 + (b^2*x^5)/5
```

fricas [A] time = 0.89, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^2x^5+\frac{2}{3}\,abx^3+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

giac [A] time = 0.15, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^{2}x^{5}+\frac{2}{3}\,abx^{3}+a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

maple [A] time = 0.05, size = 22, normalized size = 0.88

$$\frac{1}{5}b^{2}x^{5}+\frac{2}{3}ab\,x^{3}+a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^2/x^2,x)
```

```
[Out] a^2*x+2/3*a*b*x^3+1/5*b^2*x^5
```

maxima [A] time = 1.30, size = 21, normalized size = 0.84

$$\frac{1}{5}\,b^{2}x^{5}+\frac{2}{3}\,abx^{3}+a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*x^5 + 2/3*a*b*x^3 + a^2*x
```

mupad [B] time = 0.03, size = 21, normalized size = 0.84

$$a^{2}\,x+\frac{2\,a\,b\,x^{3}}{3}+\frac{b^{2}\,x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^2/x^2,x)
```

```
[Out] a^2*x + (b^2*x^5)/5 + (2*a*b*x^3)/3
```

sympy [A] time = 0.08, size = 22, normalized size = 0.88

$$a^{2}x+\frac{2abx^{3}}{3}+\frac{b^{2}x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**2/x**2,x)
```

```
[Out] a**2*x + 2*a*b*x**3/3 + b**2*x**5/5
```

3.11 $\int \left(-4x+3x^3\right)^6\,dx$

Optimal. Leaf size=46

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

```
[Out] 4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729/
19*x^19
```

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(-4*x + 3*x^3)^6,x]
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \left(-4x+3x^3\right)^6\,dx &= \int x^6\left(-4+3x^2\right)^6\,dx\\
&= \int \left(4096x^6-18432x^8+34560x^{10}-34560x^{12}+19440x^{14}-5832x^{16}+729x^{18}\right)dx\\
&= \frac{4096x^7}{7}-2048x^9+\frac{34560x^{11}}{11}-\frac{34560x^{13}}{13}+1296x^{15}-\frac{5832x^{17}}{17}+\frac{729x^{19}}{19}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 46, normalized size = 1.00

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^9+\frac{4096x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4*x + 3*x^3)^6,x]
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

fricas [A] time = 0.54, size = 36, normalized size = 0.78

$$\frac{729}{19}x^{19}-\frac{5832}{17}x^{17}+1296x^{15}-\frac{34560}{13}x^{13}+\frac{34560}{11}x^{11}-2048x^9+\frac{4096}{7}x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="fricas")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

giac [A] time = 0.15, size = 36, normalized size = 0.78

$$\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^{9}+\frac{4096}{7}\,x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="giac")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

maple [A] time = 0.04, size = 37, normalized size = 0.80

$$\frac{729}{19}x^{19}-\frac{5832}{17}x^{17}+1296x^{15}-\frac{34560}{13}x^{13}+\frac{34560}{11}x^{11}-2048x^{9}+\frac{4096}{7}x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-4*x)^6,x)
```

```
[Out] 4096/7*x^7-2048*x^9+34560/11*x^11-34560/13*x^13+1296*x^15-5832/17*x^17+729/
19*x^19
```

maxima [A] time = 1.33, size = 36, normalized size = 0.78

$$\frac{729}{19}\,x^{19}-\frac{5832}{17}\,x^{17}+1296\,x^{15}-\frac{34560}{13}\,x^{13}+\frac{34560}{11}\,x^{11}-2048\,x^{9}+\frac{4096}{7}\,x^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-4*x)^6,x, algorithm="maxima")
```

```
[Out] 729/19*x^19 - 5832/17*x^17 + 1296*x^15 - 34560/13*x^13 + 34560/11*x^11 - 20
48*x^9 + 4096/7*x^7
```

mupad [B] time = 0.04, size = 36, normalized size = 0.78

$$\frac{729\,x^{19}}{19}-\frac{5832\,x^{17}}{17}+1296\,x^{15}-\frac{34560\,x^{13}}{13}+\frac{34560\,x^{11}}{11}-2048\,x^{9}+\frac{4096\,x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x - 3*x^3)^6,x)
```

```
[Out] (4096*x^7)/7 - 2048*x^9 + (34560*x^11)/11 - (34560*x^13)/13 + 1296*x^15 - (
5832*x^17)/17 + (729*x^19)/19
```

sympy [A] time = 0.07, size = 42, normalized size = 0.91

$$\frac{729x^{19}}{19}-\frac{5832x^{17}}{17}+1296x^{15}-\frac{34560x^{13}}{13}+\frac{34560x^{11}}{11}-2048x^{9}+\frac{4096x^{7}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-4*x)**6,x)
```

```
[Out] 729*x**19/19 - 5832*x**17/17 + 1296*x**15 - 34560*x**13/13 + 34560*x**11/11
 - 2048*x**9 + 4096*x**7/7
```

3.12 $\int \frac{x^4}{ax+bx^3}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] 1/2*x^2/b-1/2*a*ln(b*x^2+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 43}

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3),x]
```

```
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{ax+bx^3}\,dx &= \int \frac{x^3}{a+bx^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+bx}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)\,dx,x,x^2\right)\\
&= \frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 1.00

$$\frac{x^2}{2b}-\frac{a\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3),x]
[Out] x^2/(2*b) - (a*Log[a + b*x^2])/(2*b^2)
```

fricas [A] time = 0.78, size = 22, normalized size = 0.81

$$\frac{bx^2 - a\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="fricas")
[Out] 1/2*(b*x^2 - a*log(b*x^2 + a))/b^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.89

$$\frac{x^2}{2\,b} - \frac{a\log\left(\left|bx^2 + a\right|\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="giac")
[Out] 1/2*x^2/b - 1/2*a*log(abs(b*x^2 + a))/b^2
```

maple [A] time = 0.04, size = 24, normalized size = 0.89

$$\frac{x^2}{2b} - \frac{a\ln\left(b\,x^2 + a\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x),x)
[Out] 1/2/b*x^2-1/2*a*ln(b*x^2+a)/b^2
```

maxima [A] time = 1.32, size = 23, normalized size = 0.85

$$\frac{x^2}{2\,b} - \frac{a\log\left(bx^2 + a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x),x, algorithm="maxima")
[Out] 1/2*x^2/b - 1/2*a*log(b*x^2 + a)/b^2
```

mupad [B] time = 0.04, size = 22, normalized size = 0.81

$$-\frac{a\,\ln\left(b\,x^2 + a\right) - b\,x^2}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3),x)
[Out] -(a*log(a + b*x^2) - b*x^2)/(2*b^2)
```

sympy [A] time = 0.15, size = 20, normalized size = 0.74

$$-\frac{a\log\left(a + bx^2\right)}{2b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x),x)
```

```
[Out] -a*log(a + b*x**2)/(2*b**2) + x**2/(2*b)
```

3.13 $\int \frac{x^3}{ax+bx^3}\,dx$

Optimal. Leaf size=31

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

```
[Out] x/b-arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 321, 205}

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{ax+bx^3}\,dx &= \int \frac{x^2}{a+bx^2}\,dx\\ &= \frac{x}{b}-\frac{a\int \frac{1}{a+bx^2}\,dx}{b}\\ &= \frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{x}{b}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2)
```

fricas [A] time = 0.83, size = 82, normalized size = 2.65

$$\left[\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,x}{2\,b},\,-\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-x}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 2*x)/b,
-(sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - x)/b]
```

giac [A] time = 0.15, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

maple [A] time = 0.04, size = 27, normalized size = 0.87

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x),x)
```

```
[Out] 1/b*x-a/b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.94, size = 26, normalized size = 0.84

$$-\frac{a\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -a*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + x/b
```

mupad [B] time = 4.91, size = 23, normalized size = 0.74

$$\frac{x}{b}-\frac{\sqrt{a}\;\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3),x)
```

```
[Out] x/b - (a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/b^(3/2)
```

sympy [**B**] time = 0.17, size = 56, normalized size = 1.81

$$\frac{\sqrt{-\frac{a}{b^3}}\log\left(-b\sqrt{-\frac{a}{b^3}}+x\right)}{2}-\frac{\sqrt{-\frac{a}{b^3}}\log\left(b\sqrt{-\frac{a}{b^3}}+x\right)}{2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x),x)
```

```
[Out] sqrt(-a/b**3)*log(-b*sqrt(-a/b**3) + x)/2 - sqrt(-a/b**3)*log(b*sqrt(-a/b**
3) + x)/2 + x/b
```

3.14 $\int \frac{x^2}{ax+bx^3} dx$

Optimal. Leaf size=15

$$\frac{\log(a+bx^2)}{2b}$$

```
[Out] 1/2*ln(b*x^2+a)/b
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 260}

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{ax+bx^3}\,dx &= \int \frac{x}{a+bx^2}\,dx\\ &= \frac{\log(a+bx^2)}{2b}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(a+bx^2)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3),x]
```

```
[Out] Log[a + b*x^2]/(2*b)
```

fricas [A] time = 0.88, size = 13, normalized size = 0.87

$$\frac{\log(bx^2+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="fricas")
[Out] 1/2*log(b*x^2 + a)/b
```

giac [A] time = 0.19, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|bx^2+a\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="giac")
[Out] 1/2*log(abs(b*x^2 + a))/b
```

maple [A] time = 0.04, size = 14, normalized size = 0.93

$$\frac{\ln\left(b\,x^2+a\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x),x)
[Out] 1/2*ln(b*x^2+a)/b
```

maxima [A] time = 1.42, size = 13, normalized size = 0.87

$$\frac{\log\left(bx^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 + a)/b
```

mupad [B] time = 4.92, size = 13, normalized size = 0.87

$$\frac{\ln\left(b\,x^2+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3),x)
[Out] log(a + b*x^2)/(2*b)
```

sympy [A] time = 0.13, size = 10, normalized size = 0.67

$$\frac{\log\left(a+bx^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x),x)
[Out] log(a + b*x**2)/(2*b)
```

3.15 $\int \frac{x}{ax+bx^3}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

```
[Out] arctan(x*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1584, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{ax+bx^3}\,dx &= \int \frac{1}{a+bx^2}\,dx\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.59, size = 67, normalized size = 2.79

$$\left[-\frac{\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab},\ \frac{\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

giac [A] time = 0.15, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.04, size = 16, normalized size = 0.67

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.95, size = 15, normalized size = 0.62

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 0.04, size = 16, normalized size = 0.67

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3),x)
```

```
[Out] atan((b^(1/2)*x)/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [B] time = 0.15, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

3.16 $\int \frac{1}{ax+bx^3}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}$$

```
[Out] ln(x)/a-1/2*ln(b*x^2+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{ax+bx^3}\,dx &= \int \frac{1}{x\left(a+bx^2\right)}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)}{2a} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,x^2\right)}{2a} \\
&= \frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}
\end{aligned}$$

Mathematica [**A**] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log\left(a+bx^2\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-1),x]
[Out] Log[x]/a - Log[a + b*x^2]/(2*a)
```

fricas [**A**] time = 0.80, size = 18, normalized size = 0.82

$$-\frac{\log\left(bx^2+a\right) - 2\,\log(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="fricas")
[Out] -1/2*(log(b*x^2 + a) - 2*log(x))/a
```

giac [**A**] time = 0.15, size = 24, normalized size = 1.09

$$\frac{\log\left(x^2\right)}{2\,a} - \frac{\log\left(\left|bx^2+a\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="giac")
[Out] 1/2*log(x^2)/a - 1/2*log(abs(b*x^2 + a))/a
```

maple [**A**] time = 0.05, size = 21, normalized size = 0.95

$$\frac{\ln(x)}{a} - \frac{\ln\left(b\,x^2+a\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x),x)
[Out] 1/a*ln(x)-1/2*ln(b*x^2+a)/a
```

maxima [**A**] time = 1.34, size = 20, normalized size = 0.91

$$-\frac{\log\left(bx^2+a\right)}{2\,a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 + a)/a + log(x)/a
```

mupad [B] time = 0.06, size = 18, normalized size = 0.82

$$-\frac{\ln\left(b\,x^2+a\right)-2\,\ln(x)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3),x)
[Out] -(log(a + b*x^2) - 2*log(x))/(2*a)
```

sympy [A] time = 0.21, size = 15, normalized size = 0.68

$$\frac{\log{\left(x \right)}}{a} - \frac{\log{\left(\frac{a}{b} + x^{2} \right)}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x),x)
[Out] log(x)/a - log(a/b + x**2)/(2*a)
```

3.17 $\int \frac{1}{x(ax+bx^3)}\,dx$

Optimal. Leaf size=34

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

```
[Out] -1/a/x-arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 325, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)),x]
```

```
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(ax+bx^3\right)}\,dx &= \int \frac{1}{x^2\left(a+bx^2\right)}\,dx\\ &= -\frac{1}{ax}-\frac{b\int \frac{1}{a+bx^2}\,dx}{a}\\ &= -\frac{1}{ax}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 34, normalized size = 1.00

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)),x]
[Out] -(1/(a*x)) - (Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2)
```

fricas [A] time = 0.72, size = 82, normalized size = 2.41

$$\left[\frac{x\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-2}{2\,ax}, -\frac{x\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+1}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="fricas")
[Out] [1/2*(x*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 2)/(a*
x), -(x*sqrt(b/a)*arctan(x*sqrt(b/a)) + 1)/(a*x)]
```

giac [A] time = 0.15, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="giac")
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

maple [A] time = 0.05, size = 30, normalized size = 0.88

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x),x)
[Out] -1/a*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/a/x
```

maxima [A] time = 2.96, size = 29, normalized size = 0.85

$$-\frac{b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x),x, algorithm="maxima")
[Out] -b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/(a*x)
```

mupad [B] time = 4.96, size = 26, normalized size = 0.76

$$-\frac{1}{a\,x}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3)),x)
```

```
[Out] - 1/(a*x) - (b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

sympy [B] time = 0.22, size = 65, normalized size = 1.91

$$\frac{\sqrt{-\frac{b}{a^3}}\log\left(-\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^3}}\log\left(\frac{a^2\sqrt{-\frac{b}{a^3}}}{b}+x\right)}{2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x),x)
```

```
[Out] sqrt(-b/a**3)*log(-a**2*sqrt(-b/a**3)/b + x)/2 - sqrt(-b/a**3)*log(a**2*sqr
t(-b/a**3)/b + x)/2 - 1/(a*x)
```

3.18 $\int \frac{1}{x^2(ax+bx^3)}\,dx$

Optimal. Leaf size=35

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2-b*ln(x)/a^2+1/2*b*ln(b*x^2+a)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 44}

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)),x]
```

```
[Out] -1/(2*a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(ax+bx^3\right)}\,dx &= \int \frac{1}{x^3\left(a+bx^2\right)}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx,x,x^2\right)\\ &= -\frac{1}{2ax^2}-\frac{b\log(x)}{a^2}+\frac{b\log\left(a+bx^2\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{b\log\left(a+bx^2\right)}{2a^2}-\frac{b\log(x)}{a^2}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)),x]
```

```
[Out] -1/2*1/(a*x^2) - (b*Log[x])/a^2 + (b*Log[a + b*x^2])/(2*a^2)
```

fricas [A] time = 0.60, size = 33, normalized size = 0.94

$$\frac{bx^2 \log\left(bx^2 + a\right) - 2\,bx^2 \log(x) - a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + a) - 2*b*x^2*log(x) - a)/(a^2*x^2)
```

giac [A] time = 0.17, size = 43, normalized size = 1.23

$$-\frac{b\log\left(x^2\right)}{2\,a^2} + \frac{b\log\left(\left|bx^2 + a\right|\right)}{2\,a^2} + \frac{bx^2 - a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2)/a^2 + 1/2*b*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 - a)/(a^2*
x^2)
```

maple [A] time = 0.05, size = 32, normalized size = 0.91

$$-\frac{b\ln(x)}{a^2} + \frac{b\ln\left(b\,x^2 + a\right)}{2a^2} - \frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x),x)
```

```
[Out] -1/2/a/x^2-1/a^2*b*ln(x)+1/2*b*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.27, size = 31, normalized size = 0.89

$$\frac{b\log\left(bx^2 + a\right)}{2\,a^2} - \frac{b\log(x)}{a^2} - \frac{1}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + a)/a^2 - b*log(x)/a^2 - 1/2/(a*x^2)
```

mupad [B] time = 0.06, size = 31, normalized size = 0.89

$$\frac{b\,\ln\left(b\,x^2 + a\right)}{2\,a^2} - \frac{1}{2\,a\,x^2} - \frac{b\,\ln(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3)),x)
```

```
[Out] (b*log(a + b*x^2))/(2*a^2) - 1/(2*a*x^2) - (b*log(x))/a^2
```

sympy [A] time = 0.29, size = 31, normalized size = 0.89

$$-\frac{1}{2ax^2} - \frac{b\log(x)}{a^2} + \frac{b\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x),x)
```

```
[Out] -1/(2*a*x**2) - b*log(x)/a**2 + b*log(a/b + x**2)/(2*a**2)
```

3.19 $\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx$

Optimal. Leaf size=43

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3+b/a^2/x+b^(3/2)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)
```

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 325, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a*x + b*x^3)),x]
```

```
[Out] -1/(3*a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(ax+bx^3\right)}\,dx &= \int \frac{1}{x^4\left(a+bx^2\right)}\,dx\\
&= -\frac{1}{3ax^3}-\frac{b\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{a}\\
&= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^2\int \frac{1}{a+bx^2}\,dx}{a^2}\\
&= -\frac{1}{3ax^3}+\frac{b}{a^2x}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 43, normalized size = 1.00

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}}+\frac{b}{a^2x}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a*x + b*x^3)),x]
[Out] -1/3*1/(a*x^3) + b/(a^2*x) + (b^(3/2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(5/2)
```

fricas [A] time = 0.64, size = 106, normalized size = 2.47

$$\left[\frac{3\,bx^3\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+6\,bx^2-2\,a}{6\,a^2x^3},\frac{3\,bx^3\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+3\,bx^2-a}{3\,a^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="fricas")
[Out] [1/6*(3*b*x^3*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) +
6*b*x^2 - 2*a)/(a^2*x^3), 1/3*(3*b*x^3*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3*b*
x^2 - a)/(a^2*x^3)]
```

giac [A] time = 0.15, size = 40, normalized size = 0.93

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="giac")
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

maple [A] time = 0.05, size = 39, normalized size = 0.91

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\;a^2}+\frac{b}{a^2x}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x),x)
[Out] 1/a^2*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/3/a/x^3+b/a^2/x
```

maxima [A] time = 2.97, size = 40, normalized size = 0.93

$$\frac{b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}+\frac{3\,bx^2-a}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x),x, algorithm="maxima")
[Out] b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/3*(3*b*x^2 - a)/(a^2*x^3)
```

mupad [**B**] time = 4.94, size = 37, normalized size = 0.86

$$\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}}-\frac{\frac{1}{3\,a}-\frac{b\,x^2}{a^2}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^3)),x)
```

```
[Out] (b^(3/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(5/2) - (1/(3*a) - (b*x^2)/a^2)/x^3
```

sympy [**B**] time = 0.25, size = 87, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(-\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^5}}\log\left(\frac{a^3\sqrt{-\frac{b^3}{a^5}}}{b^2}+x\right)}{2}+\frac{-a+3bx^2}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x),x)
```

```
[Out] -sqrt(-b**3/a**5)*log(-a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + sqrt(-b**3/a**5)
*log(a**3*sqrt(-b**3/a**5)/b**2 + x)/2 + (-a + 3*b*x**2)/(3*a**2*x**3)
```

3.20 $\int \frac{1}{x^4(ax+bx^3)}\,dx$

Optimal. Leaf size=49

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+b^2*ln(x)/a^3-1/2*b^2*ln(b*x^2+a)/a^3
```

Rubi [A] time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 44}

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a*x + b*x^3)),x]
```

```
[Out] -1/(4*a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2*a
^3)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(ax+bx^3\right)}\,dx &= \int \frac{1}{x^5\left(a+bx^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(a+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{4ax^4}+\frac{b}{2a^2x^2}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log\left(a+bx^2\right)}{2a^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.00

$$-\frac{b^2\log\left(a+bx^2\right)}{2a^3}+\frac{b^2\log(x)}{a^3}+\frac{b}{2a^2x^2}-\frac{1}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a*x + b*x^3)),x]
```

```
[Out] -1/4*1/(a*x^4) + b/(2*a^2*x^2) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x^2])/(2
*a^3)
```

fricas [A] time = 0.92, size = 45, normalized size = 0.92

$$-\frac{2\,b^2x^4\log\left(bx^2+a\right)-4\,b^2x^4\log(x)-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*b^2*x^4*log(b*x^2 + a) - 4*b^2*x^4*log(x) - 2*a*b*x^2 + a^2)/(a^3*x
^4)
```

giac [A] time = 0.16, size = 57, normalized size = 1.16

$$\frac{b^2\log\left(x^2\right)}{2\,a^3}-\frac{b^2\log\left(\left|bx^2+a\right|\right)}{2\,a^3}-\frac{3\,b^2x^4-2\,abx^2+a^2}{4\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="giac")
```

```
[Out] 1/2*b^2*log(x^2)/a^3 - 1/2*b^2*log(abs(b*x^2 + a))/a^3 - 1/4*(3*b^2*x^4 - 2
*a*b*x^2 + a^2)/(a^3*x^4)
```

maple [A] time = 0.05, size = 44, normalized size = 0.90

$$\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(b\,x^2+a\right)}{2a^3}+\frac{b}{2a^2x^2}-\frac{1}{4a\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^3+a*x),x)
```

```
[Out] -1/4/a/x^4+1/2*b/a^2/x^2+1/a^3*b^2*ln(x)-1/2*b^2*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.39, size = 44, normalized size = 0.90

$$-\frac{b^2\log\left(bx^2+a\right)}{2\,a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,bx^2-a}{4\,a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^3+a*x),x, algorithm="maxima")
```

```
[Out] -1/2*b^2*log(b*x^2 + a)/a^3 + b^2*log(x)/a^3 + 1/4*(2*b*x^2 - a)/(a^2*x^4)
```

mupad [B] time = 0.06, size = 46, normalized size = 0.94

$$\frac{b^2\,\ln(x)}{a^3}-\frac{b^2\,\ln\left(b\,x^2+a\right)}{2\,a^3}-\frac{\frac{1}{4\,a}-\frac{b\,x^2}{2\,a^2}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^3)),x)
```

```
[Out] (b^2*log(x))/a^3 - (b^2*log(a + b*x^2))/(2*a^3) - (1/(4*a) - (b*x^2)/(2*a^2
))/x^4
```

<u>**sympy**</u> **[A]** time = 0.34, size = 42, normalized size = 0.86

$$\frac{-a+2bx^2}{4a^2x^4}+\frac{b^2\log{(x)}}{a^3}-\frac{b^2\log{\left(\frac{a}{b}+x^2\right)}}{2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**3+a*x),x)
```

```
[Out] (-a + 2*b*x**2)/(4*a**2*x**4) + b**2*log(x)/a**3 - b**2*log(a/b + x**2)/(2*
a**3)
```

3.21 $\int \frac{x^2}{\left(ax+bx^3\right)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

```
[Out] 1/2*x/a/(b*x^2+a)+1/2*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3)^2,x]
```

```
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(ax+bx^3\right)^2}\,dx &= \int \frac{1}{\left(a+bx^2\right)^2}\,dx\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\int \frac{1}{a+bx^2}\,dx}{2a}\\ &= \frac{x}{2a\left(a+bx^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{x}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3)^2,x]
[Out] x/(2*a*(a + b*x^2)) + ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt[b])
```

fricas [A] time = 0.68, size = 120, normalized size = 2.67

$$\left[\frac{2\,abx-\left(bx^2+a\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\left(a^2b^2x^2+a^3b\right)},\frac{abx+\left(bx^2+a\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\left(a^2b^2x^2+a^3b\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
[Out] [1/4*(2*a*b*x - (b*x^2 + a)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)))/(a^2*b^2*x^2 + a^3*b), 1/2*(a*b*x + (b*x^2 + a)*sqrt(a*b)*arctan
(sqrt(a*b)*x/a))/(a^2*b^2*x^2 + a^3*b)]
```

giac [A] time = 0.17, size = 35, normalized size = 0.78

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}+\frac{x}{2\left(bx^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="giac")
[Out] 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*x/((b*x^2 + a)*a)
```

maple [A] time = 0.05, size = 36, normalized size = 0.80

$$\frac{x}{2\left(b\,x^2+a\right)a}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x)^2,x)
[Out] 1/2*x/a/(b*x^2+a)+1/2/a/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)
```

maxima [A] time = 2.98, size = 35, normalized size = 0.78

$$\frac{x}{2\left(abx^2+a^2\right)}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
[Out] 1/2*x/(a*b*x^2 + a^2) + 1/2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a)
```

mupad [B] time = 4.96, size = 33, normalized size = 0.73

$$\frac{x}{2\,a\,\left(b\,x^2+a\right)}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3)^2,x)
```

```
[Out] x/(2*a*(a + b*x^2)) + atan((b^(1/2)*x)/a^(1/2))/(2*a^(3/2)*b^(1/2))
```

sympy [B] time = 0.24, size = 78, normalized size = 1.73

$$\frac{x}{2a^2+2abx^2}-\frac{\sqrt{-\frac{1}{a^3b}}\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^3b}}\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*b*x**2) - sqrt(-1/(a**3*b))*log(-a**2*sqrt(-1/(a**3*b)) + x
)/4 + sqrt(-1/(a**3*b))*log(a**2*sqrt(-1/(a**3*b)) + x)/4
```

3.22 $\int \frac{x}{\left(ax+bx^3\right)^2}\,dx$

Optimal. Leaf size=38

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

```
[Out] 1/2/a/(b*x^2+a)+ln(x)/a^2-1/2*ln(b*x^2+a)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 44}

$$-\frac{\log\left(a+bx^2\right)}{2a^2}+\frac{\log(x)}{a^2}+\frac{1}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3)^2,x]
```

```
[Out] 1/(2*a*(a + b*x^2)) + Log[x]/a^2 - Log[a + b*x^2]/(2*a^2)
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(ax+bx^3\right)^2}\,dx &= \int \frac{1}{x\left(a+bx^2\right)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right)dx,x,x^2\right)\\
&= \frac{1}{2a\left(a+bx^2\right)}+\frac{\log(x)}{a^2}-\frac{\log\left(a+bx^2\right)}{2a^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.87

$$\frac{\frac{a}{a+bx^2} - \log\left(a+bx^2\right) + 2\log(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3)^2,x]
```

```
[Out] (a/(a + b*x^2) + 2*Log[x] - Log[a + b*x^2])/(2*a^2)
```

fricas [A] time = 0.72, size = 47, normalized size = 1.24

$$-\frac{\left(bx^2+a\right)\log\left(bx^2+a\right) - 2\left(bx^2+a\right)\log(x) - a}{2\left(a^2bx^2+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((b*x^2 + a)*log(b*x^2 + a) - 2*(b*x^2 + a)*log(x) - a)/(a^2*b*x^2 + a
^3)
```

giac [A] time = 0.16, size = 47, normalized size = 1.24

$$\frac{\log\left(x^2\right)}{2\,a^2} - \frac{\log\left(\left|bx^2+a\right|\right)}{2\,a^2} + \frac{bx^2+2\,a}{2\left(bx^2+a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(x^2)/a^2 - 1/2*log(abs(b*x^2 + a))/a^2 + 1/2*(b*x^2 + 2*a)/((b*x^2
+ a)*a^2)
```

maple [A] time = 0.05, size = 35, normalized size = 0.92

$$\frac{1}{2\left(b\,x^2+a\right)a} + \frac{\ln(x)}{a^2} - \frac{\ln\left(b\,x^2+a\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x)^2,x)
```

```
[Out] 1/2/a/(b*x^2+a)+1/a^2*ln(x)-1/2*ln(b*x^2+a)/a^2
```

maxima [A] time = 1.40, size = 34, normalized size = 0.89

$$\frac{1}{2\left(abx^2+a^2\right)} - \frac{\log\left(bx^2+a\right)}{2\,a^2} + \frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(a*b*x^2 + a^2) - 1/2*log(b*x^2 + a)/a^2 + log(x)/a^2
```

mupad [B] time = 0.05, size = 34, normalized size = 0.89

$$\frac{\ln(x)}{a^2} + \frac{1}{2\,a\,\left(b\,x^2+a\right)} - \frac{\ln\left(b\,x^2+a\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3)^2,x)
```

```
[Out] log(x)/a^2 + 1/(2*a*(a + b*x^2)) - log(a + b*x^2)/(2*a^2)
```

sympy **[A]** time = 0.33, size = 34, normalized size = 0.89

$$\frac{1}{2a^2 + 2abx^2} + \frac{\log(x)}{a^2} - \frac{\log\left(\frac{a}{b} + x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**2,x)
```

```
[Out] 1/(2*a**2 + 2*a*b*x**2) + log(x)/a**2 - log(a/b + x**2)/(2*a**2)
```

3.23 $\int \frac{1}{\left(ax+bx^3\right)^2}\,dx$

Optimal. Leaf size=57

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

```
[Out] -3/2/a^2/x+1/2/a/x/(b*x^2+a)-3/2*arctan(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(5/2)
```

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1593, 290, 325, 205}

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3}{2a^2x}+\frac{1}{2ax\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-2),x]
```

```
[Out] -3/(2*a^2*x) + 1/(2*a*x*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(2*a^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\left(ax+bx^3\right)^2}\,dx = \int \frac{1}{x^2\left(a+bx^2\right)^2}\,dx$$
$$= \frac{1}{2ax\left(a+bx^2\right)} + \frac{3\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax\left(a+bx^2\right)} - \frac{(3b)\int \frac{1}{a+bx^2}\,dx}{2a^2}$$
$$= -\frac{3}{2a^2x} + \frac{1}{2ax\left(a+bx^2\right)} - \frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}}$$

Mathematica [A] time = 0.04, size = 54, normalized size = 0.95

$$-\frac{3\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}} - \frac{bx}{2a^2\left(a+bx^2\right)} - \frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-2),x]
```

```
[Out] -(1/(a^2*x)) - (b*x)/(2*a^2*(a + b*x^2)) - (3*Sqrt[b]*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(2*a^(5/2))
```

fricas [A] time = 0.77, size = 136, normalized size = 2.39

$$\left[-\frac{6\,bx^2 - 3\left(bx^3+ax\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) + 4\,a}{4\left(a^2bx^3+a^3x\right)},\ -\frac{3\,bx^2 + 3\left(bx^3+ax\right)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right) + 2\,a}{2\left(a^2bx^3+a^3x\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(6*b*x^2 - 3*(b*x^3 + a*x)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) -
 a)/(b*x^2 + a)) + 4*a)/(a^2*b*x^3 + a^3*x), -1/2*(3*b*x^2 + 3*(b*x^3 + a*x
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 2*a)/(a^2*b*x^3 + a^3*x)]
```

giac [A] time = 0.15, size = 47, normalized size = 0.82

$$-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2} - \frac{3\,bx^2+2\,a}{2\left(bx^3+ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] -3/2*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(3*b*x^2 + 2*a)/((b*x^3
+ a*x)*a^2)
```

maple [A] time = 0.06, size = 46, normalized size = 0.81

$$-\frac{bx}{2\left(b\,x^2+a\right)a^2} - \frac{3b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^2} - \frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x)^2,x)
```

```
[Out] -1/2/a^2*b*x/(b*x^2+a)-3/2/a^2*b/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1/a^
2/x
```

maxima [A] time = 2.96, size = 49, normalized size = 0.86

$$-\frac{3\,bx^{2}+2\,a}{2\left(a^{2}bx^{3}+a^{3}x\right)}-\frac{3\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*b*x^2 + 2*a)/(a^2*b*x^3 + a^3*x) - 3/2*b*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^2)
```

mupad [B] time = 4.98, size = 44, normalized size = 0.77

$$-\frac{\frac{1}{a}+\frac{3\,b\,x^{2}}{2\,a^{2}}}{b\,x^{3}+a\,x}-\frac{3\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3)^2,x)
```

```
[Out] - (1/a + (3*b*x^2)/(2*a^2))/(a*x + b*x^3) - (3*b^(1/2)*atan((b^(1/2)*x)/a^(
1/2)))/(2*a^(5/2))
```

sympy [A] time = 0.33, size = 92, normalized size = 1.61

$$\frac{3\sqrt{-\frac{b}{a^{5}}}\log\left(-\frac{a^{3}\sqrt{-\frac{b}{a^{5}}}}{b}+x\right)}{4}-\frac{3\sqrt{-\frac{b}{a^{5}}}\log\left(\frac{a^{3}\sqrt{-\frac{b}{a^{5}}}}{b}+x\right)}{4}+\frac{-2a-3bx^{2}}{2a^{3}x+2a^{2}bx^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**2,x)
```

```
[Out] 3*sqrt(-b/a**5)*log(-a**3*sqrt(-b/a**5)/b + x)/4 - 3*sqrt(-b/a**5)*log(a**3
*sqrt(-b/a**5)/b + x)/4 + (-2*a - 3*b*x**2)/(2*a**3*x + 2*a**2*b*x**3)
```

3.24 $\int \frac{1}{x(ax+bx^3)^2} dx$

Optimal. Leaf size=49

$$\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{1}{2a^2x^2}$$

```
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2*b*ln(x)/a^3+b*ln(b*x^2+a)/a^3
```

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1584, 266, 44}

$$-\frac{b}{2a^2\left(a+bx^2\right)}+\frac{b\log\left(a+bx^2\right)}{a^3}-\frac{2b\log(x)}{a^3}-\frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)^2),x]
```

```
[Out] -1/(2*a^2*x^2) - b/(2*a^2*(a + b*x^2)) - (2*b*Log[x])/a^3 + (b*Log[a + b*x^
2])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(ax+bx^3\right)^2}\,dx &= \int \frac{1}{x^3\left(a+bx^2\right)^2}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(a+bx)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx,x,x^2\right)\\
&= -\frac{1}{2a^2x^2}-\frac{b}{2a^2\left(a+bx^2\right)}-\frac{2b\log(x)}{a^3}+\frac{b\log\left(a+bx^2\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 41, normalized size = 0.84

$$-\frac{a\left(\frac{b}{a+bx^2}+\frac{1}{x^2}\right)-2b\log\left(a+bx^2\right)+4b\log(x)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)^2),x]
[Out] -1/2*(a*(x^(-2) + b/(a + b*x^2)) + 4*b*Log[x] - 2*b*Log[a + b*x^2])/a^3
```

fricas [A] time = 0.79, size = 73, normalized size = 1.49

$$-\frac{2\,abx^2+a^2-2\left(b^2x^4+abx^2\right)\log\left(bx^2+a\right)+4\left(b^2x^4+abx^2\right)\log(x)}{2\left(a^3bx^4+a^4x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="fricas")
[Out] -1/2*(2*a*b*x^2 + a^2 - 2*(b^2*x^4 + a*b*x^2)*log(b*x^2 + a) + 4*(b^2*x^4 +
 a*b*x^2)*log(x))/(a^3*b*x^4 + a^4*x^2)
```

giac [A] time = 0.21, size = 51, normalized size = 1.04

$$-\frac{b\log\left(x^2\right)}{a^3}+\frac{b\log\left(\left|bx^2+a\right|\right)}{a^3}-\frac{2\,bx^2+a}{2\left(bx^4+ax^2\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="giac")
[Out] -b*log(x^2)/a^3 + b*log(abs(b*x^2 + a))/a^3 - 1/2*(2*b*x^2 + a)/((b*x^4 + a
*x^2)*a^2)
```

maple [A] time = 0.05, size = 46, normalized size = 0.94

$$-\frac{b}{2\left(b\,x^2+a\right)a^2}-\frac{2b\ln(x)}{a^3}+\frac{b\ln\left(b\,x^2+a\right)}{a^3}-\frac{1}{2a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x)^2,x)
[Out] -1/2/a^2/x^2-1/2*b/a^2/(b*x^2+a)-2/a^3*b*ln(x)+b*ln(b*x^2+a)/a^3
```

maxima [A] time = 1.36, size = 50, normalized size = 1.02

$$-\frac{2\,bx^2+a}{2\left(a^2bx^4+a^3x^2\right)}+\frac{b\log\left(bx^2+a\right)}{a^3}-\frac{2\,b\log(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^2,x, algorithm="maxima")
[Out] -1/2*(2*b*x^2 + a)/(a^2*b*x^4 + a^3*x^2) + b*log(b*x^2 + a)/a^3 - 2*b*log(x
)/a^3
```

mupad [B] time = 0.05, size = 51, normalized size = 1.04

$$\frac{b\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{\frac{1}{2\,a}+\frac{b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{2\,b\,\ln(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3)^2),x)
```

```
[Out] (b*log(a + b*x^2))/a^3 - (1/(2*a) + (b*x^2)/a^2)/(a*x^2 + b*x^4) - (2*b*log
(x))/a^3
```

sympy [A] time = 0.41, size = 51, normalized size = 1.04

$$\frac{-a-2bx^2}{2a^3x^2+2a^2bx^4}-\frac{2b\log{(x)}}{a^3}+\frac{b\log{\left(\frac{a}{b}+x^2\right)}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**2,x)
```

```
[Out] (-a - 2*b*x**2)/(2*a**3*x**2 + 2*a**2*b*x**4) - 2*b*log(x)/a**3 + b*log(a/b
 + x**2)/a**3
```

3.25 $\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx$

Optimal. Leaf size=68

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

```
[Out] -5/6/a^2/x^3+5/2*b/a^3/x+1/2/a/x^3/(b*x^2+a)+5/2*b^(3/2)*arctan(x*b^(1/2)/a
^(1/2))/a^(7/2)
```

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1584, 290, 325, 205}

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}+\frac{5b}{2a^3x}-\frac{5}{6a^2x^3}+\frac{1}{2ax^3\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] -5/(6*a^2*x^3) + (5*b)/(2*a^3*x) + 1/(2*a*x^3*(a + b*x^2)) + (5*b^(3/2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax+bx^3\right)^2}\,dx = \int \frac{1}{x^4\left(a+bx^2\right)^2}\,dx$$
$$= \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{5\int \frac{1}{x^4\left(a+bx^2\right)}\,dx}{2a}$$
$$= -\frac{5}{6a^2x^3} + \frac{1}{2ax^3\left(a+bx^2\right)} - \frac{(5b)\int \frac{1}{x^2\left(a+bx^2\right)}\,dx}{2a^2}$$
$$= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{\left(5b^2\right)\int \frac{1}{a+bx^2}\,dx}{2a^3}$$
$$= -\frac{5}{6a^2x^3} + \frac{5b}{2a^3x} + \frac{1}{2ax^3\left(a+bx^2\right)} + \frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}}$$

Mathematica [A] time = 0.04, size = 67, normalized size = 0.99

$$\frac{5b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}} + \frac{b^2x}{2a^3\left(a+bx^2\right)} + \frac{2b}{a^3x} - \frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)^2),x]
```

```
[Out] -1/3*1/(a^2*x^3) + (2*b)/(a^3*x) + (b^2*x)/(2*a^3*(a + b*x^2)) + (5*b^(3/2)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2))
```

fricas [A] time = 1.03, size = 172, normalized size = 2.53

$$\left[\frac{30\,b^2x^4 + 20\,abx^2 + 15\left(b^2x^5 + abx^3\right)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right) - 4\,a^2}{12\left(a^3bx^5 + a^4x^3\right)}, \frac{15\,b^2x^4 + 10\,abx^2 + 15\left(b^2x^5 + abx^3\right)\sqrt{}}{6\left(a^3bx^5 + a^4x^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*b^2*x^4 + 20*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(-b/a)*log((b*x
^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 4*a^2)/(a^3*b*x^5 + a^4*x^3), 1/6
*(15*b^2*x^4 + 10*a*b*x^2 + 15*(b^2*x^5 + a*b*x^3)*sqrt(b/a)*arctan(x*sqrt(
b/a)) - 2*a^2)/(a^3*b*x^5 + a^4*x^3)]
```

giac [A] time = 0.17, size = 59, normalized size = 0.87

$$\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\,a^3} + \frac{b^2x}{2\left(bx^2+a\right)a^3} + \frac{6\,bx^2 - a}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="giac")
```

```
[Out] 5/2*b^2*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/2*b^2*x/((b*x^2 + a)*a^3)
 + 1/3*(6*b*x^2 - a)/(a^3*x^3)
```

maple [A] time = 0.06, size = 59, normalized size = 0.87

$$\frac{b^2 x}{2\left(b\,x^2+a\right)a^3}+\frac{5b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}\;a^3}+\frac{2b}{a^3x}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x)^2,x)
```

```
[Out] 1/2/a^3*b^2*x/(b*x^2+a)+5/2/a^3*b^2/(a*b)^(1/2)*arctan(1/(a*b)^(1/2)*b*x)-1
/3/a^2/x^3+2*b/a^3/x
```

maxima [A] time = 2.92, size = 64, normalized size = 0.94

$$\frac{15\,b^2x^4+10\,abx^2-2\,a^2}{6\left(a^3bx^5+a^4x^3\right)}+\frac{5\,b^2\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(15*b^2*x^4 + 10*a*b*x^2 - 2*a^2)/(a^3*b*x^5 + a^4*x^3) + 5/2*b^2*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

mupad [B] time = 5.03, size = 58, normalized size = 0.85

$$\frac{\frac{5\,b\,x^2}{3\,a^2}-\frac{1}{3\,a}+\frac{5\,b^2\,x^4}{2\,a^3}}{b\,x^5+a\,x^3}+\frac{5\,b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3)^2),x)
```

```
[Out] ((5*b*x^2)/(3*a^2) - 1/(3*a) + (5*b^2*x^4)/(2*a^3))/(a*x^3 + b*x^5) + (5*b^
(3/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(7/2))
```

sympy [A] time = 0.39, size = 114, normalized size = 1.68

$$-\frac{5\sqrt{-\frac{b^3}{a^7}}\,\log\left(-\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{5\sqrt{-\frac{b^3}{a^7}}\,\log\left(\frac{a^4\sqrt{-\frac{b^3}{a^7}}}{b^2}+x\right)}{4}+\frac{-2a^2+10abx^2+15b^2x^4}{6a^4x^3+6a^3bx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**2,x)
```

```
[Out] -5*sqrt(-b**3/a**7)*log(-a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + 5*sqrt(-b**3/a
**7)*log(a**4*sqrt(-b**3/a**7)/b**2 + x)/4 + (-2*a**2 + 10*a*b*x**2 + 15*b*
*2*x**4)/(6*a**4*x**3 + 6*a**3*b*x**5)
```

3.26 $\int \frac{x^5}{x-x^3} \, dx$

Optimal. Leaf size=13

$$-\frac{x^3}{3}-x+\tanh^{-1}(x)$$

```
[Out] -x-1/3*x^3+arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 302, 206}

$$-\frac{x^3}{3}-x+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(x - x^3),x]
```

```
[Out] -x - x^3/3 + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{x-x^3} \, dx &= \int \frac{x^4}{1-x^2} \, dx \\
&= \int \left(-1-x^2+\frac{1}{1-x^2}\right) \, dx \\
&= -x-\frac{x^3}{3}+\int \frac{1}{1-x^2} \, dx \\
&= -x-\frac{x^3}{3}+\tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 29, normalized size = 2.23

$$-\frac{x^3}{3}-x-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(x - x^3),x]
[Out] -x - x^3/3 - Log[1 - x]/2 + Log[1 + x]/2
```

fricas [A] time = 0.84, size = 21, normalized size = 1.62

$$-\frac{1}{3}x^3 - x + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="fricas")
[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

giac [B] time = 0.17, size = 23, normalized size = 1.77

$$-\frac{1}{3}x^3 - x + \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="giac")
[Out] -1/3*x^3 - x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [A] time = 0.05, size = 22, normalized size = 1.69

$$-\frac{x^3}{3} - x - \frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^3+x),x)
[Out] -1/3*x^3-x-1/2*ln(x-1)+1/2*ln(x+1)
```

maxima [A] time = 1.38, size = 21, normalized size = 1.62

$$-\frac{1}{3}x^3 - x + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^3+x),x, algorithm="maxima")
[Out] -1/3*x^3 - x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 4.98, size = 11, normalized size = 0.85

$$\operatorname{atanh}(x) - x - \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x - x^3),x)
[Out] atanh(x) - x - x^3/3
```

sympy [B] time = 0.11, size = 19, normalized size = 1.46

$$-\frac{x^3}{3} - x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**3+x),x)
[Out] -x**3/3 - x - log(x - 1)/2 + log(x + 1)/2
```

3.27 $\int \frac{x^4}{x-x^3} \, dx$

Optimal. Leaf size=20

$$-\frac{x^2}{2}-\frac{1}{2} \log \left(1-x^2\right)$$

```
[Out] -1/2*x^2-1/2*ln(-x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 43}

$$-\frac{x^2}{2}-\frac{1}{2} \log \left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(x - x^3),x]
```

```
[Out] -x^2/2 - Log[1 - x^2]/2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{x-x^3} \, dx &= \int \frac{x^3}{1-x^2} \, dx\\
&= \frac{1}{2} \text{Subst}\left(\int \frac{x}{1-x} \, dx,x,x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right) \, dx,x,x^2\right)\\
&= -\frac{x^2}{2}-\frac{1}{2} \log \left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$-\frac{x^2}{2}-\frac{1}{2} \log \left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(x - x^3),x]
[Out] -1/2*x^2 - Log[-1 + x^2]/2
```

fricas [A] time = 0.88, size = 14, normalized size = 0.70

$$-\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*x^2 - 1/2*log(x^2 - 1)
```

giac [A] time = 0.15, size = 15, normalized size = 0.75

$$-\frac{1}{2}x^2 - \frac{1}{2}\log\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="giac")
[Out] -1/2*x^2 - 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.04, size = 19, normalized size = 0.95

$$-\frac{x^2}{2} - \frac{\ln(x-1)}{2} - \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^3+x),x)
[Out] -1/2*x^2-1/2*ln(x-1)-1/2*ln(x+1)
```

maxima [A] time = 1.32, size = 18, normalized size = 0.90

$$-\frac{1}{2}x^2 - \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^3+x),x, algorithm="maxima")
[Out] -1/2*x^2 - 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.04, size = 14, normalized size = 0.70

$$-\frac{\ln\left(x^2 - 1\right)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x - x^3),x)
[Out] - log(x^2 - 1)/2 - x^2/2
```

sympy [A] time = 0.08, size = 14, normalized size = 0.70

$$-\frac{x^2}{2} - \frac{\log\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**3+x),x)
[Out] -x**2/2 - log(x**2 - 1)/2
```

3.28 $\int \frac{x^3}{x-x^3} \, dx$

Optimal. Leaf size=6

$$\tanh^{-1}(x) - x$$

```
[Out] -x+arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 321, 206}

$$\tanh^{-1}(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(x - x^3),x]
```

```
[Out] -x + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{x-x^3} \, dx &= \int \frac{x^2}{1-x^2} \, dx \\
&= -x + \int \frac{1}{1-x^2} \, dx \\
&= -x + \tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 22, normalized size = 3.67

$$-x - \frac{1}{2}\log(1-x) + \frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(x - x^3),x]
```

```
[Out] -x - Log[1 - x]/2 + Log[1 + x]/2
```

fricas [B] time = 0.92, size = 16, normalized size = 2.67

$$-x + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="fricas")
[Out] -x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

giac [B] time = 0.17, size = 18, normalized size = 3.00

$$-x + \frac{1}{2} \log(|x+1|) - \frac{1}{2} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="giac")
[Out] -x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [B] time = 0.05, size = 17, normalized size = 2.83

$$-x - \frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-x^3+x),x)
[Out] -x-1/2*ln(x-1)+1/2*ln(x+1)
```

maxima [B] time = 1.29, size = 16, normalized size = 2.67

$$-x + \frac{1}{2} \log(x+1) - \frac{1}{2} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-x^3+x),x, algorithm="maxima")
[Out] -x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.06, size = 6, normalized size = 1.00

$$\operatorname{atanh}(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x - x^3),x)
[Out] atanh(x) - x
```

sympy [B] time = 0.11, size = 14, normalized size = 2.33

$$-x - \frac{\log(x-1)}{2} + \frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-x**3+x),x)
[Out] -x - log(x - 1)/2 + log(x + 1)/2
```

3.29 $\int \frac{x^2}{x-x^3} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] -1/2*ln(-x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1584, 260}

$$-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(x - x^3),x]
```

```
[Out] -Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{x-x^3} \, dx &= \int \frac{x}{1-x^2} \, dx \\ &= -\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(x - x^3),x]
```

```
[Out] -1/2*Log[1 - x^2]
```

fricas [A] time = 0.98, size = 8, normalized size = 0.67

$$-\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 - 1)
```

giac [**A**] time = 0.15, size = 15, normalized size = 1.25

$$-\frac{1}{2}\log(|x+1|)-\frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="giac")
[Out] -1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [**A**] time = 0.05, size = 14, normalized size = 1.17

$$-\frac{\ln(x-1)}{2}-\frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^3+x),x)
[Out] -1/2*ln(x-1)-1/2*ln(x+1)
```

maxima [**A**] time = 1.29, size = 13, normalized size = 1.08

$$-\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^3+x),x, algorithm="maxima")
[Out] -1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [**B**] time = 0.03, size = 8, normalized size = 0.67

$$-\frac{\ln\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x - x^3),x)
[Out] -log(x^2 - 1)/2
```

sympy [**A**] time = 0.10, size = 8, normalized size = 0.67

$$-\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**3+x),x)
[Out] -log(x**2 - 1)/2
```

3.30 $\int \frac{x}{x-x^3} dx$

Optimal. Leaf size=2

$$\tanh^{-1}(x)$$

```
[Out] arctanh(x)
```

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1584, 206}

$$\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(x - x^3),x]
```

```
[Out] ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{x-x^3} dx &= \int \frac{1}{1-x^2} dx \\ &= \tanh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.00, size = 19, normalized size = 9.50

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - x^3),x]
```

```
[Out] -1/2*Log[1 - x] + Log[1 + x]/2
```

fricas [B] time = 0.93, size = 13, normalized size = 6.50

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

giac [**B**] time = 0.17, size = 15, normalized size = 7.50

$$\frac{1}{2}\log(|x+1|)-\frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="giac")
[Out] 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [**A**] time = 0.04, size = 3, normalized size = 1.50

$$\operatorname{arctanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^3+x),x)
[Out] arctanh(x)
```

maxima [**B**] time = 1.28, size = 13, normalized size = 6.50

$$\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^3+x),x, algorithm="maxima")
[Out] 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [**B**] time = 0.03, size = 2, normalized size = 1.00

$$\operatorname{atanh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x - x^3),x)
[Out] atanh(x)
```

sympy [**B**] time = 0.12, size = 12, normalized size = 6.00

$$-\frac{\log(x-1)}{2}+\frac{\log(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**3+x),x)
[Out] -log(x - 1)/2 + log(x + 1)/2
```

3.31 $\int \frac{1}{x-x^3} \, dx$

Optimal. Leaf size=15

$$\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] ln(x)-1/2*ln(-x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1593, 266, 36, 31, 29}

$$\log(x)-\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 - x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x-x^3} \, dx &= \int \frac{1}{x\left(1-x^2\right)} \, dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x} \, dx, x, x^2\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x} \, dx, x, x^2\right) \\
&= \log(x)-\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [**A**] time = 0.00, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{2}\log\left(1 - x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^3)^(-1),x]
[Out] Log[x] - Log[1 - x^2]/2
```

fricas [**A**] time = 0.94, size = 11, normalized size = 0.73

$$-\frac{1}{2}\log\left(x^2 - 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*log(x^2 - 1) + log(x)
```

giac [**A**] time = 0.16, size = 16, normalized size = 1.07

$$\frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="giac")
[Out] 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

maple [**A**] time = 0.06, size = 16, normalized size = 1.07

$$\ln(x) - \frac{\ln(x-1)}{2} - \frac{\ln(x+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^3+x),x)
[Out] -1/2*ln(x-1)-1/2*ln(x+1)+ln(x)
```

maxima [**A**] time = 1.32, size = 15, normalized size = 1.00

$$-\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^3+x),x, algorithm="maxima")
[Out] -1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

mupad [**B**] time = 4.96, size = 11, normalized size = 0.73

$$\ln(x) - \frac{\ln\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^3),x)
[Out] log(x) - log(x^2 - 1)/2
```

<u>**sympy**</u> **[A]** time = 0.11, size = 10, normalized size = 0.67

$$\log(x) - \frac{\log(x^2 - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**3+x),x)
[Out] log(x) - log(x**2 - 1)/2
```

3.32 $\int \frac{1}{x(x-x^3)}\,dx$

Optimal. Leaf size=8

$$\tanh^{-1}(x) - \frac{1}{x}$$

[Out] -1/x+arctanh(x)

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 325, 206}

$$\tanh^{-1}(x) - \frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(x - x^3)),x]
```

```
[Out] -x^(-1) + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(x-x^3\right)}\,dx &= \int \frac{1}{x^2\left(1-x^2\right)}\,dx\\
&= -\frac{1}{x} + \int \frac{1}{1-x^2}\,dx\\
&= -\frac{1}{x} + \tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 24, normalized size = 3.00

$$-\frac{1}{x} - \frac{1}{2}\log(1-x) + \frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(x - x^3)),x]
```

```
[Out] -x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

fricas [B] time = 0.92, size = 20, normalized size = 2.50

$$\frac{x\log(x+1) - x\log(x-1) - 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="fricas")
```

```
[Out] 1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

giac [B] time = 0.15, size = 20, normalized size = 2.50

$$-\frac{1}{x} + \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="giac")
```

```
[Out] -1/x + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [B] time = 0.05, size = 19, normalized size = 2.38

$$-\frac{\ln(x-1)}{2} + \frac{\ln(x+1)}{2} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^3+x),x)
```

```
[Out] -1/2*ln(x-1)-1/x+1/2*ln(x+1)
```

maxima [B] time = 1.33, size = 18, normalized size = 2.25

$$-\frac{1}{x} + \frac{1}{2}\log(x+1) - \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^3+x),x, algorithm="maxima")
```

```
[Out] -1/x + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 0.03, size = 8, normalized size = 1.00

$$\operatorname{atanh}(x) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(x - x^3)),x)
```

```
[Out] atanh(x) - 1/x
```

sympy [B] time = 0.13, size = 15, normalized size = 1.88

$$-\frac{\log(x-1)}{2} + \frac{\log(x+1)}{2} - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - 1/x
```

3.33 $\int \frac{1}{x^2(x-x^3)}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

```
[Out] -1/2/x^2+ln(x)-1/2*ln(-x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 44}

$$-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(x - x^3)),x]
```

```
[Out] -1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(x-x^3\right)}\,dx &= \int \frac{1}{x^3\left(1-x^2\right)}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1-x)x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^2}+\frac{1}{x}\right)dx,x,x^2\right)\\
&= -\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(x - x^3)),x]
[Out] -1/2*1/x^2 + Log[x] - Log[1 - x^2]/2
```

fricas [A] time = 0.68, size = 24, normalized size = 1.09

$$-\frac{x^2 \log\left(x^2-1\right)-2\,x^2 \log(x)+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="fricas")
[Out] -1/2*(x^2*log(x^2 - 1) - 2*x^2*log(x) + 1)/x^2
```

giac [A] time = 0.15, size = 26, normalized size = 1.18

$$-\frac{x^2+1}{2\,x^2}+\frac{1}{2}\,\log\left(x^2\right)-\frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="giac")
[Out] -1/2*(x^2 + 1)/x^2 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.05, size = 21, normalized size = 0.95

$$\ln(x)-\frac{\ln\left(x-1\right)}{2}-\frac{\ln\left(x+1\right)}{2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^3+x),x)
[Out] -1/2*ln(x-1)-1/2*ln(x+1)-1/2/x^2+ln(x)
```

maxima [A] time = 1.29, size = 20, normalized size = 0.91

$$-\frac{1}{2\,x^2}-\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^3+x),x, algorithm="maxima")
[Out] -1/2/x^2 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

mupad [B] time = 0.03, size = 16, normalized size = 0.73

$$\ln(x)-\frac{\ln\left(x^2-1\right)}{2}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(x - x^3)),x)
[Out] log(x) - log(x^2 - 1)/2 - 1/(2*x^2)
```

sympy [A] time = 0.11, size = 17, normalized size = 0.77

$$\log\left(x\right)-\frac{\log\left(x^2-1\right)}{2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**3+x),x)
[Out] log(x) - log(x**2 - 1)/2 - 1/(2*x**2)
```

3.34 $\int \frac{1}{x^3(x-x^3)}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)$$

```
[Out] -1/3/x^3-1/x+arctanh(x)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 325, 206}

$$-\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(x - x^3)),x]
```

```
[Out] -1/(3*x^3) - x^(-1) + ArcTanh[x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(x-x^3\right)}\,dx &= \int \frac{1}{x^4\left(1-x^2\right)}\,dx\\
&= -\frac{1}{3x^3}+\int \frac{1}{x^2\left(1-x^2\right)}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{x}+\int \frac{1}{1-x^2}\,dx\\
&= -\frac{1}{3x^3}-\frac{1}{x}+\tanh^{-1}(x)
\end{aligned}$$

Mathematica [B] time = 0.00, size = 31, normalized size = 2.07

$$-\frac{1}{3x^3}-\frac{1}{x}-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(x - x^3)),x]
[Out] -1/3*1/x^3 - x^(-1) - Log[1 - x]/2 + Log[1 + x]/2
```

fricas [B] time = 0.59, size = 30, normalized size = 2.00

$$\frac{3\,x^3\log\left(x+1\right)-3\,x^3\log\left(x-1\right)-6\,x^2-2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="fricas")
[Out] 1/6*(3*x^3*log(x + 1) - 3*x^3*log(x - 1) - 6*x^2 - 2)/x^3
```

giac [B] time = 0.15, size = 27, normalized size = 1.80

$$-\frac{3\,x^2+1}{3\,x^3}+\frac{1}{2}\,\log\left(|x+1|\right)-\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="giac")
[Out] -1/3*(3*x^2 + 1)/x^3 + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

maple [A] time = 0.06, size = 24, normalized size = 1.60

$$-\frac{\ln\left(x-1\right)}{2}+\frac{\ln\left(x+1\right)}{2}-\frac{1}{x}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-x^3+x),x)
[Out] -1/2*ln(x-1)+1/2*ln(x+1)-1/3/x^3-1/x
```

maxima [A] time = 1.26, size = 25, normalized size = 1.67

$$-\frac{3\,x^2+1}{3\,x^3}+\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-x^3+x),x, algorithm="maxima")
[Out] -1/3*(3*x^2 + 1)/x^3 + 1/2*log(x + 1) - 1/2*log(x - 1)
```

mupad [B] time = 4.92, size = 13, normalized size = 0.87

$$\mathrm{atanh}(x)-\frac{x^2+\frac{1}{3}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(x - x^3)),x)
[Out] atanh(x) - (x^2 + 1/3)/x^3
```

sympy [A] time = 0.14, size = 24, normalized size = 1.60

$$-\frac{\log\left(x-1\right)}{2}+\frac{\log\left(x+1\right)}{2}-\frac{3x^2+1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-x**3+x),x)
```

```
[Out] -log(x - 1)/2 + log(x + 1)/2 - (3*x**2 + 1)/(3*x**3)
```

3.35 $\int \frac{1}{x^4(x-x^3)}\,dx$

Optimal. Leaf size=29

$$-\frac{1}{4x^4}-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

```
[Out] -1/4/x^4-1/2/x^2+ln(x)-1/2*ln(-x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1584, 266, 44}

$$-\frac{1}{2x^2}-\frac{1}{4x^4}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(x - x^3)),x]
```

```
[Out] -1/(4*x^4) - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(x-x^3\right)}\,dx &= \int \frac{1}{x^5\left(1-x^2\right)}\,dx\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-x)x^3}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{1-x}+\frac{1}{x^3}+\frac{1}{x^2}+\frac{1}{x}\right)dx,x,x^2\right)\\
&= -\frac{1}{4x^4}-\frac{1}{2x^2}+\log(x)-\frac{1}{2}\log\left(1-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 29, normalized size = 1.00

$$-\frac{1}{4x^4}-\frac{1}{2x^2}-\frac{1}{2}\log\left(1-x^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(x - x^3)),x]
[Out] -1/4*1/x^4 - 1/(2*x^2) + Log[x] - Log[1 - x^2]/2
```

fricas [A] time = 0.56, size = 30, normalized size = 1.03

$$-\frac{2\,x^4\log\left(x^2-1\right)-4\,x^4\log(x)+2\,x^2+1}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="fricas")
[Out] -1/4*(2*x^4*log(x^2 - 1) - 4*x^4*log(x) + 2*x^2 + 1)/x^4
```

giac [A] time = 0.16, size = 33, normalized size = 1.14

$$-\frac{3\,x^4+2\,x^2+1}{4\,x^4}+\frac{1}{2}\,\log\left(x^2\right)-\frac{1}{2}\,\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="giac")
[Out] -1/4*(3*x^4 + 2*x^2 + 1)/x^4 + 1/2*log(x^2) - 1/2*log(abs(x^2 - 1))
```

maple [A] time = 0.05, size = 26, normalized size = 0.90

$$\ln(x)-\frac{\ln\left(x-1\right)}{2}-\frac{\ln\left(x+1\right)}{2}-\frac{1}{2x^2}-\frac{1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(-x^3+x),x)
[Out] -1/2*ln(x-1)-1/2*ln(x+1)-1/4/x^4-1/2/x^2+ln(x)
```

maxima [A] time = 1.35, size = 27, normalized size = 0.93

$$-\frac{2\,x^2+1}{4\,x^4}-\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(x-1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(-x^3+x),x, algorithm="maxima")
[Out] -1/4*(2*x^2 + 1)/x^4 - 1/2*log(x + 1) - 1/2*log(x - 1) + log(x)
```

mupad [B] time = 0.03, size = 23, normalized size = 0.79

$$\ln(x)-\frac{\ln\left(x^2-1\right)}{2}-\frac{\frac{x^2}{2}+\frac{1}{4}}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(x - x^3)),x)
[Out] log(x) - log(x^2 - 1)/2 - (x^2/2 + 1/4)/x^4
```

sympy [A] time = 0.13, size = 22, normalized size = 0.76

$$\log\left(x\right)-\frac{\log\left(x^2-1\right)}{2}-\frac{2x^2+1}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(-x**3+x),x)
```

```
[Out] log(x) - log(x**2 - 1)/2 - (2*x**2 + 1)/(4*x**4)
```

3.36 $\int \frac{1}{x+bx^3}\,dx$

Optimal. Leaf size=15

$$\log(x) - \frac{1}{2}\log\left(bx^2+1\right)$$

```
[Out] ln(x)-1/2*ln(b*x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1593, 266, 36, 29, 31}

$$\log(x) - \frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + b*x^3)^(-1),x]
```

```
[Out] Log[x] - Log[1 + b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x+bx^3}\,dx &= \int \frac{1}{x\left(1+bx^2\right)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(1+bx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{1+bx}\,dx, x, x^2\right)\\
&= \log(x) - \frac{1}{2}\log\left(1+bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\log(x) - \frac{1}{2}\log\left(bx^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + b*x^3)^(-1),x]
[Out] Log[x] - Log[1 + b*x^2]/2
```

fricas [A] time = 0.55, size = 13, normalized size = 0.87

$$-\frac{1}{2}\log\left(bx^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="fricas")
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

giac [A] time = 0.15, size = 18, normalized size = 1.20

$$\frac{1}{2}\log\left(x^2\right) - \frac{1}{2}\log\left(\left|bx^2+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="giac")
[Out] 1/2*log(x^2) - 1/2*log(abs(b*x^2 + 1))
```

maple [A] time = 0.04, size = 14, normalized size = 0.93

$$\ln(x) - \frac{\ln\left(b\,x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+x),x)
[Out] ln(x)-1/2*ln(b*x^2+1)
```

maxima [A] time = 1.29, size = 13, normalized size = 0.87

$$-\frac{1}{2}\log\left(bx^2+1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+x),x, algorithm="maxima")
[Out] -1/2*log(b*x^2 + 1) + log(x)
```

mupad [B] time = 4.95, size = 14, normalized size = 0.93

$$\ln(x) - \frac{\ln\left(\frac{3\,b\,x^2}{2}+\frac{3}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + b*x^3),x)
[Out] log(x) - log((3*b*x^2)/2 + 3/2)/2
```

sympy [**A**] time = 0.15, size = 12, normalized size = 0.80

$$\log(x) - \frac{\log\left(x^2 + \frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+x),x)
```

```
[Out] log(x) - log(x**2 + 1/b)/2
```

3.37 $\int \frac{1}{-x+bx^3} \, dx$

Optimal. Leaf size=18

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

```
[Out] -ln(x)+1/2*ln(-b*x^2+1)
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + b*x^3)^(-1),x]
```

```
[Out] -Log[x] + Log[1 - b*x^2]/2
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-x+bx^3} \, dx &= \int \frac{1}{x\left(-1+bx^2\right)} \, dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(-1+bx)} \, dx, x, x^2\right) \\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x} \, dx, x, x^2\right)\right)+\frac{1}{2}b\operatorname{Subst}\left(\int \frac{1}{-1+bx} \, dx, x, x^2\right) \\
&= -\log(x)+\frac{1}{2}\log\left(1-bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{1}{2}\log\left(1-bx^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + b*x^3)^(-1),x]
[Out] -Log[x] + Log[1 - b*x^2]/2
```

fricas [A] time = 0.59, size = 15, normalized size = 0.83

$$\frac{1}{2}\log\left(bx^2-1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="fricas")
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

giac [A] time = 0.15, size = 18, normalized size = 1.00

$$-\frac{1}{2}\log\left(x^2\right)+\frac{1}{2}\log\left(\left|bx^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="giac")
[Out] -1/2*log(x^2) + 1/2*log(abs(b*x^2 - 1))
```

maple [A] time = 0.05, size = 16, normalized size = 0.89

$$-\ln(x)+\frac{\ln\left(b\,x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3-x),x)
[Out] 1/2*ln(b*x^2-1)-ln(x)
```

maxima [A] time = 1.26, size = 15, normalized size = 0.83

$$\frac{1}{2}\log\left(bx^2-1\right)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3-x),x, algorithm="maxima")
[Out] 1/2*log(b*x^2 - 1) - log(x)
```

mupad [B] time = 0.05, size = 16, normalized size = 0.89

$$\frac{\ln\left(\frac{3}{2}-\frac{3\,b\,x^2}{2}\right)}{2}-\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(x - b*x^3),x)
[Out] log(3/2 - (3*b*x^2)/2)/2 - log(x)
```

sympy [A] time = 0.14, size = 12, normalized size = 0.67

$$-\log(x) + \frac{\log\left(x^2 - \frac{1}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3-x),x)
```

```
[Out] -log(x) + log(x**2 - 1/b)/2
```

3.38 $\int x^3\sqrt{ax+bx^3}\,dx$

Optimal. Leaf size=163

$$\frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}-\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}$$

```
[Out] -20/231*a^2*(b*x^3+a*x)^(1/2)/b^2+4/77*a*x^2*(b*x^3+a*x)^(1/2)/b+2/11*x^4*(
b*x^3+a*x)^(1/2)+10/231*a^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)
^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4
)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^
(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2021, 2024, 2011, 329, 220}

$$-\frac{20a^2\sqrt{ax+bx^3}}{231b^2}+\frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}+\frac{2}{11}x^4\sqrt{ax+bx^3}+\frac{4ax^2\sqrt{ax+bx^3}}{77b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a*x + b*x^3],x]
```

```
[Out] (-20*a^2*Sqrt[a*x + b*x^3])/(231*b^2) + (4*a*x^2*Sqrt[a*x + b*x^3])/(77*b)
+ (2*x^4*Sqrt[a*x + b*x^3])/11 + (10*a^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)
*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt
[x])/a^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{ax+bx^3}\,dx = \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{1}{11}(2a)\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} - \frac{\left(10a^2\right)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{77b}$$

$$= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{\left(10a^3\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{231b^2}$$

$$= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{\left(10a^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+b}}}{231b^2\sqrt{ax+bx^3}}$$

$$= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{\left(20a^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\right.}{231b^2\sqrt{ax+b}}$$

$$= -\frac{20a^2\sqrt{ax+bx^3}}{231b^2} + \frac{4ax^2\sqrt{ax+bx^3}}{77b} + \frac{2}{11}x^4\sqrt{ax+bx^3} + \frac{10a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{}{(\sqrt{}}}}{231b^{9/4}\sqrt{}}$$

Mathematica [C] time = 0.06, size = 95, normalized size = 0.58

$$\frac{2\sqrt{x\left(a+bx^2\right)}\left(\sqrt{\frac{bx^2}{a}+1}\left(-5a^2+2abx^2+7b^2x^4\right)+5a^2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{77b^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*(Sqrt[1 + (b*x^2)/a]*(-5*a^2 + 2*a*b*x^2 + 7*b^2*x^4
) + 5*a^2*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(77*b^2*Sqrt[1
+ (b*x^2)/a])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3+ax}\,x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3+ax}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

maple [A] time = 0.10, size = 168, normalized size = 1.03

$$\frac{2\sqrt{b\,x^{3}+ax}\;x^{4}}{11}+\frac{4\sqrt{b\,x^{3}+ax}\;a\,x^{2}}{77b}+\frac{10\sqrt{-ab}\;\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\;\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\;\sqrt{-\frac{bx}{\sqrt{-ab}}}\;a^{3}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{231\sqrt{b\,x^{3}+ax}\;b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/11*x^4*(b*x^3+a*x)^(1/2)+4/77*a*x^2*(b*x^3+a*x)^(1/2)/b-20/231*a^2*(b*x^3
+a*x)^(1/2)/b^2+10/231*a^3/b^3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*
x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1
/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3 + ax}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{b\,x^3 + a\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x^3*(a*x + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{x\left(a + bx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(x*(a + b*x**2)), x)
```

3.39 $\int x^2\sqrt{ax+bx^3}\,dx$

Optimal. Leaf size=281

$$-\frac{2a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] -4/15*a^2*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+4/45*a*
x*(b*x^3+a*x)^(1/2)/b+2/9*x^3*(b*x^3+a*x)^(1/2)+4/15*a^(9/4)*(cos(2*arctan(
b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*E
llipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(
1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(
1/2)-2/15*a^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{4a^2x\left(a+bx^2\right)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{7/4}\sqrt{ax+bx^3}}+\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a*x + b*x^3],x]
```

```
[Out] (-4*a^2*x*(a + b*x^2))/(15*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (4*a*x*Sqrt[a*x + b*x^3])/(45*b) + (2*x^3*Sqrt[a*x + b*x^3])/9 + (4*a^(9
/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]
*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(7/4)*Sqrt[a*x
+ b*x^3]) - (2*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt
[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(1
5*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^2\sqrt{ax+bx^3}\,dx = \frac{2}{9}x^3\sqrt{ax+bx^3} + \frac{1}{9}(2a)\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{4ax\sqrt{ax+bx^3}}{45b} + \frac{2}{9}x^3\sqrt{ax+bx^3} - \frac{\left(2a^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{15b}$$

$$= \frac{4ax\sqrt{ax+bx^3}}{45b} + \frac{2}{9}x^3\sqrt{ax+bx^3} - \frac{\left(2a^2\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15b\sqrt{ax+bx^3}}$$

$$= \frac{4ax\sqrt{ax+bx^3}}{45b} + \frac{2}{9}x^3\sqrt{ax+bx^3} - \frac{\left(4a^2\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{15b\sqrt{ax+bx^3}}$$

$$= \frac{4ax\sqrt{ax+bx^3}}{45b} + \frac{2}{9}x^3\sqrt{ax+bx^3} - \frac{\left(4a^{5/2}\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{15b^{3/2}\sqrt{ax+bx^3}} + \frac{(4a}{}$$

$$= -\frac{4a^2x\left(a+bx^2\right)}{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} + \frac{4ax\sqrt{ax+bx^3}}{45b} + \frac{2}{9}x^3\sqrt{ax+bx^3} + \frac{4a^{9/4}\sqrt{x}\,\left(\sqrt{a}+\sqrt{b}\right.}{}$$

Mathematica [C] time = 0.06, size = 80, normalized size = 0.28

$$\frac{2x\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{9b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometri
c2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)]))/(9*b*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3+ax}\,x^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3+ax}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

maple [A] time = 0.06, size = 197, normalized size = 0.70

$$\frac{2\sqrt{b\,x^3+ax}\;x^3}{9}+\frac{4\sqrt{b\,x^3+ax}\;ax}{45b}-\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\right.}{b}\right.}{15\sqrt{b\,x^3+ax}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/9*x^3*(b*x^3+a*x)^(1/2)+4/45*a*x*(b*x^3+a*x)^(1/2)/b-2/15*a^2/b^2*(-a*b)^
(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^
(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-
a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3+ax}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{b x^3 + a x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3)^(1/2),x)
[Out] int(x^2*(a*x + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{x \left(a + bx^2\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**(1/2),x)
[Out] Integral(x**2*sqrt(x*(a + b*x**2)), x)
```

3.40 $\int x\sqrt{ax+bx^3}\,dx$

Optimal. Leaf size=137

$$-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}+\frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}$$

```
[Out] 4/21*a*(b*x^3+a*x)^(1/2)/b+2/7*x^2*(b*x^3+a*x)^(1/2)-2/21*a^(7/4)*(cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a
*x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2021, 2024, 2011, 329, 220}

$$-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}+\frac{2}{7}x^2\sqrt{ax+bx^3}+\frac{4a\sqrt{ax+bx^3}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x + b*x^3],x]
```

```
[Out] (4*a*Sqrt[a*x + b*x^3])/(21*b) + (2*x^2*Sqrt[a*x + b*x^3])/7 - (2*a^(7/4)*S
qrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(5/4)*Sqrt[a*x + b*x
^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{ax+bx^3}\,dx &= \frac{2}{7}x^2\sqrt{ax+bx^3}+\frac{1}{7}(2a)\int\frac{x^2}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{\left(2a^2\right)\int\frac{1}{\sqrt{ax+bx^3}}\,dx}{21b}\\
&= \frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{\left(2a^2\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{21b\sqrt{ax+bx^3}}\\
&= \frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{21b\sqrt{ax+bx^3}}\\
&= \frac{4a\sqrt{ax+bx^3}}{21b}+\frac{2}{7}x^2\sqrt{ax+bx^3}-\frac{2a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{5/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 79, normalized size = 0.58

$$\frac{2\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)\sqrt{\frac{bx^2}{a}+1}-a\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{7b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)*Sqrt[1 + (b*x^2)/a] - a*Hypergeometric2
F1[-1/2, 1/4, 5/4, -((b*x^2)/a)]))/(7*b*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3+ax}\,x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{bx^3+ax}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x, x)
```

maple [A] time = 0.07, size = 146, normalized size = 1.07

$$\frac{2\sqrt{b\,x^{3}+a x}\;x^{2}}{7}-\frac{2\sqrt{-a b}\,\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}}\,\sqrt{-\frac{b x}{\sqrt{-a b}}}\;a^{2}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a b}}{b}\right)b}{\sqrt{-a b}}},\frac{\sqrt{2}}{2}\right)}{21\sqrt{b\,x^{3}+a x}\;b^{2}}+\frac{4\sqrt{b\,x^{3}+a x}\;a}{21 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/7*x^2*(b*x^3+a*x)^(1/2)+4/21*a*(b*x^3+a*x)^(1/2)/b-2/21*a^2/b^2*(-a*b)^(1
/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^{3}+ax}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{b\,x^{3}+a\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x*(a*x + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x\left(a+bx^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x*(a + b*x**2)), x)
```

3.41 $\int \sqrt{ax+bx^3}\,dx$

Optimal. Leaf size=255

$$\frac{2a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}-\frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{5b^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 4/5*a*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+2/5*x*(b*x^
3+a*x)^(1/2)-4/5*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*
b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)+2/5*a^(5/4)*(cos(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellipt
icF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))
*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2004, 2032, 329, 305, 220, 1196}

$$\frac{2a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+bx^3}}-\frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{5b^{3/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3],x]
```

```
[Out] (4*a*x*(a + b*x^2))/(5*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) + (
2*x*Sqrt[a*x + b*x^3])/5 - (4*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(5*b^(3/4)*Sqrt[a*x + b*x^3]) + (2*a^(5/4)*Sqrt[x]*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^
(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
```

```
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt{ax+bx^3}\,dx = \frac{2}{5}x\sqrt{ax+bx^3}+\frac{1}{5}(2a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$
$$= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}$$
$$= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{\left(4a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}$$
$$= \frac{2}{5}x\sqrt{ax+bx^3}+\frac{\left(4a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{b}\sqrt{ax+bx^3}}-\frac{\left(4a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{S}}{5\sqrt{b}}$$
$$= \frac{4ax\left(a+bx^2\right)}{5\sqrt{b}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{2}{5}x\sqrt{ax+bx^3}-\frac{4a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\right.}{5b^{3/4}\sqrt{ax+bx^3}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.20

$$\frac{2x\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 3/4, 7/4, -((b*x^2)/a)])/(
3*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{bx^3+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3 + ax}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x), x)
```

maple [A] time = 0.07, size = 175, normalized size = 0.69

$$\frac{2\sqrt{b\,x^3+ax}\,x}{5}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{EllipticF}\left(\sqrt{\left(\right.}\right.}{b}\right.}{5\sqrt{b\,x^3+ax}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/5*x*(b*x^3+a*x)^(1/2)+2/5*a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/
2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b
*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b
)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^3 + ax}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x), x)
```

mupad [B] time = 5.05, size = 40, normalized size = 0.16

$$\frac{2\,x\,\sqrt{b\,x^3+a\,x}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{b\,x^2}{a}\right)}{3\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(1/2),x)
```

```
[Out] (2*x*(a*x + b*x^3)^(1/2)*hypergeom([-1/2, 3/4], 7/4, -(b*x^2)/a))/(3*((b*x^
2)/a + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**3), x)
```

3.42 $\int \frac{\sqrt{ax+bx^3}}{x}\,dx$

Optimal. Leaf size=113

$$\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

```
[Out] 2/3*(b*x^3+a*x)^(1/2)+2/3*a^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2
)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a
^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2021, 2011, 329, 220}

$$\frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\,\sqrt{ax+bx^3}}+\frac{2}{3}\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x,x]
```

```
[Out] (2*Sqrt[a*x + b*x^3])/3 + (2*a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(3*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax+bx^3}}{x}\,dx = \frac{2}{3}\sqrt{ax+bx^3} + \frac{1}{3}(2a)\int \frac{1}{\sqrt{ax+bx^3}}\,dx$$
$$= \frac{2}{3}\sqrt{ax+bx^3} + \frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}}$$
$$= \frac{2}{3}\sqrt{ax+bx^3} + \frac{\left(4a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{3\sqrt{ax+bx^3}}$$
$$= \frac{2}{3}\sqrt{ax+bx^3} + \frac{2a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{b}\sqrt{ax+bx^3}}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.42

$$\frac{2\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x,x]
[Out] (2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((b*x^2)/a)])/Sqr
t[1 + (b*x^2)/a]
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a*x)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+ax}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)/x, x)
```

maple [A] time = 0.07, size = 124, normalized size = 1.10

$$\frac{2\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^3+ax}\,b}+\frac{2\sqrt{b\,x^3+ax}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x,x)
```

```
[Out] 2/3*(b*x^3+a*x)^(1/2)+2/3*a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x
)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/
2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3 + ax}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^3 + a\,x}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(1/2)/x,x)
```

```
[Out] int((a*x + b*x^3)^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(a + bx^2\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x, x)
```

3.43 $\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$

Optimal. Leaf size=248

$$-\frac{2\sqrt{ax+bx^3}}{x}+\frac{4\sqrt{b}\,x\left(a+bx^2\right)}{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}}-\frac{4\sqrt[4]{a}\ \cdots}{\cdots}$$

```
[Out] 4*x*(b*x^2+a)*b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x)^(
1/2)/x-4*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/c
os(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*
b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)+2*a^(1/4)*b^(1/4)*(cos(2*arctan(b^(1/4)
*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Elliptic
F(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x
^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2020, 2032, 329, 305, 220, 1196}

$$\frac{4\sqrt{b}\,x\left(a+bx^2\right)}{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{x}+\frac{2\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt{ax+bx^3}}-\frac{4\sqrt[4]{a}\ \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x^2,x]
```

```
[Out] (4*Sqrt[b]*x*(a + b*x^2))/((Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*Sq
rt[a*x + b*x^3])/x - (4*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[
(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a
^(1/4)], 1/2])/Sqrt[a*x + b*x^3] + (2*a^(1/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sq
rt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1
/4)*Sqrt[x])/a^(1/4)], 1/2])/Sqrt[a*x + b*x^3]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx = -\frac{2\sqrt{ax+bx^3}}{x} + (2b)\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$

$$= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{\left(4b\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{x} + \frac{\left(4\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}} - \frac{\left(4\sqrt{a}\sqrt{b}\sqrt{x}\sqrt{a+bx \cdots\right.}{\cdots}$$

$$= \frac{4\sqrt{b}\,x\left(a+bx^2\right)}{\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt{ax+bx^3}}{x} - \frac{4\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\cdots\right.}{\sqrt{ax+bx^3}}$$

Mathematica [C] time = 0.01, size = 51, normalized size = 0.21

$$-\frac{2\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^2,x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-1/2, -1/4, 3/4, -((b*x^2)/a)])/(
x*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a*x)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+ax}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

maple [A] time = 0.08, size = 177, normalized size = 0.71

$$-\frac{2\left(b\,x^2+a\right)}{\sqrt{\left(b\,x^2+a\right)x}}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\operatorname{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\operatorname{Elliptic}}{}\right.}{\sqrt{b\,x^3+ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x^2,x)
[Out] -2*(b*x^2+a)/(x*(b*x^2+a))^(1/2)+2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2
)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)
^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+ax}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a\,x}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(1/2)/x^2,x)
```

```
[Out] int((a*x + b*x^3)^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(a + bx^2\right)}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x**2, x)
```

3.44 $\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx$

Optimal. Leaf size=116

$$\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*b^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(
b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+
a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2020, 2011, 329, 220}

$$\frac{2b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x^3,x]
```

```
[Out] (-2*Sqrt[a*x + b*x^3])/(3*x^2) + (2*b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*a^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx &= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{1}{3}(2b)\int \frac{1}{\sqrt{ax+bx^3}}\,dx\\ &= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{\left(2b\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}}\\ &= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{\left(4b\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{ax+bx^3}}\\ &= -\frac{2\sqrt{ax+bx^3}}{3x^2}+\frac{2b^{3/4}\sqrt{x}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{a}\,\sqrt{ax+bx^3}}\end{aligned}$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.46

$$-\frac{2\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{4},-\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^3,x]
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/4, -1/2, 1/4, -((b*x^2)/a)])/(
3*x^2*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a*x)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+ax}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="giac")
[Out] integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

maple [A] time = 0.04, size = 123, normalized size = 1.06

$$\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^3+ax}}-\frac{2\sqrt{b\,x^3+ax}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x^3,x)
```

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/x^2+2/3*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2
)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*
x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1
/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3 + ax}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^3 + a\,x}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(1/2)/x^3,x)
```

```
[Out] int((a*x + b*x^3)^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(a + bx^2\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(x*(a + b*x**2))/x**3, x)
```

3.45 $\int \frac{\sqrt{ax+bx^3}}{x^4} dx$

Optimal. Leaf size=283

$$\frac{2b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}-\frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{5a^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 4/5*b^(3/2)*x*(b*x^2+a)/a/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2/5*(b*x^3+
a*x)^(1/2)/x^3-4/5*b*(b*x^3+a*x)^(1/2)/a/x-4/5*b^(5/4)*(cos(2*arctan(b^(1/4
)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellipti
cE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*
x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+2
/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/
2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{2b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+bx^3}}-\frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{5a^{3/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^3]/x^4,x]
```

```
[Out] (4*b^(3/2)*x*(a + b*x^2))/(5*a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (
2*Sqrt[a*x + b*x^3])/(5*x^3) - (4*b*Sqrt[a*x + b*x^3])/(5*a*x) - (4*b^(5/4)
*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[a*x + b*
x^3]) + (2*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
+ Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(
3/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx = -\frac{2\sqrt{ax+bx^3}}{5x^3}+\frac{1}{5}(2b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{\left(2b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5a}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{\left(2b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5a\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{\left(4b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5a\sqrt{ax+bx^3}}$$

$$= -\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}+\frac{\left(4b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{a}\sqrt{ax+bx^3}}-\frac{\left(4b^{3/\cdots\right.}{\cdots}$$

$$= \frac{4b^{3/2}x\left(a+bx^2\right)}{5a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{5x^3}-\frac{4b\sqrt{ax+bx^3}}{5ax}-\frac{4b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{\cdots}{(\sqrt{\cdots}}}}{5a^{3/4}\sqrt{\cdots}}$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.19

$$-\frac{2\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^3]/x^4,x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((b*x^2)/a)])/
(5*x^3*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{bx^3+ax}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

maple [A] time = 0.04, size = 201, normalized size = 0.71

$$-\frac{4\left(b\,x^2+a\right)b}{5\sqrt{\left(b\,x^2+a\right)x}\;a}+\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\;\mathrm{Elliptic}}{}\right.}{5\sqrt{b\,x^3+ax}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(1/2)/x^4,x)
```

```
[Out] -2/5*(b*x^3+a*x)^(1/2)/x^3-4/5*(b*x^2+a)*b/a/((b*x^2+a)*x)^(1/2)+2/5*b/a*(-
a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)
/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-
a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2
))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2
^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{bx^3+ax}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(1/2)/x^4,x, algorithm="maxima")
[Out] integrate(sqrt(b*x^3 + a*x)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\,x^3+a\,x}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(1/2)/x^4,x)
[Out] int((a*x + b*x^3)^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x\left(a+bx^2\right)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(1/2)/x**4,x)
[Out] Integral(sqrt(x*(a + b*x**2))/x**4, x)
```

3.46 $\int x^2 \left(ax+bx^3\right)^{3/2} dx$

Optimal. Leaf size=186

$$\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}-\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{2}{15}x^3\left(ax+bx^3\right)^3$$

```
[Out] 2/15*x^3*(b*x^3+a*x)^(3/2)-8/231*a^3*(b*x^3+a*x)^(1/2)/b^2+8/385*a^2*x^2*(b
*x^3+a*x)^(1/2)/b+4/55*a*x^4*(b*x^3+a*x)^(1/2)+4/231*a^(15/4)*(cos(2*arctan
(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*
EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+a*x)^
(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2021, 2024, 2011, 329, 220}

$$-\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{4a^{15/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{231b^{9/4}\sqrt{ax+bx^3}}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-8*a^3*Sqrt[a*x + b*x^3])/(231*b^2) + (8*a^2*x^2*Sqrt[a*x + b*x^3])/(385*b
) + (4*a*x^4*Sqrt[a*x + b*x^3])/55 + (2*x^3*(a*x + b*x^3)^(3/2))/15 + (4*a^
(15/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(231*b^(9/4)*Sqrt[
a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
```

```
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int x^2\left(ax+bx^3\right)^{3/2}\,dx = \frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{1}{5}(2a)\int x^3\sqrt{ax+bx^3}\,dx$$

$$= \frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{1}{55}\left(4a^2\right)\int\frac{x^4}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int\frac{x^2}{\sqrt{ax+bx^3}}\,dx}{77b}$$

$$= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{\left(4a^{4}\right.}{}$$

$$= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{\left(4a^{4}\right.}{}$$

$$= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{\left(8a^{4}\right.}{}$$

$$= -\frac{8a^3\sqrt{ax+bx^3}}{231b^2}+\frac{8a^2x^2\sqrt{ax+bx^3}}{385b}+\frac{4}{55}ax^4\sqrt{ax+bx^3}+\frac{2}{15}x^3\left(ax+bx^3\right)^{3/2}+\frac{4a^{15}}{}$$

Mathematica [C] time = 0.09, size = 94, normalized size = 0.51

$$\frac{2\sqrt{x\left(a+bx^2\right)}\left(5a^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)-\left(5a-11bx^2\right)\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}\right)}{165b^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*(-((5*a - 11*b*x^2)*(a + b*x^2)^2*Sqrt[1 + (b*x^2)/a
]) + 5*a^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(165*b^2*Sqrt[
1 + (b*x^2)/a])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^5+ax^3\right)\sqrt{bx^3+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

[Out] integral((b*x^5 + a*x^3)*sqrt(b*x^3 + a*x), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + ax\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

maple [A] time = 0.07, size = 188, normalized size = 1.01

$$\frac{2\sqrt{b\,x^3+ax}\;b\,x^6}{15}+\frac{34\sqrt{b\,x^3+ax}\;a\,x^4}{165}+\frac{8\sqrt{b\,x^3+ax}\;a^2x^2}{385b}+\frac{4\sqrt{-ab}\;\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\;\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\;\sqrt{-\frac{bx}{\sqrt{-ab}}}\;a^4\,\mathrm{Ellipt}}{231\sqrt{b\,x^3+ax}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x)^(3/2),x)
```

```
[Out] 2/15*b*x^6*(b*x^3+a*x)^(1/2)+34/165*a*x^4*(b*x^3+a*x)^(1/2)+8/385*a^2*x^2*(
b*x^3+a*x)^(1/2)/b-8/231*a^3*(b*x^3+a*x)^(1/2)/b^2+4/231*a^4/b^3*(-a*b)^(1/
2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+
(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + ax\right)^{\frac{3}{2}} x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(b\,x^3 + a\,x\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^2*(a*x + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(x\left(a + bx^2\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**2*(x*(a + b*x**2))**(3/2), x)
```

3.47 $\int x \left(ax+bx^3\right)^{3/2} dx$

Optimal. Leaf size=304

$$-\frac{4a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}+\frac{8a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{65b^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] 2/13*x^2*(b*x^3+a*x)^(3/2)-8/65*a^3*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))
/(b*x^3+a*x)^(1/2)+8/195*a^2*x*(b*x^3+a*x)^(1/2)/b+4/39*a*x^3*(b*x^3+a*x)^(
1/2)+8/65*a^(13/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1
/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2
))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)-4/65*a^(13/4)*(cos(2*arctan(b^(1/4)*x
^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(
sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(
1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{8a^3x\left(a+bx^2\right)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{4a^{13/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{65b^{7/4}\sqrt{ax+bx^3}}+\frac{8a^{13/4}\sqrt{x}\left(\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-8*a^3*x*(a + b*x^2))/(65*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (8*a^2*x*Sqrt[a*x + b*x^3])/(195*b) + (4*a*x^3*Sqrt[a*x + b*x^3])/39 + (
2*x^2*(a*x + b*x^3)^(3/2))/13 + (8*a^(13/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a*x + b*x^3]) - (4*a^(13/4)*Sqrt[x]*(Sq
rt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*Ar
cTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(65*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x\left(ax+bx^3\right)^{3/2}\,dx = \frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{1}{13}(6a)\int x^2\sqrt{ax+bx^3}\,dx$$

$$= \frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}+\frac{1}{39}\left(4a^2\right)\int\frac{x^3}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int\frac{x}{\sqrt{ax+bx^3}}\,dx}{65b}$$

$$= \frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\sqrt{x}\sqrt{a+bx^2}\right)\int\frac{\sqrt{x}}{\sqrt{a+bx^2}}}{65b\sqrt{ax+bx^3}}$$

$$= \frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(8a^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\right.}{65b\sqrt{ax+b}}$$

$$= \frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx^3\right)^{3/2}-\frac{\left(8a^{7/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\right.}{65b^{3/2}\sqrt{ax+}}$$

$$= -\frac{8a^3x\left(a+bx^2\right)}{65b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{8a^2x\sqrt{ax+bx^3}}{195b}+\frac{4}{39}ax^3\sqrt{ax+bx^3}+\frac{2}{13}x^2\left(ax+bx\right.$$

Mathematica [C] time = 0.05, size = 84, normalized size = 0.28

$$\frac{2x\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)\right)}{13b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*x*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeom
etric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)]))/(13*b*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4+ax^2\right)\sqrt{bx^3+ax}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a*x^2)*sqrt(b*x^3 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^3+ax\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)*x, x)
```

maple [A] time = 0.08, size = 217, normalized size = 0.71

$$\frac{2\sqrt{b\,x^3+ax}\;b\,x^5}{13}+\frac{10\sqrt{b\,x^3+ax}\;a\,x^3}{39}+\frac{8\sqrt{b\,x^3+ax}\;a^2x}{195b}-\frac{4\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-\cdots}}{\cdots}\cdots\right.}{65\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x)^(3/2),x)
```

```
[Out] 2/13*b*x^5*(b*x^3+a*x)^(1/2)+10/39*a*x^3*(b*x^3+a*x)^(1/2)+8/195*a^2*x*(b*x
^3+a*x)^(1/2)/b-4/65*a^3/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)
^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(
-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)
/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^3+ax\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(b\,x^3+a\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^3)^(3/2),x)
[Out] int(x*(a*x + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x)**(3/2),x)
[Out] Integral(x*(x*(a + b*x**2))**(3/2), x)
```

3.48 $\int \left(ax+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=158

$$-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}}+\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{12}{77}ax^2\sqrt{ax+}$$

```
[Out] 2/11*x*(b*x^3+a*x)^(3/2)+8/77*a^2*(b*x^3+a*x)^(1/2)/b+12/77*a*x^2*(b*x^3+a*
x)^(1/2)-4/77*a^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^
(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2004, 2021, 2024, 2011, 329, 220}

$$-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77b^{5/4}\sqrt{ax+bx^3}}+\frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2),x]
```

```
[Out] (8*a^2*Sqrt[a*x + b*x^3])/(77*b) + (12*a*x^2*Sqrt[a*x + b*x^3])/77 + (2*x*(
a*x + b*x^3)^(3/2))/11 - (4*a^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(77*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(ax+bx^3\right)^{3/2}\,dx &= \frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{1}{11}(6a)\int x\sqrt{ax+bx^3}\,dx\\
&= \frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}+\frac{1}{77}\left(12a^2\right)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{77b}\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(4a^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,d}{77b\sqrt{ax+bx^3}}\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{\left(8a^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+}}\right.}{77b\sqrt{ax+bx^3}}\\
&= \frac{8a^2\sqrt{ax+bx^3}}{77b}+\frac{12}{77}ax^2\sqrt{ax+bx^3}+\frac{2}{11}x\left(ax+bx^3\right)^{3/2}-\frac{4a^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\right.}}}{77b^{5/4}\sqrt{ax-}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 83, normalized size = 0.53

$$\frac{2\sqrt{x\left(a+bx^2\right)}\left(\left(a+bx^2\right)^2\sqrt{\frac{bx^2}{a}+1}-a^2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)\right)}{11b\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*Sqrt[x*(a + b*x^2)]*((a + b*x^2)^2*Sqrt[1 + (b*x^2)/a] - a^2*Hypergeomet
ric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)]))/(11*b*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+ax\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + ax\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2), x)
```

maple [A] time = 0.07, size = 166, normalized size = 1.05

$$\frac{2\sqrt{b x^3+ax}\, b x^4}{11}+\frac{26\sqrt{b x^3+ax}\, a x^2}{77}-\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\, a^3 \operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{77\sqrt{b x^3+ax}\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2),x)
```

```
[Out] 2/11*b*x^4*(b*x^3+a*x)^(1/2)+26/77*a*x^2*(b*x^3+a*x)^(1/2)+8/77*a^2*(b*x^3+
a*x)^(1/2)/b-4/77*a^3/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^
(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1
/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1
/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + ax\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2), x)
```

mupad [B] time = 5.00, size = 40, normalized size = 0.25

$$\frac{2\,x\,\left(b\,x^3+a\,x\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{5}{4};\ \frac{9}{4};\ -\frac{b\,x^2}{a}\right)}{5\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2),x)
```

```
[Out] (2*x*(a*x + b*x^3)^(3/2)*hypergeom([-3/2, 5/4], 9/4, -(b*x^2)/a))/(5*((b*x^
2)/a + 1)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax + bx^3\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**3)**(3/2), x)
```

3.49 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=275

$$\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{15b^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 2/9*(b*x^3+a*x)^(3/2)+8/15*a^2*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x
^3+a*x)^(1/2)+4/15*a*x*(b*x^3+a*x)^(1/2)-8/15*a^(9/4)*(cos(2*arctan(b^(1/4)
*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Elliptic
E(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x
^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)+4/
15*a^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/
2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1
/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2021, 2004, 2032, 329, 305, 220, 1196}

$$\frac{4a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15b^{3/4}\sqrt{ax+bx^3}}-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{15b^{3/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x,x]
```

```
[Out] (8*a^2*x*(a + b*x^2))/(15*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
 + (4*a*x*Sqrt[a*x + b*x^3])/15 + (2*(a*x + b*x^3)^(3/2))/9 - (8*a^(9/4)*Sqr
t[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipt
icE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/4)*Sqrt[a*x + b*x^3
]) + (4*a^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + S
qrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*b^(3/
4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x}\,dx = \frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{1}{3}(2a)\int \sqrt{ax+bx^3}\,dx$$

$$= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{1}{15}\left(4a^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$

$$= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{\left(4a^2\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15\sqrt{ax+bx^3}}$$

$$= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{\left(8a^2\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{ax+bx^3}}$$

$$= \frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}+\frac{\left(8a^{5/2}\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{15\sqrt{b}\,\sqrt{ax+bx^3}} -$$

$$= \frac{8a^2x\left(a+bx^2\right)}{15\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{4}{15}ax\sqrt{ax+bx^3}+\frac{2}{9}\left(ax+bx^3\right)^{3/2}-\frac{8a^{9/4}\sqrt{x}\left(\sqrt{a}+\right.}{}$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.19

$$\frac{2ax\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x,x]
```

```
[Out] (2*a*x*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((b*x^2)/a)])
/(3*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{bx^3+ax}\left(bx^2+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x, x)
```

maple [A] time = 0.07, size = 195, normalized size = 0.71

$$\frac{2\sqrt{b\,x^3+ax}\;b\,x^3}{9}+\frac{22\sqrt{b\,x^3+ax}\;ax}{45}+\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\ldots\right)}{b}\ldots\right)}{15\sqrt{b\,x^3+ax}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x,x)
```

```
[Out] 2/9*b*x^3*(b*x^3+a*x)^(1/2)+22/45*a*x*(b*x^3+a*x)^(1/2)+4/15*a^2*(-a*b)^(1/
2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(
1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a
*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\,x\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x,x)
[Out] int((a*x + b*x^3)^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x,x)
[Out] Integral((x*(a + b*x**2))**(3/2)/x, x)
```

3.50 $\int \frac{(ax+bx^3)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=134

$$\frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\,\sqrt{ax+bx^3}}+\frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}$$

```
[Out] 2/7*(b*x^3+a*x)^(3/2)/x+4/7*a*(b*x^3+a*x)^(1/2)+4/7*a^(7/4)*(cos(2*arctan(b
^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*El
lipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1
/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(1/4)/(b*x^3+a*x)^(1
/2)
```

Rubi [A] time = 0.13, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2021, 2011, 329, 220}

$$\frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{b}\,\sqrt{ax+bx^3}}+\frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (4*a*Sqrt[a*x + b*x^3])/7 + (2*(a*x + b*x^3)^(3/2))/(7*x) + (4*a^(7/4)*Sqrt
[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ellipti
cF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] && !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax+bx^3\right)^{3/2}}{x^2}\,dx &= \frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{1}{7}(6a)\int \frac{\sqrt{ax+bx^3}}{x}\,dx\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{1}{7}\left(4a^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{\left(4a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{7\sqrt{ax+bx^3}}\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{\left(8a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{7\sqrt{ax+bx^3}}\\
&= \frac{4}{7}a\sqrt{ax+bx^3}+\frac{2\left(ax+bx^3\right)^{3/2}}{7x}+\frac{4a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{7\sqrt[4]{b}\sqrt{ax+bx^3}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.37

$$\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, 1/4, 5/4, -((b*x^2)/a)])/S
qrt[1 + (b*x^2)/a]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

maple [A] time = 0.08, size = 144, normalized size = 1.07

$$\frac{2\sqrt{b\,x^3+ax}\;b\,x^2}{7}+\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a^2\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b\,x^3+ax}\;b}+\frac{6\sqrt{b\,x^3+ax}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^2,x)
```

```
[Out] 2/7*b*x^2*(b*x^3+a*x)^(1/2)+6/7*a*(b*x^3+a*x)^(1/2)+4/7*a^2*(-a*b)^(1/2)/b*
((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/
2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a
*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + ax\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3 + a\,x\right)^{3/2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^2,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a + bx^2\right)\right)^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**2, x)
```

3.51 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=274

$$\frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}-\frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\text{t}\right.}{5\sqrt{ax+bx^3}}$$

```
[Out] -2*(b*x^3+a*x)^(3/2)/x^2+24/5*a*x*(b*x^2+a)*b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*
x^3+a*x)^(1/2)+12/5*b*x*(b*x^3+a*x)^(1/2)-24/5*a^(5/4)*b^(1/4)*(cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)+1
2/5*a^(5/4)*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/
2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2020, 2004, 2032, 329, 305, 220, 1196}

$$\frac{12a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}-\frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\text{t}\right.}{5\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (24*a*Sqrt[b]*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) +
(12*b*x*Sqrt[a*x + b*x^3])/5 - (2*(a*x + b*x^3)^(3/2))/x^2 - (24*a^(5/4)*b^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b*x^3
]) + (12*a^(5/4)*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sq
rt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/
(5*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax+bx^3\right)^{3/2}}{x^3}\,dx &= -\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+(6b)\int \sqrt{ax+bx^3}\,dx \\
&= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{1}{5}(12ab)\int \frac{x}{\sqrt{ax+bx^3}}\,dx \\
&= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{\left(12ab\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}} \\
&= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{\left(24ab\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}} \\
&= \frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}+\frac{\left(24a^{3/2}\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}} \\
&= \frac{24a\sqrt{b}\,x\left(a+bx^2\right)}{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{12}{5}bx\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{x^2}-\frac{24a^{5/4}\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 52, normalized size = 0.19

$$-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{1}{4};\frac{3}{4};-\frac{bx^2}{a}\right)}{x\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -1/4, 3/4, -((b*x^2)/a)])
/(x*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

maple [A] time = 0.07, size = 194, normalized size = 0.71

$$\frac{2\sqrt{b\,x^3+ax}\;bx}{5}-\frac{2\left(b\,x^2+a\right)a}{\sqrt{\left(b\,x^2+a\right)x}}+\frac{12\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\ldots\right)}{b}\ldots\right)}{5\sqrt{b\,x^3+ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^3,x)
```

```
[Out] -2*(b*x^2+a)*a/((b*x^2+a)*x)^(1/2)+2/5*b*x*(b*x^3+a*x)^(1/2)+12/5*a*(-a*b)^
(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*
b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^
(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-
a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2
)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^3+a\,x\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^3,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**3,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**3, x)
```

3.52 $\int \frac{(ax+bx^3)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=134

$$\frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{4}{3}b\sqrt{ax+bx^3}$$

```
[Out] -2/3*(b*x^3+a*x)^(3/2)/x^3+4/3*b*(b*x^3+a*x)^(1/2)+4/3*a^(3/4)*b^(3/4)*(cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a
^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(
1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)
^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2020, 2021, 2011, 329, 220}

$$\frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3\sqrt{ax+bx^3}}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{4}{3}b\sqrt{ax+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (4*b*Sqrt[a*x + b*x^3])/3 - (2*(a*x + b*x^3)^(3/2))/(3*x^3) + (4*a^(3/4)*b^
(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(3*Sqrt[a*x + b*x^3
])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax+bx^3\right)^{3/2}}{x^4}\,dx &= -\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+(2b)\int \frac{\sqrt{ax+bx^3}}{x}\,dx\\
&= \frac{4}{3}b\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{1}{3}(4ab)\int \frac{1}{\sqrt{ax+bx^3}}\,dx\\
&= \frac{4}{3}b\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{\left(4ab\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3\sqrt{ax+bx^3}}\\
&= \frac{4}{3}b\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{\left(8ab\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3\sqrt{ax+bx^3}}\\
&= \frac{4}{3}b\sqrt{ax+bx^3}-\frac{2\left(ax+bx^3\right)^{3/2}}{3x^3}+\frac{4a^{3/4}b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{3\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.40

$$-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{bx^2}{a}\right)}{3x^2\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((b*x^2)/a)])
/(3*x^2*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

maple [A] time = 0.07, size = 139, normalized size = 1.04

$$\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^{3}+ax}}+\frac{2\sqrt{b\,x^{3}+ax}\;b}{3}-\frac{2\sqrt{b\,x^{3}+ax}\;a}{3x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^4,x)
```

```
[Out] -2/3*a*(b*x^3+a*x)^(1/2)/x^2+2/3*b*(b*x^3+a*x)^(1/2)+4/3*a*(-a*b)^(1/2)*((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)
^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+ax\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^{3}+a\,x\right)^{3/2}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^4,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a+bx^{2}\right)\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**4,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**4, x)
```

3.53 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=277

$$\frac{24b^{3/2}x\left(a+bx^2\right)}{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{12\sqrt[4]{a}\,b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}-\frac{24\sqrt[4]{a}\,b^{5/4}\sqrt{x}\left(\sqrt{a}+\right.}{}$$

```
[Out] -2/5*(b*x^3+a*x)^(3/2)/x^4+24/5*b^(3/2)*x*(b*x^2+a)/(a^(1/2)+x*b^(1/2))/(b*
x^3+a*x)^(1/2)-12/5*b*(b*x^3+a*x)^(1/2)/x-24/5*a^(1/4)*b^(5/4)*(cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/(b*x^3+a*x)^(1/2)+1
2/5*a^(1/4)*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/
2))^2)^(1/2)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2020, 2032, 329, 305, 220, 1196}

$$\frac{24b^{3/2}x\left(a+bx^2\right)}{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{12\sqrt[4]{a}\,b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5\sqrt{ax+bx^3}}-\frac{24\sqrt[4]{a}\,b^{5/4}\sqrt{x}\left(\sqrt{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (24*b^(3/2)*x*(a + b*x^2))/(5*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (1
2*b*Sqrt[a*x + b*x^3])/(5*x) - (2*(a*x + b*x^3)^(3/2))/(5*x^4) - (24*a^(1/4
)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]
*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*Sqrt[a*x + b
*x^3]) + (12*a^(1/4)*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)
/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/
2])/(5*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x^5}\,dx = -\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{1}{5}(6b)\int \frac{\sqrt{ax+bx^3}}{x^2}\,dx$$

$$= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{1}{5}\left(12b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx$$

$$= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{\left(12b^2\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5\sqrt{ax+bx^3}}$$

$$= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{\left(24b^2\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5\sqrt{ax+bx^3}}$$

$$= -\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}+\frac{\left(24\sqrt{a}\,b^{3/2}\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{\right.}{5\sqrt{ax+bx^3}}$$

$$= \frac{24b^{3/2}x\left(a+bx^2\right)}{5\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{12b\sqrt{ax+bx^3}}{5x}-\frac{2\left(ax+bx^3\right)^{3/2}}{5x^4}-\frac{24\sqrt[4]{a}\,b^{5/4}\sqrt{x}\,\left(\sqrt{a}+\sqrt{\right.}{}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.19

$$-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{3}{2},-\frac{5}{4};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^3\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-3/2, -5/4, -1/4, -((b*x^2)/a)]
)/(5*x^3*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

maple [A] time = 0.08, size = 196, normalized size = 0.71

$$-\frac{14\left(b\,x^2+a\right)b}{5\sqrt{\left(b\,x^2+a\right)x}}+\frac{12\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\ \mathrm{EllipticF}}{}\right.}{5\sqrt{b\,x^3+ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^5,x)
```

```
[Out] -2/5*a*(b*x^3+a*x)^(1/2)/x^3-14/5*(b*x^2+a)*b/((b*x^2+a)*x)^(1/2)+12/5*b*(-
a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)
/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-
a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2
))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2
^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(b\,x^3+a\,x\right)}^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^5,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^5, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**5,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**5, x)
```

3.54 $\int \frac{(ax+bx^3)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=137

$$\frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}-\frac{4b\sqrt{ax+bx^3}}{7x^2}$$

```
[Out] -2/7*(b*x^3+a*x)^(3/2)/x^5-4/7*b*(b*x^3+a*x)^(1/2)/x^2+4/7*b^(7/4)*(cos(2*a
rctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)
+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^3+
a*x)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2020, 2011, 329, 220}

$$\frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{7\sqrt[4]{a}\sqrt{ax+bx^3}}-\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-4*b*Sqrt[a*x + b*x^3])/(7*x^2) - (2*(a*x + b*x^3)^(3/2))/(7*x^5) + (4*b^(
7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2
]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(7*a^(1/4)*Sqrt[a*x
+ b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x^6}\,dx = -\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}+\frac{1}{7}(6b)\int \frac{\sqrt{ax+bx^3}}{x^3}\,dx$$
$$= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}+\frac{1}{7}\left(4b^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx$$
$$= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}+\frac{\left(4b^2\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{7\sqrt{ax+bx^3}}$$
$$= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}+\frac{\left(8b^2\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{7\sqrt{ax+bx^3}}$$
$$= -\frac{4b\sqrt{ax+bx^3}}{7x^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{7x^5}+\frac{4b^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)\ldots\right.}{7\sqrt[4]{a}\,\sqrt{ax+bx^3}}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.39

$$-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{7}{4},-\frac{3}{2};-\frac{3}{4};-\frac{bx^2}{a}\right)}{7x^4\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^6,x]
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-7/4, -3/2, -3/4, -((b*x^2)/a)]
)/(7*x^4*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

maple [A] time = 0.08, size = 142, normalized size = 1.04

$$\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,b\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{7\sqrt{b\,x^{3}+ax}}-\frac{6\sqrt{b\,x^{3}+ax}\,b}{7x^{2}}-\frac{2\sqrt{b\,x^{3}+ax}\,a}{7x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^6,x)
```

```
[Out] -2/7*a*(b*x^3+a*x)^(1/2)/x^4-6/7*b*(b*x^3+a*x)^(1/2)/x^2+4/7*b*(-a*b)^(1/2)
*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1
/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+ax\right)^{\frac{3}{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^{3}+a\,x\right)^{3/2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^6,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a+bx^{2}\right)\right)^{\frac{3}{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**6,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**6, x)
```

3.55 $\int \frac{(ax+bx^3)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=306

$$\frac{4b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}-\frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\right.\right.}{\sqrt[4]{}}\right.}{15a^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] -2/9*(b*x^3+a*x)^(3/2)/x^6+8/15*b^(5/2)*x*(b*x^2+a)/a/(a^(1/2)+x*b^(1/2))/(
b*x^3+a*x)^(1/2)-4/15*b*(b*x^3+a*x)^(1/2)/x^3-8/15*b^2*(b*x^3+a*x)^(1/2)/a/
x-8/15*b^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2
)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+4/15*b^(9/4)*(cos(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*
((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{15a^{3/4}\sqrt{ax+bx^3}}-\frac{8b^{9/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\right.\right.}{\sqrt[4]{}}\right.}{15a^{3/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (8*b^(5/2)*x*(a + b*x^2))/(15*a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) -
(4*b*Sqrt[a*x + b*x^3])/(15*x^3) - (8*b^2*Sqrt[a*x + b*x^3])/(15*a*x) - (2*
(a*x + b*x^3)^(3/2))/(9*x^6) - (8*b^(9/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a*x + b*x^3]) + (4*b^(9/4)*Sqrt[x]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(15*a^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(ax+bx^3\right)^{3/2}}{x^7}\,dx = -\frac{2\left(ax+bx^3\right)^{3/2}}{9x^6} + \frac{1}{3}(2b)\int \frac{\sqrt{ax+bx^3}}{x^4}\,dx$$
$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{2\left(ax+bx^3\right)^{3/2}}{9x^6} + \frac{1}{15}\left(4b^2\right)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$$
$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2\left(ax+bx^3\right)^{3/2}}{9x^6} + \frac{\left(4b^3\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{15a}$$
$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2\left(ax+bx^3\right)^{3/2}}{9x^6} + \frac{\left(4b^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{15a\sqrt{ax+bx^3}}$$
$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2\left(ax+bx^3\right)^{3/2}}{9x^6} + \frac{\left(8b^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+b}}\right.}{15a\sqrt{ax+bx^3}}$$
$$= -\frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2\left(ax+bx^3\right)^{3/2}}{9x^6} + \frac{\left(8b^{5/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+}}\right.}{15\sqrt{a}\sqrt{ax+bx^3}}$$
$$= \frac{8b^{5/2}x\left(a+bx^2\right)}{15a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{4b\sqrt{ax+bx^3}}{15x^3} - \frac{8b^2\sqrt{ax+bx^3}}{15ax} - \frac{2\left(ax+bx^3\right)^{3/2}}{9x^6} - \frac{8b^{9/}}{}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.18

$$-\frac{2a\sqrt{x\left(a+bx^2\right)}\,{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{bx^2}{a}\right)}{9x^5\sqrt{\frac{bx^2}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((b*x^2)/a)]
)/(9*x^5*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^6, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="giac")
```

[Out] integrate((b*x^3 + a*x)^(3/2)/x^7, x)

maple [A] time = 0.08, size = 223, normalized size = 0.73

$$-\frac{8\left(b x^{2}+a\right) b^{2}}{15\sqrt{\left(b x^{2}+a\right) x}\, a}+\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{Ellipti}\cdots}{\cdots}\right.}{15\sqrt{b x^{3}+ax}\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^7,x)
```

```
[Out] -2/9*a*(b*x^3+a*x)^(1/2)/x^5-22/45*b*(b*x^3+a*x)^(1/2)/x^3-8/15*(b*x^2+a)*b
^2/a/((b*x^2+a)*x)^(1/2)+4/15/a*b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/
2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+ax\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(3/2)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\,x^{3}+a\,x\right)^{3/2}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^7,x)
```

```
[Out] int((a*x + b*x^3)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a+bx^{2}\right)\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**7,x)
```

```
[Out] Integral((x*(a + b*x**2))**(3/2)/x**7, x)
```

3.56 $\int \frac{\left(ax+bx^3\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=163

$$-\frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}-\frac{12b\sqrt{ax+bx^3}}{77x^4}$$

```
[Out] -2/11*(b*x^3+a*x)^(3/2)/x^7-12/77*b*(b*x^3+a*x)^(1/2)/x^4-8/77*b^2*(b*x^3+a
*x)^(1/2)/a/x^2-4/77*b^(11/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1
/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x
^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/
2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2020, 2025, 2011, 329, 220}

$$-\frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{77a^{5/4}\sqrt{ax+bx^3}}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-12*b*Sqrt[a*x + b*x^3])/(77*x^4) - (8*b^2*Sqrt[a*x + b*x^3])/(77*a*x^2) -
 (2*(a*x + b*x^3)^(3/2))/(11*x^7) - (4*b^(11/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*
x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sq
rt[x])/a^(1/4)], 1/2])/(77*a^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
```

```
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax+bx^3\right)^{3/2}}{x^8}\,dx &= -\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}+\frac{1}{11}(6b)\int \frac{\sqrt{ax+bx^3}}{x^5}\,dx\\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}+\frac{1}{77}\left(12b^2\right)\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx\\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}-\frac{\left(4b^3\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{77a}\\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}-\frac{\left(4b^3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{77a\sqrt{ax+bx^3}}\\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}-\frac{\left(8b^3\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\right.}{77a\sqrt{ax+bx^3}}\\
&= -\frac{12b\sqrt{ax+bx^3}}{77x^4}-\frac{8b^2\sqrt{ax+bx^3}}{77ax^2}-\frac{2\left(ax+bx^3\right)^{3/2}}{11x^7}-\frac{4b^{11/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}}{77a^{5/4}\sqrt{ax+bx}}
\end{aligned}
$$

Mathematica [C] time = 0.02, size = 54, normalized size = 0.33

$$
-\frac{2a\sqrt{x\left(a+bx^2\right)}\;{}_2F_1\left(-\frac{11}{4},-\frac{3}{2};-\frac{7}{4};-\frac{bx^2}{a}\right)}{11x^6\sqrt{\frac{bx^2}{a}+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-2*a*Sqrt[x*(a + b*x^2)]*Hypergeometric2F1[-11/4, -3/2, -7/4, -((b*x^2)/a)
])/(11*x^6*Sqrt[1 + (b*x^2)/a])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{\sqrt{bx^3+ax}\left(bx^2+a\right)}{x^7},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*(b*x^2 + a)/x^7, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="giac")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

maple [A] time = 0.08, size = 169, normalized size = 1.04

$$-\frac{4\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,b^2\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{77\sqrt{b\,x^3+ax}\,a}-\frac{8\sqrt{b\,x^3+ax}\,b^2}{77a\,x^2}-\frac{26\sqrt{b\,x^3+a}}{77x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x)^(3/2)/x^8,x)
[Out] -2/11*a*(b*x^3+a*x)^(1/2)/x^6-26/77*b*(b*x^3+a*x)^(1/2)/x^4-8/77*b^2*(b*x^3
+a*x)^(1/2)/a/x^2-4/77/a*b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*
b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)
^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2
),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3+ax\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x)^(3/2)/x^8,x, algorithm="maxima")
[Out] integrate((b*x^3 + a*x)^(3/2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\,x\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^3)^(3/2)/x^8,x)
[Out] int((a*x + b*x^3)^(3/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x)**(3/2)/x**8,x)
[Out] Integral((x*(a + b*x**2))**(3/2)/x**8, x)
```

3.57 $\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=140

$$\frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}}-\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}$$

```
[Out] -10/21*a*(b*x^3+a*x)^(1/2)/b^2+2/7*x^2*(b*x^3+a*x)^(1/2)/b+5/21*a^(7/4)*(co
s(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^
(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b
*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2024, 2011, 329, 220}

$$\frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{21b^{9/4}\sqrt{ax+bx^3}}-\frac{10a\sqrt{ax+bx^3}}{21b^2}+\frac{2x^2\sqrt{ax+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x + b*x^3],x]
```

```
[Out] (-10*a*Sqrt[a*x + b*x^3])/(21*b^2) + (2*x^2*Sqrt[a*x + b*x^3])/(7*b) + (5*a
^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)
^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(21*b^(9/4)*Sqrt[a
*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
```

```
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx &= \frac{2x^2\sqrt{ax+bx^3}}{7b} - \frac{(5a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{7b} \\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{\left(5a^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{21b^2} \\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{\left(5a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{21b^2\sqrt{ax+bx^3}} \\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{\left(10a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{21b^2\sqrt{ax+bx^3}} \\
&= -\frac{10a\sqrt{ax+bx^3}}{21b^2} + \frac{2x^2\sqrt{ax+bx^3}}{7b} + \frac{5a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right.\right.}{21b^{9/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 80, normalized size = 0.57

$$\frac{2x\left(5a^2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-5a^2-2abx^2+3b^2x^4\right)}{21b^2\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x + b*x^3],x]
[Out] (2*x*(-5*a^2 - 2*a*b*x^2 + 3*b^2*x^4 + 5*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(21*b^2*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\,x^3}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a*x)*x^3/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/sqrt(b*x^3 + a*x), x)
```

maple [A] time = 0.07, size = 149, normalized size = 1.06

$$\frac{2\sqrt{b\,x^{3}+ax}\;x^{2}}{7b}+\frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\;a^{2}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{21\sqrt{b\,x^{3}+ax}\;b^{3}}-\frac{10\sqrt{b\,x^{3}+ax}\;a}{21b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/7*x^2*(b*x^3+a*x)^(1/2)/b-10/21*a*(b*x^3+a*x)^(1/2)/b^2+5/21*a^2/b^3*(-a*
b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(
-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*Elliptic
F(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x^4/(a*x + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(a + b*x**2)), x)
```

3.58 $\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=258

$$-\frac{3a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{5b^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] -6/5*a*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)+2/5*x*(b*x
^3+a*x)^(1/2)/b+6/5*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2
)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(
1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)
+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)-3/5*a^(5/4)*(cos(2*arctan(b^
(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ell
ipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/
2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/
2)
```

Rubi [A] time = 0.21, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2024, 2032, 329, 305, 220, 1196}

$$-\frac{3a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{7/4}\sqrt{ax+bx^3}}+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{5b^{7/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x + b*x^3],x]
```

```
[Out] (-6*a*x*(a + b*x^2))/(5*b^(3/2)*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) +
(2*x*Sqrt[a*x + b*x^3])/(5*b) + (6*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sq
rt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x]
)/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3]) - (3*a^(5/4)*Sqrt[x]*(Sqrt[
a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTa
n[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx &= \frac{2x\sqrt{ax+bx^3}}{5b}-\frac{(3a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5b}\\ &= \frac{2x\sqrt{ax+bx^3}}{5b}-\frac{\left(3a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5b\sqrt{ax+bx^3}}\\ &= \frac{2x\sqrt{ax+bx^3}}{5b}-\frac{\left(6a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5b\sqrt{ax+bx^3}}\\ &= \frac{2x\sqrt{ax+bx^3}}{5b}-\frac{\left(6a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5b^{3/2}\sqrt{ax+bx^3}}+\frac{\left(6a^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Sub}}{5b^{3/2}\sqrt{a}}\\ &= -\frac{6ax\left(a+bx^2\right)}{5b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{2x\sqrt{ax+bx^3}}{5b}+\frac{6a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\,\text{t}\right.}{5b^{7/4}\sqrt{ax+bx^3}}\end{aligned}$$

Mathematica [C] time = 0.03, size = 66, normalized size = 0.26

$$\frac{2x^2\left(-a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{5b\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x^2*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4,
-((b*x^2)/a)]))/(5*b*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\,x^2}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^2/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^3 + a*x), x)
```

maple [A] time = 0.07, size = 178, normalized size = 0.69

$$\frac{2\sqrt{b\,x^3+ax}\;x}{5b}-\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\;\text{EllipticF}\left(\cdots\right.}{\cdots}\right.}{5\sqrt{b\,x^3+ax}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/5*x*(b*x^3+a*x)^(1/2)/b-3/5*a/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/
2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(b*x^3 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x^3/(a*x + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x*(a + b*x**2)), x)
```

3.59 $\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=116

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

```
[Out] 2/3*(b*x^3+a*x)^(1/2)/b-1/3*a^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(
1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/
(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2024, 2011, 329, 220}

$$\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x + b*x^3])/(3*b) - (a^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(3*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx&=\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{3b}\\&=\frac{2\sqrt{ax+bx^3}}{3b}-\frac{\left(a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{3b\sqrt{ax+bx^3}}\\&=\frac{2\sqrt{ax+bx^3}}{3b}-\frac{\left(2a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3b\sqrt{ax+bx^3}}\\&=\frac{2\sqrt{ax+bx^3}}{3b}-\frac{a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3b^{5/4}\sqrt{ax+bx^3}}\end{aligned}$$

Mathematica [C] time = 0.04, size = 64, normalized size = 0.55

$$\frac{2x\left(-a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+a+bx^2\right)}{3b\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x + b*x^3],x]
[Out] (2*x*(a + b*x^2 - a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -(
(b*x^2)/a)]))/(3*b*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\,x}{bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a*x)*x/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(b*x^3 + a*x), x)
```

maple [A] time = 0.06, size = 127, normalized size = 1.09

$$-\frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^3+ax}\,b^2}+\frac{2\sqrt{b\,x^3+ax}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x)^(1/2),x)
```

```
[Out] 2/3*(b*x^3+a*x)^(1/2)/b-1/3*a/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(
1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)
*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)
^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^3 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^3 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\sqrt{b\,x^3 + a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x^2/(a*x + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x\left(a + bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x*(a + b*x**2)), x)
```

3.60 $\int \frac{x}{\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=229

$$\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right.}{b^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 2*x*(b*x^2+a)/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*a^(1/4)*(cos(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^
(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1
/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(3/4)/(b*x
^3+a*x)^(1/2)+a^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a
^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(
1/2))^2)^(1/2)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2032, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right.}{b^{3/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x*(a + b*x^2))/(Sqrt[b]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*a^
(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^
2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(3/4)*Sqrt[a*x +
 b*x^3]) + (a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a]
 + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(b^(3
/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
```

```
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax+bx^3}}\,dx = \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}}$$

$$= \frac{\left(2\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{b}\sqrt{ax+bx^3}} - \frac{\left(2\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a}}}{\sqrt{a+bx^4}}\,d\right.}{\sqrt{b}\sqrt{ax+bx^3}}$$

$$= \frac{2x\left(a+bx^2\right)}{\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}} - \frac{2\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+bx^3}}$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.23

$$\frac{2x^2\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{bx^2}{a}\right)}{3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x + b*x^3],x]
```

```
[Out] (2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/2, 3/4, 7/4, -((b*x^2)/a)])/
(3*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{bx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(b*x^3 + a*x), x)
```

maple [A] time = 0.07, size = 158, normalized size = 0.69

$$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\ \mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right)}{\sqrt{b\,x^3+ax}\ b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x)^(1/2),x)
```

```
[Out] (-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-
2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^
(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1
/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^3 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3)^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**2)), x)
```

3.61 $\int \frac{1}{\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax+bx^3}}$$

```
[Out] (cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*
(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(1/4)
/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2011, 329, 220}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + b*x^3],x]
```

```
[Out] (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*El
lipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(1/4)*b^(1/4)*Sqrt[a*
x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt{ax+bx^3}}\,dx = \frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{\sqrt{ax+bx^3}}$$
$$= \frac{\left(2\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^3}}$$
$$= \frac{\sqrt{x}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt[4]{b}\,\sqrt{ax+bx^3}}$$

Mathematica [C] time = 0.02, size = 49, normalized size = 0.53

$$\frac{2x\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)}{\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^3],x]
[Out] (2*x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/Sq
rt[x*(a + b*x^2)]
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{bx^3+ax}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(b*x^3 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*x^3 + a*x), x)
```

maple [A] time = 0.07, size = 108, normalized size = 1.17

$$\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{\sqrt{b\,x^3+ax}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x)^(1/2),x)
```

```
[Out] (-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*El
lipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x), x)
```

mupad [B] time = 5.03, size = 40, normalized size = 0.43

$$\frac{2\,x\,\sqrt{\frac{b\,x^2}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{b\,x^2}{a}\right)}{\sqrt{b\,x^3+a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3)^(1/2),x)
```

```
[Out] (2*x*((b*x^2)/a + 1)^(1/2)*hypergeom([1/4, 1/2], 5/4, -(b*x^2)/a))/(a*x + b
*x^3)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**3), x)
```

3.62 $\int \frac{1}{x\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=253

$$\frac{\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right.}{a^{3/4}\sqrt{ax+bx^3}}$$

```
[Out] 2*x*(b*x^2+a)*b^(1/2)/a/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2*(b*x^3+a*x)
^(1/2)/a/x-2*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^
(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1
/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)+b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(
2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)
*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$\frac{\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}\sqrt{ax+bx^3}}-\frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)\right.}{a^{3/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (2*Sqrt[b]*x*(a + b*x^2))/(a*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (2*
Sqrt[a*x + b*x^3])/(a*x) - (2*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3]) + (b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt
[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4
)*Sqrt[x])/a^(1/4)], 1/2])/(a^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{ax+bx^3}}\,dx &= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{b\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{a}\\
&= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(2b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{ax}+\frac{\left(2\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a}\sqrt{ax+bx^3}}-\frac{\left(2\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)}{\sqrt{a}}\\
&= \frac{2\sqrt{b}\,x\left(a+bx^2\right)}{a\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{ax}-\frac{2\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan\right.}{a^{3/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.19

$$-\frac{2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((b*x^2)/a)])/Sq
rt[x*(a + b*x^2)]
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{bx^4+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b*x^4 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

maple [A] time = 0.07, size = 182, normalized size = 0.72

$$-\frac{2\left(b\,x^2+a\right)}{\sqrt{\left(b\,x^2+a\right)x}\;a}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\;\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}\right)}{\sqrt{b\,x^3+ax}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x)^(1/2),x)
```

```
[Out] -2*(b*x^2+a)/a/((b*x^2+a)*x)^(1/2)+1/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a
*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^
(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^
(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-
a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\sqrt{b\,x^3+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x\left(a + bx^{2}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x*(a + b*x**2))), x)
```

3.63 $\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=119

$$-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3*b^(3/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1
/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^
2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2025, 2011, 329, 220}

$$-\frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^3])/(3*a*x^2) - (b^(3/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*S
qrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x
])/a^(1/4)], 1/2])/(3*a^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx = -\frac{2\sqrt{ax+bx^3}}{3ax^2} - \frac{b\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{3a}$$
$$= -\frac{2\sqrt{ax+bx^3}}{3ax^2} - \frac{\left(b\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{3a\sqrt{ax+bx^3}}$$
$$= -\frac{2\sqrt{ax+bx^3}}{3ax^2} - \frac{\left(2b\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{3a\sqrt{ax+bx^3}}$$
$$= -\frac{2\sqrt{ax+bx^3}}{3ax^2} - \frac{b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{3a^{5/4}\sqrt{ax+bx^3}}$$

Mathematica [C] time = 0.02, size = 53, normalized size = 0.45

$$-\frac{2\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3x\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x + b*x^3]),x]
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 1/2, 1/4, -((b*x^2)/a)])/(3
*x*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{bx^5+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a*x)/(b*x^5 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

maple [A] time = 0.07, size = 129, normalized size = 1.08

$$-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\;\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{3\sqrt{b\,x^3+ax}\;a} - \frac{2\sqrt{b\,x^3+ax}}{3a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x)^(1/2),x)
```

```
[Out] -2/3*(b*x^3+a*x)^(1/2)/a/x^2-1/3/a*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2
)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3 + ax}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\, \sqrt{b\, x^3 + a\, x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{x \left(a + bx^2\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x*(a + b*x**2))), x)
```

3.64 $\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx$

Optimal. Leaf size=286

$$-\frac{3b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}+\frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}E\left(2\tan^{-1}\left(\right.\right.}{5a^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] -6/5*b^(3/2)*x*(b*x^2+a)/a^2/(a^(1/2)+x*b^(1/2))/(b*x^3+a*x)^(1/2)-2/5*(b*x
^3+a*x)^(1/2)/a/x^3+6/5*b*(b*x^3+a*x)^(1/2)/a^2/x+6/5*b^(5/4)*(cos(2*arctan
(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*
EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^
(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^
(1/2)-3/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2)
)^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2025, 2032, 329, 305, 220, 1196}

$$-\frac{6b^{3/2}x\left(a+bx^2\right)}{5a^2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{3b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+bx^3}}+\frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-6*b^(3/2)*x*(a + b*x^2))/(5*a^2*(Sqrt[a] + Sqrt[b]*x)*Sqrt[a*x + b*x^3])
- (2*Sqrt[a*x + b*x^3])/(5*a*x^3) + (6*b*Sqrt[a*x + b*x^3])/(5*a^2*x) + (6*
b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x
)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[a
*x + b*x^3]) - (3*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(S
qrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])
/(5*a^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx &= -\frac{2\sqrt{ax+bx^3}}{5ax^3}-\frac{(3b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}-\frac{\left(3b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{5a^2}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}-\frac{\left(3b^2\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{5a^2\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}-\frac{\left(6b^2\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5a^2\sqrt{ax+bx^3}}\\
&= -\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}-\frac{\left(6b^{3/2}\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{5a^{3/2}\sqrt{ax+bx^3}}+\frac{\left(6b^{3/\cdots\right.}{\cdots}\\
&= -\frac{6b^{3/2}x\left(a+bx^2\right)}{5a^2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{2\sqrt{ax+bx^3}}{5ax^3}+\frac{6b\sqrt{ax+bx^3}}{5a^2x}+\frac{6b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\cdots}}{5a^{7/\cdots}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.19

$$
-\frac{2\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(-\frac{5}{4},\frac{1}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5x^2\sqrt{x\left(a+bx^2\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x + b*x^3]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 1/2, -1/4, -((b*x^2)/a)])/(
5*x^2*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}}{bx^6 + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b*x^6 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3 + ax}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

maple [A] time = 0.07, size = 204, normalized size = 0.71

$$\frac{6\left(b\,x^2 + a\right)b}{5\sqrt{\left(b\,x^2 + a\right)x}\;a^2} - \frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x + \frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x - \frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\mathrm{EllipticE}\left(\sqrt{\frac{\left(x + \frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{b} + \frac{\sqrt{-ab}\;\mathrm{Ellip}}{}\right.}{5\sqrt{b\,x^3 + ax}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x)^(1/2),x)
```

```
[Out] -2/5*(b*x^3+a*x)^(1/2)/a/x^3+6/5*(b*x^2+a)/a^2*b/((b*x^2+a)*x)^(1/2)-3/5/a^
2*b*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*
(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*
2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)
,1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3 + ax}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{b\,x^3 + a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^3)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x\left(a+bx^2\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x*(a + b*x**2))), x)
```

3.65 $\int \frac{x^7}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

```
[Out] -x^5/b/(b*x^3+a*x)^(1/2)-15/7*a*(b*x^3+a*x)^(1/2)/b^3+9/7*x^2*(b*x^3+a*x)^(
1/2)/b^2+15/14*a^(7/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos
(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/
a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^
(1/2))^2)^(1/2)/b^(13/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2022, 2024, 2011, 329, 220}

$$\frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{14b^{13/4}\sqrt{ax+bx^3}}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}-\frac{15a\sqrt{ax+bx^3}}{7b^3}-\frac{x^5}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^5/(b*Sqrt[a*x + b*x^3])) - (15*a*Sqrt[a*x + b*x^3])/(7*b^3) + (9*x^2*Sq
rt[a*x + b*x^3])/(7*b^2) + (15*a^(7/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(
a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^
(1/4)], 1/2])/(14*b^(13/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && !Int
```

```
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(ax+bx^3\right)^{3/2}}\,dx &= -\frac{x^5}{b\sqrt{ax+bx^3}}+\frac{9\int \frac{x^4}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x^5}{b\sqrt{ax+bx^3}}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}-\frac{(45a)\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{14b^2}\\
&= -\frac{x^5}{b\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}+\frac{\left(15a^2\right)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{14b^3}\\
&= -\frac{x^5}{b\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}+\frac{\left(15a^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{14b^3\sqrt{ax+bx^3}}\\
&= -\frac{x^5}{b\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}+\frac{\left(15a^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,d\right.}{7b^3\sqrt{ax+bx^3}}\\
&= -\frac{x^5}{b\sqrt{ax+bx^3}}-\frac{15a\sqrt{ax+bx^3}}{7b^3}+\frac{9x^2\sqrt{ax+bx^3}}{7b^2}+\frac{15a^{7/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F}{14b^{13/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 80, normalized size = 0.50

$$\frac{x\left(15a^2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-15a^2-6abx^2+2b^2x^4\right)}{7b^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x*(-15*a^2 - 6*a*b*x^2 + 2*b^2*x^4 + 15*a^2*Sqrt[1 + (b*x^2)/a]*Hypergeome
tric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]))/(7*b^3*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\,x^5}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^5/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

maple [A] time = 0.10, size = 172, normalized size = 1.07

$$-\frac{a^2 x}{\sqrt{\left(x^2 + \frac{a}{b}\right) bx}\, b^3} + \frac{2\sqrt{b\,x^3 + ax}\, x^2}{7b^2} + \frac{15\sqrt{-ab}\,\sqrt{\frac{\left(x + \frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x - \frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\, a^2\, \mathrm{EllipticF}\left(\sqrt{\frac{\left(x + \frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{14\sqrt{b\,x^3 + ax}\, b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -x/b^3*a^2/((x^2+a/b)*x*b)^(1/2)+2/7*x^2*(b*x^3+a*x)^(1/2)/b^2-8/7*a*(b*x^3
+a*x)^(1/2)/b^3+15/14*a^2/b^4*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)
*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x
)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/
2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/(b*x^3 + a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7}{\left(b\,x^3 + a\,x\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^7/(a*x + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{\left(x\left(a + bx^2\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**7/(x*(a + b*x**2))**(3/2), x)
```

3.66 $\int \frac{x^6}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=279

$$-\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}}+\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{ax+bx^3}}$$

```
[Out] -x^4/b/(b*x^3+a*x)^(1/2)-21/5*a*x*(b*x^2+a)/b^(5/2)/(a^(1/2)+x*b^(1/2))/(b*
x^3+a*x)^(1/2)+7/5*x*(b*x^3+a*x)^(1/2)/b^2+21/5*a^(5/4)*(cos(2*arctan(b^(1/
4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*Ellipt
icE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))
*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(11/4)/(b*x^3+a*x)^(1/2)
-21/10*a^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))
),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2
)^(1/2)/b^(11/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2022, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10b^{11/4}\sqrt{ax+bx^3}}+\frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5b^{11/4}\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^4/(b*Sqrt[a*x + b*x^3])) - (21*a*x*(a + b*x^2))/(5*b^(5/2)*(Sqrt[a] + S
qrt[b]*x)*Sqrt[a*x + b*x^3]) + (7*x*Sqrt[a*x + b*x^3])/(5*b^2) + (21*a^(5/4
)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*E
llipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(5*b^(11/4)*Sqrt[a*x +
b*x^3]) - (21*a^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[
a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10
*b^(11/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^6}{\left(ax+bx^3\right)^{3/2}}\,dx = -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7\int \frac{x^3}{\sqrt{ax+bx^3}}\,dx}{2b}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{(21a)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{10b^2}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{\left(21a\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{10b^2\sqrt{ax+bx^3}}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{\left(21a\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5b^2\sqrt{ax+bx^3}}$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} - \frac{\left(21a^{3/2}\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{5b^{5/2}\sqrt{ax+bx^3}} + \cdots$$

$$= -\frac{x^4}{b\sqrt{ax+bx^3}} - \frac{21ax\left(a+bx^2\right)}{5b^{5/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} + \frac{7x\sqrt{ax+bx^3}}{5b^2} + \frac{21a^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.24

$$\frac{2x^2\left(7a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-7a+bx^2\right)}{5b^2\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*(-7*a + b*x^2 + 7*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2,
7/4, -((b*x^2)/a)]))/(5*b^2*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\,x^4}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^4/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

maple [A] time = 0.07, size = 200, normalized size = 0.72

$$\frac{a\,x^2}{\sqrt{\left(x^2+\frac{a}{b}\right)bx\,b^2}}+\frac{2\sqrt{b\,x^3+ax}\,x}{5b^2}-\frac{21\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right.}{b}\right.}{10\sqrt{b\,x^3+ax}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x)^(3/2),x)
```

```
[Out] x^2/b^2*a/((x^2+a/b)*b*x)^(1/2)+2/5*x*(b*x^3+a*x)^(1/2)/b^2-21/10*a/b^3*(-a
*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/
(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a
*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)
)+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^
(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/(b*x^3 + a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(b\,x^3+a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^6/(a*x + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**6/(x*(a + b*x**2))**(3/2), x)
```

3.67 $\int \frac{x^5}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

```
[Out] -x^3/b/(b*x^3+a*x)^(1/2)+5/3*(b*x^3+a*x)^(1/2)/b^2-5/6*a^(3/4)*(cos(2*arcta
n(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))
*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b
^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^3+a*x)
^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2022, 2024, 2011, 329, 220}

$$-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6b^{9/4}\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{x^3}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^3/(b*Sqrt[a*x + b*x^3])) + (5*Sqrt[a*x + b*x^3])/(3*b^2) - (5*a^(3/4)*S
qrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Elli
pticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*b^(9/4)*Sqrt[a*x + b*x^
3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
```

```
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(ax+bx^3\right)^{3/2}}\,dx &= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\int \frac{x^2}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{(5a)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{6b^2}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{\left(5a\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{6b^2\sqrt{ax+bx^3}}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{\left(5a\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3b^2\sqrt{ax+bx^3}}\\
&= -\frac{x^3}{b\sqrt{ax+bx^3}}+\frac{5\sqrt{ax+bx^3}}{3b^2}-\frac{5a^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\right.}{6b^{9/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 67, normalized size = 0.49

$$\frac{x\left(-5a\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+5a+2bx^2\right)}{3b^2\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (x*(5*a + 2*b*x^2 - 5*a*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4
, -((b*x^2)/a)]))/(3*b^2*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\,x^3}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^3/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

maple [A] time = 0.07, size = 147, normalized size = 1.07

$$\frac{ax}{\sqrt{\left(x^2 + \frac{a}{b}\right) bx}\, b^2} - \frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,a\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{6\sqrt{b\,x^3 + ax}\, b^3} + \frac{2\sqrt{b\,x^3 + ax}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x)^(3/2),x)
```

```
[Out] x/b^2*a/((x^2+a/b)*b*x)^(1/2)+2/3*(b*x^3+a*x)^(1/2)/b^2-5/6*a/b^3*(-a*b)^(1
/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(b*x^3 + a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(b\,x^3 + a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^5/(a*x + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(x\left(a + bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**5/(x*(a + b*x**2))**(3/2), x)
```

3.68 $\int \frac{x^4}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=253

$$\frac{3x\left(a+bx^2\right)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\right.}{}$$

```
[Out] -x^2/b/(b*x^3+a*x)^(1/2)+3*x*(b*x^2+a)/b^(3/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a
*x)^(1/2)-3*a^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*
arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(
1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/
2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)+3/2*a^(1/4)*(cos(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(s
in(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1
/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/b^(7/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2022, 2032, 329, 305, 220, 1196}

$$\frac{3x\left(a+bx^2\right)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}+\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2b^{7/4}\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x^2/(b*Sqrt[a*x + b*x^3])) + (3*x*(a + b*x^2))/(b^(3/2)*(Sqrt[a] + Sqrt[b
]*x)*Sqrt[a*x + b*x^3]) - (3*a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
+ b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1
/4)], 1/2])/(b^(7/4)*Sqrt[a*x + b*x^3]) + (3*a^(1/4)*Sqrt[x]*(Sqrt[a] + Sqr
t[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/
4)*Sqrt[x])/a^(1/4)], 1/2])/(2*b^(7/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\left(ax+bx^3\right)^{3/2}}\,dx &= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{3\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2b}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{\left(3\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2b\sqrt{ax+bx^3}}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{\left(3\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{ax+bx^3}}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{\left(3\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b^{3/2}\sqrt{ax+bx^3}}-\frac{\left(3\sqrt{a}\sqrt{x}\sqrt{a+bx^2}\right)\mathrm{S}}{b^{3/2}\sqrt{}}\\
&= -\frac{x^2}{b\sqrt{ax+bx^3}}+\frac{3x\left(a+bx^2\right)}{b^{3/2}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{a}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\,\mathrm{ta}\right.}{b^{7/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.23

$$-\frac{2x^2\left(\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)-1\right)}{b\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^2*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2
)/a)]))/(b*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3 + ax}\,x^2}{b^2x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)*x^2/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

maple [A] time = 0.07, size = 182, normalized size = 0.72

$$-\frac{x^2}{\sqrt{\left(x^2 + \frac{a}{b}\right) bx}\; b} + \frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\ \mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{b} + \frac{\sqrt{-ab}\ \mathrm{Ellipti}\cdots}{\cdots}\right)}{2\sqrt{b\,x^3 + ax}\; b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -x^2/b/((x^2+a/b)*b*x)^(1/2)+3/2/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b
)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1
/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1
/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(b*x^3 + a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(b\,x^3 + a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^4/(a*x + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(x*(a + b*x**2))**(3/2), x)
```

3.69 $\int \frac{x^3}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

```
[Out] -x/b/(b*x^3+a*x)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)
/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1
/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+
x*b^(1/2))^2)^(1/2)/a^(1/4)/b^(5/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2022, 2011, 329, 220}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{ax+bx^3}}-\frac{x}{b\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x + b*x^3)^(3/2),x]
```

```
[Out] -(x/(b*Sqrt[a*x + b*x^3])) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2
)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1
/2])/(2*a^(1/4)*b^(5/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rubi steps

$$\int \frac{x^3}{\left(ax+bx^3\right)^{3/2}}\,dx = -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{2b}$$

$$= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^2}}\,dx}{2b\sqrt{ax+bx^3}}$$

$$= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\,\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{b\sqrt{ax+bx^3}}$$

$$= -\frac{x}{b\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}\,b^{5/4}\sqrt{ax+bx^3}}$$

Mathematica [C] time = 0.03, size = 54, normalized size = 0.47

$$\frac{x\left(\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)-1\right)}{b\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x + b*x^3)^(3/2),x]
[Out] (x*(-1 + Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)]
))/(b*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}\,x}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a*x)*x/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

maple [A] time = 0.07, size = 130, normalized size = 1.13

$$-\frac{x}{\sqrt{\left(x^2+\frac{a}{b}\right)bx}\;b}+\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\;\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{2\sqrt{b\,x^3+ax}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -x/b/((x^2+a/b)*b*x)^(1/2)+1/2/b^2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^
(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2
)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b
)^(1/2),1/2*2^(1/2))
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(b*x^3 + a*x)^(3/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{{\left(b\,x^3+a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^3)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(x*(a + b*x**2))**(3/2), x)
```

3.70 $\int \frac{x^2}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}}+\frac{-}{a}$$

```
[Out] x^2/a/(b*x^3+a*x)^(1/2)-x*(b*x^2+a)/a/b^(1/2)/(a^(1/2)+x*b^(1/2))/(b*x^3+a*
x)^(1/2)+(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1
/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*
2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2
)/a^(3/4)/b^(3/4)/(b*x^3+a*x)^(1/2)-1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/
4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan
(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2
+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2023, 2032, 329, 305, 220, 1196}

$$-\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}}+\frac{-}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x + b*x^3)^(3/2),x]
```

```
[Out] x^2/(a*Sqrt[a*x + b*x^3]) - (x*(a + b*x^2))/(a*Sqrt[b]*(Sqrt[a] + Sqrt[b]*x
)*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqr
t[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(
a^(3/4)*b^(3/4)*Sqrt[a*x + b*x^3]) - (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a
 + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(
1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeqQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(ax+bx^3\right)^{3/2}}\,dx &= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2a}\\
&= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2a\sqrt{ax+bx^3}}\\
&= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{a\sqrt{ax+bx^3}}\\
&= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{ax+bx^3}}+\frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int\right.}{\sqrt{a}\sqrt{b}\sqrt{ax-}}\\
&= \frac{x^2}{a\sqrt{ax+bx^3}}-\frac{x\left(a+bx^2\right)}{a\sqrt{b}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{ax+bx^3}}+\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}E\left(2\tan\right.}{a^{3/4}b^{3/4}\sqrt{ax+bx^3}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.22

$$\frac{2x^2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{bx^2}{a}\right)}{3a\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x + b*x^3)^(3/2),x]
```

```
[Out] (2*x^2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[3/4, 3/2, 7/4, -((b*x^2)/a)])/
(3*a*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^4+2\,abx^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

maple [A] time = 0.07, size = 184, normalized size = 0.72

$$\frac{x^2}{\sqrt{\left(x^2+\frac{a}{b}\right)bx\,a}}-\frac{\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{b}+\frac{\sqrt{-ab}\,\mathrm{EllipticF}\left(\sqrt{\cdots}\right.}{b}\right.}{2\sqrt{b\,x^3+ax}\;ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x)^(3/2),x)
```

```
[Out] x^2/a/((x^2+a/b)*b*x)^(1/2)-1/2/a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)
^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/
2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/
2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b
)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(b*x^3 + a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(b\,x^3+a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x^2/(a*x + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(x*(a + b*x**2))**(3/2), x)
```

3.71 $\int \frac{x}{\left(ax+bx^3\right)^{3/2}} \, dx$

Optimal. Leaf size=114

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\,\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

```
[Out] x/a/(b*x^3+a*x)^(1/2)+1/2*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/
cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/
2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x
*b^(1/2))^2)^(1/2)/a^(5/4)/b^(1/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2023, 2011, 329, 220}

$$\frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\,\sqrt{ax+bx^3}}+\frac{x}{a\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x + b*x^3)^(3/2),x]
```

```
[Out] x/(a*Sqrt[a*x + b*x^3]) + (Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(
Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2]
)/(2*a^(5/4)*b^(1/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rubi steps

$$\int \frac{x}{\left(ax+bx^3\right)^{3/2}}\,dx = \frac{x}{a\sqrt{ax+bx^3}} + \frac{\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{2a}$$

$$= \frac{x}{a\sqrt{ax+bx^3}} + \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{2a\sqrt{ax+bx^3}}$$

$$= \frac{x}{a\sqrt{ax+bx^3}} + \frac{\left(\sqrt{x}\sqrt{a+bx^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{a\sqrt{ax+bx^3}}$$

$$= \frac{x}{a\sqrt{ax+bx^3}} + \frac{\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{5/4}\sqrt[4]{b}\sqrt{ax+bx^3}}$$

Mathematica [C] time = 0.03, size = 54, normalized size = 0.47

$$\frac{x\sqrt{\frac{bx^2}{a}+1}\;{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{bx^2}{a}\right)+x}{a\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x + b*x^3)^(3/2),x]
[Out] (x + x*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[1/4, 1/2, 5/4, -((b*x^2)/a)])/
(a*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^5+2\,abx^3+a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^5 + 2*a*b*x^3 + a^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x/(b*x^3 + a*x)^(3/2), x)
```

maple [A] time = 0.07, size = 132, normalized size = 1.16

$$\frac{x}{\sqrt{\left(x^2+\frac{a}{b}\right)bx}\;a} + \frac{\sqrt{-ab}\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\sqrt{-\frac{bx}{\sqrt{-ab}}}\;\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{2\sqrt{b\,x^3+ax}\;ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x)^(3/2),x)
```

```
[Out] x/a/((x^2+a/b)*b*x)^(1/2)+1/2/a*(-a*b)^(1/2)/b*((x+(-a*b)^(1/2)/b)/(-a*b)^(
1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)
*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)
^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3 + ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^3 + a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(b\,x^3+a\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^3)^(3/2),x)
```

```
[Out] int(x/(a*x + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x\left(a + bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(x/(x*(a + b*x**2))**(3/2), x)
```

3.72 $\int \frac{1}{\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=273

$$\frac{3\sqrt[4]{b}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{b}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt[4]{}}\right.\right.}{a^{7/4}\sqrt{ax+bx^3}}$$

```
[Out] 1/a/(b*x^3+a*x)^(1/2)+3*x*(b*x^2+a)*b^(1/2)/a^2/(a^(1/2)+x*b^(1/2))/(b*x^3+
a*x)^(1/2)-3*(b*x^3+a*x)^(1/2)/a^2/x-3*b^(1/4)*(cos(2*arctan(b^(1/4)*x^(1/2
)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2
*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*
((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^3+a*x)^(1/2)+3/2*b^(1/
4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2
))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(7
/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {2006, 2025, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt{b}\,x\left(a+bx^2\right)}{a^2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{3\sqrt{ax+bx^3}}{a^2x}+\frac{3\sqrt[4]{b}\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2a^{7/4}\sqrt{ax+bx^3}}-\frac{3\sqrt[4]{b}\,\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^3)^(-3/2),x]
```

```
[Out] 1/(a*Sqrt[a*x + b*x^3]) + (3*Sqrt[b]*x*(a + b*x^2))/(a^2*(Sqrt[a] + Sqrt[b]
*x)*Sqrt[a*x + b*x^3]) - (3*Sqrt[a*x + b*x^3])/(a^2*x) - (3*b^(1/4)*Sqrt[x]
*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[
2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(a^(7/4)*Sqrt[a*x + b*x^3]) + (3
*b^(1/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*
x)^2]*EllipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(2*a^(7/4)*Sqrt[
a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\left(ax+bx^3\right)^{3/2}}\,dx = \frac{1}{a\sqrt{ax+bx^3}} + \frac{3\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{2a}$$

$$= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{(3b)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{2a^2}$$

$$= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{\left(3b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{2a^2\sqrt{ax+bx^3}}$$

$$= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{\left(3b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{a^2\sqrt{ax+bx^3}}$$

$$= \frac{1}{a\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} + \frac{\left(3\sqrt{b}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx, x, \sqrt{x}\right)}{a^{3/2}\sqrt{ax+bx^3}} - \frac{\left(3\sqrt{b}\sqrt{x}\right.}{}$$

$$= \frac{1}{a\sqrt{ax+bx^3}} + \frac{3\sqrt{b}\,x\left(a+bx^2\right)}{a^2\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{3\sqrt{ax+bx^3}}{a^2x} - \frac{3\sqrt[4]{b}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+}{\left(\sqrt{a}+\right.}}}{a^{7/4}\sqrt{ax}}$$

Mathematica [C] time = 0.03, size = 51, normalized size = 0.19

$$-\frac{2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{2};\frac{3}{4};-\frac{bx^2}{a}\right)}{a\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^3)^(-3/2),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-1/4, 3/2, 3/4, -((b*x^2)/a)])/(a
*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^6+2\,abx^4+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^6 + 2*a*b*x^4 + a^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x)^(-3/2), x)
```

maple [A] time = 0.07, size = 206, normalized size = 0.75

$$-\frac{b\,x^2}{\sqrt{\left(x^2+\frac{a}{b}\right)bx}\,a^2}-\frac{2\left(b\,x^2+a\right)}{\sqrt{\left(b\,x^2+a\right)x}\,a^2}+\frac{3\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\text{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\right.}{b}\right.}{2\sqrt{b\,x^3+ax}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -x^2*b/a^2/((x^2+a/b)*b*x)^(1/2)-2*(b*x^2+a)/a^2/((b*x^2+a)*x)^(1/2)+3/2/a^
2*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2
)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-
2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^
(1/2))+(-a*b)^(1/2)/b*EllipticF(((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1
/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x)^(-3/2), x)
```

mupad [B] time = 5.17, size = 40, normalized size = 0.15

$$-\frac{2\,x\,{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{2};\ \frac{3}{4};\ -\frac{b\,x^2}{a}\right)}{{\left(b\,x^3+a\,x\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^3)^(3/2),x)
```

```
[Out] -(2*x*((b*x^2)/a + 1)^(3/2)*hypergeom([-1/4, 3/2], 3/4, -(b*x^2)/a))/(a*x +
 b*x^3)^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+bx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**3)**(-3/2), x)
```

3.73 $\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}+\frac{1}{ax\sqrt{ax+bx^3}}$$

```
[Out] 1/a/x/(b*x^3+a*x)^(1/2)-5/3*(b*x^3+a*x)^(1/2)/a^2/x^2-5/6*b^(3/4)*(cos(2*ar
ctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
)))*EllipticF(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+
x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(9/4)/(b*x^3+a
*x)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2023, 2025, 2011, 329, 220}

$$-\frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\,\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6a^{9/4}\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}+\frac{1}{ax\sqrt{ax+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] 1/(a*x*Sqrt[a*x + b*x^3]) - (5*Sqrt[a*x + b*x^3])/(3*a^2*x^2) - (5*b^(3/4)*
Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*Ell
ipticF[2*ArcTan[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(6*a^(9/4)*Sqrt[a*x + b*x
^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
```

```
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(ax+bx^3\right)^{3/2}}\,dx &= \frac{1}{ax\sqrt{ax+bx^3}}+\frac{5\int \frac{1}{x^2\sqrt{ax+bx^3}}\,dx}{2a}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{(5b)\int \frac{1}{\sqrt{ax+bx^3}}\,dx}{6a^2}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{\left(5b\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^2}}\,dx}{6a^2\sqrt{ax+bx^3}}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{\left(5b\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^4}}\,dx,x,\sqrt{x}\right)}{3a^2\sqrt{ax+bx^3}}\\
&= \frac{1}{ax\sqrt{ax+bx^3}}-\frac{5\sqrt{ax+bx^3}}{3a^2x^2}-\frac{5b^{3/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}x\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{6a^{9/4}\sqrt{ax+bx^3}}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.40

$$-\frac{2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{bx^2}{a}\right)}{3ax\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((b*x^2)/a)])/(3
*a*x*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^7+2\,abx^5+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^7 + 2*a*b*x^5 + a^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

maple [A] time = 0.07, size = 150, normalized size = 1.08

$$-\frac{bx}{\sqrt{\left(x^2+\frac{a}{b}\right)bx}\,a^2}-\frac{5\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{6\sqrt{b\,x^3+ax}\,a^2}-\frac{2\sqrt{b\,x^3+ax}}{3a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x)^(3/2),x)
```

```
[Out] -x*b/a^2/((x^2+a/b)*b*x)^(1/2)-2/3*(b*x^3+a*x)^(1/2)/a^2/x^2-5/6/a^2*(-a*b)
^(1/2)*((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a
*b)^(1/2)*b)^(1/2)*(-1/(-a*b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*EllipticF(
((x+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(b\,x^3+a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(x*(a + b*x**2))**(3/2)), x)
```

3.74 $\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=306

$$-\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}+\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+bx^3}}$$

```
[Out] 1/a/x^2/(b*x^3+a*x)^(1/2)-21/5*b^(3/2)*x*(b*x^2+a)/a^3/(a^(1/2)+x*b^(1/2))/
(b*x^3+a*x)^(1/2)-7/5*(b*x^3+a*x)^(1/2)/a^2/x^3+21/5*b*(b*x^3+a*x)^(1/2)/a^
3/x+21/5*b^(5/4)*(cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*x^(1/2)/a^(1/4
))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))
^2)^(1/2)/a^(11/4)/(b*x^3+a*x)^(1/2)-21/10*b^(5/4)*(cos(2*arctan(b^(1/4)*x^
(1/2)/a^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*x^(1/2)/a^(1/4)))*EllipticF(s
in(2*arctan(b^(1/4)*x^(1/2)/a^(1/4))),1/2*2^(1/2))*(a^(1/2)+x*b^(1/2))*x^(1
/2)*((b*x^2+a)/(a^(1/2)+x*b^(1/2))^2)^(1/2)/a^(11/4)/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2023, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{21b^{3/2}x\left(a+bx^2\right)}{5a^3\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}}-\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{\frac{a+bx^2}{\left(\sqrt{a}+\sqrt{b}\,x\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\sqrt{x}}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{10a^{11/4}\sqrt{ax+bx^3}}+\frac{21b^{5/4}\sqrt{x}\left(\sqrt{a}+\sqrt{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x + b*x^3)^(3/2)),x]
```

```
[Out] 1/(a*x^2*Sqrt[a*x + b*x^3]) - (21*b^(3/2)*x*(a + b*x^2))/(5*a^3*(Sqrt[a] +
Sqrt[b]*x)*Sqrt[a*x + b*x^3]) - (7*Sqrt[a*x + b*x^3])/(5*a^2*x^3) + (21*b*S
qrt[a*x + b*x^3])/(5*a^3*x) + (21*b^(5/4)*Sqrt[x]*(Sqrt[a] + Sqrt[b]*x)*Sqr
t[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticE[2*ArcTan[(b^(1/4)*Sqrt[x])
/a^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[a*x + b*x^3]) - (21*b^(5/4)*Sqrt[x]*(Sqrt
[a] + Sqrt[b]*x)*Sqrt[(a + b*x^2)/(Sqrt[a] + Sqrt[b]*x)^2]*EllipticF[2*ArcT
an[(b^(1/4)*Sqrt[x])/a^(1/4)], 1/2])/(10*a^(11/4)*Sqrt[a*x + b*x^3])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax+bx^3\right)^{3/2}}\,dx = \frac{1}{ax^2\sqrt{ax+bx^3}} + \frac{7\int \frac{1}{x^3\sqrt{ax+bx^3}}\,dx}{2a}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} - \frac{(21b)\int \frac{1}{x\sqrt{ax+bx^3}}\,dx}{10a^2}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^2\right)\int \frac{x}{\sqrt{ax+bx^3}}\,dx}{10a^3}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^2\sqrt{x}\sqrt{a+bx^2}\right)\int \frac{\sqrt{x}}{\sqrt{a+bx^2}}\,dx}{10a^3\sqrt{ax+bx^3}}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^2\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^4}}\right.}{5a^3\sqrt{ax+bx^3}}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} - \frac{\left(21b^{3/2}\sqrt{x}\sqrt{a+bx^2}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}}\right.}{5a^{5/2}\sqrt{ax+bx^3}}$$

$$= \frac{1}{ax^2\sqrt{ax+bx^3}} - \frac{21b^{3/2}x\left(a+bx^2\right)}{5a^3\left(\sqrt{a}+\sqrt{b}\,x\right)\sqrt{ax+bx^3}} - \frac{7\sqrt{ax+bx^3}}{5a^2x^3} + \frac{21b\sqrt{ax+bx^3}}{5a^3x} + \frac{21b^{5/4}}{}$$

Mathematica [C] time = 0.02, size = 56, normalized size = 0.18

$$-\frac{2\sqrt{\frac{bx^2}{a}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};-\frac{bx^2}{a}\right)}{5ax^2\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x + b*x^3)^(3/2)),x]
[Out] (-2*Sqrt[1 + (b*x^2)/a]*Hypergeometric2F1[-5/4, 3/2, -1/4, -((b*x^2)/a)])/(
5*a*x^2*Sqrt[x*(a + b*x^2)])
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^3+ax}}{b^2x^8+2\,abx^6+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^3 + a*x)/(b^2*x^8 + 2*a*b*x^6 + a^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

maple [A] time = 0.08, size = 228, normalized size = 0.75

$$\frac{b^2x^2}{\sqrt{\left(x^2+\frac{a}{b}\right)bx}\,a^3}+\frac{16\left(bx^2+a\right)b}{5\sqrt{\left(bx^2+a\right)x}\,a^3}-\frac{21\sqrt{-ab}\,\sqrt{\frac{\left(x+\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x-\frac{\sqrt{-ab}}{b}\right)b}{\sqrt{-ab}}}\,\sqrt{-\frac{bx}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x+\frac{\sqrt{-a}}{b}\right.}{\sqrt{-ab}}}\right.}{b}\right.}{10\sqrt{b\,x^3+ax}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x)^(3/2),x)
```

```
[Out] x^2*b^2/a^3/((x^2+a/b)*b*x)^(1/2)-2/5*(b*x^3+a*x)^(1/2)/a^2/x^3+16/5*(b*x^2
+a)*b/a^3/((b*x^2+a)*x)^(1/2)-21/10/a^3*b*(-a*b)^(1/2)*((x+(-a*b)^(1/2)/b)/
(-a*b)^(1/2)*b)^(1/2)*(-2*(x-(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2)*(-1/(-a*
b)^(1/2)*b*x)^(1/2)/(b*x^3+a*x)^(1/2)*(-2*(-a*b)^(1/2)/b*EllipticE(((x+(-a*
b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/b*EllipticF(((x
+(-a*b)^(1/2)/b)/(-a*b)^(1/2)*b)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\left(b\,x^3+a\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(x\left(a+bx^2\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x*(a + b*x**2))**(3/2)), x)
```

3.75 $\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=159

$$-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}+\frac{9\sqrt{x}\,\sqrt{ax+bx^3}}{2b^5}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}-\frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}}-\frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -1/7*x^(25/2)/b/(b*x^3+a*x)^(7/2)-9/35*x^(19/2)/b^2/(b*x^3+a*x)^(5/2)-3/5*x
^(13/2)/b^3/(b*x^3+a*x)^(3/2)-9/2*a*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x)^(1/
2))/b^(11/2)-3*x^(7/2)/b^4/(b*x^3+a*x)^(1/2)+9/2*x^(1/2)*(b*x^3+a*x)^(1/2)/
b^5
```

Rubi [A] time = 0.25, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2022, 2024, 2029, 206}

$$-\frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}}-\frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}}+\frac{9\sqrt{x}\,\sqrt{ax+bx^3}}{2b^5}-\frac{9a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax+bx^3}}\right)}{2b^{11/2}}-\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(25/2)/(7*b*(a*x + b*x^3)^(7/2)) - (9*x^(19/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (3*x^(13/2))/(5*b^3*(a*x + b*x^3)^(3/2)) - (3*x^(7/2))/(b^4*Sqrt[a*x
 + b*x^3]) + (9*Sqrt[x]*Sqrt[a*x + b*x^3])/(2*b^5) - (9*a*ArcTanh[(Sqrt[b]*
x^(3/2))/Sqrt[a*x + b*x^3]])/(2*b^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{29/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} + \frac{9\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} + \frac{9\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{5b^2}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} + \frac{3\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\int \frac{x^{5/2}}{\sqrt{ax+b}}}{b^4}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax}}{2b^5}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax}}{2b^5}$$

$$= -\frac{x^{25/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{9x^{19/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{3x^{13/2}}{5b^3\left(ax+bx^3\right)^{3/2}} - \frac{3x^{7/2}}{b^4\sqrt{ax+bx^3}} + \frac{9\sqrt{x}\sqrt{ax}}{2b^5}$$

Mathematica [A] time = 0.21, size = 130, normalized size = 0.82

$$\frac{\sqrt{x}\left(\sqrt{b}\,x\left(315a^4+1050a^3bx^2+1218a^2b^2x^4+528ab^3x^6+35b^4x^8\right)-\frac{315\sqrt{a}\left(a+bx^2\right)^4\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{\frac{bx^2}{a}+1}}\right)}{70b^{11/2}\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(29/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(Sqrt[b]*x*(315*a^4 + 1050*a^3*b*x^2 + 1218*a^2*b^2*x^4 + 528*a*b^
3*x^6 + 35*b^4*x^8) - (315*Sqrt[a]*(a + b*x^2)^4*ArcSinh[(Sqrt[b]*x)/Sqrt[a
]])/Sqrt[1 + (b*x^2)/a]))/(70*b^(11/2)*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

fricas [A] time = 0.70, size = 376, normalized size = 2.36

$$\left[\frac{315\left(ab^4x^8+4a^2b^3x^6+6a^3b^2x^4+4a^4bx^2+a^5\right)\sqrt{b}\log\left(2bx^2-2\sqrt{bx^3+ax}\sqrt{b}\sqrt{x}+a\right)+2\left(35b^5x^8+528\right.}{140\left(b^{10}x^8+4ab^9x^6+6a^2b^8x^4+4a^3b^7x^2+a^4b\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a^4*b*x^2 + a^5)
*sqrt(b)*log(2*b*x^2 - 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) + 2*(35*b^5
*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*b)*sqr
t(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a^3*b^7
*x^2 + a^4*b^6), 1/70*(315*(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 + 4*a
^4*b*x^2 + a^5)*sqrt(-b)*arctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (
35*b^5*x^8 + 528*a*b^4*x^6 + 1218*a^2*b^3*x^4 + 1050*a^3*b^2*x^2 + 315*a^4*
b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^10*x^8 + 4*a*b^9*x^6 + 6*a^2*b^8*x^4 + 4*a
^3*b^7*x^2 + a^4*b^6)]
```

giac [A] time = 0.33, size = 100, normalized size = 0.63

$$\frac{\left(\left(\left(x^{2}\left(\frac{35\,x^{2}}{b}+\frac{528\,a}{b^{2}}\right)+\frac{1218\,a^{2}}{b^{3}}\right)x^{2}+\frac{1050\,a^{3}}{b^{4}}\right)x^{2}+\frac{315\,a^{4}}{b^{5}}\right)x}{70\,\left(bx^{2}+a\right)^{\frac{7}{2}}}+\frac{9\,a\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{2\,b^{\frac{11}{2}}}-\frac{9\,a\log\left(|a|\right)}{4\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/70*(((x^2*(35*x^2/b + 528*a/b^2) + 1218*a^2/b^3)*x^2 + 1050*a^3/b^4)*x^2
+ 315*a^4/b^5)*x/(b*x^2 + a)^(7/2) + 9/2*a*log(abs(-sqrt(b)*x + sqrt(b*x^2
+ a)))/b^(11/2) - 9/4*a*log(abs(a))/b^(11/2)
```

maple [A] time = 0.09, size = 212, normalized size = 1.33

$$-\frac{\sqrt{\left(b\,x^{2}+a\right)x}\;\left(-35b^{\frac{9}{2}}x^{9}-528a\,b^{\frac{7}{2}}x^{7}+315\sqrt{b\,x^{2}+a}\;a\,b^{3}x^{6}\ln\left(\sqrt{b}\;x+\sqrt{b\,x^{2}+a}\right)-1218a^{2}b^{\frac{5}{2}}x^{5}+945\sqrt{b\,x^{2}}\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(29/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/70*((b*x^2+a)*x)^(1/2)/b^(11/2)*(-35*x^9*b^(9/2)+315*ln(b^(1/2)*x+(b*x^2
+a)^(1/2))*x^6*a*b^3*(b*x^2+a)^(1/2)-528*b^(7/2)*x^7*a+945*ln(b^(1/2)*x+(b*
x^2+a)^(1/2))*x^4*a^2*b^2*(b*x^2+a)^(1/2)-1218*b^(5/2)*x^5*a^2+945*ln(b^(1/
2)*x+(b*x^2+a)^(1/2))*x^2*a^3*b*(b*x^2+a)^(1/2)-1050*b^(3/2)*x^3*a^3+315*ln
(b^(1/2)*x+(b*x^2+a)^(1/2))*a^4*(b*x^2+a)^(1/2)-315*b^(1/2)*x*a^4)/x^(1/2)/
(b*x^2+a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{29}{2}}}{\left(bx^{3}+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(29/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(29/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{29/2}}{\left(b\,x^{3}+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(29/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(29/2)/(a*x + b*x^3)^(9/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(29/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.76 $\int \frac{x^{27/2}}{\left(ax+bx^3\right)^{9/2}} \, dx$

Optimal. Leaf size=126

$$\frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -1/7*x^(23/2)/b/(b*x^3+a*x)^(7/2)-8/35*x^(17/2)/b^2/(b*x^3+a*x)^(5/2)-16/35
*x^(11/2)/b^3/(b*x^3+a*x)^(3/2)-64/35*x^(5/2)/b^4/(b*x^3+a*x)^(1/2)+128/35*
(b*x^3+a*x)^(1/2)/b^5/x^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}}+\frac{128\sqrt{ax+bx^3}}{35b^5\sqrt{x}}-\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(23/2)/(7*b*(a*x + b*x^3)^(7/2)) - (8*x^(17/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (16*x^(11/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (64*x^(5/2))/(35*b^4*Sq
rt[a*x + b*x^3]) + (128*Sqrt[a*x + b*x^3])/(35*b^5*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{27/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} + \frac{8\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}$$

$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} + \frac{48\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}$$

$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}} + \frac{64\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35b^3}$$

$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{128\int}{}$$

$$= -\frac{x^{23/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{8x^{17/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{16x^{11/2}}{35b^3\left(ax+bx^3\right)^{3/2}} - \frac{64x^{5/2}}{35b^4\sqrt{ax+bx^3}} + \frac{128\sqrt{}}{35}$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.61

$$\frac{\sqrt{x}\left(128a^4+448a^3bx^2+560a^2b^2x^4+280ab^3x^6+35b^4x^8\right)}{35b^5\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(27/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(128*a^4 + 448*a^3*b*x^2 + 560*a^2*b^2*x^4 + 280*a*b^3*x^6 + 35*b^
4*x^8))/(35*b^5*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

fricas [A] time = 0.65, size = 108, normalized size = 0.86

$$\frac{\left(35\,b^4x^8+280\,ab^3x^6+560\,a^2b^2x^4+448\,a^3bx^2+128\,a^4\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(b^9x^9+4\,ab^8x^7+6\,a^2b^7x^5+4\,a^3b^6x^3+a^4b^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(35*b^4*x^8 + 280*a*b^3*x^6 + 560*a^2*b^2*x^4 + 448*a^3*b*x^2 + 128*a^
4)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^9*x^9 + 4*a*b^8*x^7 + 6*a^2*b^7*x^5 + 4*a^3
*b^6*x^3 + a^4*b^5*x)
```

giac [A] time = 0.21, size = 80, normalized size = 0.63

$$\frac{\sqrt{bx^2+a}}{b^5} - \frac{128\,\sqrt{a}}{35\,b^5} + \frac{140\left(bx^2+a\right)^3a - 70\left(bx^2+a\right)^2a^2 + 28\left(bx^2+a\right)a^3 - 5\,a^4}{35\left(bx^2+a\right)^{\frac{7}{2}}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] sqrt(b*x^2 + a)/b^5 - 128/35*sqrt(a)/b^5 + 1/35*(140*(b*x^2 + a)^3*a - 70*(
b*x^2 + a)^2*a^2 + 28*(b*x^2 + a)*a^3 - 5*a^4)/((b*x^2 + a)^(7/2)*b^5)
```

maple [A] time = 0.04, size = 70, normalized size = 0.56

$$\frac{\left(b\,x^{2}+a\right)\left(35x^{8}b^{4}+280a\,x^{6}b^{3}+560a^{2}x^{4}b^{2}+448a^{3}x^{2}b+128a^{4}\right)x^{\frac{9}{2}}}{35\left(b\,x^{3}+ax\right)^{\frac{9}{2}}b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(27/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/35*(b*x^2+a)*(35*b^4*x^8+280*a*b^3*x^6+560*a^2*b^2*x^4+448*a^3*b*x^2+128*
a^4)*x^(9/2)/b^5/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{27}{2}}}{\left(bx^{3}+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(27/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(27/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{27/2}}{\left(b\,x^{3}+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(27/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(27/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(27/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.77 $\int \frac{x^{25/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=130

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}-\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -1/7*x^(21/2)/b/(b*x^3+a*x)^(7/2)-1/5*x^(15/2)/b^2/(b*x^3+a*x)^(5/2)-1/3*x^
(9/2)/b^3/(b*x^3+a*x)^(3/2)+arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x)^(1/2))/b^(9
/2)-x^(3/2)/b^4/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2022, 2029, 206}

$$-\frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{3/2}}{b^4\sqrt{ax+bx^3}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax+bx^3}}\right)}{b^{9/2}}-\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(21/2)/(7*b*(a*x + b*x^3)^(7/2)) - x^(15/2)/(5*b^2*(a*x + b*x^3)^(5/2))
- x^(9/2)/(3*b^3*(a*x + b*x^3)^(3/2)) - x^(3/2)/(b^4*Sqrt[a*x + b*x^3]) + A
rcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x + b*x^3]]/b^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{25/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} + \frac{\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{b}$$

$$= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} + \frac{\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{b^2}$$

$$= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} + \frac{\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{b^3}$$

$$= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\int \frac{\sqrt{x}}{\sqrt{ax+bx^3}}\,dx}{b^4}$$

$$= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\text{Subst}\left(\int \frac{1}{1-b\cdots}\right.}{\cdots}$$

$$= -\frac{x^{21/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{x^{15/2}}{5b^2\left(ax+bx^3\right)^{5/2}} - \frac{x^{9/2}}{3b^3\left(ax+bx^3\right)^{3/2}} - \frac{x^{3/2}}{b^4\sqrt{ax+bx^3}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x\cdots}{\sqrt{ax+\cdots}}\right.}{b^{9/2}}$$

Mathematica [A] time = 0.29, size = 120, normalized size = 0.92

$$\frac{\sqrt{x}\left(105\sqrt{a}\left(a+bx^2\right)^3\sqrt{\frac{bx^2}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\sqrt{b}\,x\left(105a^3+350a^2bx^2+406ab^2x^4+176b^3x^6\right)\right)}{105b^{9/2}\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(25/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*(-(Sqrt[b]*x*(105*a^3 + 350*a^2*b*x^2 + 406*a*b^2*x^4 + 176*b^3*x^
6)) + 105*Sqrt[a]*(a + b*x^2)^3*Sqrt[1 + (b*x^2)/a]*ArcSinh[(Sqrt[b]*x)/Sqr
t[a]]))/(105*b^(9/2)*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

fricas [A] time = 0.82, size = 348, normalized size = 2.68

$$\left[\frac{105\left(b^4x^8+4\,ab^3x^6+6\,a^2b^2x^4+4\,a^3bx^2+a^4\right)\sqrt{b}\,\log\left(2\,bx^2+2\,\sqrt{bx^3+ax}\,\sqrt{b}\,\sqrt{x}+a\right)-2\left(176\,b^4x^6+406\,ab^3\cdots\right.}{210\left(b^9x^8+4\,ab^8x^6+6\,a^2b^7x^4+4\,a^3b^6x^2+a^4b^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqr
t(b)*log(2*b*x^2 + 2*sqrt(b*x^3 + a*x)*sqrt(b)*sqrt(x) + a) - 2*(176*b^4*x^
6 + 406*a*b^3*x^4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))
/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*
(105*(b^4*x^8 + 4*a*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4)*sqrt(-b)*a
rctan(sqrt(b*x^3 + a*x)*sqrt(-b)/(b*x^(3/2))) + (176*b^4*x^6 + 406*a*b^3*x^
4 + 350*a^2*b^2*x^2 + 105*a^3*b)*sqrt(b*x^3 + a*x)*sqrt(x))/(b^9*x^8 + 4*a*
b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

giac [A] time = 0.30, size = 86, normalized size = 0.66

$$-\frac{\left(2\left(x^2\left(\frac{88\,x^2}{b}+\frac{203\,a}{b^2}\right)+\frac{175\,a^2}{b^3}\right)x^2+\frac{105\,a^3}{b^4}\right)x}{105\left(bx^2+a\right)^{\frac{7}{2}}}-\frac{\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}+\frac{\log\left(|a|\right)}{2\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(2*(x^2*(88*x^2/b + 203*a/b^2) + 175*a^2/b^3)*x^2 + 105*a^3/b^4)*x/(
b*x^2 + a)^(7/2) - log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2) + 1/2*log
(abs(a))/b^(9/2)
```

maple [A] time = 0.08, size = 198, normalized size = 1.52

$$\frac{\sqrt{\left(b\,x^2+a\right)x}\,\left(-176b^{\frac{7}{2}}x^7+105\sqrt{b\,x^2+a}\;b^3x^6\ln\left(\sqrt{b}\;x+\sqrt{b\,x^2+a}\right)-406a\,b^{\frac{5}{2}}x^5+315\sqrt{b\,x^2+a}\;a\,b^2x^4\ln\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(25/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/105*((b*x^2+a)*x)^(1/2)/b^(9/2)*(105*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*x^6*b^
3*(b*x^2+a)^(1/2)-176*x^7*b^(7/2)+315*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*x^4*a*b
^2*(b*x^2+a)^(1/2)-406*b^(5/2)*x^5*a+315*ln(b^(1/2)*x+(b*x^2+a)^(1/2))*x^2*
a^2*b*(b*x^2+a)^(1/2)-350*b^(3/2)*x^3*a^2+105*ln(b^(1/2)*x+(b*x^2+a)^(1/2))
*a^3*(b*x^2+a)^(1/2)-105*b^(1/2)*x*a^3)/x^(1/2)/(b*x^2+a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{25}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(25/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(25/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{25/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(25/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(25/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(25/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.78 $\int \frac{x^{23/2}}{(ax+bx^3)^{9/2}}\, dx$

Optimal. Leaf size=101

$$-\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -1/7*x^(19/2)/b/(b*x^3+a*x)^(7/2)-6/35*x^(13/2)/b^2/(b*x^3+a*x)^(5/2)-8/35*
x^(7/2)/b^3/(b*x^3+a*x)^(3/2)-16/35*x^(1/2)/b^4/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2015, 2014}

$$-\frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}}-\frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}-\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(19/2)/(7*b*(a*x + b*x^3)^(7/2)) - (6*x^(13/2))/(35*b^2*(a*x + b*x^3)^(5
/2)) - (8*x^(7/2))/(35*b^3*(a*x + b*x^3)^(3/2)) - (16*Sqrt[x])/(35*b^4*Sqrt
[a*x + b*x^3])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{23/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}} + \frac{6\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}$$

$$= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}} + \frac{24\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}$$

$$= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}} + \frac{16\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35b^3}$$

$$= -\frac{x^{19/2}}{7b\left(ax+bx^3\right)^{7/2}} - \frac{6x^{13/2}}{35b^2\left(ax+bx^3\right)^{5/2}} - \frac{8x^{7/2}}{35b^3\left(ax+bx^3\right)^{3/2}} - \frac{16\sqrt{x}}{35b^4\sqrt{ax+bx^3}}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.65

$$-\frac{\sqrt{x}\left(16a^3+56a^2bx^2+70ab^2x^4+35b^3x^6\right)}{35b^4\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(23/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/35*(Sqrt[x]*(16*a^3 + 56*a^2*b*x^2 + 70*a*b^2*x^4 + 35*b^3*x^6))/(b^4*(a
 + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

fricas [A] time = 0.60, size = 97, normalized size = 0.96

$$-\frac{\left(35\,b^3x^6+70\,ab^2x^4+56\,a^2bx^2+16\,a^3\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(b^8x^9+4\,ab^7x^7+6\,a^2b^6x^5+4\,a^3b^5x^3+a^4b^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(35*b^3*x^6 + 70*a*b^2*x^4 + 56*a^2*b*x^2 + 16*a^3)*sqrt(b*x^3 + a*x)
*sqrt(x)/(b^8*x^9 + 4*a*b^7*x^7 + 6*a^2*b^6*x^5 + 4*a^3*b^5*x^3 + a^4*b^4*x
)
```

giac [A] time = 0.21, size = 64, normalized size = 0.63

$$\frac{16}{35\,\sqrt{a}\,b^4} - \frac{35\left(bx^2+a\right)^3 - 35\left(bx^2+a\right)^2 a + 21\left(bx^2+a\right)a^2 - 5\,a^3}{35\left(bx^2+a\right)^{\frac{7}{2}} b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 16/35/(sqrt(a)*b^4) - 1/35*(35*(b*x^2 + a)^3 - 35*(b*x^2 + a)^2*a + 21*(b*x
^2 + a)*a^2 - 5*a^3)/((b*x^2 + a)^(7/2)*b^4)
```

maple [A] time = 0.05, size = 59, normalized size = 0.58

$$-\frac{\left(b\,x^2+a\right)\left(35b^3x^6+70a\,b^2x^4+56a^2b\,x^2+16a^3\right)x^{\frac{9}{2}}}{35\left(b\,x^3+ax\right)^{\frac{9}{2}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*(b*x^2+a)*(35*b^3*x^6+70*a*b^2*x^4+56*a^2*b*x^2+16*a^3)*x^(9/2)/b^4/(
b*x^3+a*x)^(9/2)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{23}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(23/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(23/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{23/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(23/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(23/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(23/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.79 $\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}} \, dx$

Optimal. Leaf size=25

$$\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] 1/7*x^(21/2)/a/(b*x^3+a*x)^(7/2)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(21/2)/(a*x + b*x^3)^(9/2),x]
[Out] x^(21/2)/(7*a*(a*x + b*x^3)^(7/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{21/2}}{\left(ax+bx^3\right)^{9/2}} \, dx = \frac{x^{21/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{x^{21/2}}{7a\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(21/2)/(a*x + b*x^3)^(9/2),x]
[Out] x^(21/2)/(7*a*(x*(a + b*x^2))^(7/2))
```

fricas [B] time = 0.49, size = 61, normalized size = 2.44

$$\frac{\sqrt{bx^3+ax}\, x^{\frac{13}{2}}}{7\left(ab^4x^8+4\,a^2b^3x^6+6\,a^3b^2x^4+4\,a^4bx^2+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] 1/7*sqrt(b*x^3 + a*x)*x^(13/2)/(a*b^4*x^8 + 4*a^2*b^3*x^6 + 6*a^3*b^2*x^4 +
 4*a^4*b*x^2 + a^5)
```

giac [A] time = 0.29, size = 17, normalized size = 0.68

$$\frac{x^7}{7\left(bx^2+a\right)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] 1/7*x^7/((b*x^2 + a)^(7/2)*a)
```

maple [A] time = 0.05, size = 27, normalized size = 1.08

$$\frac{\left(b\,x^2+a\right)x^{\frac{23}{2}}}{7\left(b\,x^3+ax\right)^{\frac{9}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(b*x^3+a*x)^(9/2),x)
[Out] 1/7*(b*x^2+a)/a*x^(23/2)/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{21}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(21/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(21/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{21/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(21/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(21/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(21/2)/(b*x**3+a*x)**(9/2),x)
[Out] Timed out
```

3.80 $\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=76

$$-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -1/7*x^(15/2)/b/(b*x^3+a*x)^(7/2)-4/35*x^(9/2)/b^2/(b*x^3+a*x)^(5/2)-8/105*
x^(3/2)/b^3/(b*x^3+a*x)^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}-\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(15/2)/(7*b*(a*x + b*x^3)^(7/2)) - (4*x^(9/2))/(35*b^2*(a*x + b*x^3)^(5/
2)) - (8*x^(3/2))/(105*b^3*(a*x + b*x^3)^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{19/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{4\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\
&= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}+\frac{8\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35b^2}\\
&= -\frac{x^{15/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{4x^{9/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{8x^{3/2}}{105b^3\left(ax+bx^3\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 55, normalized size = 0.72

$$-\frac{\sqrt{x}\left(8a^2+28abx^2+35b^2x^4\right)}{105b^3\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(19/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/105*(Sqrt[x]*(8*a^2 + 28*a*b*x^2 + 35*b^2*x^4))/(b^3*(a + b*x^2)^3*Sqrt[
x*(a + b*x^2)])
```

fricas [A] time = 0.70, size = 86, normalized size = 1.13

$$-\frac{\left(35\,b^2x^4+28\,abx^2+8\,a^2\right)\sqrt{bx^3+ax}\,\sqrt{x}}{105\left(b^7x^9+4\,ab^6x^7+6\,a^2b^5x^5+4\,a^3b^4x^3+a^4b^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^2*x^4 + 28*a*b*x^2 + 8*a^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(b^7*x^9
 + 4*a*b^6*x^7 + 6*a^2*b^5*x^5 + 4*a^3*b^4*x^3 + a^4*b^3*x)
```

giac [A] time = 0.20, size = 50, normalized size = 0.66

$$\frac{8}{105\,a^{\frac{3}{2}}b^3}-\frac{35\left(bx^2+a\right)^2-42\left(bx^2+a\right)a+15\,a^2}{105\left(bx^2+a\right)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 8/105/(a^(3/2)*b^3) - 1/105*(35*(b*x^2 + a)^2 - 42*(b*x^2 + a)*a + 15*a^2)/
((b*x^2 + a)^(7/2)*b^3)
```

maple [A] time = 0.05, size = 48, normalized size = 0.63

$$-\frac{\left(b\,x^2+a\right)\left(35b^2x^4+28ab\,x^2+8a^2\right)x^{\frac{9}{2}}}{105\left(b\,x^3+ax\right)^{\frac{9}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/105*(b*x^2+a)*(35*b^2*x^4+28*a*b*x^2+8*a^2)*x^(9/2)/b^3/(b*x^3+a*x)^(9/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{19}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(19/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(19/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{19/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(19/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(19/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(19/2)/(b*x**3+a*x)**(9/2),x)
[Out] Timed out
```

3.81 $\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=51

$$\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] 1/7*x^(17/2)/a/(b*x^3+a*x)^(7/2)+2/35*x^(15/2)/a^2/(b*x^3+a*x)^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(17/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(17/2)/(7*a*(a*x + b*x^3)^(7/2)) + (2*x^(15/2))/(35*a^2*(a*x + b*x^3)^(5/
2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{17/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\ &= \frac{x^{17/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{2x^{15/2}}{35a^2\left(ax+bx^3\right)^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.86

$$\frac{x^{9/2}\sqrt{x\left(a+bx^2\right)}\left(7a+2bx^2\right)}{35a^2\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(17/2)/(a*x + b*x^3)^(9/2),x]
[Out] (x^(9/2)*Sqrt[x*(a + b*x^2)]*(7*a + 2*b*x^2))/(35*a^2*(a + b*x^2)^4)
```

fricas [A] time = 0.67, size = 76, normalized size = 1.49

$$\frac{\left(2 \, b x^{6} + 7 \, a x^{4}\right) \sqrt{b x^{3} + a x} \sqrt{x}}{35 \, \left(a^{2} b^{4} x^{8} + 4 \, a^{3} b^{3} x^{6} + 6 \, a^{4} b^{2} x^{4} + 4 \, a^{5} b x^{2} + a^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] 1/35*(2*b*x^6 + 7*a*x^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^2*b^4*x^8 + 4*a^3*b^3
*x^6 + 6*a^4*b^2*x^4 + 4*a^5*b*x^2 + a^6)
```

giac [A] time = 0.29, size = 29, normalized size = 0.57

$$\frac{x^{5} \left(\frac{2 \, b x^{2}}{a^{2}} + \frac{7}{a}\right)}{35 \, \left(b x^{2} + a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] 1/35*x^5*(2*b*x^2/a^2 + 7/a)/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.06, size = 37, normalized size = 0.73

$$\frac{\left(b \,x^{2}+a \right) \left(2 b \,x^{2}+7 a \right) x^{\frac{19}{2}}}{35 \left(b \,x^{3}+a x \right)^{\frac{9}{2}} a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(b*x^3+a*x)^(9/2),x)
[Out] 1/35*(b*x^2+a)*x^(19/2)*(2*b*x^2+7*a)/a^2/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{17}{2}}}{\left(b x^{3} + a x\right)^{\frac{9}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(17/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(17/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{17/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(17/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(17/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(17/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.82 $\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=51

$$-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -1/7*x^(11/2)/b/(b*x^3+a*x)^(7/2)-2/35*x^(5/2)/b^2/(b*x^3+a*x)^(5/2)
```

Rubi [A] time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}-\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -x^(11/2)/(7*b*(a*x + b*x^3)^(7/2)) - (2*x^(5/2))/(35*b^2*(a*x + b*x^3)^(5/
2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{15/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= -\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}+\frac{2\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7b}\\
&= -\frac{x^{11/2}}{7b\left(ax+bx^3\right)^{7/2}}-\frac{2x^{5/2}}{35b^2\left(ax+bx^3\right)^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.86

$$-\frac{\sqrt{x}\left(2a+7bx^2\right)}{35b^2\left(a+bx^2\right)^3\sqrt{x\left(a+bx^2\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(15/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/35*(Sqrt[x]*(2*a + 7*b*x^2))/(b^2*(a + b*x^2)^3*Sqrt[x*(a + b*x^2)])
```

fricas [A] time = 0.61, size = 75, normalized size = 1.47

$$-\frac{\sqrt{bx^3+ax}\left(7\,bx^2+2\,a\right)\sqrt{x}}{35\left(b^6x^9+4\,ab^5x^7+6\,a^2b^4x^5+4\,a^3b^3x^3+a^4b^2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*sqrt(b*x^3 + a*x)*(7*b*x^2 + 2*a)*sqrt(x)/(b^6*x^9 + 4*a*b^5*x^7 + 6*
a^2*b^4*x^5 + 4*a^3*b^3*x^3 + a^4*b^2*x)
```

giac [A] time = 0.30, size = 33, normalized size = 0.65

$$-\frac{7\,bx^2+2\,a}{35\left(bx^2+a\right)^{\frac{7}{2}}b^2}+\frac{2}{35\,a^{\frac{5}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*(7*b*x^2 + 2*a)/((b*x^2 + a)^(7/2)*b^2) + 2/35/(a^(5/2)*b^2)
```

maple [A] time = 0.06, size = 37, normalized size = 0.73

$$-\frac{\left(b\,x^2+a\right)\left(7b\,x^2+2a\right)x^{\frac{9}{2}}}{35\left(b\,x^3+ax\right)^{\frac{9}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*(b*x^2+a)*(7*b*x^2+2*a)*x^(9/2)/b^2/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{15}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(15/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(15/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{x^{15/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(15/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(15/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(15/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.83 $\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=76

$$\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] 1/7*x^(13/2)/a/(b*x^3+a*x)^(7/2)+4/35*x^(11/2)/a^2/(b*x^3+a*x)^(5/2)+8/105*
x^(9/2)/a^3/(b*x^3+a*x)^(3/2)
```

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}+\frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(13/2)/(7*a*(a*x + b*x^3)^(7/2)) + (4*x^(11/2))/(35*a^2*(a*x + b*x^3)^(5/
2)) + (8*x^(9/2))/(105*a^3*(a*x + b*x^3)^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{13/2}}{\left(ax+bx^3\right)^{9/2}}\,dx &= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}\\
&= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}\\
&= \frac{x^{13/2}}{7a\left(ax+bx^3\right)^{7/2}}+\frac{4x^{11/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{9/2}}{105a^3\left(ax+bx^3\right)^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.72

$$\frac{x^{5/2}\sqrt{x\left(a+bx^2\right)}\left(35a^2+28abx^2+8b^2x^4\right)}{105a^3\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(5/2)*Sqrt[x*(a + b*x^2)]*(35*a^2 + 28*a*b*x^2 + 8*b^2*x^4))/(105*a^3*(a
 + b*x^2)^4)
```

fricas [A] time = 0.65, size = 87, normalized size = 1.14

$$\frac{\left(8\,b^2x^6+28\,abx^4+35\,a^2x^2\right)\sqrt{bx^3+ax}\,\sqrt{x}}{105\left(a^3b^4x^8+4\,a^4b^3x^6+6\,a^5b^2x^4+4\,a^6bx^2+a^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*b^2*x^6 + 28*a*b*x^4 + 35*a^2*x^2)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^3*
b^4*x^8 + 4*a^4*b^3*x^6 + 6*a^5*b^2*x^4 + 4*a^6*b*x^2 + a^7)
```

giac [A] time = 0.28, size = 43, normalized size = 0.57

$$\frac{\left(4\,x^2\left(\frac{2\,b^2x^2}{a^3}+\frac{7\,b}{a^2}\right)+\frac{35}{a}\right)x^3}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(4*x^2*(2*b^2*x^2/a^3 + 7*b/a^2) + 35/a)*x^3/(b*x^2 + a)^(7/2)
```

maple [A] time = 0.05, size = 48, normalized size = 0.63

$$\frac{\left(b\,x^2+a\right)\left(8b^2x^4+28ab\,x^2+35a^2\right)x^{\frac{15}{2}}}{105\left(b\,x^3+ax\right)^{\frac{9}{2}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/105*(b*x^2+a)*x^(15/2)*(8*b^2*x^4+28*a*b*x^2+35*a^2)/a^3/(b*x^3+a*x)^(9/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{13}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(13/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{13/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(13/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.84 $\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{9/2}} \, dx$

Optimal. Leaf size=25

$$-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

```
[Out] -1/7*x^(7/2)/b/(b*x^3+a*x)^(7/2)
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/(a*x + b*x^3)^(9/2),x]
[Out] -x^(7/2)/(7*b*(a*x + b*x^3)^(7/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{11/2}}{\left(ax+bx^3\right)^{9/2}} \, dx = -\frac{x^{7/2}}{7b\left(ax+bx^3\right)^{7/2}}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$-\frac{x^{7/2}}{7b\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/(a*x + b*x^3)^(9/2),x]
[Out] -1/7*x^(7/2)/(b*(x*(a + b*x^2))^(7/2))
```

fricas [B] time = 0.43, size = 63, normalized size = 2.52

$$-\frac{\sqrt{bx^3+ax}\,\sqrt{x}}{7\left(b^5x^9+4\,ab^4x^7+6\,a^2b^3x^5+4\,a^3b^2x^3+a^4bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
[Out] -1/7*sqrt(b*x^3 + a*x)*sqrt(x)/(b^5*x^9 + 4*a*b^4*x^7 + 6*a^2*b^3*x^5 + 4*a
^3*b^2*x^3 + a^4*b*x)
```

giac [A] time = 0.20, size = 23, normalized size = 0.92

$$-\frac{1}{7\left(bx^2+a\right)^{\frac{7}{2}}b}+\frac{1}{7\,a^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
[Out] -1/7/((b*x^2 + a)^(7/2)*b) + 1/7/(a^(7/2)*b)
```

maple [A] time = 0.04, size = 27, normalized size = 1.08

$$-\frac{\left(b\,x^2+a\right)x^{\frac{9}{2}}}{7\left(b\,x^3+ax\right)^{\frac{9}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^3+a*x)^(9/2),x)
[Out] -1/7*(b*x^2+a)/b*x^(9/2)/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
[Out] integrate(x^(11/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^{11/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a*x + b*x^3)^(9/2),x)
[Out] int(x^(11/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**3+a*x)**(9/2),x)
[Out] Integral(x**(11/2)/(x*(a + b*x**2))**(9/2), x)
```

3.85 $\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=101

$$\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] 1/7*x^(9/2)/a/(b*x^3+a*x)^(7/2)+6/35*x^(7/2)/a^2/(b*x^3+a*x)^(5/2)+8/35*x^(
5/2)/a^3/(b*x^3+a*x)^(3/2)+16/35*x^(3/2)/a^4/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}+\frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(9/2)/(7*a*(a*x + b*x^3)^(7/2)) + (6*x^(7/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (8*x^(5/2))/(35*a^3*(a*x + b*x^3)^(3/2)) + (16*x^(3/2))/(35*a^4*Sqrt[a*
x + b*x^3])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{9/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{6\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{24\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}$$

$$= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}} + \frac{16\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{3/2}}\,dx}{35a^3}$$

$$= \frac{x^{9/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{6x^{7/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{8x^{5/2}}{35a^3\left(ax+bx^3\right)^{3/2}} + \frac{16x^{3/2}}{35a^4\sqrt{ax+bx^3}}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.65

$$\frac{\sqrt{x}\sqrt{x\left(a+bx^2\right)}\left(35a^3+70a^2bx^2+56ab^2x^4+16b^3x^6\right)}{35a^4\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[x*(a + b*x^2)]*(35*a^3 + 70*a^2*b*x^2 + 56*a*b^2*x^4 + 16*b^3
*x^6))/(35*a^4*(a + b*x^2)^4)
```

fricas [A] time = 0.53, size = 95, normalized size = 0.94

$$\frac{\left(16\,b^3x^6+56\,ab^2x^4+70\,a^2bx^2+35\,a^3\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*b^3*x^6 + 56*a*b^2*x^4 + 70*a^2*b*x^2 + 35*a^3)*sqrt(b*x^3 + a*x)*
sqrt(x)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

giac [A] time = 0.25, size = 55, normalized size = 0.54

$$\frac{\left(2\left(4\,x^2\left(\frac{2\,b^3x^2}{a^4}+\frac{7\,b^2}{a^3}\right)+\frac{35\,b}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*b^3*x^2/a^4 + 7*b^2/a^3) + 35*b/a^2)*x^2 + 35/a)*x/(b*x^2
 + a)^(7/2)
```

maple [A] time = 0.04, size = 59, normalized size = 0.58

$$\frac{\left(b\,x^2+a\right)\left(16b^3x^6+56a\,b^2x^4+70a^2b\,x^2+35a^3\right)x^{\frac{11}{2}}}{35\left(b\,x^3+ax\right)^{\frac{9}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/35*(b*x^2+a)*x^(11/2)*(16*b^3*x^6+56*a*b^2*x^4+70*a^2*b*x^2+35*a^3)/a^4/(
b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\left(bx^3 + ax\right)^{\frac{9}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(9/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{9/2}}{\left(b\,x^3 + a\,x\right)^{9/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(9/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Timed out
```

3.86 $\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] 1/7*x^(7/2)/a/(b*x^3+a*x)^(7/2)+1/5*x^(5/2)/a^2/(b*x^3+a*x)^(5/2)+1/3*x^(3/
2)/a^3/(b*x^3+a*x)^(3/2)-arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(1/2))/a^(9/2)
+x^(1/2)/a^4/(b*x^3+a*x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2023, 2029, 206}

$$\frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}}+\frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{a^{9/2}}+\frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(7/2)/(7*a*(a*x + b*x^3)^(7/2)) + x^(5/2)/(5*a^2*(a*x + b*x^3)^(5/2)) + x
^(3/2)/(3*a^3*(a*x + b*x^3)^(3/2)) + Sqrt[x]/(a^4*Sqrt[a*x + b*x^3]) - ArcT
anh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]]/a^(9/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{7/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{a}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{5/2}}\,dx}{a^2}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{3/2}}\,dx}{a^3}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} + \frac{\int \frac{1}{\sqrt{x}\sqrt{ax+bx}}}{a^4}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\text{Subst}\left(\int \frac{}{1-}\right.}{}$$

$$= \frac{x^{7/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{x^{5/2}}{5a^2\left(ax+bx^3\right)^{5/2}} + \frac{x^{3/2}}{3a^3\left(ax+bx^3\right)^{3/2}} + \frac{\sqrt{x}}{a^4\sqrt{ax+bx^3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{a}}\right.}{a^{9/2}}$$

Mathematica [C] time = 0.01, size = 43, normalized size = 0.33

$$\frac{x^{7/2}\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (x^(7/2)*Hypergeometric2F1[-7/2, 1, -5/2, 1 + (b*x^2)/a])/(7*a*(x*(a + b*x^
2))^(7/2))
```

fricas [A] time = 0.50, size = 360, normalized size = 2.77

$$\left[\frac{105\left(b^4x^9+4\,ab^3x^7+6\,a^2b^2x^5+4\,a^3bx^3+a^4x\right)\sqrt{a}\,\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}\,\sqrt{a}\,\sqrt{x}}{x^3}\right)+2\left(105\,ab^3x^6+350\,a^2b^2x^4\right.}{210\left(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*x)*s
qrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3) + 2*(
105*a*b^3*x^6 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x
)*sqrt(x))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9
*x), 1/105*(105*(b^4*x^9 + 4*a*b^3*x^7 + 6*a^2*b^2*x^5 + 4*a^3*b*x^3 + a^4*
x)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (105*a*b^3*x^6
 + 350*a^2*b^2*x^4 + 406*a^3*b*x^2 + 176*a^4)*sqrt(b*x^3 + a*x)*sqrt(x))/(a
^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

giac [A] time = 0.27, size = 114, normalized size = 0.88

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}-\frac{105\,\sqrt{a}\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+176\,\sqrt{-a}}{105\,\sqrt{-a}\,a^{\frac{9}{2}}}+\frac{105\,\left(bx^2+a\right)^3+35\,\left(bx^2+a\right)^2a+21\,\left(bx^2+a\right)a^2+15\,a^3}{105\,\left(bx^2+a\right)^{\frac{7}{2}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) - 1/105*(105*sqrt(a)*arctan
(sqrt(a)/sqrt(-a)) + 176*sqrt(-a))/(sqrt(-a)*a^(9/2)) + 1/105*(105*(b*x^2 +
 a)^3 + 35*(b*x^2 + a)^2*a + 21*(b*x^2 + a)*a^2 + 15*a^3)/((b*x^2 + a)^(7/2
)*a^4)
```

maple [B] time = 0.05, size = 217, normalized size = 1.67

$$-\frac{\sqrt{\left(b\,x^2+a\right)x}\,\left(105\sqrt{b\,x^2+a}\;b^3x^6\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)-105\sqrt{a}\;b^3x^6+315\sqrt{b\,x^2+a}\;a\,b^2x^4\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)\right.}{105\left(b\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/105*((b*x^2+a)*x)^(1/2)/a^(9/2)*(105*ln(2*((b*x^2+a)^(1/2)*a^(1/2)+a)/x)
*x^6*b^3*(b*x^2+a)^(1/2)-105*a^(1/2)*x^6*b^3+315*ln(2*((b*x^2+a)^(1/2)*a^(1
/2)+a)/x)*x^4*a*b^2*(b*x^2+a)^(1/2)-350*a^(3/2)*x^4*b^2+315*ln(2*((b*x^2+a)
^(1/2)*a^(1/2)+a)/x)*x^2*a^2*b*(b*x^2+a)^(1/2)-406*a^(5/2)*x^2*b+105*ln(2*(
(b*x^2+a)^(1/2)*a^(1/2)+a)/x)*a^3*(b*x^2+a)^(1/2)-176*a^(7/2))/x^(1/2)/(b*x
^2+a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{7}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{7/2}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(7/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{7}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(7/2)/(x*(a + b*x**2))**(9/2), x)
```

3.87 $\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}+\frac{64}{35a^4\sqrt{x}\,\sqrt{ax+bx^3}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] 1/7*x^(5/2)/a/(b*x^3+a*x)^(7/2)+8/35*x^(3/2)/a^2/(b*x^3+a*x)^(5/2)+16/35*x^
(1/2)/a^3/(b*x^3+a*x)^(3/2)+64/35/a^4/x^(1/2)/(b*x^3+a*x)^(1/2)-128/35*(b*x
^3+a*x)^(1/2)/a^5/x^(3/2)
```

Rubi [A] time = 0.19, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 2014}

$$\frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}}+\frac{64}{35a^4\sqrt{x}\,\sqrt{ax+bx^3}}-\frac{128\sqrt{ax+bx^3}}{35a^5x^{3/2}}+\frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(5/2)/(7*a*(a*x + b*x^3)^(7/2)) + (8*x^(3/2))/(35*a^2*(a*x + b*x^3)^(5/2)
) + (16*Sqrt[x])/(35*a^3*(a*x + b*x^3)^(3/2)) + 64/(35*a^4*Sqrt[x]*Sqrt[a*x
 + b*x^3]) - (128*Sqrt[a*x + b*x^3])/(35*a^5*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{8\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{48\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}$$

$$= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}} + \frac{64\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{3/2}}\,dx}{35a^3}$$

$$= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}} + 12\cdots$$

$$= \frac{x^{5/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{8x^{3/2}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{16\sqrt{x}}{35a^3\left(ax+bx^3\right)^{3/2}} + \frac{64}{35a^4\sqrt{x}\sqrt{ax+bx^3}} - 128\cdots$$

Mathematica [A] time = 0.04, size = 77, normalized size = 0.61

$$-\frac{\sqrt{x\left(a+bx^2\right)}\left(35a^4+280a^3bx^2+560a^2b^2x^4+448ab^3x^6+128b^4x^8\right)}{35a^5x^{3/2}\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/35*(Sqrt[x*(a + b*x^2)]*(35*a^4 + 280*a^3*b*x^2 + 560*a^2*b^2*x^4 + 448*
a*b^3*x^6 + 128*b^4*x^8))/(a^5*x^(3/2)*(a + b*x^2)^4)
```

fricas [A] time = 0.90, size = 110, normalized size = 0.87

$$-\frac{\left(128\,b^4x^8+448\,ab^3x^6+560\,a^2b^2x^4+280\,a^3bx^2+35\,a^4\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(a^5b^4x^{10}+4\,a^6b^3x^8+6\,a^7b^2x^6+4\,a^8bx^4+a^9x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(128*b^4*x^8 + 448*a*b^3*x^6 + 560*a^2*b^2*x^4 + 280*a^3*b*x^2 + 35*a
^4)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^5*b^4*x^10 + 4*a^6*b^3*x^8 + 6*a^7*b^2*x^6
 + 4*a^8*b*x^4 + a^9*x^2)
```

giac [A] time = 0.27, size = 90, normalized size = 0.71

$$-\frac{\left(\left(x^2\left(\frac{93\,b^4x^2}{a^5}+\frac{308\,b^3}{a^4}\right)+\frac{350\,b^2}{a^3}\right)x^2+\frac{140\,b}{a^2}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((x^2*(93*b^4*x^2/a^5 + 308*b^3/a^4) + 350*b^2/a^3)*x^2 + 140*b/a^2)*
x/(b*x^2 + a)^(7/2) + 2*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

maple [A] time = 0.05, size = 70, normalized size = 0.56

$$-\frac{\left(b\,x^{2}+a\right)\left(128x^{8}b^{4}+448a\,x^{6}b^{3}+560a^{2}x^{4}b^{2}+280a^{3}x^{2}b+35a^{4}\right)x^{\frac{7}{2}}}{35\left(b\,x^{3}+ax\right)^{\frac{9}{2}}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*(b*x^2+a)*x^(7/2)*(128*b^4*x^8+448*a*b^3*x^6+560*a^2*b^2*x^4+280*a^3*
b*x^2+35*a^4)/a^5/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(bx^{3}+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\left(b\,x^{3}+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(x\left(a+bx^{2}\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(5/2)/(x*(a + b*x**2))**(9/2), x)
```

3.88 $\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=159

$$\frac{9b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}+\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

```
[Out] 1/7*x^(3/2)/a/(b*x^3+a*x)^(7/2)+9/2*b*arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(
1/2))/a^(11/2)+3/5/a^3/(b*x^3+a*x)^(3/2)/x^(1/2)+9/35*x^(1/2)/a^2/(b*x^3+a*
x)^(5/2)+3/a^4/x^(3/2)/(b*x^3+a*x)^(1/2)-9/2*(b*x^3+a*x)^(1/2)/a^5/x^(5/2)
```

Rubi [A] time = 0.24, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2023, 2025, 2029, 206}

$$\frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}}+\frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}}+\frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}}-\frac{9\sqrt{ax+bx^3}}{2a^5x^{5/2}}+\frac{9b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{2a^{11/2}}+\frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] x^(3/2)/(7*a*(a*x + b*x^3)^(7/2)) + (9*Sqrt[x])/(35*a^2*(a*x + b*x^3)^(5/2)
) + 3/(5*a^3*Sqrt[x]*(a*x + b*x^3)^(3/2)) + 3/(a^4*x^(3/2)*Sqrt[a*x + b*x^3
]) - (9*Sqrt[a*x + b*x^3])/(2*a^5*x^(5/2)) + (9*b*ArcTanh[(Sqrt[a]*Sqrt[x])
/Sqrt[a*x + b*x^3]])/(2*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{9\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{5/2}}\,dx}{5a^2}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{3/2}}\,dx}{a^3}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} + \frac{9\int \frac{}{x^5}}{}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax}}{2a^5}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax}}{2a^5}$$

$$= \frac{x^{3/2}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{9\sqrt{x}}{35a^2\left(ax+bx^3\right)^{5/2}} + \frac{3}{5a^3\sqrt{x}\left(ax+bx^3\right)^{3/2}} + \frac{3}{a^4x^{3/2}\sqrt{ax+bx^3}} - \frac{9\sqrt{ax}}{2a^5}$$

Mathematica [C] time = 0.02, size = 44, normalized size = 0.28

$$-\frac{bx^{7/2}\,{}_2F_1\left(-\frac{7}{2},2;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a^2\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(a*x + b*x^3)^(9/2),x]
```

```
[Out] -1/7*(b*x^(7/2)*Hypergeometric2F1[-7/2, 2, -5/2, 1 + (b*x^2)/a])/(a^2*(x*(a
 + b*x^2))^(7/2))
```

fricas [A] time = 0.60, size = 396, normalized size = 2.49

$$\left[\frac{315\left(b^5x^{11}+4\,ab^4x^9+6\,a^2b^3x^7+4\,a^3b^2x^5+a^4bx^3\right)\sqrt{a}\,\log\left(\frac{bx^3+2\,ax+2\,\sqrt{bx^3+ax}\,\sqrt{a}\,\sqrt{x}}{x^3}\right)-2\left(315\,ab^4x^8+1050\,a^2b^3\right.}{140\left(a^6b^4x^{11}+4\,a^7b^3x^9+6\,a^8b^2x^7+4\,a^9bx^5+a^{10}x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/140*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b
*x^3)*sqrt(a)*log((b*x^3 + 2*a*x + 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))/x^3
) - 2*(315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4 + 528*a^4*b*x^2
+ 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*
b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3), -1/70*(315*(b^5*x^11 + 4*a*b^4*x^9 + 6*a
```

```
^2*b^3*x^7 + 4*a^3*b^2*x^5 + a^4*b*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x)*s
qrt(-a)/(a*sqrt(x))) + (315*a*b^4*x^8 + 1050*a^2*b^3*x^6 + 1218*a^3*b^2*x^4
 + 528*a^4*b*x^2 + 35*a^5)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^6*b^4*x^11 + 4*a^7
*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)]
```

giac [A] time = 0.25, size = 104, normalized size = 0.65

$$-\frac{9\,b\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a^5}-\frac{\sqrt{bx^2+a}}{2\,a^5x^2}-\frac{140\left(bx^2+a\right)^3b+35\left(bx^2+a\right)^2ab+14\left(bx^2+a\right)a^2b+5\,a^3b}{35\left(bx^2+a\right)^{\frac{7}{2}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -9/2*b*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^5) - 1/2*sqrt(b*x^2 + a
)/(a^5*x^2) - 1/35*(140*(b*x^2 + a)^3*b + 35*(b*x^2 + a)^2*a*b + 14*(b*x^2
+ a)*a^2*b + 5*a^3*b)/((b*x^2 + a)^(7/2)*a^5)
```

maple [A] time = 0.06, size = 234, normalized size = 1.47

$$\frac{\sqrt{\left(b\,x^2+a\right)x}\,\left(315\sqrt{b\,x^2+a}\;b^4x^8\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)-315\sqrt{a}\;b^4x^8+945\sqrt{b\,x^2+a}\;a\,b^3x^6\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] 1/70*((b*x^2+a)*x)^(1/2)/a^(11/2)*(315*ln(2*(a+(b*x^2+a)^(1/2)*a^(1/2))/x)*
x^8*b^4*(b*x^2+a)^(1/2)-315*a^(1/2)*x^8*b^4+945*ln(2*(a+(b*x^2+a)^(1/2)*a^(
1/2))/x)*x^6*a*b^3*(b*x^2+a)^(1/2)-1050*a^(3/2)*x^6*b^3+945*ln(2*(a+(b*x^2+
a)^(1/2)*a^(1/2))/x)*x^4*a^2*b^2*(b*x^2+a)^(1/2)-1218*a^(5/2)*x^4*b^2+315*l
n(2*(a+(b*x^2+a)^(1/2)*a^(1/2))/x)*x^2*a^3*b*(b*x^2+a)^(1/2)-528*a^(7/2)*x^
2*b-35*a^(9/2))/x^(5/2)/(b*x^2+a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(bx^3+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(x**(3/2)/(x*(a + b*x**2))**(9/2), x)
```

3.89 $\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=152

$$\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{\sqrt{\cdots}}{7a\left(ax+\cdots\right.}$$

```
[Out] 16/21/a^3/x^(3/2)/(b*x^3+a*x)^(3/2)+2/7/a^2/(b*x^3+a*x)^(5/2)/x^(1/2)+1/7*x
^(1/2)/a/(b*x^3+a*x)^(7/2)+32/7/a^4/x^(5/2)/(b*x^3+a*x)^(1/2)-128/21*(b*x^3
+a*x)^(1/2)/a^5/x^(7/2)+256/21*b*(b*x^3+a*x)^(1/2)/a^6/x^(3/2)
```

Rubi [A] time = 0.23, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{256b\sqrt{ax+bx^3}}{21a^6x^{3/2}}-\frac{128\sqrt{ax+bx^3}}{21a^5x^{7/2}}+\frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}}+\frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}}+\frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}}+\frac{\sqrt{\cdots}}{7a\left(ax+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] Sqrt[x]/(7*a*(a*x + b*x^3)^(7/2)) + 2/(7*a^2*Sqrt[x]*(a*x + b*x^3)^(5/2)) +
 16/(21*a^3*x^(3/2)*(a*x + b*x^3)^(3/2)) + 32/(7*a^4*x^(5/2)*Sqrt[a*x + b*x
^3]) - (128*Sqrt[a*x + b*x^3])/(21*a^5*x^(7/2)) + (256*b*Sqrt[a*x + b*x^3])
/(21*a^6*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(ax+bx^3\right)^{9/2}}\,dx = \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{10\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{5/2}}\,dx}{7a^2}$$

$$= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32\int \frac{1}{x^{5/2}\left(ax+bx^3\right)^{3/2}}\,dx}{7a^3}$$

$$= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} + \cdots 1$$

$$= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} - \cdots 1$$

$$= \frac{\sqrt{x}}{7a\left(ax+bx^3\right)^{7/2}} + \frac{2}{7a^2\sqrt{x}\left(ax+bx^3\right)^{5/2}} + \frac{16}{21a^3x^{3/2}\left(ax+bx^3\right)^{3/2}} + \frac{32}{7a^4x^{5/2}\sqrt{ax+bx^3}} - \cdots 1$$

Mathematica [A] time = 0.05, size = 88, normalized size = 0.58

$$\frac{\sqrt{x\left(a+bx^2\right)}\left(-7a^5+70a^4bx^2+560a^3b^2x^4+1120a^2b^3x^6+896ab^4x^8+256b^5x^{10}\right)}{21a^6x^{7/2}\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(a*x + b*x^3)^(9/2),x]
```

```
[Out] (Sqrt[x*(a + b*x^2)]*(-7*a^5 + 70*a^4*b*x^2 + 560*a^3*b^2*x^4 + 1120*a^2*b^
3*x^6 + 896*a*b^4*x^8 + 256*b^5*x^10))/(21*a^6*x^(7/2)*(a + b*x^2)^4)
```

fricas [A] time = 0.90, size = 121, normalized size = 0.80

$$\frac{\left(256\,b^5x^{10}+896\,ab^4x^8+1120\,a^2b^3x^6+560\,a^3b^2x^4+70\,a^4bx^2-7\,a^5\right)\sqrt{bx^3+ax}\,\sqrt{x}}{21\left(a^6b^4x^{12}+4\,a^7b^3x^{10}+6\,a^8b^2x^8+4\,a^9bx^6+a^{10}x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/21*(256*b^5*x^10 + 896*a*b^4*x^8 + 1120*a^2*b^3*x^6 + 560*a^3*b^2*x^4 + 7
0*a^4*b*x^2 - 7*a^5)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^6*b^4*x^12 + 4*a^7*b^3*x^
10 + 6*a^8*b^2*x^8 + 4*a^9*b*x^6 + a^10*x^4)
```

giac [A] time = 0.32, size = 147, normalized size = 0.97

$$\frac{\left(\left(x^2\left(\frac{158\,b^5x^2}{a^6}+\frac{511\,b^4}{a^5}\right)+\frac{560\,b^3}{a^4}\right)x^2+\frac{210\,b^2}{a^3}\right)x}{21\left(bx^2+a\right)^{\frac{7}{2}}} - \frac{4\left(6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{3}{2}}-15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{3}{2}}+7\,a^2b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 1/21*((x^2*(158*b^5*x^2/a^6 + 511*b^4/a^5) + 560*b^3/a^4)*x^2 + 210*b^2/a^3
)*x/(b*x^2 + a)^(7/2) - 4/3*(6*(sqrt(b)*x - sqrt(b*x^2 + a))^4*b^(3/2) - 15
*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a*b^(3/2) + 7*a^2*b^(3/2))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^3*a^5)
```

maple [A] time = 0.04, size = 81, normalized size = 0.53

$$-\frac{\left(b\,x^{2}+a\right)\left(-256b^{5}x^{10}-896a\,b^{4}x^{8}-1120a^{2}b^{3}x^{6}-560a^{3}b^{2}x^{4}-70a^{4}b\,x^{2}+7a^{5}\right)x^{\frac{3}{2}}}{21\left(b\,x^{3}+ax\right)^{\frac{9}{2}}a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/21*(b*x^2+a)*x^(3/2)*(-256*b^5*x^10-896*a*b^4*x^8-1120*a^2*b^3*x^6-560*a
^3*b^2*x^4-70*a^4*b*x^2+7*a^5)/a^6/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(bx^{3}+ax\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(b*x^3 + a*x)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{{\left(b\,x^3+a\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^3)^(9/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^3)^(9/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(x\left(a+bx^{2}\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(sqrt(x)/(x*(a + b*x**2))**(9/2), x)
```

3.90 $\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=189

$$-\frac{99b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}}-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}+\frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}}+\frac{}{35a^2x^{3/2}\left(\right.}$$

```
[Out] 11/35/a^2/x^(3/2)/(b*x^3+a*x)^(5/2)+33/35/a^3/x^(5/2)/(b*x^3+a*x)^(3/2)-99/
8*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^3+a*x)^(1/2))/a^(13/2)+1/7/a/(b*x^3+a*x)
^(7/2)/x^(1/2)+33/5/a^4/x^(7/2)/(b*x^3+a*x)^(1/2)-33/4*(b*x^3+a*x)^(1/2)/a^
5/x^(9/2)+99/8*b*(b*x^3+a*x)^(1/2)/a^6/x^(5/2)
```

Rubi [A] time = 0.29, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2029, 206}

$$-\frac{99b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^3}}\right)}{8a^{13/2}}+\frac{99b\sqrt{ax+bx^3}}{8a^6x^{5/2}}-\frac{33\sqrt{ax+bx^3}}{4a^5x^{9/2}}+\frac{33}{5a^4x^{7/2}\sqrt{ax+bx^3}}+\frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}}+\frac{}{35a^2x^{3/2}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] 1/(7*a*Sqrt[x]*(a*x + b*x^3)^(7/2)) + 11/(35*a^2*x^(3/2)*(a*x + b*x^3)^(5/2
)) + 33/(35*a^3*x^(5/2)*(a*x + b*x^3)^(3/2)) + 33/(5*a^4*x^(7/2)*Sqrt[a*x +
 b*x^3]) - (33*Sqrt[a*x + b*x^3])/(4*a^5*x^(9/2)) + (99*b*Sqrt[a*x + b*x^3]
)/(8*a^6*x^(5/2)) - (99*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^3]])/(
8*a^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
```

```
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{99\int \frac{1}{x^{5/2}\left(ax+bx^3\right)^{5/2}}\,dx}{35a^2}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33\int \frac{}{x^{7/2}(a}}{5}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{a}}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{a}}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{a}}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{a}}$$

$$= \frac{1}{7a\sqrt{x}\left(ax+bx^3\right)^{7/2}} + \frac{11}{35a^2x^{3/2}\left(ax+bx^3\right)^{5/2}} + \frac{33}{35a^3x^{5/2}\left(ax+bx^3\right)^{3/2}} + \frac{33}{5a^4x^{7/2}\sqrt{a}}$$

Mathematica [C] time = 0.03, size = 46, normalized size = 0.24

$$\frac{b^2x^{7/2}\,{}_2F_1\left(-\frac{7}{2},3;-\frac{5}{2};\frac{bx^2}{a}+1\right)}{7a^3\left(x\left(a+bx^2\right)\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] (b^2*x^(7/2)*Hypergeometric2F1[-7/2, 3, -5/2, 1 + (b*x^2)/a])/(7*a^3*(x*(a
+ b*x^2))^(7/2))
```

fricas [A] time = 0.69, size = 422, normalized size = 2.23

$$\left[\frac{3465\left(b^6x^{13}+4\,ab^5x^{11}+6\,a^2b^4x^9+4\,a^3b^3x^7+a^4b^2x^5\right)\sqrt{a}\,\log\left(\frac{bx^3+2\,ax-2\,\sqrt{bx^3+ax}\,\sqrt{a}\,\sqrt{x}}{x^3}\right)+2\left(3465\,ab^5x^{10}+1\right.}{560\left(a^7b^4x^{13}+4\,a^8b^3x^{11}+6\,a^9b^2x^9+4\,a^{10}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/560*(3465*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4
*b^2*x^5)*sqrt(a)*log((b*x^3 + 2*a*x - 2*sqrt(b*x^3 + a*x)*sqrt(a)*sqrt(x))
/x^3) + 2*(3465*a*b^5*x^10 + 11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a
^4*b^2*x^4 + 385*a^5*b*x^2 - 70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^
13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5), 1/280*(3465
*(b^6*x^13 + 4*a*b^5*x^11 + 6*a^2*b^4*x^9 + 4*a^3*b^3*x^7 + a^4*b^2*x^5)*sq
rt(-a)*arctan(sqrt(b*x^3 + a*x)*sqrt(-a)/(a*sqrt(x))) + (3465*a*b^5*x^10 +
11550*a^2*b^4*x^8 + 13398*a^3*b^3*x^6 + 5808*a^4*b^2*x^4 + 385*a^5*b*x^2 -
70*a^6)*sqrt(b*x^3 + a*x)*sqrt(x))/(a^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b
^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)]
```

giac [A] time = 0.27, size = 138, normalized size = 0.73

$$\frac{99\,b^2\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^6}+\frac{350\,\left(bx^2+a\right)^3b^2+70\,\left(bx^2+a\right)^2ab^2+21\,\left(bx^2+a\right)a^2b^2+5\,a^3b^2}{35\,\left(bx^2+a\right)^{\frac{7}{2}}a^6}+\frac{19\,\left(bx^2+a\right)^{\frac{3}{2}}b^2-21\,\sqrt{}}{8\,a^6b^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] 99/8*b^2*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^6) + 1/35*(350*(b*x^2
 + a)^3*b^2 + 70*(b*x^2 + a)^2*a*b^2 + 21*(b*x^2 + a)*a^2*b^2 + 5*a^3*b^2)/
((b*x^2 + a)^(7/2)*a^6) + 1/8*(19*(b*x^2 + a)^(3/2)*b^2 - 21*sqrt(b*x^2 + a
)*a*b^2)/(a^6*b^2*x^4)
```

maple [A] time = 0.06, size = 247, normalized size = 1.31

$$-\frac{\sqrt{\left(b\,x^2+a\right)x}\,\left(3465\sqrt{b\,x^2+a}\;b^5x^{10}\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\;\sqrt{a}}{x}\right)-3465\sqrt{a}\;b^5x^{10}+10395\sqrt{b\,x^2+a}\;a\,b^4x^8\ln\left(\frac{2a+2\sqrt{b\,x^2}}{x}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/280*((b*x^2+a)*x)^(1/2)/a^(13/2)*(3465*ln(2*(a+(b*x^2+a)^(1/2)*a^(1/2))/
x)*x^10*b^5*(b*x^2+a)^(1/2)-3465*a^(1/2)*x^10*b^5+10395*ln(2*(a+(b*x^2+a)^(
1/2)*a^(1/2))/x)*x^8*a*b^4*(b*x^2+a)^(1/2)-11550*a^(3/2)*x^8*b^4+10395*ln(2
*(a+(b*x^2+a)^(1/2)*a^(1/2))/x)*x^6*a^2*b^3*(b*x^2+a)^(1/2)-13398*a^(5/2)*x
^6*b^3+3465*ln(2*(a+(b*x^2+a)^(1/2)*a^(1/2))/x)*x^4*a^3*b^2*(b*x^2+a)^(1/2)
-5808*a^(7/2)*x^4*b^2-385*a^(9/2)*x^2*b+70*a^(11/2))/x^(9/2)/(b*x^2+a)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax\right)^{\frac{9}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(9/2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x}\,\left(b\,x^3+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x + b*x^3)^(9/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\left(x\left(a+bx^2\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(x*(a + b*x**2))**(9/2)), x)
```

3.91 $\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx$

Optimal. Leaf size=180

$$-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}+\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{12}{35a^2x^{5/2}\left(ax+\right.}$$

```
[Out] 1/7/a/x^(3/2)/(b*x^3+a*x)^(7/2)+12/35/a^2/x^(5/2)/(b*x^3+a*x)^(5/2)+8/7/a^3
/x^(7/2)/(b*x^3+a*x)^(3/2)+64/7/a^4/x^(9/2)/(b*x^3+a*x)^(1/2)-384/35*(b*x^3
+a*x)^(1/2)/a^5/x^(11/2)+512/35*b*(b*x^3+a*x)^(1/2)/a^6/x^(7/2)-1024/35*b^2
*(b*x^3+a*x)^(1/2)/a^7/x^(3/2)
```

Rubi [A] time = 0.28, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$-\frac{1024b^2\sqrt{ax+bx^3}}{35a^7x^{3/2}}+\frac{512b\sqrt{ax+bx^3}}{35a^6x^{7/2}}-\frac{384\sqrt{ax+bx^3}}{35a^5x^{11/2}}+\frac{64}{7a^4x^{9/2}\sqrt{ax+bx^3}}+\frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}}+\frac{12}{35a^2x^{5/2}\left(ax+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] 1/(7*a*x^(3/2)*(a*x + b*x^3)^(7/2)) + 12/(35*a^2*x^(5/2)*(a*x + b*x^3)^(5/2
)) + 8/(7*a^3*x^(7/2)*(a*x + b*x^3)^(3/2)) + 64/(7*a^4*x^(9/2)*Sqrt[a*x + b
*x^3]) - (384*Sqrt[a*x + b*x^3])/(35*a^5*x^(11/2)) + (512*b*Sqrt[a*x + b*x^
3])/(35*a^6*x^(7/2)) - (1024*b^2*Sqrt[a*x + b*x^3])/(35*a^7*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(ax+bx^3\right)^{9/2}}\,dx = \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12\int \frac{1}{x^{5/2}\left(ax+bx^3\right)^{7/2}}\,dx}{7a}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{24\int \frac{1}{x^{7/2}\left(ax+bx^3\right)^{5/2}}\,dx}{7a^2}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64\int \frac{1}{x^{9/2}\left(ax\cdots\right.}}{7a\cdots}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax\cdots}}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax\cdots}}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax\cdots}}$$

$$= \frac{1}{7ax^{3/2}\left(ax+bx^3\right)^{7/2}} + \frac{12}{35a^2x^{5/2}\left(ax+bx^3\right)^{5/2}} + \frac{8}{7a^3x^{7/2}\left(ax+bx^3\right)^{3/2}} + \frac{64}{7a^4x^{9/2}\sqrt{ax\cdots}}$$

Mathematica [A] time = 0.04, size = 99, normalized size = 0.55

$$-\frac{\sqrt{x\left(a+bx^2\right)}\left(7a^6-28a^5bx^2+280a^4b^2x^4+2240a^3b^3x^6+4480a^2b^4x^8+3584ab^5x^{10}+1024b^6x^{12}\right)}{35a^7x^{11/2}\left(a+bx^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x]
```

```
[Out] -1/35*(Sqrt[x*(a + b*x^2)]*(7*a^6 - 28*a^5*b*x^2 + 280*a^4*b^2*x^4 + 2240*a
^3*b^3*x^6 + 4480*a^2*b^4*x^8 + 3584*a*b^5*x^10 + 1024*b^6*x^12))/(a^7*x^(1
1/2)*(a + b*x^2)^4)
```

fricas [A] time = 1.11, size = 132, normalized size = 0.73

$$-\frac{\left(1024\,b^6x^{12}+3584\,ab^5x^{10}+4480\,a^2b^4x^8+2240\,a^3b^3x^6+280\,a^4b^2x^4-28\,a^5bx^2+7\,a^6\right)\sqrt{bx^3+ax}\,\sqrt{x}}{35\left(a^7b^4x^{14}+4\,a^8b^3x^{12}+6\,a^9b^2x^{10}+4\,a^{10}bx^8+a^{11}x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/35*(1024*b^6*x^12 + 3584*a*b^5*x^10 + 4480*a^2*b^4*x^8 + 2240*a^3*b^3*x^
6 + 280*a^4*b^2*x^4 - 28*a^5*b*x^2 + 7*a^6)*sqrt(b*x^3 + a*x)*sqrt(x)/(a^7*
b^4*x^14 + 4*a^8*b^3*x^12 + 6*a^9*b^2*x^10 + 4*a^10*b*x^8 + a^11*x^6)
```

giac [A] time = 0.41, size = 202, normalized size = 1.12

$$-\frac{\left(\left(2\,x^2\left(\frac{281\,b^6x^2}{a^7}+\frac{896\,b^5}{a^6}\right)+\frac{1925\,b^4}{a^5}\right)x^2+\frac{700\,b^3}{a^4}\right)x}{35\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{4\left(25\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8b^{\frac{5}{2}}-120\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6ab^{\frac{5}{2}}+\cdots\right.}{5\left(\left(\sqrt{b}\,x\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="giac")
```

```
[Out] -1/35*((2*x^2*(281*b^6*x^2/a^7 + 896*b^5/a^6) + 1925*b^4/a^5)*x^2 + 700*b^3
/a^4)*x/(b*x^2 + a)^(7/2) + 4/5*(25*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2)
 - 120*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(5/2) + 210*(sqrt(b)*x - sqrt(b*
x^2 + a))^4*a^2*b^(5/2) - 140*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/2) +
 33*a^4*b^(5/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

maple [A] time = 0.05, size = 92, normalized size = 0.51

$$-\frac{\left(b\,x^{2}+a\right)\left(1024 b^{6} x^{12}+3584 b^{5} x^{10} a+4480 x^{8} b^{4} a^{2}+2240 b^{3} x^{6} a^{3}+280 b^{2} x^{4} a^{4}-28 b\,x^{2} a^{5}+7 a^{6}\right)}{35\left(b\,x^{3}+a x\right)^{\frac{9}{2}} a^{7} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^3+a*x)^(9/2),x)
```

```
[Out] -1/35*(b*x^2+a)*(1024*b^6*x^12+3584*a*b^5*x^10+4480*a^2*b^4*x^8+2240*a^3*b^
3*x^6+280*a^4*b^2*x^4-28*a^5*b*x^2+7*a^6)/x^(1/2)/a^7/(b*x^3+a*x)^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{3}+ax\right)^{\frac{9}{2}} x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x)^(9/2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\left(b\,x^{3}+a\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x + b*x^3)^(9/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\left(x\left(a+bx^{2}\right)\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**3+a*x)**(9/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(x*(a + b*x**2))**(9/2)), x)
```

3.92 $\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=55

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

```
[Out] -1/3*a*arctanh(x^2*b^(1/2)/(b*x^4+a*x)^(1/2))/b^(3/2)+1/3*x*(b*x^4+a*x)^(1/
2)/b
```

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2024, 2029, 206}

$$\frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*Sqrt[a*x + b*x^4])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/
(3*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{ax+bx^4}}\,dx &= \frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\int \frac{x}{\sqrt{ax+bx^4}}\,dx}{2b}\\
&= \frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^2}{\sqrt{ax+bx^4}}\right)}{3b}\\
&= \frac{x\sqrt{ax+bx^4}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax+bx^4}}\right)}{3b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.47

$$\frac{\sqrt{b}\,x^2\left(a+bx^3\right)-a\sqrt{x}\,\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^{3/2}\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x^3) - a*Sqrt[x]*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/
2))/Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x*(a + b*x^3)])
```

fricas [A] time = 0.96, size = 133, normalized size = 2.42

$$\left[\frac{4\sqrt{bx^4+ax}\,bx+a\sqrt{b}\,\log\left(-8\,b^2x^6-8\,abx^3-a^2+4\left(2\,bx^4+ax\right)\sqrt{bx^4+ax}\,\sqrt{b}\right)}{12\,b^2},\ \frac{2\sqrt{bx^4+ax}\,bx+a\sqrt{-b}\,\text{arct}}{6\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*sqrt(b*x^4 + a*x)*b*x + a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2
 + 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)*sqrt(b)))/b^2, 1/6*(2*sqrt(b*x^4 + a
*x)*b*x + a*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b*x^3 + a)))/
b^2]
```

giac [A] time = 0.30, size = 45, normalized size = 0.82

$$\frac{\sqrt{bx^4+ax}\,x}{3\,b}+\frac{a\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\sqrt{-b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^4 + a*x)*x/b + 1/3*a*arctan(sqrt(b + a/x^3)/sqrt(-b))/(sqrt(-b
)*b)
```

maple [C] time = 0.09, size = 997, normalized size = 18.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 1/3*x*(b*x^4+a*x)^(1/2)/b-a*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*x/(-1/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b))^(1/2)*(x
-(-a*b^2)^(1/3)/b)^2*((-a*b^2)^(1/3)/b*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/
(x-(-a*b^2)^(1/3)/b))^(1/2)*((-a*b^2)^(1/3)/b*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)/(x-(-a*b^2)^(1/3)/b))^(1/2)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)/(-a*b^2)^(1/3)/(b*x*(x-(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b))^(1/2)*((-a*b^2)^(1/3)/b*EllipticF(((-3/2*(-a*b^2)^(1/3)/
```

```
b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*x/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b))^(1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)/(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3
)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2))-(-a*b^2)^(1/3)/b*EllipticPi(((-
3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*x/(-1/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b))^(1/2),(-1/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1/3
)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^4 + ax}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^4 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^4}{\sqrt{b\,x^4 + a\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a*x + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x\left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x*(a + b*x**3)), x)
```

3.93 $\int \frac{x}{\sqrt{ax+bx^4}} dx$

Optimal. Leaf size=32

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

```
[Out] 2/3*arctanh(x^2*b^(1/2)/(b*x^4+a*x)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2029, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x + b*x^4]])/(3*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{ax+bx^4}} dx &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x^2}{\sqrt{ax+bx^4}}\right) \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{ax+bx^4}}\right)}{3\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.91

$$\frac{2\sqrt{x}\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{b} x^{3/2}}{\sqrt{a+bx^3}}\right)}{3\sqrt{b}\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*S
qrt[b]*Sqrt[x*(a + b*x^3)])
```

fricas [A] time = 0.89, size = 94, normalized size = 2.94

$$\left[\frac{\log\left(-8\,b^{2}x^{6}-8\,abx^{3}-a^{2}-4\left(2\,bx^{4}+ax\right)\sqrt{bx^{4}+ax}\,\sqrt{b}\right)}{6\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{bx^{4}+ax}\,\sqrt{-b}\,x}{2\,bx^{3}+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^4 + a*x)*sqrt(b*x^4 + a*x)
*sqrt(b))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^4 + a*x)*sqrt(-b)*x/(2*b
*x^3 + a))/b]
```

giac [A] time = 0.21, size = 23, normalized size = 0.72

$$-\frac{2\,\arctan\left(\frac{\sqrt{b+\frac{a}{x^{3}}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b)
```

maple [C] time = 0.09, size = 979, normalized size = 30.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 2*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*((-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2)*(x-(-a*b^2)^(1/3)/b)^2*((-a*
b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/b)^(1/2
)*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(
-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/
b)^(1/2)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-a*b^2)^(1
/3)*b/((x-(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*b*x)^(1/2)
*((-a*b^2)^(1/3)/b*EllipticF(((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2
)^(1/3)/b)*x)^(1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*
(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1/3)/b
+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b))^(1/2))-(-a*b^2)^(1/3)/b*EllipticPi(((-3/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2),(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b),(
(3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(1/2*(-a*b^2)^(1/3)/b
-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^{4}+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^4 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{b\,x^4 + a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(a + b*x**3)), x)
```

3.94 $\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

[Out] -2/3*(b*x^4+a*x)^(1/2)/a/x^2

Rubi [A] time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2014}

$$-\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a*x + b*x^4]),x]

[Out] (-2*Sqrt[a*x + b*x^4])/(3*a*x^2)

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{3ax^2}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.00

$$-\frac{2\sqrt{x\left(a+bx^3\right)}}{3ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*Sqrt[a*x + b*x^4]),x]

[Out] (-2*Sqrt[x*(a + b*x^3)])/(3*a*x^2)

fricas [A] time = 0.80, size = 19, normalized size = 0.83

$$-\frac{2\sqrt{bx^4+ax}}{3\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")

[Out] -2/3*sqrt(b*x^4 + a*x)/(a*x^2)

giac [A] time = 0.22, size = 14, normalized size = 0.61

$$-\frac{2\sqrt{b+\frac{a}{x^3}}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b + a/x^3)/a
```

maple [A] time = 0.04, size = 27, normalized size = 1.17

$$-\frac{2\left(b\,x^{3}+a\right)}{3\sqrt{b\,x^{4}+ax}\;ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a*x)^(1/2),x)
```

```
[Out] -2/3/x*(b*x^3+a)/a/(b*x^4+a*x)^(1/2)
```

maxima [A] time = 1.46, size = 26, normalized size = 1.13

$$-\frac{2\left(bx^{4}+ax\right)}{3\sqrt{bx^{3}+a}\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

mupad [B] time = 5.13, size = 19, normalized size = 0.83

$$-\frac{2\sqrt{b\,x^{4}+a\,x}}{3\,a\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^4)^(1/2)),x)
```

```
[Out] -(2*(a*x + b*x^4)^(1/2))/(3*a*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{2}\sqrt{x\left(a+bx^{3}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x*(a + b*x**3))), x)
```

3.95 $\int \frac{1}{x^5 \sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=48

$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

```
[Out] -2/9*(b*x^4+a*x)^(1/2)/a/x^5+4/9*b*(b*x^4+a*x)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.07, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}-\frac{2\sqrt{ax+bx^4}}{9ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(9*a*x^5) + (4*b*Sqrt[a*x + b*x^4])/(9*a^2*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx &= -\frac{2\sqrt{ax+bx^4}}{9ax^5}-\frac{(2b)\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax+bx^4}}{9ax^5}+\frac{4b\sqrt{ax+bx^4}}{9a^2x^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.65

$$-\frac{2\left(a-2bx^3\right)\sqrt{x\left(a+bx^3\right)}}{9a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*(a - 2*b*x^3)*Sqrt[x*(a + b*x^3)])/(9*a^2*x^5)
```

fricas [A] time = 1.02, size = 29, normalized size = 0.60

$$\frac{2 \sqrt{bx^4 + ax} \left(2 bx^3 - a\right)}{9 a^2 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^4 + a*x)*(2*b*x^3 - a)/(a^2*x^5)
```

giac [A] time = 0.19, size = 30, normalized size = 0.62

$$-\frac{2 \left(b + \frac{a}{x^3}\right)^{\frac{3}{2}}}{9 a^2} + \frac{2 \sqrt{b + \frac{a}{x^3}}\, b}{3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] -2/9*(b + a/x^3)^(3/2)/a^2 + 2/3*sqrt(b + a/x^3)*b/a^2
```

maple [A] time = 0.04, size = 35, normalized size = 0.73

$$-\frac{2 \left(b x^3 + a\right) \left(-2b x^3 + a\right)}{9 \sqrt{b x^4 + ax}\, a^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(b*x^4+a*x)^(1/2),x)
[Out] -2/9*(b*x^3+a)*(-2*b*x^3+a)/x^4/a^2/(b*x^4+a*x)^(1/2)
```

maxima [A] time = 1.51, size = 38, normalized size = 0.79

$$\frac{2 \left(2 b^2 x^7 + abx^4 - a^2 x\right)}{9 \sqrt{bx^3 + a}\, a^2 x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

mupad [B] time = 5.13, size = 27, normalized size = 0.56

$$-\frac{2 \sqrt{b x^4 + a x}\, \left(a - 2 b x^3\right)}{9 a^2 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a*x + b*x^4)^(1/2)),x)
[Out] -(2*(a*x + b*x^4)^(1/2)*(a - 2*b*x^3))/(9*a^2*x^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \sqrt{x \left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(b*x**4+a*x)**(1/2),x)
[Out] Integral(1/(x**5*sqrt(x*(a + b*x**3))), x)
```

3.96 $\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=74

$$-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

```
[Out] -2/15*(b*x^4+a*x)^(1/2)/a/x^8+8/45*b*(b*x^4+a*x)^(1/2)/a^2/x^5-16/45*b^2*(b
*x^4+a*x)^(1/2)/a^3/x^2
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2016, 2014}

$$-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{2\sqrt{ax+bx^4}}{15ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(15*a*x^8) + (8*b*Sqrt[a*x + b*x^4])/(45*a^2*x^5) -
(16*b^2*Sqrt[a*x + b*x^4])/(45*a^3*x^2)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^8\sqrt{ax+bx^4}}\,dx &= -\frac{2\sqrt{ax+bx^4}}{15ax^8}-\frac{(4b)\int \frac{1}{x^5\sqrt{ax+bx^4}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}+\frac{\left(8b^2\right)\int \frac{1}{x^2\sqrt{ax+bx^4}}\,dx}{15a^2}\\
&= -\frac{2\sqrt{ax+bx^4}}{15ax^8}+\frac{8b\sqrt{ax+bx^4}}{45a^2x^5}-\frac{16b^2\sqrt{ax+bx^4}}{45a^3x^2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 0.59

$$-\frac{2\sqrt{x\left(a+bx^3\right)}\left(3a^2-4abx^3+8b^2x^6\right)}{45a^3x^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^8*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x*(a + b*x^3)]*(3*a^2 - 4*a*b*x^3 + 8*b^2*x^6))/(45*a^3*x^8)
```

fricas [A] time = 0.76, size = 40, normalized size = 0.54

$$-\frac{2\left(8\,b^2x^6-4\,abx^3+3\,a^2\right)\sqrt{bx^4+ax}}{45\,a^3x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/45*(8*b^2*x^6 - 4*a*b*x^3 + 3*a^2)*sqrt(b*x^4 + a*x)/(a^3*x^8)
```

giac [A] time = 0.20, size = 47, normalized size = 0.64

$$-\frac{2\sqrt{b+\frac{a}{x^3}}\,b^2}{3\,a^3}-\frac{2\left(3\left(b+\frac{a}{x^3}\right)^{\frac{5}{2}}-10\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}b\right)}{45\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b + a/x^3)*b^2/a^3 - 2/45*(3*(b + a/x^3)^(5/2) - 10*(b + a/x^3)^(
3/2)*b)/a^3
```

maple [A] time = 0.05, size = 48, normalized size = 0.65

$$-\frac{2\left(b\,x^3+a\right)\left(8b^2x^6-4ab\,x^3+3a^2\right)}{45\sqrt{b\,x^4+ax}\;a^3x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^8/(b*x^4+a*x)^(1/2),x)
```

```
[Out] -2/45*(b*x^3+a)*(8*b^2*x^6-4*a*b*x^3+3*a^2)/x^7/a^3/(b*x^4+a*x)^(1/2)
```

maxima [A] time = 1.49, size = 50, normalized size = 0.68

$$-\frac{2\left(8\,b^3x^{10}+4\,ab^2x^7-a^2bx^4+3\,a^3x\right)}{45\,\sqrt{bx^3+a}\,a^3x^{\frac{17}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^8/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/45*(8*b^3*x^10 + 4*a*b^2*x^7 - a^2*b*x^4 + 3*a^3*x)/(sqrt(b*x^3 + a)*a^3
*x^(17/2))
```

mupad [B] time = 5.27, size = 40, normalized size = 0.54

$$-\frac{2\,\sqrt{b\,x^4+a\,x}\,\left(3\,a^2-4\,a\,b\,x^3+8\,b^2\,x^6\right)}{45\,a^3\,x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8*(a*x + b*x^4)^(1/2)),x)
```

```
[Out] -(2*(a*x + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^6 - 4*a*b*x^3))/(45*a^3*x^8)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^8 \sqrt{x\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**8/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**8*sqrt(x*(a + b*x**3))), x)
```

3.97 $\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=224

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[Out] 1/2*(b*x^4+a*x)^(1/2)/b-1/12*a^(2/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1
/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/b/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*
x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2024, 2011, 329, 225}

$$\frac{\sqrt{ax+bx^4}}{2b}-\frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x + b*x^4],x]
```

```
[Out] Sqrt[a*x + b*x^4]/(2*b) - (a^(2/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) -
a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Ell
ipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{\sqrt{ax+bx^4}}\,dx &= \frac{\sqrt{ax+bx^4}}{2b} - \frac{a\int \frac{1}{\sqrt{ax+bx^4}}\,dx}{4b}\\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{\left(a\sqrt{x}\,\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^3}}\,dx}{4b\sqrt{ax+bx^4}}\\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{\left(a\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{2b\sqrt{ax+bx^4}}\\
&= \frac{\sqrt{ax+bx^4}}{2b} - \frac{a^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)\middle|\frac{1}{4}\left(2+\sqrt{3}\right.\right.}{4\sqrt[4]{3}\,b\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+bx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 64, normalized size = 0.29

$$\frac{x\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+a+bx^3\right)}{2b\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b
*x^3)/a)]))/(2*b*Sqrt[x*(a + b*x^3)])
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+ax}\,x^2}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a*x)*x^2/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^4+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x), x)
```

maple [C] time = 0.10, size = 688, normalized size = 3.07

$$-\frac{\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)b}}\sqrt{\cdots}\cdots}{2\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 1/2*(b*x^4+a*x)^(1/2)/b-1/2*a*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*
b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2)*
(x-(-a*b^2)^(1/3)/b)^2*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/
(x-(-a*b^2)^(1/3)/b)/b)^(1/2)*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)/(x-(-a*b^2)^(1/3)/b)/b)^(1/2)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)/(-a*b^2)^(1/3)/((x-(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)
/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)*b*x)^(1/2)*EllipticF(((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x
-(-a*b^2)^(1/3)/b)*x)^(1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^4 + ax}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{b\,x^4 + a\,x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x^3/(a*x + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x*(a + b*x**3)), x)
```

3.98 $\int \frac{1}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=197

$$\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

```
[Out] 1/3*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(
1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(1/3
)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^
(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2011, 329, 225}

$$\frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x + b*x^4],x]
```

```
[Out] (x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt
[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4]])/(3^
(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{ax+bx^4}}\,dx &= \frac{\left(\sqrt{x}\,\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^3}}\,dx}{\sqrt{ax+bx^4}} \\
&= \frac{\left(2\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^4}} \\
&= \frac{x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+bx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 49, normalized size = 0.25

$$
\frac{2x\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x + b*x^4],x]
[Out] (2*x*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)])/Sq
rt[x*(a + b*x^3)]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$
\text{integral}\left(\frac{1}{\sqrt{bx^4+ax}}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(1/sqrt(b*x^4 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\sqrt{bx^4+ax}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*x^4 + a*x), x)
```

maple [C] time = 0.09, size = 671, normalized size = 3.41

$$\frac{2\left(\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(x+\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)}{\left(-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)b}}\sqrt{\cdots}}{\left(-\frac{3\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\left(x-\frac{\left(-a\,b^{2}\right)^{\frac{1}{3}}}{b}\right)\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 2*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*((-3/2*(-a*b^2)^(1/
3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2)*(x-(-a*b^2)^(1/3)/b)^2*((-a*
b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/b)^(1/2
)*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(
-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/
b)^(1/2)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-a*b^2)^(1
/3)*b/((x-(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*b*x)^(1/2)
*EllipticF(((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-
a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2
),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(1/2*(-a*b^2)^(1/3
)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4 + ax}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^4 + a*x), x)
```

mupad [B] time = 5.16, size = 40, normalized size = 0.20

$$\frac{2\,x\,\sqrt{\frac{b\,x^3}{a}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{7}{6};\ -\frac{b\,x^3}{a}\right)}{\sqrt{b\,x^4+a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^4)^(1/2),x)
```

```
[Out] (2*x*((b*x^3)/a + 1)^(1/2)*hypergeom([1/6, 1/2], 7/6, -(b*x^3)/a))/(a*x + b
*x^4)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**4), x)
```

3.99 $\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=225

$$-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

```
[Out] -2/5*(b*x^4+a*x)^(1/2)/a/x^3-2/15*b*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)
*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/
3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/
4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(
1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2025, 2011, 329, 225}

$$-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt{ax+bx^4}}{5ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^4])/(5*a*x^3) - (2*b*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2
]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt
[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^(1/3)*x*(a^(
1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]
)
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{ax+bx^4}}\,dx &= -\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{(2b)\int \frac{1}{\sqrt{ax+bx^4}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{\left(2b\sqrt{x}\,\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\,\sqrt{a+bx^3}}\,dx}{5a\sqrt{ax+bx^4}}\\
&= -\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{\left(4b\sqrt{x}\,\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{5a\sqrt{ax+bx^4}}\\
&= -\frac{2\sqrt{ax+bx^4}}{5ax^3}-\frac{2bx\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\Big(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x}\right)|\frac{1}{4}\Big(2+}{5\sqrt[4]{3}\,a^{4/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax+bx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.24

$$-\frac{2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};-\frac{bx^3}{a}\right)}{5x^2\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(5
*x^2*Sqrt[x*(a + b*x^3)])
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+ax}}{bx^7+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a*x)/(b*x^7 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+ax}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

maple [C] time = 0.09, size = 696, normalized size = 3.09

$$-\frac{4\left(\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)\sqrt{\frac{\left(-\frac{3\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)x}{\left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)}}\left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)^{2}\sqrt{\frac{\left(-a b^{2}\right)^{\frac{1}{3}}\left(x+\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)}{\left(-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}-\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)b}}\sqrt{}}{5\left(-\frac{3\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}+\frac{i\sqrt{3}\left(-a b^{2}\right)^{\frac{1}{3}}}{2b}\right)\left(-a b^{2}\right)^{\frac{1}{3}}\sqrt{\left(x-\frac{\left(-a b^{2}\right)^{\frac{1}{3}}}{b}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a*x)^(1/2),x)
```

```
[Out] -2/5*(b*x^4+a*x)^(1/2)/a/x^3-4/5/a*b^2*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(
-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*
x)^(1/2)*(x-(-a*b^2)^(1/3)/b)^2*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/b)^(1/2)*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3
)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/b)^(1/2)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*
3^(1/2)*(-a*b^2)^(1/3)/b)/(-a*b^2)^(1/3)/((x-(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b
^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3
^(1/2)*(-a*b^2)^(1/3)/b)*b*x)^(1/2)*EllipticF(((-3/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^{4}+ax}\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3}\sqrt{b\,x^{4}+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a*x)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(x*(a + b*x**3))), x)
```

3.100 $\int \frac{x^5}{\sqrt{ax+bx^4}} \, dx$

Optimal. Leaf size=503

$$\frac{5\left(1-\sqrt{3}\right)a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}+\frac{5\sqrt[4]{3}\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

```
[Out] -5/8*a*x*(b*x^3+a)*(1+3^(1/2))/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x
^4+a*x)^(1/2)+1/4*x^2*(b*x^4+a*x)^(1/2)/b+5/8*3^(1/4)*a^(4/3)*x*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*E
llipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^
2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*x^4+a*x)^(1/2)/(b^(1
/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+5/48*a^(
4/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3
)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3
)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3
^(3/4)/b^(5/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2024, 2032, 329, 308, 225, 1881}

$$\frac{5\left(1-\sqrt{3}\right)a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax+bx^4}}+\frac{5\sqrt[4]{3}\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a*x + b*x^4],x]
```

```
[Out] (-5*(1 + Sqrt[3])*a*x*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1
/3)*x)*Sqrt[a*x + b*x^4]) + (x^2*Sqrt[a*x + b*x^4])/(4*b) + (5*3^(1/4)*a^(4
/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - S
qrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/
(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) + (5*(1 - Sqrt[3])*a^(4/3)*x*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/
(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(16*3^(1/4)*b^(5/3)
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a*x + b*x^4])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{ax+bx^4}}\,dx = \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{(5a)\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx}{8b}$$
$$= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{\left(5a\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{8b\sqrt{ax+bx^4}}$$
$$= \frac{x^2\sqrt{ax+bx^4}}{4b} - \frac{\left(5a\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{4b\sqrt{ax+bx^4}}$$
$$= \frac{x^2\sqrt{ax+bx^4}}{4b} + \frac{\left(5a\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax+bx^4}} + \frac{\left(5\left(1-\sqrt{3}\right)a^{5/3}\right.}{}$$
$$= -\frac{5\left(1+\sqrt{3}\right)ax\left(a+bx^3\right)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}} + \frac{x^2\sqrt{ax+bx^4}}{4b} + \frac{5\sqrt[4]{3}\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\cdots}{\left(\sqrt[3]{a}+\cdots\right.}}}{8b^{5/3}\sqrt{\cdots}}$$

Mathematica [C] time = 0.03, size = 66, normalized size = 0.13

$$\frac{x^3\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+a+bx^3\right)}{4b\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a*x + b*x^4],x]
[Out] (x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -
((b*x^3)/a)]))/(4*b*Sqrt[x*(a + b*x^3)])
```

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+ax}\,x^4}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a*x)*x^4/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{bx^4+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(x^5/sqrt(b*x^4 + a*x), x)
```

maple [C] time = 0.09, size = 1079, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^4+a*x)^(1/2),x)
```

```
[Out] 1/4*x^2*(b*x^4+a*x)^(1/2)/b-5/8*a/b*(x*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)
+(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*((-3/2*(-a*b^2)^(1/3
)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-
a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2)*(x-(-a*b^2)^(1/3)/b)^2*((-a*b
^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a
*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/b)^(1/2)
*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-
1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/b
)^(1/2)*(((-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(-a*b^2)^(
1/3)/b+(-a*b^2)^(2/3)/b^2)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1
/3)/b)/(-a*b^2)^(1/3)*b*EllipticF(((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a
*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-
a*b^2)^(1/3)/b)*x)^(1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3
)/b)*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1
/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b))^(1/2))+(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*EllipticE(((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2
*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(
1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(1/2*(-a*b^2)^(
1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*
(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1
/2))/(-a*b^2)^(1/3)*b))/((x-(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I
*3^(1/2)*(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(
1/3)/b)*b*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/sqrt(b*x^4 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{b\,x^4 + a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x + b*x^4)^(1/2),x)
```

```
[Out] int(x^5/(a*x + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{x\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(x**5/sqrt(x*(a + b*x**3)), x)
```

3.101 $\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=474

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax+bx^4}}-\frac{\sqrt[4]{3}\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{}$$

```
[Out] x*(b*x^3+a)*(1+3^(1/2))/b^(2/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^4+a*x)
^(1/2)-3^(1/4)*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)
))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((
a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(
1/2)/b^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(
1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b
^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2
)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3
)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2032, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax+bx^4}}-\frac{\sqrt[4]{3}\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x + b*x^4],x]
```

```
[Out] ((1 + Sqrt[3])*x*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)*
Sqrt[a*x + b*x^4]) - (3^(1/4)*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
 - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*
EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3
])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/
3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sq
rt[3])*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x +
b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/
3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sq
rt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/
3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x + b*x^4])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
```

```
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx = \frac{\left(\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax+bx^4}}$$

$$= \frac{\left(2\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax+bx^4}}$$

$$= -\frac{\left(\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax+bx^4}} - \frac{\left(\left(1-\sqrt{3}\right)a^{2/3}\sqrt{x}\sqrt{a+bx^3}\right)\text{Su}}{b^{2/3}\sqrt{ax+}}$$

$$= \frac{\left(1+\sqrt{3}\right)x\left(a+bx^3\right)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}} - \frac{\sqrt[4]{3}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E\left(\text{co}\right.}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}}$$

Mathematica [C] time = 0.01, size = 53, normalized size = 0.11

$$\frac{2x^3\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{5\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x + b*x^4],x]
```

```
[Out] (2*x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/a)])
/(5*Sqrt[x*(a + b*x^3)])
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+ax}\,x}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^4 + a*x)*x/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^4+ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^4 + a*x), x)
```

maple [C] time = 0.10, size = 1054, normalized size = 2.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a*x)^(1/2),x)
```

```
[Out] ((x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1
/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*x+(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)
*(-a*b^2)^(1/3)/b)*((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/
(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)
*x)^(1/2)*(x-(-a*b^2)^(1/3)/b)^2*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b+1/
2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/b)^(1/2)*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/
3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(
-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/b)^(1/2)*(((-1/2*(-a*b^2)^(1/3)/b+1/2
*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(-a*b^2)^(1/3)/b+(-a*b^2)^(2/3)/b^2)/(-3/2*(-a
*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-a*b^2)^(1/3)*b*EllipticF(((
-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/
b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2),((3/2*(-a*b
^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2))+(1/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*EllipticE(((-3/2*(-a*b^2)^(1/3)/
b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*
```

```
b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^
(1/2)*(-a*b^2)^(1/3)/b)*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)/(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3
)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b))^(1/2))/(-a*b^2)^(1/3)*b))/((x-(-a*b^2)
^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(x+1/2*(-
a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*b*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^4 + ax}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/sqrt(b*x^4 + a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{b\,x^4 + a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^4)^(1/2),x)
[Out] int(x^2/(a*x + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a*x)**(1/2),x)
[Out] Integral(x**2/sqrt(x*(a + b*x**3)), x)
```

3.102 $\int \frac{1}{x\sqrt{ax+bx^4}}\,dx$

Optimal. Leaf size=497

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt[4]{3}\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{}$$

```
[Out] 2*b^(1/3)*x*(b*x^3+a)*(1+3^(1/2))/a/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^4+
a*x)^(1/2)-2*(b*x^4+a*x)^(1/2)/a/x-2*3^(1/4)*b^(1/3)*x*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE(
(1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^
(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/3*b^(1/3)*x*(a^
(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3
^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(
1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(
2/3)/(b*x^4+a*x)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1
+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 497, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2025, 2032, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax+bx^4}}-\frac{2\sqrt[4]{3}\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x + b*x^4]),x]
```

```
[Out] (2*(1 + Sqrt[3])*b^(1/3)*x*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)*Sqrt[a*x + b*x^4]) - (2*Sqrt[a*x + b*x^4])/(a*x) - (2*3^(1/4)*b^(1/3)*x
*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^
(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3
])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(a^(2
/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*Sqrt[a*x + b*x^4]) - ((1 - Sqrt[3])*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/
3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x
 + b*x^4])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x\sqrt{ax+bx^4}}\,dx = -\frac{2\sqrt{ax+bx^4}}{ax} + \frac{(2b)\int \frac{x^2}{\sqrt{ax+bx^4}}\,dx}{a}$$

$$= -\frac{2\sqrt{ax+bx^4}}{ax} + \frac{\left(2b\sqrt{x}\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{a\sqrt{ax+bx^4}}$$

$$= -\frac{2\sqrt{ax+bx^4}}{ax} + \frac{\left(4b\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{a\sqrt{ax+bx^4}}$$

$$= -\frac{2\sqrt{ax+bx^4}}{ax} - \frac{\left(2\sqrt[3]{b}\sqrt{x}\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{a\sqrt{ax+bx^4}} - \frac{\left(2\left(1-\sqrt{3}\right)\right.}{}$$

$$= \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\left(a+bx^3\right)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax+bx^4}} - \frac{2\sqrt{ax+bx^4}}{ax} - \frac{2\sqrt[4]{3}\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right.\right.}}}{a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\right.}{\left(\sqrt[3]{a}+\left(1\right.\right.}}}$$

Mathematica [C] time = 0.01, size = 48, normalized size = 0.10

$$-\frac{2\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\frac{bx^3}{a}\right)}{\sqrt{x\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x + b*x^4]),x]
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)/a)])/Sq
rt[x*(a + b*x^3)]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^4+ax}}{bx^5+ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^4 + a*x)/(b*x^5 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+ax}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

maple [C] time = 0.09, size = 1083, normalized size = 2.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a*x)^(1/2),x)
```

```
[Out] -2*(b*x^3+a)/a/(x*(b*x^3+a))^(1/2)+2/a*b*((x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)
/b)*x+(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*((-3/2*(-a*b^2)
^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/
2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)*x)^(1/2)*(x-(-a*b^2)^(1/3)/b)^2*(
(-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(-1/
2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)/b)^
(1/2)*((-a*b^2)^(1/3)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/
b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)
/b)/b)^(1/2)*(((-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(-a*b
^2)^(1/3)/b+(-a*b^2)^(2/3)/b^2)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^
2)^(1/3)/b)/(-a*b^2)^(1/3)*b*EllipticF(((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2
)*(-a*b^2)^(1/3)/b)/(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/
(x-(-a*b^2)^(1/3)/b)*x)^(1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)
^(1/3)/b)*(1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^
2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1
/2)*(-a*b^2)^(1/3)/b))^(1/2))+(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^
(1/3)/b)*EllipticE(((-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/
(-1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(x-(-a*b^2)^(1/3)/b)
*x)^(1/2),((3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(1/2*(-a*b
^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/(1/2*(-a*b^2)^(1/3)/b+1/2*I*3^(
1/2)*(-a*b^2)^(1/3)/b)/(3/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b
))^(1/2))/(-a*b^2)^(1/3)*b))/((x-(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b+
1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b
^2)^(1/3)/b)*b*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4 + ax}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\sqrt{b\,x^4 + a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^4)^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x*(a + b*x**3))), x)
```

3.103 $\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=174

$$-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{11/2}}+\frac{63b^4\sqrt{ax+b\sqrt{x}}}{64a^5}-\frac{21b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{21b^2x\sqrt{ax+b\sqrt{x}}}{40a^3}-\frac{9bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^2}+\frac{2x}{}$$

```
[Out] -63/64*b^5*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(11/2)+63/64*b^
4*(b*x^(1/2)+a*x)^(1/2)/a^5+21/40*b^2*x*(b*x^(1/2)+a*x)^(1/2)/a^3-9/20*b*x^
(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+2/5*x^2*(b*x^(1/2)+a*x)^(1/2)/a-21/32*b^3*x
^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^4
```

Rubi [A] time = 0.15, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 670, 640, 620, 206}

$$\frac{63b^4\sqrt{ax+b\sqrt{x}}}{64a^5}-\frac{21b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{21b^2x\sqrt{ax+b\sqrt{x}}}{40a^3}-\frac{63b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{11/2}}-\frac{9bx^{3/2}\sqrt{ax+b\sqrt{x}}}{20a^2}+\frac{2x^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (63*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^5) - (21*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] +
a*x])/(32*a^4) + (21*b^2*x*Sqrt[b*Sqrt[x] + a*x])/(40*a^3) - (9*b*x^(3/2)*S
qrt[b*Sqrt[x] + a*x])/(20*a^2) + (2*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a) - (63*
b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(64*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{b\sqrt{x}+ax}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{(9b)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{5a}\\
&= -\frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} + \frac{\left(63b^2\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{40a^2}\\
&= \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} - \frac{\left(21b^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\right.}{16a^3}\\
&= -\frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} + \frac{2x^2\sqrt{b\sqrt{x}+ax}}{5a} +\\
&= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} +\\
&= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} +\\
&= \frac{63b^4\sqrt{b\sqrt{x}+ax}}{64a^5} - \frac{21b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{21b^2x\sqrt{b\sqrt{x}+ax}}{40a^3} - \frac{9bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^2} +
\end{aligned}
$$

Mathematica [A] time = 0.19, size = 151, normalized size = 0.87

$$
\frac{\left(a\sqrt{x}+b\right)\left(\sqrt{a}\sqrt{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\left(128a^4x^2-144a^3bx^{3/2}+168a^2b^2x-210ab^3\sqrt{x}+315b^4\right)-315b^{9/2}\sqrt[4]{x}\sinh^{-1}\left(\right.\right.}{320a^{11/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\sqrt{ax+b\sqrt{x}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] ((b + a*Sqrt[x])*(Sqrt[a]*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[x]*(315*b^4 - 210*a*
b^3*Sqrt[x] + 168*a^2*b^2*x - 144*a^3*b*x^(3/2) + 128*a^4*x^2) - 315*b^(9/2
)*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]]))/(320*a^(11/2)*Sqrt[1 + (a*Sq
rt[x])/b]*Sqrt[b*Sqrt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

[Out] Timed out

giac [A] time = 0.29, size = 111, normalized size = 0.64

$$\frac{1}{320}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\sqrt{x}\left(\frac{8\sqrt{x}}{a}-\frac{9b}{a^2}\right)+\frac{21b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right)\sqrt{x}+\frac{315b^4}{a^5}\right)+\frac{63b^5\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{\right.\right.\right.}{128a^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/320*sqrt(a*x + b*sqrt(x))*(2*(4*(2*sqrt(x)*(8*sqrt(x)/a - 9*b/a^2) + 21*b
^2/a^3)*sqrt(x) - 105*b^3/a^4)*sqrt(x) + 315*b^4/a^5) + 63/128*b^5*log(abs(
-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(11/2)
```

maple [A] time = 0.09, size = 223, normalized size = 1.28

$$\frac{\sqrt{ax+b\sqrt{x}}\left(-640a\,b^5\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+325a\,b^5\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-1300\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^(1/2)+a*x)^(1/2),x)
```

```
[Out] 1/640*(b*x^(1/2)+a*x)^(1/2)*(256*x*(b*x^(1/2)+a*x)^(3/2)*a^(9/2)-544*a^(7/2
)*x^(1/2)*(b*x^(1/2)+a*x)^(3/2)*b-1300*a^(5/2)*x^(1/2)*(b*x^(1/2)+a*x)^(1/2
)*b^3+880*a^(5/2)*(b*x^(1/2)+a*x)^(3/2)*b^2+1280*a^(3/2)*(x^(1/2)*(b+a*x^(1
/2)))^(1/2)*b^4-650*a^(3/2)*(b*x^(1/2)+a*x)^(1/2)*b^4+325*ln(1/2*(2*(b*x^(1
/2)+a*x)^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*a*b^5-640*a*ln(1/2*(2*(x^(1/
2)*(b+a*x^(1/2)))^(1/2)*a^(1/2)+2*a*x^(1/2)+b)/a^(1/2))*b^5)/(x^(1/2)*(b+a*
x^(1/2)))^(1/2)/a^(13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*sqrt(x)), x)
```

3.104 $\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=116

$$-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{7/2}}+\frac{5b^2\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{5b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^2}+\frac{2x\sqrt{ax+b\sqrt{x}}}{3a}$$

```
[Out] -5/4*b^3*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(7/2)+5/4*b^2*(b*
x^(1/2)+a*x)^(1/2)/a^3+2/3*x*(b*x^(1/2)+a*x)^(1/2)/a-5/6*b*x^(1/2)*(b*x^(1/
2)+a*x)^(1/2)/a^2
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2018, 670, 640, 620, 206}

$$\frac{5b^2\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{7/2}}-\frac{5b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^2}+\frac{2x\sqrt{ax+b\sqrt{x}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (5*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^3) - (5*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
/(6*a^2) + (2*x*Sqrt[b*Sqrt[x] + a*x])/(3*a) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt
[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{3a}$$

$$= -\frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} + \frac{(5b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}$$

$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x\right.}{8a^3}$$

$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{(5b^3)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, -\right.}{4a^3}$$

$$= \frac{5b^2\sqrt{b\sqrt{x}+ax}}{4a^3} - \frac{5b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^2} + \frac{2x\sqrt{b\sqrt{x}+ax}}{3a} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{4a^{7/2}}$$

Mathematica [A] time = 0.12, size = 129, normalized size = 1.11

$$\frac{5b^4\left(\frac{a\sqrt{x}}{b}+1\right)\left(\frac{16a^3x^{3/2}}{15b^3} - \frac{4a^2x}{3b^2} + \frac{2a\sqrt{x}}{b} - \frac{2\sqrt{a}\sqrt[4]{x}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{a\sqrt{x}}{b}+1}}\right)}{8a^4\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (5*b^4*(1 + (a*Sqrt[x])/b)*((2*a*Sqrt[x])/b - (4*a^2*x)/(3*b^2) + (16*a^3*x
^(3/2))/(15*b^3) - (2*Sqrt[a]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(
Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b])))/(8*a^4*Sqrt[(b + a*Sqrt[x])*Sqrt[x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.31, size = 83, normalized size = 0.72

$$\frac{1}{12}\sqrt{ax+b\sqrt{x}}\left(2\sqrt{x}\left(\frac{4\sqrt{x}}{a} - \frac{5b}{a^2}\right) + \frac{15b^2}{a^3}\right) + \frac{5b^3\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x} - \sqrt{ax+b\sqrt{x}}\right) - b\right|\right)}{8a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)*(4*sqrt(x)/a - 5*b/a^2) + 15*b^2/a^3)
 + 5/8*b^3*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b
))/a^(7/2)
```

maple [B] time = 0.05, size = 181, normalized size = 1.56

$$-\frac{\sqrt{ax+b\sqrt{x}}\left(24a\,b^{3}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-9a\,b^{3}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+36\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b\sqrt{x}-\right.}{24\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] -1/24*(a*x+b*x^(1/2))^(1/2)/a^(9/2)*(36*x^(1/2)*(a*x+b*x^(1/2))^(1/2)*a^(5/
2)*b-16*(a*x+b*x^(1/2))^(3/2)*a^(5/2)-48*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3
/2)*b^2+18*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b^2+24*a*ln(1/2*(2*a*x^(1/2)+b+2*(
(a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^3-9*ln(1/2*(2*a*x^(1/2)+b+
2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a*b^3)/((a*x^(1/2)+b)*x^(1/2))^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*x + b*sqrt(x)), x)
```

3.105 $\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=56

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

```
[Out] -2*b*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(3/2)+2*(b*x^(1/2)+a*
x)^(1/2)/a
```

Rubi [A] time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2010, 2013, 620, 206}

$$\frac{2\sqrt{ax+b\sqrt{x}}}{a}-\frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (2*Sqrt[b*Sqrt[x] + a*x])/a - (2*b*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x]
 + a*x]])/a^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt
[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n -
 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sqrt{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{b\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx}{2a}$$
$$= \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$
$$= \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{(2b)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a}$$
$$= \frac{2\sqrt{b\sqrt{x}+ax}}{a} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.05, size = 88, normalized size = 1.57

$$\frac{2\sqrt{a}\sqrt{x}\left(a\sqrt{x}+b\right)-2b^{3/2}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{a^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (2*Sqrt[a]*(b + a*Sqrt[x])*Sqrt[x] - 2*b^(3/2)*Sqrt[1 + (a*Sqrt[x])/b]*x^(1
/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(a^(3/2)*Sqrt[b*Sqrt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.28, size = 54, normalized size = 0.96

$$\frac{b\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{a^{\frac{3}{2}}}+\frac{2\sqrt{ax+b\sqrt{x}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(3/2
) + 2*sqrt(a*x + b*sqrt(x))/a
```

maple [A] time = 0.05, size = 83, normalized size = 1.48

$$\frac{\sqrt{ax+b\sqrt{x}}\left(-b\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\right)}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] (a*x+b*x^(1/2))^(1/2)*(2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)-b*ln(1/2*(2*
a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2)))/((a*x^(1/2)+
b)*x^(1/2))^(1/2)/a^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*sqrt(x)), x)
```

mupad [B] time = 5.24, size = 72, normalized size = 1.29

$$\frac{4\,x\,\left(\frac{3\,\sqrt{b}\,\sqrt{b+a\,\sqrt{x}}}{2\,a\,\sqrt{x}}+\frac{b^{3/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,x^{1/4}\,1{}\mathrm{i}}{\sqrt{b}}\right)\,3{}\mathrm{i}}{2\,a^{3/2}\,x^{3/4}}\right)\,\sqrt{\frac{a\,\sqrt{x}}{b}+1}}{3\,\sqrt{a\,x+b\,\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] (4*x*((3*b^(1/2)*(b + a*x^(1/2))^(1/2))/(2*a*x^(1/2)) + (b^(3/2)*asin((a^(1
/2)*x^(1/4)*1i)/b^(1/2))*3i)/(2*a^(3/2)*x^(3/4)))*((a*x^(1/2))/b + 1)^(1/2)
)/(3*(a*x + b*x^(1/2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*sqrt(x)), x)
```

3.106 $\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=25

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

[Out] -4*(b*x^(1/2)+a*x)^(1/2)/b/x^(1/2)

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{b\sqrt{x}}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[b*Sqrt[x] + a*x]),x]

[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(b*Sqrt[x])

fricas [A] time = 0.87, size = 19, normalized size = 0.76

$$-\frac{4\sqrt{ax+b\sqrt{x}}}{b\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")

```
[Out] -4*sqrt(a*x + b*sqrt(x))/(b*sqrt(x))
```

giac [A] time = 0.18, size = 25, normalized size = 1.00

$$\frac{4}{\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
```

maple [C] time = 0.06, size = 159, normalized size = 6.36

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(-abx\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+abx\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{3}{2}}x+}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\,b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] (a*x+b*x^(1/2))^(1/2)*(2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*x-4*(a*x+b*x
^(1/2))^(3/2)*a^(1/2)+2*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*x-ln(1/2*(2*a*x^(1/2)
+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x*a*b+ln(1/2*(2*a*x^(1
/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x*a*b)/((a*x^(1/2)+b)*x^(1/
2))^(1/2)/b^2/x/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a*x + b*sqrt(x))), x)
```

3.107 $\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=84

$$-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{15b^3\sqrt{x}}+\frac{16a\sqrt{ax+b\sqrt{x}}}{15b^2x}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

```
[Out] -4/5*(b*x^(1/2)+a*x)^(1/2)/b/x^(3/2)+16/15*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x-32
/15*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{15b^3\sqrt{x}}+\frac{16a\sqrt{ax+b\sqrt{x}}}{15b^2x}-\frac{4\sqrt{ax+b\sqrt{x}}}{5bx^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(5*b*x^(3/2)) + (16*a*Sqrt[b*Sqrt[x] + a*x])/(15
*b^2*x) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(15*b^3*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx &= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}-\frac{(4a)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{5b}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}+\frac{\left(8a^2\right)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{15b^2}\\
&= -\frac{4\sqrt{b\sqrt{x}+ax}}{5bx^{3/2}}+\frac{16a\sqrt{b\sqrt{x}+ax}}{15b^2x}-\frac{32a^2\sqrt{b\sqrt{x}+ax}}{15b^3\sqrt{x}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.57

$$-\frac{4\sqrt{ax+b\sqrt{x}}\,\left(8a^2x-4ab\sqrt{x}+3b^2\right)}{15b^3x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(3*b^2 - 4*a*b*Sqrt[x] + 8*a^2*x))/(15*b^3*x^(3/2
))
```

fricas [A] time = 1.03, size = 42, normalized size = 0.50

$$\frac{4\left(4\,abx-\left(8\,a^2x+3\,b^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{15\,b^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/15*(4*a*b*x - (8*a^2*x + 3*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^3*x^2)
```

giac [A] time = 0.23, size = 84, normalized size = 1.00

$$\frac{4\left(20\,a\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+15\,\sqrt{a}\,b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+3\,b^2\right)}{15\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/15*(20*a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 15*sqrt(a)*b*(sqrt
(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 3*b^2)/(sqrt(a)*sqrt(x) - sqrt(a*x +
 b*sqrt(x)))^5
```

maple [C] time = 0.07, size = 218, normalized size = 2.60

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(-15a^3b\,x^{\frac{7}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+15a^3b\,x^{\frac{7}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+30\sqrt{\left(a\sqrt{x}+b\right)}\right.}{15\sqrt{\left(a\sqrt{x}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/15*(a*x+b*x^(1/2))^(1/2)*(30*x^(7/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(7/2
)-60*x^(5/2)*(a*x+b*x^(1/2))^(3/2)*a^(5/2)+30*x^(7/2)*(a*x+b*x^(1/2))^(1/2)
*a^(7/2)-15*x^(7/2)*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a
^(1/2))/a^(1/2))*a^3*b+15*x^(7/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(
1/2)*a^(1/2))/a^(1/2))*a^3*b-12*x^(3/2)*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*b^2+2
8*a^(3/2)*(a*x+b*x^(1/2))^(3/2)*b*x^2)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^4/x^
(7/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{ax+b\sqrt{x}}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \sqrt{a x + b \sqrt{x}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{ax + b\sqrt{x}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*sqrt(x))), x)
```

3.108 $\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=142

$$-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{315b^5\sqrt{x}}+\frac{256a^3\sqrt{ax+b\sqrt{x}}}{315b^4x}-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{105b^3x^{3/2}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{63b^2x^2}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

```
[Out] -4/9*(b*x^(1/2)+a*x)^(1/2)/b/x^(5/2)+32/63*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^2-
64/105*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(3/2)+256/315*a^3*(b*x^(1/2)+a*x)^(1
/2)/b^4/x-512/315*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{105b^3x^{3/2}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{315b^5\sqrt{x}}+\frac{256a^3\sqrt{ax+b\sqrt{x}}}{315b^4x}+\frac{32a\sqrt{ax+b\sqrt{x}}}{63b^2x^2}-\frac{4\sqrt{ax+b\sqrt{x}}}{9bx^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(9*b*x^(5/2)) + (32*a*Sqrt[b*Sqrt[x] + a*x])/(63
*b^2*x^2) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(105*b^3*x^(3/2)) + (256*a^3*Sqr
t[b*Sqrt[x] + a*x])/(315*b^4*x) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(315*b^5*
Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} - \frac{(8a)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{9b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} + \frac{\left(16a^2\right)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{21b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} - \frac{\left(64a^3\right)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{105b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} + \frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x} + \frac{\left(128a^4\right)}{}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{9bx^{5/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{63b^2x^2} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{105b^3x^{3/2}} + \frac{256a^3\sqrt{b\sqrt{x}+ax}}{315b^4x} - \frac{512a^4\sqrt{b}}{315b}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.51

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(128a^4x^2-64a^3bx^{3/2}+48a^2b^2x-40ab^3\sqrt{x}+35b^4\right)}{315b^5x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(35*b^4 - 40*a*b^3*Sqrt[x] + 48*a^2*b^2*x - 64*a^
3*b*x^(3/2) + 128*a^4*x^2))/(315*b^5*x^(5/2))
```

fricas [A] time = 0.88, size = 64, normalized size = 0.45

$$\frac{4\left(64\,a^3bx^2+40\,ab^3x-\left(128\,a^4x^2+48\,a^2b^2x+35\,b^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{315\,b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/315*(64*a^3*b*x^2 + 40*a*b^3*x - (128*a^4*x^2 + 48*a^2*b^2*x + 35*b^4)*sq
rt(x))*sqrt(a*x + b*sqrt(x))/(b^5*x^3)
```

giac [A] time = 0.19, size = 146, normalized size = 1.03

$$\frac{4\left(1008\,a^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+1680\,a^{\frac{3}{2}}b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+1080\,ab^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+\right.}{315\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(1008*a^2*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 1680*a^(3/2)*
b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 1080*a*b^2*(sqrt(a)*sqrt(x)
 - sqrt(a*x + b*sqrt(x)))^2 + 315*sqrt(a)*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x +
 b*sqrt(x))) + 35*b^4)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^9
```

maple [C] time = 0.06, size = 262, normalized size = 1.85

$$-\frac{\sqrt{ax+b\sqrt{x}}\,\left(315a^5b\,x^{\frac{11}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-315a^5b\,x^{\frac{11}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-630\sqrt{ax+b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] -1/315*(a*x+b*x^(1/2))^(1/2)*(1260*(a*x+b*x^(1/2))^(3/2)*a^(9/2)*x^(9/2)-63
0*(a*x+b*x^(1/2))^(1/2)*a^(11/2)*x^(11/2)+315*ln(1/2*(2*a*x^(1/2)+b+2*((a*x
^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(11/2)*a^5*b-630*a^(11/2)*x^(1
1/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)-315*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/
2))^(1/2)*a^(1/2))/a^(1/2))*x^(11/2)*a^5*b+492*(a*x+b*x^(1/2))^(3/2)*a^(5/2
)*x^(7/2)*b^2+140*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*x^(5/2)*b^4-748*a^(7/2)*(a*
x+b*x^(1/2))^(3/2)*b*x^4-300*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*x^3*b^3)/((a*x^(
1/2)+b)*x^(1/2))^(1/2)/b^6/x^(11/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*sqrt(x))), x)
```

3.109 $\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=200

$$-\frac{4096a^6\sqrt{ax+b\sqrt{x}}}{3003b^7\sqrt{x}}+\frac{2048a^5\sqrt{ax+b\sqrt{x}}}{3003b^6x}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{1001b^5x^{3/2}}+\frac{1280a^3\sqrt{ax+b\sqrt{x}}}{3003b^4x^2}-\frac{160a^2\sqrt{ax+b\sqrt{x}}}{429b^3x^{5/2}}+\frac{48a\ldots}{1\ldots}$$

```
[Out] -4/13*(b*x^(1/2)+a*x)^(1/2)/b/x^(7/2)+48/143*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^
3-160/429*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(5/2)+1280/3003*a^3*(b*x^(1/2)+a*
x)^(1/2)/b^4/x^2-512/1001*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(3/2)+2048/3003*a
^5*(b*x^(1/2)+a*x)^(1/2)/b^6/x-4096/3003*a^6*(b*x^(1/2)+a*x)^(1/2)/b^7/x^(1
/2)
```

Rubi [A] time = 0.30, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2016, 2014}

$$-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{1001b^5x^{3/2}}+\frac{1280a^3\sqrt{ax+b\sqrt{x}}}{3003b^4x^2}-\frac{160a^2\sqrt{ax+b\sqrt{x}}}{429b^3x^{5/2}}-\frac{4096a^6\sqrt{ax+b\sqrt{x}}}{3003b^7\sqrt{x}}+\frac{2048a^5\sqrt{ax+b\sqrt{x}}}{3003b^6x}+\frac{48a\ldots}{1\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(13*b*x^(7/2)) + (48*a*Sqrt[b*Sqrt[x] + a*x])/(1
43*b^2*x^3) - (160*a^2*Sqrt[b*Sqrt[x] + a*x])/(429*b^3*x^(5/2)) + (1280*a^3
*Sqrt[b*Sqrt[x] + a*x])/(3003*b^4*x^2) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(1
001*b^5*x^(3/2)) + (2048*a^5*Sqrt[b*Sqrt[x] + a*x])/(3003*b^6*x) - (4096*a^
6*Sqrt[b*Sqrt[x] + a*x])/(3003*b^7*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} - \frac{(12a)\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{13b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} + \frac{(120a^2)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{143b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} - \frac{(320a^3)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{429b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} + \frac{(64}{}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} - \frac{512}{}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} - \frac{512}{}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{13bx^{7/2}} + \frac{48a\sqrt{b\sqrt{x}+ax}}{143b^2x^3} - \frac{160a^2\sqrt{b\sqrt{x}+ax}}{429b^3x^{5/2}} + \frac{1280a^3\sqrt{b\sqrt{x}+ax}}{3003b^4x^2} - \frac{512}{}$$

Mathematica [A] time = 0.06, size = 96, normalized size = 0.48

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(1024a^6x^3-512a^5bx^{5/2}+384a^4b^2x^2-320a^3b^3x^{3/2}+280a^2b^4x-252ab^5\sqrt{x}+231b^6\right)}{3003b^7x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x]*(231*b^6 - 252*a*b^5*Sqrt[x] + 280*a^2*b^4*x - 32
0*a^3*b^3*x^(3/2) + 384*a^4*b^2*x^2 - 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(3
003*b^7*x^(7/2))
```

fricas [A] time = 0.66, size = 86, normalized size = 0.43

$$\frac{4\left(512\,a^5bx^3+320\,a^3b^3x^2+252\,ab^5x-\left(1024\,a^6x^3+384\,a^4b^2x^2+280\,a^2b^4x+231\,b^6\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3003\,b^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3003*(512*a^5*b*x^3 + 320*a^3*b^3*x^2 + 252*a*b^5*x - (1024*a^6*x^3 + 384
*a^4*b^2*x^2 + 280*a^2*b^4*x + 231*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^7
*x^4)
```

giac [A] time = 0.20, size = 208, normalized size = 1.04

$$\frac{4\left(27456\,a^3\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^6+72072\,a^{\frac{5}{2}}b\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5+80080\,a^2b^2\left(\sqrt{a}\,\sqrt{x}-\sqrt{ax+b\sqrt{}}\right.\right.}{3003}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3003*(27456*a^3*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^6 + 72072*a^(5/
2)*b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 80080*a^2*b^2*(sqrt(a)*s
qrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 48048*a^(3/2)*b^3*(sqrt(a)*sqrt(x) - sq
rt(a*x + b*sqrt(x)))^3 + 16380*a*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x
)))^2 + 3003*sqrt(a)*b^5*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + 231*b^
6)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^13
```

maple [C] time = 0.07, size = 306, normalized size = 1.53

$$\sqrt{ax+b\sqrt{x}}\left(-3003a^7 b x^{\frac{15}{2}} \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\ \sqrt{a}}{2\sqrt{a}}\right)+3003a^7 b x^{\frac{15}{2}} \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\ \sqrt{a}}{2\sqrt{a}}\right)+6006\sqrt{\left(a\sqrt{x}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/3003*(a*x+b*x^(1/2))^(1/2)*(6006*x^(15/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a
^(15/2)-12012*x^(13/2)*(a*x+b*x^(1/2))^(3/2)*a^(13/2)+6006*x^(15/2)*(a*x+b*
x^(1/2))^(1/2)*a^(15/2)-3003*x^(15/2)*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b
)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^7*b+3003*x^(15/2)*ln(1/2*(2*a*x^(1/2)+
b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^7*b-5868*x^(11/2)*(a*x+b*x^(1
/2))^(3/2)*a^(9/2)*b^2-3052*x^(9/2)*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*b^4+7916*
a^(11/2)*(a*x+b*x^(1/2))^(3/2)*b*x^6-924*x^(7/2)*(a*x+b*x^(1/2))^(3/2)*a^(1
/2)*b^6+4332*x^5*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*b^3+1932*(a*x+b*x^(1/2))^(3/
2)*a^(3/2)*x^4*b^5)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^8/x^(15/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{a\,x+b\,\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^4*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{ax+b\sqrt{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a*x + b*sqrt(x))), x)
```

3.110 $\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{693b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{13/2}}+\frac{693b^4\sqrt{ax+b\sqrt{x}}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{231b^2x\sqrt{ax+b\sqrt{x}}}{40a^4}-\frac{99bx^{3/2}\sqrt{ax+}}{20a^3}$$

```
[Out] -693/64*b^5*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(13/2)-4*x^3/a
/(b*x^(1/2)+a*x)^(1/2)+693/64*b^4*(b*x^(1/2)+a*x)^(1/2)/a^6+231/40*b^2*x*(b
*x^(1/2)+a*x)^(1/2)/a^4-99/20*b*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^3+22/5*x^2*
(b*x^(1/2)+a*x)^(1/2)/a^2-231/32*b^3*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^5
```

Rubi [A] time = 0.18, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{693b^4\sqrt{ax+b\sqrt{x}}}{64a^6}-\frac{231b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{231b^2x\sqrt{ax+b\sqrt{x}}}{40a^4}-\frac{693b^5\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{64a^{13/2}}-\frac{99bx^{3/2}\sqrt{ax+b}}{20a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^3)/(a*Sqrt[b*Sqrt[x] + a*x]) + (693*b^4*Sqrt[b*Sqrt[x] + a*x])/(64*a^
6) - (231*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(32*a^5) + (231*b^2*x*Sqrt[b*S
qrt[x] + a*x])/(40*a^4) - (99*b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(20*a^3) + (
22*x^2*Sqrt[b*Sqrt[x] + a*x])/(5*a^2) - (693*b^5*ArcTanh[(Sqrt[a]*Sqrt[x])/
Sqrt[b*Sqrt[x] + a*x]])/(64*a^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^3}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{x^7}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{22\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} - \frac{(99b)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{5a^2}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} + \frac{\left(693b^2\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,d\right.}{40a^3}$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} + \frac{22x^2\sqrt{b\sqrt{x}+ax}}{5a^2} - (231$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} - \frac{99bx^{3/2}\sqrt{b\sqrt{x}+ax}}{20a^3} +$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} - 9$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} - 9$$

$$= -\frac{4x^3}{a\sqrt{b\sqrt{x}+ax}} + \frac{693b^4\sqrt{b\sqrt{x}+ax}}{64a^6} - \frac{231b^3\sqrt{x}\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{231b^2x\sqrt{b\sqrt{x}+ax}}{40a^4} - 9$$

Mathematica [C] time = 0.06, size = 64, normalized size = 0.32

$$\frac{4x^{7/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{13}{2};\frac{15}{2};-\frac{a\sqrt{x}}{b}\right)}{13b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(7/2)*Hypergeometric2F1[3/2, 13/2, 15/2, -((a*
Sqrt[x])/b)])/(13*b*Sqrt[b*Sqrt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [B] time = 0.06, size = 549, normalized size = 2.79

$$\frac{\sqrt{ax+b\sqrt{x}}\;\left(-4480a^3b^5x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)+1015a^3b^5x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)-8960a^2b^6\sqrt{x}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/640*(a*x+b*x^(1/2))^(1/2)*(256*(a*x+b*x^(1/2))^(3/2)*a^(13/2)*x^2-352*(a*
x+b*x^(1/2))^(3/2)*a^(11/2)*x^(3/2)*b-4060*(a*x+b*x^(1/2))^(1/2)*a^(9/2)*x^
(3/2)*b^3+528*(a*x+b*x^(1/2))^(3/2)*a^(9/2)*x*b^2+3136*(a*x+b*x^(1/2))^(3/2
)*a^(7/2)*x^(1/2)*b^3-10150*(a*x+b*x^(1/2))^(1/2)*a^(7/2)*x*b^4+8960*a^(7/2
)*x*((a*x^(1/2)+b)*x^(1/2))^(1/2)*b^4+2000*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*b^
4-8120*(a*x+b*x^(1/2))^(1/2)*a^(5/2)*x^(1/2)*b^5+17920*a^(5/2)*x^(1/2)*((a*
x^(1/2)+b)*x^(1/2))^(1/2)*b^5-2560*a^(5/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)*b^
4-2030*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b^6+8960*a^(3/2)*((a*x^(1/2)+b)*x^(1/2
))^(1/2)*b^6+2030*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^
(1/2))*x^(1/2)*a^2*b^6+1015*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a
^(1/2))/a^(1/2))*x*a^3*b^5-8960*a^2*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*
x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(1/2)*b^6-4480*a^3*ln(1/2*(2*a*x^(1/2)+b
+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x*b^5+1015*ln(1/2*(2*a*x
^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a*b^7-4480*a*ln(1/2*(2*a
*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^7)/a^(15/2)/
((a*x^(1/2)+b)*x^(1/2))^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^3/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(a\,x + b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(1/2))^(3/2),x)
[Out] int(x^3/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(x**3/(a*x + b*sqrt(x))**(3/2), x)
```

3.111 $\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{9/2}}+\frac{35b^2\sqrt{ax+b\sqrt{x}}}{4a^4}-\frac{35b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^3}+\frac{14x\sqrt{ax+b\sqrt{x}}}{3a^2}-\frac{4x^2}{a\sqrt{ax+b\sqrt{x}}}$$

```
[Out] -35/4*b^3*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(9/2)-4*x^2/a/(b
*x^(1/2)+a*x)^(1/2)+35/4*b^2*(b*x^(1/2)+a*x)^(1/2)/a^4+14/3*x*(b*x^(1/2)+a*
x)^(1/2)/a^2-35/6*b*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^3
```

Rubi [A] time = 0.13, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{35b^2\sqrt{ax+b\sqrt{x}}}{4a^4}-\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{4a^{9/2}}-\frac{35b\sqrt{x}\sqrt{ax+b\sqrt{x}}}{6a^3}+\frac{14x\sqrt{ax+b\sqrt{x}}}{3a^2}-\frac{4x^2}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^2)/(a*Sqrt[b*Sqrt[x] + a*x]) + (35*b^2*Sqrt[b*Sqrt[x] + a*x])/(4*a^4)
 - (35*b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(6*a^3) + (14*x*Sqrt[b*Sqrt[x] + a*
x])/(3*a^2) - (35*b^3*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(4*
a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^2}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{x^5}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{14\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{(35b)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{3a^2}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx,\right.}{4a^3}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{\left(35b^3\right)\text{S}}{}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{\left(35b^3\right)\text{S}}{}$$

$$= -\frac{4x^2}{a\sqrt{b\sqrt{x}+ax}} + \frac{35b^2\sqrt{b\sqrt{x}+ax}}{4a^4} - \frac{35b\sqrt{x}\sqrt{b\sqrt{x}+ax}}{6a^3} + \frac{14x\sqrt{b\sqrt{x}+ax}}{3a^2} - \frac{35b^3\tan}{}$$

Mathematica [C] time = 0.08, size = 64, normalized size = 0.46

$$\frac{4x^{5/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{9}{2};\frac{11}{2};-\frac{a\sqrt{x}}{b}\right)}{9b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(5/2)*Hypergeometric2F1[3/2, 9/2, 11/2, -((a*S
qrt[x])/b)])/(9*b*Sqrt[b*Sqrt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [B] time = 0.06, size = 503, normalized size = 3.62

$$\sqrt{ax+b\sqrt{x}}\,\left(-120a^3b^3x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+15a^3b^3x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-240a^2b^4\sqrt{x}\,\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/24*(a*x+b*x^(1/2))^(1/2)/a^(11/2)*(16*(a*x+b*x^(1/2))^(3/2)*a^(9/2)*x-60*
(a*x+b*x^(1/2))^(1/2)*a^(9/2)*x^(3/2)*b-120*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(
1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^3*x*b^3+32*(a*x+b*x^(1/2))^(3/2)
*a^(7/2)*b*x^(1/2)-150*(a*x+b*x^(1/2))^(1/2)*a^(7/2)*x*b^2+240*a^(7/2)*x*((
a*x^(1/2)+b)*x^(1/2))^(1/2)*b^2+15*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^
(1/2)*a^(1/2))/a^(1/2))*x*a^3*b^3-240*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b
)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^2*x^(1/2)*b^4+16*(a*x+b*x^(1/2))^(3/2)
*a^(5/2)*b^2-120*(a*x+b*x^(1/2))^(1/2)*a^(5/2)*b^3*x^(1/2)+480*a^(5/2)*x^(1
/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*b^3-96*a^(5/2)*((a*x^(1/2)+b)*x^(1/2))^(3
/2)*b^2+30*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*
x^(1/2)*a^2*b^4-120*a*b^5*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(
1/2)*a^(1/2))/a^(1/2))-30*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b^4+240*((a*x^(1/2)
+b)*x^(1/2))^(1/2)*a^(3/2)*b^4+15*a*b^5*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1
/2))^(1/2)*a^(1/2))/a^(1/2)))/((a*x^(1/2)+b)*x^(1/2))^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a*x + b*sqrt(x))**(3/2), x)
```

3.112 $\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{5/2}}+\frac{6\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x}{a\sqrt{ax+b\sqrt{x}}}$$

```
[Out] -6*b*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(5/2)-4*x/a/(b*x^(1/2
)+a*x)^(1/2)+6*(b*x^(1/2)+a*x)^(1/2)/a^2
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2018, 668, 640, 620, 206}

$$\frac{6\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{5/2}}-\frac{4x}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x)/(a*Sqrt[b*Sqrt[x] + a*x]) + (6*Sqrt[b*Sqrt[x] + a*x])/a^2 - (6*b*Arc
Tanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/a^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{x^3}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(3b)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(6b)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^2}\\
&= -\frac{4x}{a\sqrt{b\sqrt{x}+ax}}+\frac{6\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{6b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.04, size = 64, normalized size = 0.83

$$\frac{4x^{3/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{5}{2};\frac{7}{2};-\frac{a\sqrt{x}}{b}\right)}{5b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*Sqrt[x] + a*x)^(3/2),x]
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^(3/2)*Hypergeometric2F1[3/2, 5/2, 7/2, -((a*Sq
rt[x])/b)])/(5*b*Sqrt[b*Sqrt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.32, size = 94, normalized size = 1.22

$$\frac{3\,b\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{a^{\frac{5}{2}}}+\frac{4\,b^2}{\left(a\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+\sqrt{a}\,b\right)a^2}+\frac{2\sqrt{ax+b\sqrt{x}}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 3*b*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(5
/2) + 4*b^2/((a*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + sqrt(a)*b)*a^2)
 + 2*sqrt(a*x + b*sqrt(x))/a^2
```

maple [B] time = 0.05, size = 236, normalized size = 3.06

$$\frac{\sqrt{ax+b\sqrt{x}}\left(-3a^2bx\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-6a\,b^2\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-3b^3\ln\left(\frac{2a\sqrt{x}+b+\cdots}{\cdots}\right.\cdots}{\sqrt{\left(a\sqrt{x}\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] (a*x+b*x^(1/2))^(1/2)/a^(5/2)*(6*x*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)-3*
x*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a
^2*b+12*x^(1/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b-6*x^(1/2)*ln(1/2*(2
*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a*b^2-4*a^(3
/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)+6*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)*b
^2-3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2)
)*b^3)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*sqrt(x))**(3/2), x)
```

3.113 $\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

[Out] 4*x^(1/2)/b/(b*x^(1/2)+a*x)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Int[(b*Sqrt[x] + a*x)^(-3/2),x]

[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx=\frac{4\sqrt{x}}{b\sqrt{b\sqrt{x}+ax}}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{4\sqrt{x}}{b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Sqrt[x] + a*x)^(-3/2),x]

[Out] (4*Sqrt[x])/(b*Sqrt[b*Sqrt[x] + a*x])

fricas [A] time = 0.88, size = 36, normalized size = 1.44

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(a\sqrt{x}-b\right)}{a^2bx-b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")

[Out] 4*sqrt(a*x + b*sqrt(x))*(a*sqrt(x) - b)/(a^2*b*x - b^3)

giac [A] time = 0.19, size = 34, normalized size = 1.36

$$\frac{4}{\left(\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 4/((sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)*sqrt(a))
```

maple [C] time = 0.06, size = 404, normalized size = 16.16

$$\sqrt{ax+b\sqrt{x}}\,\left(-a^{2}bx\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+a^{2}bx\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-2a\,b^{2}\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{(}}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] (a*x+b*x^(1/2))^(1/2)*(2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)*x+2*x*(a*x+b
*x^(1/2))^(1/2)*a^(5/2)-a^2*b*x*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1
/2))^(1/2)*a^(1/2))/a^(1/2))+x*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2
)*a^(1/2))/a^(1/2))*a^2*b+4*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b*x^(1/2)
+4*x^(1/2)*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b-2*a*b^2*x^(1/2)*ln(1/2*(2*a*x^(1
/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))+2*x^(1/2)*ln(1/2*(2
*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a*b^2-4*((a*x^(1/2)+
b)*x^(1/2))^(3/2)*a^(3/2)+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)*b^2+2*(a*
x+b*x^(1/2))^(1/2)*a^(1/2)*b^2-b^3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x
^(1/2))^(1/2)*a^(1/2))/a^(1/2))+ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/
2)*a^(1/2))/a^(1/2))*b^3)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^2/(a*x^(1/2)+b)^2
/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*sqrt(x))^(-3/2), x)
```

mupad [B] time = 5.43, size = 40, normalized size = 1.60

$$-\frac{4\,x\,\left(\frac{b}{a\,\sqrt{x}}+1\right)}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}\,\left(\sqrt{\frac{b}{a\,\sqrt{x}}+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] -(4*x*(b/(a*x^(1/2)) + 1))/((a*x + b*x^(1/2))^(3/2)*((b/(a*x^(1/2)) + 1)^(1
/2) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*sqrt(x))**(-3/2), x)
```

3.114 $\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{32a\sqrt{ax+b\sqrt{x}}}{3b^3\sqrt{x}}-\frac{16\sqrt{ax+b\sqrt{x}}}{3b^2x}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

```
[Out] 4/b/x^(1/2)/(b*x^(1/2)+a*x)^(1/2)-16/3*(b*x^(1/2)+a*x)^(1/2)/b^2/x+32/3*a*(
b*x^(1/2)+a*x)^(1/2)/b^3/x^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{32a\sqrt{ax+b\sqrt{x}}}{3b^3\sqrt{x}}-\frac{16\sqrt{ax+b\sqrt{x}}}{3b^2x}+\frac{4}{b\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]) - (16*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^3*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}} + \frac{4\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}} - \frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x} - \frac{(8a)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3b^2}$$

$$= \frac{4}{b\sqrt{x}\sqrt{b\sqrt{x}+ax}} - \frac{16\sqrt{b\sqrt{x}+ax}}{3b^2x} + \frac{32a\sqrt{b\sqrt{x}+ax}}{3b^3\sqrt{x}}$$

Mathematica [A] time = 0.08, size = 48, normalized size = 0.61

$$\frac{4\left(8a^2x+4ab\sqrt{x}-b^2\right)}{3b^3\sqrt{x}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*Sqrt[x] + a*x)^(3/2)),x]
[Out] (4*(-b^2 + 4*a*b*Sqrt[x] + 8*a^2*x))/(3*b^3*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])
```

fricas [A] time = 0.87, size = 63, normalized size = 0.80

$$-\frac{4\left(4\,a^2bx-b^3-\left(8\,a^3x-5\,ab^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{3\left(a^2b^3x^2-b^5x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
[Out] -4/3*(4*a^2*b*x - b^3 - (8*a^3*x - 5*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/
(a^2*b^3*x^2 - b^5*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

maple [C] time = 0.07, size = 524, normalized size = 6.63

$$\sqrt{ax+b\sqrt{x}}\,\left(3a^4b\,x^{\frac{7}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)-3a^4b\,x^{\frac{7}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)+6a^3b^2x^3\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{}}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/3*(a*x+b*x^(1/2))^(1/2)*(24*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*x^(5/2)-6*(a*x+
b*x^(1/2))^(1/2)*a^(9/2)*x^(7/2)+3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x
^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(7/2)*a^4*b-6*a^(9/2)*x^(7/2)*((a*x^(1/2)
+b)*x^(1/2))^(1/2)-3*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))
/a^(1/2))*x^(7/2)*a^4*b+44*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*x^2*b-12*(a*x+b*x^
(1/2))^(1/2)*a^(7/2)*x^3*b+6*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2)
)^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^2-12*a^(7/2)*x^3*((a*x^(1/2)+b)*x^(1/2)
)^(1/2)*b-12*a^(7/2)*x^(5/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)-6*ln(1/2*(2*a*x^
(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^3*a^3*b^2+16*(a*x+b*x^(
1/2))^(3/2)*a^(3/2)*x^(3/2)*b^2-6*(a*x+b*x^(1/2))^(1/2)*a^(5/2)*x^(5/2)*b^2
+3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*
x^(5/2)*a^2*b^3-6*a^(5/2)*x^(5/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*b^2-3*ln(1/
2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(5/2)*a^2*b^3-
4*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*x*b^3)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^4/a^
(1/2)/x^(5/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a\,x + b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*sqrt(x))**(3/2)), x)
```

3.115 $\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=137

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{35b^5\sqrt{x}}-\frac{256a^2\sqrt{ax+b\sqrt{x}}}{35b^4x}+\frac{192a\sqrt{ax+b\sqrt{x}}}{35b^3x^{3/2}}-\frac{32\sqrt{ax+b\sqrt{x}}}{7b^2x^2}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

```
[Out] 4/b/x^(3/2)/(b*x^(1/2)+a*x)^(1/2)-32/7*(b*x^(1/2)+a*x)^(1/2)/b^2/x^2+192/35
*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(3/2)-256/35*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x
+512/35*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{35b^5\sqrt{x}}-\frac{256a^2\sqrt{ax+b\sqrt{x}}}{35b^4x}+\frac{192a\sqrt{ax+b\sqrt{x}}}{35b^3x^{3/2}}-\frac{32\sqrt{ax+b\sqrt{x}}}{7b^2x^2}+\frac{4}{bx^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^(3/2)*Sqrt[b*Sqrt[x] + a*x]) - (32*Sqrt[b*Sqrt[x] + a*x])/(7*b^2*x^2
) + (192*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x^(3/2)) - (256*a^2*Sqrt[b*Sqrt[x
] + a*x])/(35*b^4*x) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(35*b^5*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} + \frac{8\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} - \frac{(48a)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{7b^2}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}} + \frac{\left(192a^2\right)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,d}{35b^3}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}} - \frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x} - \frac{(1}{}$$

$$= \frac{4}{bx^{3/2}\sqrt{b\sqrt{x}+ax}} - \frac{32\sqrt{b\sqrt{x}+ax}}{7b^2x^2} + \frac{192a\sqrt{b\sqrt{x}+ax}}{35b^3x^{3/2}} - \frac{256a^2\sqrt{b\sqrt{x}+ax}}{35b^4x} + \frac{51}{}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.53

$$\frac{4\left(128a^4x^2+64a^3bx^{3/2}-16a^2b^2x+8ab^3\sqrt{x}-5b^4\right)}{35b^5x^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*(-5*b^4 + 8*a*b^3*Sqrt[x] - 16*a^2*b^2*x + 64*a^3*b*x^(3/2) + 128*a^4*x^
2))/(35*b^5*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])
```

fricas [A] time = 0.75, size = 87, normalized size = 0.64

$$-\frac{4\left(64\,a^4bx^2-24\,a^2b^3x-5\,b^5-\left(128\,a^5x^2-80\,a^3b^2x-13\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\left(a^2b^5x^3-b^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/35*(64*a^4*b*x^2 - 24*a^2*b^3*x - 5*b^5 - (128*a^5*x^2 - 80*a^3*b^2*x -
13*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^5*x^3 - b^7*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

maple [C] time = 0.06, size = 570, normalized size = 4.16

$$-\frac{\sqrt{ax+b\sqrt{x}}\left(-105a^{6}b\,x^{\frac{11}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+105a^{6}b\,x^{\frac{11}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-210a^{5}b^{2}x^{5}\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] -1/35*(a*x+b*x^(1/2))^(1/2)*(210*x^(11/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(
13/2)-560*x^(9/2)*(a*x+b*x^(1/2))^(3/2)*a^(11/2)+210*x^(11/2)*(a*x+b*x^(1/2
))^(1/2)*a^(13/2)-105*x^(11/2)*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/
2))^(1/2)*a^(1/2))/a^(1/2))*a^6*b+105*x^(11/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x
+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^6*b-256*x^(7/2)*(a*x+b*x^(1/2))^(3/2)
*a^(7/2)*b^2+420*x^5*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(11/2)*b-932*x^4*(a*x+
b*x^(1/2))^(3/2)*a^(9/2)*b+420*x^5*(a*x+b*x^(1/2))^(1/2)*a^(11/2)*b-210*x^5
*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^
5*b^2+210*x^5*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2
))*a^5*b^2+140*x^(9/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)*a^(11/2)+64*x^3*(a*x+b
*x^(1/2))^(3/2)*a^(5/2)*b^3+210*x^(9/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(9/
2)*b^2+210*x^(9/2)*(a*x+b*x^(1/2))^(1/2)*a^(9/2)*b^2-105*x^(9/2)*ln(1/2*(2*
a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^4*b^3+105*x
^(9/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^4*
b^3-32*x^(5/2)*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*b^4+20*(a*x+b*x^(1/2))^(3/2)*a
^(1/2)*x^2*b^5)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^6/x^(9/2)/a^(1/2)/(a*x^(1/2
)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2}\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*x + b*sqrt(x))**(3/2)), x)
```

3.116 $\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=195

$$\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{231b^7\sqrt{x}}-\frac{2048a^4\sqrt{ax+b\sqrt{x}}}{231b^6x}+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{77b^5x^{3/2}}-\frac{1280a^2\sqrt{ax+b\sqrt{x}}}{231b^4x^2}+\frac{160a\sqrt{ax+b\sqrt{x}}}{33b^3x^{5/2}}-48\sqrt{\ }$$

```
[Out] 4/b/x^(5/2)/(b*x^(1/2)+a*x)^(1/2)-48/11*(b*x^(1/2)+a*x)^(1/2)/b^2/x^3+160/3
3*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^(5/2)-1280/231*a^2*(b*x^(1/2)+a*x)^(1/2)/b^
4/x^2+512/77*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^(3/2)-2048/231*a^4*(b*x^(1/2)+
a*x)^(1/2)/b^6/x+4096/231*a^5*(b*x^(1/2)+a*x)^(1/2)/b^7/x^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 2014}

$$\frac{512a^3\sqrt{ax+b\sqrt{x}}}{77b^5x^{3/2}}-\frac{1280a^2\sqrt{ax+b\sqrt{x}}}{231b^4x^2}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{231b^7\sqrt{x}}-\frac{2048a^4\sqrt{ax+b\sqrt{x}}}{231b^6x}+\frac{160a\sqrt{ax+b\sqrt{x}}}{33b^3x^{5/2}}-48\sqrt{\ }$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^(5/2)*Sqrt[b*Sqrt[x] + a*x]) - (48*Sqrt[b*Sqrt[x] + a*x])/(11*b^2*x^
3) + (160*a*Sqrt[b*Sqrt[x] + a*x])/(33*b^3*x^(5/2)) - (1280*a^2*Sqrt[b*Sqrt
[x] + a*x])/(231*b^4*x^2) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(77*b^5*x^(3/2)
) - (2048*a^4*Sqrt[b*Sqrt[x] + a*x])/(231*b^6*x) + (4096*a^5*Sqrt[b*Sqrt[x]
 + a*x])/(231*b^7*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} + \frac{12\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} - \frac{(120a)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{11b^2}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} + \frac{\left(320a^2\right)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{33b^3}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} - \frac{(640}{}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} + \frac{512a}{}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} + \frac{512a}{}$$

$$= \frac{4}{bx^{5/2}\sqrt{b\sqrt{x}+ax}} - \frac{48\sqrt{b\sqrt{x}+ax}}{11b^2x^3} + \frac{160a\sqrt{b\sqrt{x}+ax}}{33b^3x^{5/2}} - \frac{1280a^2\sqrt{b\sqrt{x}+ax}}{231b^4x^2} + \frac{512a}{}$$

Mathematica [A] time = 0.09, size = 96, normalized size = 0.49

$$\frac{4\left(1024a^6x^3+512a^5bx^{5/2}-128a^4b^2x^2+64a^3b^3x^{3/2}-40a^2b^4x+28ab^5\sqrt{x}-21b^6\right)}{231b^7x^{5/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (4*(-21*b^6 + 28*a*b^5*Sqrt[x] - 40*a^2*b^4*x + 64*a^3*b^3*x^(3/2) - 128*a^
4*b^2*x^2 + 512*a^5*b*x^(5/2) + 1024*a^6*x^3))/(231*b^7*x^(5/2)*Sqrt[b*Sqrt
[x] + a*x])
```

fricas [A] time = 1.06, size = 109, normalized size = 0.56

$$-\frac{4\left(512\,a^6bx^3-192\,a^4b^3x^2-68\,a^2b^5x-21\,b^7-\left(1024\,a^7x^3-640\,a^5b^2x^2-104\,a^3b^4x-49\,ab^6\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{231\left(a^2b^7x^4-b^9x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -4/231*(512*a^6*b*x^3 - 192*a^4*b^3*x^2 - 68*a^2*b^5*x - 21*b^7 - (1024*a^7
*x^3 - 640*a^5*b^2*x^2 - 104*a^3*b^4*x - 49*a*b^6)*sqrt(x))*sqrt(a*x + b*sq
rt(x))/(a^2*b^7*x^4 - b^9*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

maple [C] time = 0.07, size = 614, normalized size = 3.15

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(1155a^8b\,x^{\frac{15}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)-1155a^8b\,x^{\frac{15}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)+2310a^7b^2x^7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/231*(a*x+b*x^(1/2))^(1/2)*(-2310*a^(13/2)*x^(13/2)*((a*x^(1/2)+b)*x^(1/2)
)^(1/2)*b^2-2310*(a*x+b*x^(1/2))^(1/2)*a^(13/2)*x^(13/2)*b^2-512*(a*x+b*x^(
1/2))^(3/2)*a^(9/2)*x^5*b^3-4620*a^(15/2)*x^7*((a*x^(1/2)+b)*x^(1/2))^(1/2)
*b-4620*(a*x+b*x^(1/2))^(1/2)*a^(15/2)*x^7*b+2048*(a*x+b*x^(1/2))^(3/2)*a^(
11/2)*x^(11/2)*b^2+8716*(a*x+b*x^(1/2))^(3/2)*a^(13/2)*x^6*b-84*(a*x+b*x^(1
/2))^(3/2)*a^(1/2)*x^3*b^7+5544*(a*x+b*x^(1/2))^(3/2)*a^(15/2)*x^(13/2)-231
0*(a*x+b*x^(1/2))^(1/2)*a^(17/2)*x^(15/2)+1155*ln(1/2*(2*a*x^(1/2)+b+2*((a*
x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(15/2)*a^8*b-2310*a^(17/2)*x^
(15/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)-1155*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^
(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(15/2)*a^8*b+2310*ln(1/2*(2*a*x^(1/2)+b+2*
((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^7*a^7*b^2-924*a^(15/2)*x^
(13/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)-2310*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^
(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^7*a^7*b^2+256*(a*x+b*x^(1/2))^(3/2)*a^(7/2
)*x^(9/2)*b^4+1155*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^
(1/2))/a^(1/2))*x^(13/2)*a^6*b^3-1155*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2
))^(1/2)*a^(1/2))/a^(1/2))*x^(13/2)*a^6*b^3-160*(a*x+b*x^(1/2))^(3/2)*a^(5/
2)*x^4*b^5+112*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*x^(7/2)*b^6)/((a*x^(1/2)+b)*x^
(1/2))^(1/2)/b^8/x^(13/2)/a^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(x**3*(a*x + b*sqrt(x))**(3/2)), x)
```

3.117 $\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=204

$$\frac{231b^6\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{256a^{13/2}}-\frac{231b^5\sqrt{ax+b\sqrt{x}}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{ax+b\sqrt{x}}}{128a^5}-\frac{77b^3x\sqrt{ax+b\sqrt{x}}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{ax+b\sqrt{}}}{80a^3}$$

```
[Out] 231/256*b^6*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(13/2)-231/256
*b^5*(b*x^(1/2)+a*x)^(1/2)/a^6-77/160*b^3*x*(b*x^(1/2)+a*x)^(1/2)/a^4+33/80
*b^2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^3-11/30*b*x^2*(b*x^(1/2)+a*x)^(1/2)/a^
2+1/3*x^(5/2)*(b*x^(1/2)+a*x)^(1/2)/a+77/128*b^4*x^(1/2)*(b*x^(1/2)+a*x)^(1
/2)/a^5
```

Rubi [A] time = 0.17, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2018, 670, 640, 620, 206}

$$\frac{33b^2x^{3/2}\sqrt{ax+b\sqrt{x}}}{80a^3}-\frac{231b^5\sqrt{ax+b\sqrt{x}}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{ax+b\sqrt{x}}}{128a^5}-\frac{77b^3x\sqrt{ax+b\sqrt{x}}}{160a^4}+\frac{231b^6\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{256a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-231*b^5*Sqrt[b*Sqrt[x] + a*x])/(256*a^6) + (77*b^4*Sqrt[x]*Sqrt[b*Sqrt[x]
 + a*x])/(128*a^5) - (77*b^3*x*Sqrt[b*Sqrt[x] + a*x])/(160*a^4) + (33*b^2*x
^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(80*a^3) - (11*b*x^2*Sqrt[b*Sqrt[x] + a*x])/(
30*a^2) + (x^(5/2)*Sqrt[b*Sqrt[x] + a*x])/(3*a) + (231*b^6*ArcTanh[(Sqrt[a]
*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(256*a^(13/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
```

```
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^{5/2}}{\sqrt{b\sqrt{x}+ax}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)\\
&= \frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}-\frac{(11b)\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{6a}\\
&= -\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}+\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}+\frac{\left(33b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{20a^2}\\
&= \frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}+\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}-\frac{\left(231b^3\right)\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\right.}{160a^3}\\
&= -\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}+\frac{x^{5/2}\sqrt{b\sqrt{x}+ax}}{3a}+\frac{(7}{}\\
&= \frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5}-\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\frac{11bx^2\sqrt{b\sqrt{x}+ax}}{30a^2}+\\
&= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5}-\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\\
&= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5}-\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-\\
&= -\frac{231b^5\sqrt{b\sqrt{x}+ax}}{256a^6}+\frac{77b^4\sqrt{x}\sqrt{b\sqrt{x}+ax}}{128a^5}-\frac{77b^3x\sqrt{b\sqrt{x}+ax}}{160a^4}+\frac{33b^2x^{3/2}\sqrt{b\sqrt{x}+ax}}{80a^3}-
\end{aligned}
$$

Mathematica [A] time = 0.26, size = 164, normalized size = 0.80

$$\frac{\left(a\sqrt{x}+b\right)\left(\sqrt{a}\,\sqrt{x}\,\sqrt{\frac{a\sqrt{x}}{b}+1}\,\left(1280a^5x^{5/2}-1408a^4bx^2+1584a^3b^2x^{3/2}-1848a^2b^3x+2310ab^4\sqrt{x}-3465b^5\right)+\right.}{3840a^{13/2}\sqrt{\frac{a\sqrt{x}}{b}+1}\,\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] ((b + a*Sqrt[x])*(Sqrt[a]*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[x]*(-3465*b^5 + 2310
*a*b^4*Sqrt[x] - 1848*a^2*b^3*x + 1584*a^3*b^2*x^(3/2) - 1408*a^4*b*x^2 + 1
280*a^5*x^(5/2)) + 3465*b^(11/2)*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]]
))/(3840*a^(13/2)*Sqrt[1 + (a*Sqrt[x])/b]*Sqrt[b*Sqrt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.28, size = 125, normalized size = 0.61

$$\frac{1}{3840}\sqrt{ax+b\sqrt{x}}\left(2\left(4\left(2\left(8\sqrt{x}\left(\frac{10\sqrt{x}}{a}-\frac{11\,b}{a^2}\right)+\frac{99\,b^2}{a^3}\right)\sqrt{x}-\frac{231\,b^3}{a^4}\right)\sqrt{x}+\frac{1155\,b^4}{a^5}\right)\sqrt{x}-\frac{3465\,b^5}{a^6}\right)-\frac{231\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*sqrt(a*x + b*sqrt(x))*(2*(4*(2*(8*sqrt(x)*(10*sqrt(x)/a - 11*b/a^2)
+ 99*b^2/a^3)*sqrt(x) - 231*b^3/a^4)*sqrt(x) + 1155*b^4/a^5)*sqrt(x) - 3465
*b^5/a^6) - 231/512*b^6*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*
sqrt(x))) - b))/a^(13/2)
```

maple [A] time = 0.06, size = 245, normalized size = 1.20

$$\frac{\sqrt{ax+b\sqrt{x}}\left(7680a\,b^6\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)-4215a\,b^6\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\;\sqrt{a}}{2\sqrt{a}}\right)+2560\left(ax+b\sqrt{x}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/7680*(a*x+b*x^(1/2))^(1/2)*(2560*x^(3/2)*(a*x+b*x^(1/2))^(3/2)*a^(11/2)+8
544*x^(1/2)*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*b^2-5376*(a*x+b*x^(1/2))^(3/2)*a^
(9/2)*x*b+16860*x^(1/2)*(a*x+b*x^(1/2))^(1/2)*a^(5/2)*b^4-12240*(a*x+b*x^(1
/2))^(3/2)*a^(5/2)*b^3-15360*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b^5+8430
*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b^5+7680*a*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/
2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^6-4215*ln(1/2*(2*a*x^(1/2)+b+2*(a*
x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a*b^6)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/a
^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{5}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^{5/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{ax + b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(a*x + b*sqrt(x)), x)
```

3.118 $\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=146

$$\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{9/2}}-\frac{35b^3\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{48a^3}-\frac{7bx\sqrt{ax+b\sqrt{x}}}{12a^2}+\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a}$$

```
[Out] 35/32*b^4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(9/2)-35/32*b^3*
(b*x^(1/2)+a*x)^(1/2)/a^4-7/12*b*x*(b*x^(1/2)+a*x)^(1/2)/a^2+1/2*x^(3/2)*(b
*x^(1/2)+a*x)^(1/2)/a+35/48*b^2*x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a^3
```

Rubi [A] time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2018, 670, 640, 620, 206}

$$-\frac{35b^3\sqrt{ax+b\sqrt{x}}}{32a^4}+\frac{35b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{48a^3}+\frac{35b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{9/2}}-\frac{7bx\sqrt{ax+b\sqrt{x}}}{12a^2}+\frac{x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-35*b^3*Sqrt[b*Sqrt[x] + a*x])/(32*a^4) + (35*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] +
 a*x])/(48*a^3) - (7*b*x*Sqrt[b*Sqrt[x] + a*x])/(12*a^2) + (x^(3/2)*Sqrt[b*
Sqrt[x] + a*x])/(2*a) + (35*b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] +
a*x]])/(32*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{3/2}}{\sqrt{b\sqrt{x}+ax}}\,dx = 2\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} - \frac{(7b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a}$$

$$= -\frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{\left(35b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{24a^2}$$

$$= \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} - \frac{\left(35b^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx,\right.}{32a^3}$$

$$= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{\left(35b^4\right.}{}$$

$$= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{\left(35b^4\right.}{}$$

$$= -\frac{35b^3\sqrt{b\sqrt{x}+ax}}{32a^4} + \frac{35b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{48a^3} - \frac{7bx\sqrt{b\sqrt{x}+ax}}{12a^2} + \frac{x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a} + \frac{35b^4\,\text{t}}{}$$

Mathematica [A] time = 0.21, size = 142, normalized size = 0.97

$$-\frac{35b^5\left(\frac{a\sqrt{x}}{b}+1\right)\left(-\frac{32a^4x^2}{35b^4}+\frac{16a^3x^{3/2}}{15b^3}-\frac{4a^2x}{3b^2}+\frac{2a\sqrt{x}}{b}-\frac{2\sqrt{a}\sqrt[4]{x}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{\frac{a\sqrt{x}}{b}+1}}\right)}{64a^5\sqrt{\sqrt{x}\left(a\sqrt{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-35*b^5*(1 + (a*Sqrt[x])/b)*((2*a*Sqrt[x])/b - (4*a^2*x)/(3*b^2) + (16*a^3
*x^(3/2))/(15*b^3) - (32*a^4*x^2)/(35*b^4) - (2*Sqrt[a]*x^(1/4)*ArcSinh[(Sq
rt[a]*x^(1/4))/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b])))/(64*a^5*Sqrt[(
b + a*Sqrt[x])*Sqrt[x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.41, size = 97, normalized size = 0.66

$$\frac{1}{96}\sqrt{ax+b\sqrt{x}}\left(2\left(4\sqrt{x}\left(\frac{6\sqrt{x}}{a}-\frac{7b}{a^2}\right)+\frac{35b^2}{a^3}\right)\sqrt{x}-\frac{105b^3}{a^4}\right)-\frac{35b^4\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{64a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*sqrt(a*x + b*sqrt(x))*(2*(4*sqrt(x)*(6*sqrt(x)/a - 7*b/a^2) + 35*b^2/a
^3)*sqrt(x) - 105*b^3/a^4) - 35/64*b^4*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x)
- sqrt(a*x + b*sqrt(x))) - b))/a^(9/2)
```

maple [A] time = 0.05, size = 203, normalized size = 1.39

$$\frac{\sqrt{ax+b\sqrt{x}}\left(192ab^4\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)-87ab^4\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)+348\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}b^2\cdots\right.}{192\sqrt{\left(a\sqrt{x}+\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/192*(a*x+b*x^(1/2))^(1/2)*(96*x^(1/2)*(a*x+b*x^(1/2))^(3/2)*a^(7/2)+348*x
^(1/2)*(a*x+b*x^(1/2))^(1/2)*a^(5/2)*b^2-208*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*
b-384*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b^3+174*(a*x+b*x^(1/2))^(1/2)*a
^(3/2)*b^3+192*a*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1
/2))/a^(1/2))*b^4-87*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))
/a^(1/2))*a*b^4)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/a^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{3/2}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x + b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(a*x + b*sqrt(x)), x)
```

3.119 $\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=87

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{5/2}}-\frac{3b\sqrt{ax+b\sqrt{x}}}{2a^2}+\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a}$$

```
[Out] 3/2*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(5/2)-3/2*b*(b*x^(
1/2)+a*x)^(1/2)/a^2+x^(1/2)*(b*x^(1/2)+a*x)^(1/2)/a
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2018, 670, 640, 620, 206}

$$\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{5/2}}-\frac{3b\sqrt{ax+b\sqrt{x}}}{2a^2}+\frac{\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (-3*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a +
(3*b^2*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\
&= \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} - \frac{(3b)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{2a}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^2}\\
&= -\frac{3b\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a} + \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 102, normalized size = 1.17

$$\frac{\sqrt{a}\sqrt{x}\left(2a^2x-ab\sqrt{x}-3b^2\right)+3b^{5/2}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{2a^{5/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[b*Sqrt[x] + a*x],x]
```

```
[Out] (Sqrt[a]*Sqrt[x]*(-3*b^2 - a*b*Sqrt[x] + 2*a^2*x) + 3*b^(5/2)*Sqrt[1 + (a*S
qrt[x])/b]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b]])/(2*a^(5/2)*Sqrt[b*Sq
rt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.27, size = 69, normalized size = 0.79

$$\frac{1}{2}\sqrt{ax+b\sqrt{x}}\left(\frac{2\sqrt{x}}{a}-\frac{3\,b}{a^2}\right)-\frac{3\,b^2\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{4\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a*x + b*sqrt(x))*(2*sqrt(x)/a - 3*b/a^2) - 3/4*b^2*log(abs(-2*sqrt
(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/a^(5/2)
```

maple [B] time = 0.05, size = 160, normalized size = 1.84

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(4a\,b^{2}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-a\,b^{2}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+4\sqrt{ax+b\sqrt{x}}\,a^{\frac{5}{2}}\sqrt{x}-8\sqrt{}}{4\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/4*(a*x+b*x^(1/2))^(1/2)*(4*x^(1/2)*(a*x+b*x^(1/2))^(1/2)*a^(5/2)-8*((a*x^
(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b+2*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b+4*a*ln(
1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^2-b^
2*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a)/((a*x^
(1/2)+b)*x^(1/2))^(1/2)/a^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x}}{\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^(1/2))^(1/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(a*x + b*sqrt(x)), x)
```

3.120 $\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=34

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

```
[Out] 4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2013, 620, 206}

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/Sqrt[a]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\sqrt{b\sqrt{x}+ax}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)\\ &= 4\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)\\ &= \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 1.91

$$\frac{4\sqrt{b}\sqrt[4]{x}\sqrt{\frac{a\sqrt{x}}{b}+1}\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[b*Sqrt[x] + a*x]),x]
[Out] (4*Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b]*x^(1/4)*ArcSinh[(Sqrt[a]*x^(1/4))/Sqrt[b
]])/(Sqrt[a]*Sqrt[b*Sqrt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.26, size = 37, normalized size = 1.09

$$-\frac{2\log\left(\left|-2\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)-b\right|\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
[Out] -2*log(abs(-2*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) - b))/sqrt(
a)
```

maple [B] time = 0.05, size = 136, normalized size = 4.00

$$-\frac{\sqrt{ax+b\sqrt{x}}\left(-b\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)-b\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\sqrt{a}}{2\sqrt{a}}\right)+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}-2\sqrt{a}\right.}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\sqrt{a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(a*x+b*x^(1/2))^(1/2),x)
[Out] -(a*x+b*x^(1/2))^(1/2)*(2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)-2*(a*x+b*x^
(1/2))^(1/2)*a^(1/2)-b*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2
)*a^(1/2))/a^(1/2))-b*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2)
)/a^(1/2)))/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{ax+b\sqrt{x}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)),x)
[Out] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(1/2),x)
[Out] Integral(1/(sqrt(x)*sqrt(a*x + b*sqrt(x))), x)
```

3.121 $\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=54

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

```
[Out] -4/3*(b*x^(1/2)+a*x)^(1/2)/b/x+8/3*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{8a\sqrt{ax+b\sqrt{x}}}{3b^2\sqrt{x}}-\frac{4\sqrt{ax+b\sqrt{x}}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(3*b*x) + (8*a*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*Sqr
t[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx &= -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}-\frac{(2a)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{3b}\\ &= -\frac{4\sqrt{b\sqrt{x}+ax}}{3bx}+\frac{8a\sqrt{b\sqrt{x}+ax}}{3b^2\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 37, normalized size = 0.69

$$\frac{4\left(2a\sqrt{x}-b\right)\sqrt{ax+b\sqrt{x}}}{3b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*(-b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(3*b^2*x)
```

fricas [A] time = 1.37, size = 29, normalized size = 0.54

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(2\,a\sqrt{x}-b\right)}{3\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(a*x + b*sqrt(x))*(2*a*sqrt(x) - b)/(b^2*x)
```

giac [A] time = 0.19, size = 53, normalized size = 0.98

$$\frac{4\left(3\sqrt{a}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+b\right)}{3\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(3*sqrt(a)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x))) + b)/(sqrt(a)*sqrt
(x) - sqrt(a*x + b*sqrt(x)))^3
```

maple [C] time = 0.06, size = 194, normalized size = 3.59

$$-\frac{\sqrt{ax+b\sqrt{x}}\left(-3a^2b\,x^{\frac{5}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+3a^2b\,x^{\frac{5}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+6\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,a\right.}{3\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}\,b^3x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] -1/3*(a*x+b*x^(1/2))^(1/2)*(6*x^(5/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)
-12*x^(3/2)*(a*x+b*x^(1/2))^(3/2)*a^(3/2)+6*x^(5/2)*(a*x+b*x^(1/2))^(1/2)*a
^(5/2)-3*x^(5/2)*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1
/2))/a^(1/2))*a^2*b+3*x^(5/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)
*a^(1/2))/a^(1/2))*a^2*b+4*(a*x+b*x^(1/2))^(3/2)*b*a^(1/2)*x)/((a*x^(1/2)+b
)*x^(1/2))^(1/2)/b^3/a^(1/2)/x^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{ax+b\sqrt{x}}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{x^{3/2}\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{ax + b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(a*x + b*sqrt(x))), x)
```

3.122 $\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=112

$$\frac{64a^3\sqrt{ax+b\sqrt{x}}}{35b^4\sqrt{x}}-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{35b^3x}+\frac{24a\sqrt{ax+b\sqrt{x}}}{35b^2x^{3/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

```
[Out] -4/7*(b*x^(1/2)+a*x)^(1/2)/b/x^2+24/35*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(3/2)-
32/35*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x+64/35*a^3*(b*x^(1/2)+a*x)^(1/2)/b^4/x
^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{64a^3\sqrt{ax+b\sqrt{x}}}{35b^4\sqrt{x}}-\frac{32a^2\sqrt{ax+b\sqrt{x}}}{35b^3x}+\frac{24a\sqrt{ax+b\sqrt{x}}}{35b^2x^{3/2}}-\frac{4\sqrt{ax+b\sqrt{x}}}{7bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(7*b*x^2) + (24*a*Sqrt[b*Sqrt[x] + a*x])/(35*b^2
*x^(3/2)) - (32*a^2*Sqrt[b*Sqrt[x] + a*x])/(35*b^3*x) + (64*a^3*Sqrt[b*Sqrt
[x] + a*x])/(35*b^4*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} - \frac{(6a)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{7b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} + \frac{\left(24a^2\right)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{35b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} - \frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x} - \frac{\left(16a^3\right)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{35b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{7bx^2} + \frac{24a\sqrt{b\sqrt{x}+ax}}{35b^2x^{3/2}} - \frac{32a^2\sqrt{b\sqrt{x}+ax}}{35b^3x} + \frac{64a^3\sqrt{b\sqrt{x}+ax}}{35b^4\sqrt{x}}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 0.53

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(16a^3x^{3/2}-8a^2bx+6ab^2\sqrt{x}-5b^3\right)}{35b^4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-5*b^3 + 6*a*b^2*Sqrt[x] - 8*a^2*b*x + 16*a^3*x^(
3/2)))/(35*b^4*x^2)
```

fricas [A] time = 1.25, size = 50, normalized size = 0.45

$$-\frac{4\left(8\,a^2bx+5\,b^3-2\left(8\,a^3x+3\,ab^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{35\,b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/35*(8*a^2*b*x + 5*b^3 - 2*(8*a^3*x + 3*a*b^2)*sqrt(x))*sqrt(a*x + b*sqrt
(x))/(b^4*x^2)
```

giac [A] time = 0.21, size = 115, normalized size = 1.03

$$\frac{4\left(70\,a^{\frac{3}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3+84\,ab\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^2+35\sqrt{a}\,b^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)+5\,b^3\right.}{35\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/35*(70*a^(3/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 84*a*b*(sqrt
(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^2 + 35*sqrt(a)*b^2*(sqrt(a)*sqrt(x) -
sqrt(a*x + b*sqrt(x))) + 5*b^3)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^7
```

maple [C] time = 0.06, size = 240, normalized size = 2.14

$$\sqrt{ax+b\sqrt{x}}\left(35a^4b\,x^{\frac{9}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\ \sqrt{a}}{2\sqrt{a}}\right)-35a^4b\,x^{\frac{9}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\ \sqrt{a}}{2\sqrt{a}}\right)-70\sqrt{ax+b\sqrt{x}}\ a^{\frac{9}{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/35*(a*x+b*x^(1/2))^(1/2)*(140*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*x^(7/2)-70*(a
*x+b*x^(1/2))^(1/2)*a^(9/2)*x^(9/2)+35*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+
b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(9/2)*a^4*b-70*a^(9/2)*x^(9/2)*((a*x^
(1/2)+b)*x^(1/2))^(1/2)-35*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^
(1/2))/a^(1/2))*x^(9/2)*a^4*b+44*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*x^(5/2)*b^2-
76*a^(5/2)*(a*x+b*x^(1/2))^(3/2)*b*x^3-20*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*x^2
*b^3)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^5/x^(9/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}\,x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(a*x + b*sqrt(x))), x)
```

3.123 $\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx$

Optimal. Leaf size=170

$$\frac{1024a^5\sqrt{ax+b\sqrt{x}}}{693b^6\sqrt{x}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{693b^5x}+\frac{128a^3\sqrt{ax+b\sqrt{x}}}{231b^4x^{3/2}}-\frac{320a^2\sqrt{ax+b\sqrt{x}}}{693b^3x^2}+\frac{40a\sqrt{ax+b\sqrt{x}}}{99b^2x^{5/2}}-\frac{4\sqrt{ax+}}{11bx}$$

```
[Out] -4/11*(b*x^(1/2)+a*x)^(1/2)/b/x^3+40/99*a*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(5/2)
-320/693*a^2*(b*x^(1/2)+a*x)^(1/2)/b^3/x^2+128/231*a^3*(b*x^(1/2)+a*x)^(1/2
)/b^4/x^(3/2)-512/693*a^4*(b*x^(1/2)+a*x)^(1/2)/b^5/x+1024/693*a^5*(b*x^(1/
2)+a*x)^(1/2)/b^6/x^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{128a^3\sqrt{ax+b\sqrt{x}}}{231b^4x^{3/2}}-\frac{320a^2\sqrt{ax+b\sqrt{x}}}{693b^3x^2}+\frac{1024a^5\sqrt{ax+b\sqrt{x}}}{693b^6\sqrt{x}}-\frac{512a^4\sqrt{ax+b\sqrt{x}}}{693b^5x}+\frac{40a\sqrt{ax+b\sqrt{x}}}{99b^2x^{5/2}}-\frac{4\sqrt{ax+}}{11bx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (-4*Sqrt[b*Sqrt[x] + a*x])/(11*b*x^3) + (40*a*Sqrt[b*Sqrt[x] + a*x])/(99*b^
2*x^(5/2)) - (320*a^2*Sqrt[b*Sqrt[x] + a*x])/(693*b^3*x^2) + (128*a^3*Sqrt[
b*Sqrt[x] + a*x])/(231*b^4*x^(3/2)) - (512*a^4*Sqrt[b*Sqrt[x] + a*x])/(693*
b^5*x) + (1024*a^5*Sqrt[b*Sqrt[x] + a*x])/(693*b^6*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}\,dx = -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} - \frac{(10a)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{11b}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} + \frac{(80a^2)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{99b^2}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} - \frac{(160a^3)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{231b^3}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} + \frac{(128a}{}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} - \frac{512a^4}{6}$$

$$= -\frac{4\sqrt{b\sqrt{x}+ax}}{11bx^3} + \frac{40a\sqrt{b\sqrt{x}+ax}}{99b^2x^{5/2}} - \frac{320a^2\sqrt{b\sqrt{x}+ax}}{693b^3x^2} + \frac{128a^3\sqrt{b\sqrt{x}+ax}}{231b^4x^{3/2}} - \frac{512a^4}{6}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 0.49

$$\frac{4\sqrt{ax+b\sqrt{x}}\left(256a^5x^{5/2}-128a^4bx^2+96a^3b^2x^{3/2}-80a^2b^3x+70ab^4\sqrt{x}-63b^5\right)}{693b^6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[b*Sqrt[x] + a*x]),x]
```

```
[Out] (4*Sqrt[b*Sqrt[x] + a*x]*(-63*b^5 + 70*a*b^4*Sqrt[x] - 80*a^2*b^3*x + 96*a^
3*b^2*x^(3/2) - 128*a^4*b*x^2 + 256*a^5*x^(5/2)))/(693*b^6*x^3)
```

fricas [A] time = 1.41, size = 72, normalized size = 0.42

$$-\frac{4\left(128\,a^4bx^2+80\,a^2b^3x+63\,b^5-2\left(128\,a^5x^2+48\,a^3b^2x+35\,ab^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{693\,b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/693*(128*a^4*b*x^2 + 80*a^2*b^3*x + 63*b^5 - 2*(128*a^5*x^2 + 48*a^3*b^2
*x + 35*a*b^4)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(b^6*x^3)
```

giac [A] time = 0.24, size = 177, normalized size = 1.04

$$\frac{4\left(3696\,a^{\frac{5}{2}}\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^5+7920\,a^2b\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^4+6930\,a^{\frac{3}{2}}b^2\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^3 -}{693\left(\sqrt{a}\sqrt{x}-\sqrt{ax+b\sqrt{x}}\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4/693*(3696*a^(5/2)*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^5 + 7920*a^2*
b*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^4 + 6930*a^(3/2)*b^2*(sqrt(a)*s
qrt(x) - sqrt(a*x + b*sqrt(x)))^3 + 3080*a*b^3*(sqrt(a)*sqrt(x) - sqrt(a*x
+ b*sqrt(x)))^2 + 693*sqrt(a)*b^4*(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))
 + 63*b^5)/(sqrt(a)*sqrt(x) - sqrt(a*x + b*sqrt(x)))^11
```

maple [C] time = 0.06, size = 284, normalized size = 1.67

$$\sqrt{ax+b\sqrt{x}}\left(693a^6bx^{\frac{13}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-693a^6bx^{\frac{13}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-1386\sqrt{ax+b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(a*x+b*x^(1/2))^(1/2),x)
```

```
[Out] 1/693*(a*x+b*x^(1/2))^(1/2)*(2772*(a*x+b*x^(1/2))^(3/2)*a^(11/2)*x^(11/2)-1
386*(a*x+b*x^(1/2))^(1/2)*a^(13/2)*x^(13/2)+693*ln(1/2*(2*a*x^(1/2)+b+2*((a
*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(13/2)*a^6*b-1386*a^(13/2)*x
^(13/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)-693*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^
(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(13/2)*a^6*b+1236*(a*x+b*x^(1/2))^(3/2)*a^
(7/2)*x^(9/2)*b^2+532*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*x^(7/2)*b^4-1748*a^(9/2
)*(a*x+b*x^(1/2))^(3/2)*b*x^5-852*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*x^4*b^3-252
*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*x^3*b^5)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^7/x
^(13/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+b\sqrt{x}}\,x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*sqrt(x))*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{7/2}\,\sqrt{a\,x+b\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{ax+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(a*x + b*sqrt(x))), x)
```

3.124 $\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=171

$$\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{11/2}}-\frac{315b^3\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{105b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{16a^4}-\frac{21bx\sqrt{ax+b\sqrt{x}}}{4a^3}+\frac{9x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a^2}-\frac{\cdots}{a\sqrt{\cdots}}$$

```
[Out] 315/32*b^4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(11/2)-4*x^(5/2
)/a/(b*x^(1/2)+a*x)^(1/2)-315/32*b^3*(b*x^(1/2)+a*x)^(1/2)/a^5-21/4*b*x*(b*
x^(1/2)+a*x)^(1/2)/a^3+9/2*x^(3/2)*(b*x^(1/2)+a*x)^(1/2)/a^2+105/16*b^2*x^(
1/2)*(b*x^(1/2)+a*x)^(1/2)/a^4
```

Rubi [A] time = 0.15, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2018, 668, 670, 640, 620, 206}

$$-\frac{315b^3\sqrt{ax+b\sqrt{x}}}{32a^5}+\frac{105b^2\sqrt{x}\sqrt{ax+b\sqrt{x}}}{16a^4}+\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{32a^{11/2}}+\frac{9x^{3/2}\sqrt{ax+b\sqrt{x}}}{2a^2}-\frac{21bx\sqrt{ax+b\sqrt{x}}}{4a^3}-\frac{\cdots}{a\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^(5/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (315*b^3*Sqrt[b*Sqrt[x] + a*x])/(3
2*a^5) + (105*b^2*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/(16*a^4) - (21*b*x*Sqrt[b*
Sqrt[x] + a*x])/(4*a^3) + (9*x^(3/2)*Sqrt[b*Sqrt[x] + a*x])/(2*a^2) + (315*
b^4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[b*Sqrt[x] + a*x]])/(32*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{5/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\,\text{Subst}\left(\int \frac{x^6}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{18\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} - \frac{(63b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{4a^2}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} + \frac{\left(105b^2\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\right.}{8a^3}$$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + \frac{9x^{3/2}\sqrt{b\sqrt{x}+ax}}{2a^2} - $$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + $$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + $$

$$= -\frac{4x^{5/2}}{a\sqrt{b\sqrt{x}+ax}} - \frac{315b^3\sqrt{b\sqrt{x}+ax}}{32a^5} + \frac{105b^2\sqrt{x}\sqrt{b\sqrt{x}+ax}}{16a^4} - \frac{21bx\sqrt{b\sqrt{x}+ax}}{4a^3} + $$

Mathematica [C] time = 0.08, size = 62, normalized size = 0.36

$$\frac{4x^3\sqrt{\frac{a\sqrt{x}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{11}{2};\frac{13}{2};-\frac{a\sqrt{x}}{b}\right)}{11b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^3*Hypergeometric2F1[3/2, 11/2, 13/2, -((a*Sqrt
[x])/b)])/(11*b*Sqrt[b*Sqrt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [B] time = 0.06, size = 527, normalized size = 3.08

$$\frac{\sqrt{ax+b\sqrt{x}}\,\left(384a^{3}b^{4}x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-69a^{3}b^{4}x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+768a^{2}b^{5}\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/64*(a*x+b*x^(1/2))^(1/2)/a^(13/2)*(32*(a*x+b*x^(1/2))^(3/2)*a^(11/2)*x^(3
/2)+276*x^(3/2)*(a*x+b*x^(1/2))^(1/2)*a^(9/2)*b^2-48*(a*x+b*x^(1/2))^(3/2)*
a^(9/2)*b*x-768*x*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(7/2)*b^3+690*x*(a*x+b*x^
(1/2))^(1/2)*a^(7/2)*b^3+384*x*a^3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x
^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^4-192*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*b^2*x
^(1/2)-69*x*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))
*a^3*b^4-1536*x^(1/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)*b^4+552*(a*x+b*
x^(1/2))^(1/2)*a^(5/2)*b^4*x^(1/2)+768*x^(1/2)*a^2*ln(1/2*(2*a*x^(1/2)+b+2*
((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^5+256*((a*x^(1/2)+b)*x^(1
/2))^(3/2)*a^(5/2)*b^3-112*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*b^3-138*x^(1/2)*ln
(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^2*b^5-768*(
(a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b^5+138*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*
b^5+384*a*b^6*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)
)/a^(1/2))-69*a*b^6*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/
a^(1/2)))/((a*x^(1/2)+b)*x^(1/2))^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^(5/2)/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(x**(5/2)/(a*x + b*sqrt(x))**(3/2), x)
```

3.125 $\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=113

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{7/2}}-\frac{15b\sqrt{ax+b\sqrt{x}}}{2a^3}+\frac{5\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x^{3/2}}{a\sqrt{ax+b\sqrt{x}}}$$

```
[Out] 15/2*b^2*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(7/2)-4*x^(3/2)/a
/(b*x^(1/2)+a*x)^(1/2)-15/2*b*(b*x^(1/2)+a*x)^(1/2)/a^3+5*x^(1/2)*(b*x^(1/2
)+a*x)^(1/2)/a^2
```

Rubi [A] time = 0.10, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2018, 668, 670, 640, 620, 206}

$$\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{2a^{7/2}}-\frac{15b\sqrt{ax+b\sqrt{x}}}{2a^3}+\frac{5\sqrt{x}\sqrt{ax+b\sqrt{x}}}{a^2}-\frac{4x^{3/2}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*x^(3/2))/(a*Sqrt[b*Sqrt[x] + a*x]) - (15*b*Sqrt[b*Sqrt[x] + a*x])/(2*a^
3) + (5*Sqrt[x]*Sqrt[b*Sqrt[x] + a*x])/a^2 + (15*b^2*ArcTanh[(Sqrt[a]*Sqrt[
x])/Sqrt[b*Sqrt[x] + a*x]])/(2*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 668

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)),
 x] - Dist[(e^2*(m + p))/(c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{x^{3/2}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x^4}{\left(bx+ax^2\right)^{3/2}}\,dx,x,\sqrt{x}\right)$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}+\frac{10\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{a}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}-\frac{(15b)\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{bx+ax^2}}\,dx,x,\sqrt{x}\right)}{2a^2}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{\left(15b^2\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx,\right.}{4a^3}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{\left(15b^2\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\right.}{2a^3}$$

$$= -\frac{4x^{3/2}}{a\sqrt{b\sqrt{x}+ax}}-\frac{15b\sqrt{b\sqrt{x}+ax}}{2a^3}+\frac{5\sqrt{x}\sqrt{b\sqrt{x}+ax}}{a^2}+\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{2a^{7/2}}$$

Mathematica [C] time = 0.07, size = 62, normalized size = 0.55

$$\frac{4x^2\sqrt{\frac{a\sqrt{x}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{7}{2};\frac{9}{2};-\frac{a\sqrt{x}}{b}\right)}{7b\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*Sqrt[1 + (a*Sqrt[x])/b]*x^2*Hypergeometric2F1[3/2, 7/2, 9/2, -((a*Sqrt[x
])/b)])/(7*b*Sqrt[b*Sqrt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument ValueDone
```

maple [B] time = 0.06, size = 440, normalized size = 3.89

$$\sqrt{ax+b\sqrt{x}}\,\left(16a^3b^2x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-a^3b^2x\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+32a^2b^3\sqrt{x}\,\ln\left(\frac{2a\sqrt{x}+b+2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 1/4*(a*x+b*x^(1/2))^(1/2)/a^(9/2)*(4*x^(3/2)*(a*x+b*x^(1/2))^(1/2)*a^(9/2)-
32*x*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(7/2)*b+10*x*(a*x+b*x^(1/2))^(1/2)*a^(
7/2)*b+16*x*a^3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/
2))/a^(1/2))*b^2-x*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a
^(1/2))*a^3*b^2-64*x^(1/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)*b^2+8*(a*x
+b*x^(1/2))^(1/2)*a^(5/2)*b^2*x^(1/2)+32*x^(1/2)*a^2*ln(1/2*(2*a*x^(1/2)+b+
2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*b^3+16*((a*x^(1/2)+b)*x^(
1/2))^(3/2)*a^(5/2)*b-2*x^(1/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/
2)*a^(1/2))/a^(1/2))*a^2*b^3-32*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b^3+2
*(a*x+b*x^(1/2))^(1/2)*a^(3/2)*b^3+16*a*b^4*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(
1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))-a*b^4*ln(1/2*(2*a*x^(1/2)+b+2*(a*x
+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2)))/((a*x^(1/2)+b)*x^(1/2))^(1/2)/(a*x^(1/
2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3/2}}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x + b*x^(1/2))^(3/2),x)
[Out] int(x^(3/2)/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(x**(3/2)/(a*x + b*sqrt(x))**(3/2), x)
```

3.126 $\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}-\frac{4\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}$$

```
[Out] 4*arctanh(a^(1/2)*x^(1/2)/(b*x^(1/2)+a*x)^(1/2))/a^(3/2)-4*x^(1/2)/a/(b*x^(
1/2)+a*x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2018, 652, 620, 206}

$$\frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+b\sqrt{x}}}\right)}{a^{3/2}}-\frac{4\sqrt{x}}{a\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (-4*Sqrt[x])/(a*Sqrt[b*Sqrt[x] + a*x]) + (4*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[
b*Sqrt[x] + a*x]])/a^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))^2*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symb
ol] :> Simp[(e*(d + e*x)*(a + b*x + c*x^2)^(p + 1))/(c*(p + 1)), x] - Dist[
(e^2*(p + 2))/(c*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{
a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
0] &&  !IntegerQ[p] && LtQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x^2}{\left(bx+ax^2\right)^{3/2}}\,dx, x, \sqrt{x}\right)$$
$$= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} + \frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \sqrt{x}\right)}{a}$$
$$= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a}$$
$$= -\frac{4\sqrt{x}}{a\sqrt{b\sqrt{x}+ax}} + \frac{4\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b\sqrt{x}+ax}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.11, size = 79, normalized size = 1.32

$$\frac{4\sqrt[4]{x}\left(\sqrt{b}\sqrt{\frac{a\sqrt{x}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\sqrt[4]{x}}{\sqrt{b}}\right)-\sqrt{a}\sqrt[4]{x}\right)}{a^{3/2}\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(b*Sqrt[x] + a*x)^(3/2),x]
```

```
[Out] (4*x^(1/4)*(-(Sqrt[a]*x^(1/4)) + Sqrt[b]*Sqrt[1 + (a*Sqrt[x])/b]*ArcSinh[(S
qrt[a]*x^(1/4))/Sqrt[b]]))/(a^(3/2)*Sqrt[b*Sqrt[x] + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{%%%{1,[1]%%%},[2,2]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,3]%
%%}+%%%{1,[0,4]%%%} / %%%{%%%{1,[2]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[1,1]%%%}+%%%{%%%{1,[1]%%%},[0,2]%%%} Error: Bad A
rgument Value
```

maple [B] time = 0.05, size = 240, normalized size = 4.00

$$-\frac{2\sqrt{ax+b\sqrt{x}}\left(-a^2bx\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\ \sqrt{a}}{2\sqrt{a}}\right)-2a\,b^2\sqrt{x}\ \ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\ \sqrt{a}}{2\sqrt{a}}\right)-b^3\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{}}{2}\right.\right.}{\sqrt{\left(a\sqrt{x}+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] -2*(a*x+b*x^(1/2))^(1/2)/a^(3/2)*(2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(5/2)*x
-a^2*b*x*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(
1/2))+4*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(3/2)*b*x^(1/2)-2*a*b^2*x^(1/2)*ln(
1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))-2*((a*
x^(1/2)+b)*x^(1/2))^(3/2)*a^(3/2)+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2)*b
^2-b^3*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/
2)))/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/(a*x + b*sqrt(x))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x}}{{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x + b*x^(1/2))^(3/2),x)
```

```
[Out] int(x^(1/2)/(a*x + b*x^(1/2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(x)/(a*x + b*sqrt(x))**(3/2), x)
```

3.127 $\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{4\left(2a\sqrt{x}+b\right)}{b^2\sqrt{ax+b\sqrt{x}}}$$

```
[Out] -4*(b+2*a*x^(1/2))/b^2/(b*x^(1/2)+a*x)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2013, 613}

$$-\frac{4\left(2a\sqrt{x}+b\right)}{b^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(b + 2*a*Sqrt[x]))/(b^2*Sqrt[b*Sqrt[x] + a*x])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\left(bx+ax^2\right)^{3/2}}\,dx,x,\sqrt{x}\right)\\ &= -\frac{4\left(b+2a\sqrt{x}\right)}{b^2\sqrt{b\sqrt{x}+ax}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 1.50

$$-\frac{4\left(2a\sqrt{x}+b\right)\sqrt{ax+b\sqrt{x}}}{ab^2x+b^3\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(b + 2*a*Sqrt[x])*Sqrt[b*Sqrt[x] + a*x])/(b^3*Sqrt[x] + a*b^2*x)
```

fricas [B] time = 1.20, size = 54, normalized size = 1.80

$$\frac{4\left(abx-\left(2a^2x-b^2\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{a^2b^2x^2-b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4*(a*b*x - (2*a^2*x - b^2)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^2*x^2 - b^
4*x)
```

giac [A] time = 0.18, size = 26, normalized size = 0.87

$$-\frac{4\left(\frac{2a\sqrt{x}}{b^2}+\frac{1}{b}\right)}{\sqrt{ax+b\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -4*(2*a*sqrt(x)/b^2 + 1/b)/sqrt(a*x + b*sqrt(x))
```

maple [B] time = 0.06, size = 111, normalized size = 3.70

$$-\frac{4\sqrt{ax+b\sqrt{x}}\left(\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}a^2x-\left(\left(a\sqrt{x}+b\right)\sqrt{x}\right)^{\frac{3}{2}}a^2x+2\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}ab\sqrt{x}+\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}b^2\right)}{\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\left(a\sqrt{x}+b\right)^2b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] -4*(a*x+b*x^(1/2))^(1/2)*((a*x+b*x^(1/2))^(3/2)*x*a^2+2*(a*x+b*x^(1/2))^(3/
2)*x^(1/2)*a*b-((a*x^(1/2)+b)*x^(1/2))^(3/2)*x*a^2+(a*x+b*x^(1/2))^(3/2)*b^
2)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^3/(a*x^(1/2)+b)^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{1}{\sqrt{x}\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^(1/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(sqrt(x)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.128 $\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=107

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{5b^4\sqrt{x}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{5b^3x}-\frac{24\sqrt{ax+b\sqrt{x}}}{5b^2x^{3/2}}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

```
[Out] 4/b/x/(b*x^(1/2)+a*x)^(1/2)-24/5*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(3/2)+32/5*a*(
b*x^(1/2)+a*x)^(1/2)/b^3/x-64/5*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2015, 2016, 2014}

$$-\frac{64a^2\sqrt{ax+b\sqrt{x}}}{5b^4\sqrt{x}}-\frac{24\sqrt{ax+b\sqrt{x}}}{5b^2x^{3/2}}+\frac{32a\sqrt{ax+b\sqrt{x}}}{5b^3x}+\frac{4}{bx\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x*Sqrt[b*Sqrt[x] + a*x]) - (24*Sqrt[b*Sqrt[x] + a*x])/(5*b^2*x^(3/2))
+ (32*a*Sqrt[b*Sqrt[x] + a*x])/(5*b^3*x) - (64*a^2*Sqrt[b*Sqrt[x] + a*x])/(
5*b^4*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx\sqrt{b\sqrt{x}+ax}} + \frac{6\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx\sqrt{b\sqrt{x}+ax}} - \frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}} - \frac{(24a)\int \frac{1}{x^{3/2}\sqrt{b\sqrt{x}+ax}}\,dx}{5b^2}$$

$$= \frac{4}{bx\sqrt{b\sqrt{x}+ax}} - \frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x} + \frac{\left(16a^2\right)\int \frac{1}{x\sqrt{b\sqrt{x}+ax}}\,dx}{5b^3}$$

$$= \frac{4}{bx\sqrt{b\sqrt{x}+ax}} - \frac{24\sqrt{b\sqrt{x}+ax}}{5b^2x^{3/2}} + \frac{32a\sqrt{b\sqrt{x}+ax}}{5b^3x} - \frac{64a^2\sqrt{b\sqrt{x}+ax}}{5b^4\sqrt{x}}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.53

$$-\frac{4\left(16a^3x^{3/2}+8a^2bx-2ab^2\sqrt{x}+b^3\right)}{5b^4x\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
[Out] (-4*(b^3 - 2*a*b^2*Sqrt[x] + 8*a^2*b*x + 16*a^3*x^(3/2)))/(5*b^4*x*Sqrt[b*S
qrt[x] + a*x])
```

fricas [A] time = 1.36, size = 79, normalized size = 0.74

$$\frac{4\left(8\,a^3bx^2-3\,ab^3x-\left(16\,a^4x^2-10\,a^2b^2x-b^4\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{5\left(a^2b^4x^3-b^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
[Out] 4/5*(8*a^3*b*x^2 - 3*a*b^3*x - (16*a^4*x^2 - 10*a^2*b^2*x - b^4)*sqrt(x))*s
qrt(a*x + b*sqrt(x))/(a^2*b^4*x^3 - b^6*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

maple [C] time = 0.06, size = 548, normalized size = 5.12

$$2\sqrt{ax+b\sqrt{x}}\,\left(-5a^5b\,x^{\frac{9}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)+5a^5b\,x^{\frac{9}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right)-10a^4b^2x^4\ln\left(\frac{2a\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 2/5*(a*x+b*x^(1/2))^(1/2)*(5*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*
a^(1/2))/a^(1/2))*x^(9/2)*a^5*b-30*(a*x+b*x^(1/2))^(3/2)*a^(9/2)*x^(7/2)+10
*(a*x+b*x^(1/2))^(1/2)*a^(11/2)*x^(9/2)-5*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/
2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(9/2)*a^5*b+10*a^(11/2)*x^(9/2)*((
a*x^(1/2)+b)*x^(1/2))^(1/2)+10*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2
)*a^(1/2))/a^(1/2))*x^4*a^4*b^2-16*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*x^(5/2)*b^
2-52*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*x^3*b+20*(a*x+b*x^(1/2))^(1/2)*a^(9/2)*x
^4*b-10*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1
/2))*x^4*a^4*b^2+20*a^(9/2)*x^4*((a*x^(1/2)+b)*x^(1/2))^(1/2)*b+10*a^(9/2)*
x^(7/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)+5*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1
/2))^(1/2)*a^(1/2))/a^(1/2))*x^(7/2)*a^3*b^3+4*(a*x+b*x^(1/2))^(3/2)*a^(3/2
)*x^2*b^3+10*(a*x+b*x^(1/2))^(1/2)*a^(7/2)*x^(7/2)*b^2-5*ln(1/2*(2*a*x^(1/2
)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*x^(7/2)*a^3*b^3+10*a^
(7/2)*x^(7/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*b^2-2*(a*x+b*x^(1/2))^(3/2)*a^(
1/2)*x^(3/2)*b^4)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^5/x^(7/2)/a^(1/2)/(a*x^(1
/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{3/2}\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(3/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.129 $\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=165

$$-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{63b^6\sqrt{x}}+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{63b^5x}-\frac{128a^2\sqrt{ax+b\sqrt{x}}}{21b^4x^{3/2}}+\frac{320a\sqrt{ax+b\sqrt{x}}}{63b^3x^2}-\frac{40\sqrt{ax+b\sqrt{x}}}{9b^2x^{5/2}}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

```
[Out] 4/b/x^2/(b*x^(1/2)+a*x)^(1/2)-40/9*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(5/2)+320/63
*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^2-128/21*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x^(3/
2)+512/63*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x-1024/63*a^4*(b*x^(1/2)+a*x)^(1/2)
/b^6/x^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2015, 2016, 2014}

$$-\frac{128a^2\sqrt{ax+b\sqrt{x}}}{21b^4x^{3/2}}-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{63b^6\sqrt{x}}+\frac{512a^3\sqrt{ax+b\sqrt{x}}}{63b^5x}+\frac{320a\sqrt{ax+b\sqrt{x}}}{63b^3x^2}-\frac{40\sqrt{ax+b\sqrt{x}}}{9b^2x^{5/2}}+\frac{4}{bx^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^2*Sqrt[b*Sqrt[x] + a*x]) - (40*Sqrt[b*Sqrt[x] + a*x])/(9*b^2*x^(5/2)
) + (320*a*Sqrt[b*Sqrt[x] + a*x])/(63*b^3*x^2) - (128*a^2*Sqrt[b*Sqrt[x] +
a*x])/(21*b^4*x^(3/2)) + (512*a^3*Sqrt[b*Sqrt[x] + a*x])/(63*b^5*x) - (1024
*a^4*Sqrt[b*Sqrt[x] + a*x])/(63*b^6*Sqrt[x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{5/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx = \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} + \frac{10\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}\,dx}{b}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} - \frac{(80a)\int \frac{1}{x^{5/2}\sqrt{b\sqrt{x}+ax}}\,dx}{9b^2}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} + \frac{\left(160a^2\right)\int \frac{1}{x^2\sqrt{b\sqrt{x}+ax}}\,dx}{21b^3}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} - \frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}} - \frac{(128a}{}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} - \frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}} + \frac{512a^3}{}$$

$$= \frac{4}{bx^2\sqrt{b\sqrt{x}+ax}} - \frac{40\sqrt{b\sqrt{x}+ax}}{9b^2x^{5/2}} + \frac{320a\sqrt{b\sqrt{x}+ax}}{63b^3x^2} - \frac{128a^2\sqrt{b\sqrt{x}+ax}}{21b^4x^{3/2}} + \frac{512a^3}{}$$

Mathematica [A] time = 0.07, size = 83, normalized size = 0.50

$$-\frac{4\left(256a^5x^{5/2}+128a^4bx^2-32a^3b^2x^{3/2}+16a^2b^3x-10ab^4\sqrt{x}+7b^5\right)}{63b^6x^2\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(7*b^5 - 10*a*b^4*Sqrt[x] + 16*a^2*b^3*x - 32*a^3*b^2*x^(3/2) + 128*a^4
*b*x^2 + 256*a^5*x^(5/2)))/(63*b^6*x^2*Sqrt[b*Sqrt[x] + a*x])
```

fricas [A] time = 1.56, size = 101, normalized size = 0.61

$$\frac{4\left(128\,a^5bx^3-48\,a^3b^3x^2-17\,ab^5x-\left(256\,a^6x^3-160\,a^4b^2x^2-26\,a^2b^4x-7\,b^6\right)\sqrt{x}\right)\sqrt{ax+b\sqrt{x}}}{63\left(a^2b^6x^4-b^8x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/63*(128*a^5*b*x^3 - 48*a^3*b^3*x^2 - 17*a*b^5*x - (256*a^6*x^3 - 160*a^4*
b^2*x^2 - 26*a^2*b^4*x - 7*b^6)*sqrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^6*x^4
 - b^8*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

maple [C] time = 0.07, size = 592, normalized size = 3.59

$$4\sqrt{ax+b\sqrt{x}}\left(-63a^7bx^{\frac{13}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\ \sqrt{a}}{2\sqrt{a}}\right)+63a^7bx^{\frac{13}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\ \sqrt{a}}{2\sqrt{a}}\right)-126a^6b^2x^6\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(a*x+b*x^(1/2))^(3/2),x)
[Out] 4/63*(a*x+b*x^(1/2))^(1/2)*(126*x^6*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))
^(1/2)*a^(1/2))/a^(1/2))*a^6*b^2+63*x^(11/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)*
a^(13/2)-16*x^(7/2)*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*b^4-63*x^(11/2)*ln(1/2*(2
*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^5*b^3+63*x
^(11/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^5
*b^3+10*x^3*(a*x+b*x^(1/2))^(3/2)*a^(3/2)*b^5+126*x^(11/2)*((a*x^(1/2)+b)*x
^(1/2))^(1/2)*a^(11/2)*b^2+126*x^(11/2)*(a*x+b*x^(1/2))^(1/2)*a^(11/2)*b^2+
252*x^6*(a*x+b*x^(1/2))^(1/2)*a^(13/2)*b+32*x^4*(a*x+b*x^(1/2))^(3/2)*a^(7/
2)*b^3-508*x^5*(a*x+b*x^(1/2))^(3/2)*a^(11/2)*b-128*x^(9/2)*(a*x+b*x^(1/2))
^(3/2)*a^(9/2)*b^2+252*x^6*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(13/2)*b-7*x^(5/
2)*(a*x+b*x^(1/2))^(3/2)*a^(1/2)*b^6+126*x^(13/2)*((a*x^(1/2)+b)*x^(1/2))^(
1/2)*a^(15/2)-315*x^(11/2)*(a*x+b*x^(1/2))^(3/2)*a^(13/2)+126*x^(13/2)*(a*x
+b*x^(1/2))^(1/2)*a^(15/2)-63*x^(13/2)*ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+
b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^7*b+63*x^(13/2)*ln(1/2*(2*a*x^(1/2)+b
+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^7*b-126*x^6*ln(1/2*(2*a*x^(1/2
)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^6*b^2)/((a*x^(1/2)+
b)*x^(1/2))^(1/2)/b^7/x^(11/2)/a^(1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5/2}\,{\left(a\,x+b\,\sqrt{x}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)),x)
[Out] int(1/(x^(5/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\left(ax+b\sqrt{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**(1/2)+a*x)**(3/2),x)
[Out] Integral(1/(x**(5/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.130 $\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{8192a^6\sqrt{ax+b\sqrt{x}}}{429b^8\sqrt{x}}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{429b^7x}-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{143b^6x^{3/2}}+\frac{2560a^3\sqrt{ax+b\sqrt{x}}}{429b^5x^2}-\frac{2240a^2\sqrt{ax+b\sqrt{x}}}{429b^4x^{5/2}}+$$

```
[Out] 4/b/x^3/(b*x^(1/2)+a*x)^(1/2)-56/13*(b*x^(1/2)+a*x)^(1/2)/b^2/x^(7/2)+672/1
43*a*(b*x^(1/2)+a*x)^(1/2)/b^3/x^3-2240/429*a^2*(b*x^(1/2)+a*x)^(1/2)/b^4/x
^(5/2)+2560/429*a^3*(b*x^(1/2)+a*x)^(1/2)/b^5/x^2-1024/143*a^4*(b*x^(1/2)+a
*x)^(1/2)/b^6/x^(3/2)+4096/429*a^5*(b*x^(1/2)+a*x)^(1/2)/b^7/x-8192/429*a^6
*(b*x^(1/2)+a*x)^(1/2)/b^8/x^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2015, 2016, 2014}

$$-\frac{1024a^4\sqrt{ax+b\sqrt{x}}}{143b^6x^{3/2}}+\frac{2560a^3\sqrt{ax+b\sqrt{x}}}{429b^5x^2}-\frac{2240a^2\sqrt{ax+b\sqrt{x}}}{429b^4x^{5/2}}-\frac{8192a^6\sqrt{ax+b\sqrt{x}}}{429b^8\sqrt{x}}+\frac{4096a^5\sqrt{ax+b\sqrt{x}}}{429b^7x}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] 4/(b*x^3*Sqrt[b*Sqrt[x] + a*x]) - (56*Sqrt[b*Sqrt[x] + a*x])/(13*b^2*x^(7/2
)) + (672*a*Sqrt[b*Sqrt[x] + a*x])/(143*b^3*x^3) - (2240*a^2*Sqrt[b*Sqrt[x]
 + a*x])/(429*b^4*x^(5/2)) + (2560*a^3*Sqrt[b*Sqrt[x] + a*x])/(429*b^5*x^2)
 - (1024*a^4*Sqrt[b*Sqrt[x] + a*x])/(143*b^6*x^(3/2)) + (4096*a^5*Sqrt[b*Sq
rt[x] + a*x])/(429*b^7*x) - (8192*a^6*Sqrt[b*Sqrt[x] + a*x])/(429*b^8*Sqrt[
x])
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{7/2}\left(b\sqrt{x}+ax\right)^{3/2}}dx = \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} + \frac{14\int \frac{1}{x^4\sqrt{b\sqrt{x}+ax}}dx}{b}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} - \frac{(168a)\int \frac{1}{x^{7/2}\sqrt{b\sqrt{x}+ax}}dx}{13b^2}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} + \frac{\left(1680a^2\right)\int \frac{1}{x^3\sqrt{b\sqrt{x}+ax}}dx}{143b^3}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} - \frac{(448}{}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + \frac{256}{}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + \frac{256}{}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + \frac{256}{}$$

$$= \frac{4}{bx^3\sqrt{b\sqrt{x}+ax}} - \frac{56\sqrt{b\sqrt{x}+ax}}{13b^2x^{7/2}} + \frac{672a\sqrt{b\sqrt{x}+ax}}{143b^3x^3} - \frac{2240a^2\sqrt{b\sqrt{x}+ax}}{429b^4x^{5/2}} + \frac{256}{}$$

Mathematica [A] time = 0.07, size = 107, normalized size = 0.48

$$-\frac{4\left(2048a^7x^{7/2}+1024a^6bx^3-256a^5b^2x^{5/2}+128a^4b^3x^2-80a^3b^4x^{3/2}+56a^2b^5x-42ab^6\sqrt{x}+33b^7\right)}{429b^8x^3\sqrt{ax+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*(b*Sqrt[x] + a*x)^(3/2)),x]
```

```
[Out] (-4*(33*b^7 - 42*a*b^6*Sqrt[x] + 56*a^2*b^5*x - 80*a^3*b^4*x^(3/2) + 128*a^
4*b^3*x^2 - 256*a^5*b^2*x^(5/2) + 1024*a^6*b*x^3 + 2048*a^7*x^(7/2)))/(429*
b^8*x^3*Sqrt[b*Sqrt[x] + a*x])
```

fricas [A] time = 1.42, size = 123, normalized size = 0.55

$$\frac{4\left(1024\,a^7bx^4-384\,a^5b^3x^3-136\,a^3b^5x^2-75\,ab^7x-\left(2048\,a^8x^4-1280\,a^6b^2x^3-208\,a^4b^4x^2-98\,a^2b^6x-33\,b^8\right)\sqrt{}\right.}{429\left(a^2b^8x^5-b^{10}x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 4/429*(1024*a^7*b*x^4 - 384*a^5*b^3*x^3 - 136*a^3*b^5*x^2 - 75*a*b^7*x - (2
048*a^8*x^4 - 1280*a^6*b^2*x^3 - 208*a^4*b^4*x^2 - 98*a^2*b^6*x - 33*b^8)*s
qrt(x))*sqrt(a*x + b*sqrt(x))/(a^2*b^8*x^5 - b^10*x^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}} x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

maple [C] time = 0.09, size = 636, normalized size = 2.85

$$2\sqrt{ax + b\sqrt{x}}\,\left(-1287a^9 b\,x^{\frac{17}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{\left(a\sqrt{x}+b\right)\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right) + 1287a^9 b\,x^{\frac{17}{2}}\ln\left(\frac{2a\sqrt{x}+b+2\sqrt{ax+b\sqrt{x}}\,\sqrt{a}}{2\sqrt{a}}\right) - 2574a^8 b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(a*x+b*x^(1/2))^(3/2),x)
```

```
[Out] 2/429*(a*x+b*x^(1/2))^(1/2)*(2574*x^(15/2)*(a*x+b*x^(1/2))^(1/2)*a^(15/2)*b
^2+160*x^5*(a*x+b*x^(1/2))^(3/2)*a^(7/2)*b^5-2048*x^(13/2)*(a*x+b*x^(1/2))^
(3/2)*a^(13/2)*b^2+5148*x^8*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(17/2)*b-9244*x
^7*(a*x+b*x^(1/2))^(3/2)*a^(15/2)*b+5148*x^8*(a*x+b*x^(1/2))^(1/2)*a^(17/2)
*b-256*x^(11/2)*(a*x+b*x^(1/2))^(3/2)*a^(9/2)*b^4+512*x^6*(a*x+b*x^(1/2))^(
3/2)*a^(11/2)*b^3+2574*x^(15/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(15/2)*b^2+
2574*x^(17/2)*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(19/2)-66*x^(7/2)*(a*x+b*x^(1
/2))^(3/2)*a^(1/2)*b^8-6006*x^(15/2)*(a*x+b*x^(1/2))^(3/2)*a^(17/2)+2574*x^
(17/2)*(a*x+b*x^(1/2))^(1/2)*a^(19/2)-1287*x^(17/2)*ln(1/2*(2*a*x^(1/2)+b+2
*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^9*b+1287*x^(17/2)*ln(1/2
*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^9*b-2574*x^8*ln
(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^8*b
^2+2574*x^8*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))
*a^8*b^2+858*x^(15/2)*((a*x^(1/2)+b)*x^(1/2))^(3/2)*a^(17/2)-1287*x^(15/2)*
ln(1/2*(2*a*x^(1/2)+b+2*((a*x^(1/2)+b)*x^(1/2))^(1/2)*a^(1/2))/a^(1/2))*a^7
*b^3+1287*x^(15/2)*ln(1/2*(2*a*x^(1/2)+b+2*(a*x+b*x^(1/2))^(1/2)*a^(1/2))/a
^(1/2))*a^7*b^3-112*x^(9/2)*(a*x+b*x^(1/2))^(3/2)*a^(5/2)*b^6+84*x^4*(a*x+b
*x^(1/2))^(3/2)*a^(3/2)*b^7)/((a*x^(1/2)+b)*x^(1/2))^(1/2)/b^9/x^(15/2)/a^(
1/2)/(a*x^(1/2)+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt{x}\right)^{\frac{3}{2}} x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^(1/2)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*sqrt(x))^(3/2)*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{7/2}\,\left(a\,x+b\,\sqrt{x}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a*x + b*x^(1/2))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}} \left(ax + b\sqrt{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**(1/2)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**(7/2)*(a*x + b*sqrt(x))**(3/2)), x)
```

3.131 $\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal. Leaf size=301

$$\frac{442b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14421a^{25/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{884b^6\sqrt{ax+b\sqrt[3]{x}}}{14421a^6}+\frac{884b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{24035a^5}$$

```
[Out] -884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^6+884/24035*b^5*x^(2/3)*(b*x^(1/3)+a
*x)^(1/2)/a^5-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+476/19665*b
^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^3-28/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/
a^2+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/9*x^4*(b*x^(1/3)+a*x)^(1/2)+
442/14421*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(25/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2021, 2024, 2011, 329, 220}

$$\frac{884b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{24035a^5}-\frac{6188b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^4}+\frac{476b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{19665a^3}-\frac{28b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a^2}+\frac{442b^{27/4}\sqrt[6]{x}\left(\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-884*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^6) + (884*b^5*x^(2/3)*Sqrt[b*x^(1
/3) + a*x])/(24035*a^5) - (6188*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(216315*
a^4) + (476*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^3) - (28*b^2*x^(8/3)*Sq
rt[b*x^(1/3) + a*x])/(1311*a^2) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(207
*a) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/9 + (442*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^
(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Elliptic
F[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(25/4)*Sqrt[b*x^(1/3)
 + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int x^3\sqrt{b\sqrt[3]{x}+ax}\,dx = 3\,\text{Subst}\left(\int x^{11}\sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax} + \frac{1}{9}(2b)\,\text{Subst}\left(\int \frac{x^{12}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a} + \frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax} - \frac{(14b^2)\,\text{Subst}\left(\int \frac{x^{10}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{69a}$$
$$= -\frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2} + \frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a} + \frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+ax} + \frac{(238b^3)\,\text{Subst}\left(\int\right.}{131}$$
$$= \frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3} - \frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2} + \frac{4bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a} + \frac{2}{9}x^4\sqrt{b\sqrt[3]{x}+a}$$
$$= -\frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4} + \frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3} - \frac{28b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^2} + \frac{4bx^{10/3}\sqrt{b}}{20}$$
$$= \frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5} - \frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4} + \frac{476b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^3} - \frac{28b^2x^{8/3}\sqrt{}}{131}$$
$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6} + \frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5} - \frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4} + \frac{476b^3x^2\sqrt{b}}{1966}$$
$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6} + \frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5} - \frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4} + \frac{476b^3x^2\sqrt{b}}{1966}$$
$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6} + \frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5} - \frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4} + \frac{476b^3x^2\sqrt{b}}{1966}$$
$$= -\frac{884b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^6} + \frac{884b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{24035a^5} - \frac{6188b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^4} + \frac{476b^3x^2\sqrt{b}}{1966}$$

Mathematica [C] time = 0.18, size = 155, normalized size = 0.51

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(\sqrt{\frac{ax^{2/3}}{b}+1}\left(24035a^6x^4+2090a^5bx^{10/3}-2310a^4b^2x^{8/3}+2618a^3b^3x^2-3094a^2b^4x^{4/3}+3978ab\right.\right.}{216315a^6\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-9945*b^6 + 3978*a*b^5*x
^(2/3) - 3094*a^2*b^4*x^(4/3) + 2618*a^3*b^3*x^2 - 2310*a^4*b^2*x^(8/3) + 2
090*a^5*b*x^(10/3) + 24035*a^6*x^4) + 9945*b^6*Hypergeometric2F1[-1/2, 1/4,
 5/4, -((a*x^(2/3))/b)]))/(216315*a^6*Sqrt[1 + (a*x^(2/3))/b])
```

fricas [F] time = 2.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ax+bx^{\frac{1}{3}}}\,x^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))*x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{1}{3}}}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

maple [A] time = 0.12, size = 264, normalized size = 0.88

$$\frac{2\sqrt{ax+b\,x^{\frac{1}{3}}}\,x^4}{9}+\frac{4\sqrt{ax+b\,x^{\frac{1}{3}}}\,b\,x^{\frac{10}{3}}}{207a}-\frac{28\sqrt{ax+b\,x^{\frac{1}{3}}}\,b^2x^{\frac{8}{3}}}{1311a^2}+\frac{476\sqrt{ax+b\,x^{\frac{1}{3}}}\,b^3x^2}{19665a^3}-\frac{6188\sqrt{ax+b\,x^{\frac{1}{3}}}\,b^4x^{\frac{4}{3}}}{216315a^4}+\frac{442\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^(1/3)+a*x)^(1/2),x)
```

```
[Out] 2/9*x^4*(b*x^(1/3)+a*x)^(1/2)+4/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a-28/1
311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+476/19665*b^3*x^2*(b*x^(1/3)+a*x)
^(1/2)/a^3-6188/216315*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+884/24035*b^5*
x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-884/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^6+4
42/14421*b^7/a^7*(-a*b)^(1/2)*((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/
2)*(-2*(x^(1/3)-(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-x^(1/3)/(-a*b)^(1/2
)*a)^(1/2)/(b*x^(1/3)+a*x)^(1/2)*EllipticF(((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)
^(1/2)*a)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{1}{3}}}\,x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x^3*(a*x + b*x^(1/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{ax+b\sqrt[3]{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*x + b*x**(1/3)), x)
```

3.132 $\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal. Leaf size=411

$$\frac{22b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{19/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{44b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E}{221a^{19/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 44/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(9/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/
3)+a*x)^(1/2)-44/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+220/4641*b^3*x*(
b*x^(1/3)+a*x)^(1/2)/a^3-60/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+4/11
9*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/7*x^3*(b*x^(1/3)+a*x)^(1/2)-44/221*b^
(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^
(1/2))^2)^(1/2)/a^(19/4)/(b*x^(1/3)+a*x)^(1/2)+22/221*b^(21/4)*x^(1/6)*(cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(
1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(
19/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2018, 2021, 2024, 2032, 329, 305, 220, 1196}

$$\frac{44b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{9/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{60b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^2}+\frac{22b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan\right.}{221a^{19/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (44*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(9/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*S
qrt[b*x^(1/3) + a*x]) - (44*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^4) +
(220*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^3) - (60*b^2*x^(5/3)*Sqrt[b*x^(1/
3) + a*x])/(1547*a^2) + (4*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a) + (2*x^
3*Sqrt[b*x^(1/3) + a*x])/7 - (44*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[
(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(
a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(19/4)*Sqrt[b*x^(1/3) + a*x]) + (22
*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[
a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(221*a^(19/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x^2\sqrt{b\sqrt[3]{x}+ax}\,dx = 3\,\text{Subst}\left(\int x^8\sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{7}(2b)\,\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$
$$= \frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax}-\frac{(30b^2)\,\text{Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{119a}$$
$$= -\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax}+\frac{(330b^3)\,\text{Subst}\left(\int \ldots\right.}{1547\ldots}$$
$$= \frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a}+\frac{2}{7}x^3\sqrt{b\sqrt[3]{x}+ax}$$
$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}}}{119a}$$
$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}}}{119a}$$
$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}}}{119a}$$
$$= -\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-\frac{60b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^2}+\frac{4bx^{7/3}\sqrt{b\sqrt[3]{x}}}{119a}$$
$$= \frac{44b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{9/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}-\frac{44b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^4}+\frac{220b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^3}-$$

Mathematica [C] time = 0.13, size = 136, normalized size = 0.33

$$\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\left(\sqrt{\frac{ax^{2/3}}{b}+1}\left(663a^4x^{8/3}+78a^3bx^2-90a^2b^2x^{4/3}+110ab^3x^{2/3}-385b^4\right)+385b^4\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4}\ldots\right.\right.}{4641a^4\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(-385*b^4 + 110*a
*b^3*x^(2/3) - 90*a^2*b^2*x^(4/3) + 78*a^3*b*x^2 + 663*a^4*x^(8/3)) + 385*b
^4*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(4641*a^4*Sqrt[1 +
 (a*x^(2/3))/b])
```

fricas [F] time = 7.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ax+bx^{\frac{1}{3}}}\,x^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))*x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + bx^{\frac{1}{3}}}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

maple [A] time = 0.07, size = 273, normalized size = 0.66

$$\frac{2\sqrt{ax+b\,x^{\frac{1}{3}}}\;x^3}{7}+\frac{4\sqrt{ax+b\,x^{\frac{1}{3}}}\;b\,x^{\frac{7}{3}}}{119a}-\frac{60\sqrt{ax+b\,x^{\frac{1}{3}}}\;b^2x^{\frac{5}{3}}}{1547a^2}+\frac{220\sqrt{ax+b\,x^{\frac{1}{3}}}\;b^3x}{4641a^3}-\frac{44\sqrt{ax+b\,x^{\frac{1}{3}}}\;b^4x^{\frac{1}{3}}}{663a^4}+\frac{22\sqrt{-ab}\;\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] 2/7*x^3*(a*x+b*x^(1/3))^(1/2)+4/119*b*x^(7/3)*(a*x+b*x^(1/3))^(1/2)/a-60/15
47*b^2*x^(5/3)*(a*x+b*x^(1/3))^(1/2)/a^2+220/4641*b^3*x*(a*x+b*x^(1/3))^(1/
2)/a^3-44/663*b^4*x^(1/3)*(a*x+b*x^(1/3))^(1/2)/a^4+22/221*b^5/a^5*(-a*b)^(
1/2)*((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-2*(x^(1/3)-(-a*b)^(1
/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)/(a*x+b*x^(1/
3))^(1/2)*(-2*(-a*b)^(1/2)/a*EllipticE(((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/
2)*a)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/a*EllipticF(((x^(1/3)+(-a*b)^(1/2)/a)
/(-a*b)^(1/2)*a)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + bx^{\frac{1}{3}}}\, x^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x^2*(a*x + b*x^(1/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \sqrt{a x + b \sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*x + b*x**(1/3)), x)
```

3.133 $\int x\sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal. Leaf size=213

$$-\frac{6b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{13/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{12b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^3}-\frac{36b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^2}+\frac{4bx^{4}}{}$$

```
[Out] 12/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^3-36/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2
)/a^2+4/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/5*x^2*(b*x^(1/3)+a*x)^(1/2)-
6/77*b^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(
2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(
1/2)+b^(1/2))^2)^(1/2)/a^(13/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2018, 2021, 2024, 2011, 329, 220}

$$-\frac{36b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^2}-\frac{6b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{13/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{12b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^3}+\frac{4bx^{4}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (12*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^3) - (36*b^2*x^(2/3)*Sqrt[b*x^(1/3) +
a*x])/(385*a^2) + (4*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(55*a) + (2*x^2*Sqrt[
b*x^(1/3) + a*x])/5 - (6*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x
^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*
x^(1/6))/b^(1/4)], 1/2])/(77*a^(13/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int x\sqrt{b\sqrt[3]{x}+ax}\,dx = 3\,\text{Subst}\left(\int x^5\sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} + \frac{1}{5}(2b)\,\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{(18b^2)\,\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{55a}$$

$$= -\frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} + \frac{(18b^3)\,\text{Subst}\left(\int \frac{\ }{\sqrt{bx}}\right.}{77a^2}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{(6}{}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{(6}{}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{(}{}$$

$$= \frac{12b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^3} - \frac{36b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^2} + \frac{4bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a} + \frac{2}{5}x^2\sqrt{b\sqrt[3]{x}+ax} - \frac{6}{}$$

Mathematica [C] time = 0.10, size = 118, normalized size = 0.55

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(\sqrt{\frac{ax^{2/3}}{b}+1}\left(77a^3x^2+14a^2bx^{4/3}-18ab^2x^{2/3}+45b^3\right)-45b^3\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)\right)}{385a^3\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(Sqrt[1 + (a*x^(2/3))/b]*(45*b^3 - 18*a*b^2*x^(2/3
) + 14*a^2*b*x^(4/3) + 77*a^3*x^2) - 45*b^3*Hypergeometric2F1[-1/2, 1/4, 5/
4, -((a*x^(2/3))/b)]))/(385*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

fricas [F] time = 1.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ax+bx^{\frac{1}{3}}}\,x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))*x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{1}{3}}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

maple [A] time = 0.08, size = 198, normalized size = 0.93

$$\frac{2\sqrt{ax+b\,x^{\frac{1}{3}}}\;x^{2}}{5}+\frac{4\sqrt{ax+b\,x^{\frac{1}{3}}}\;b\,x^{\frac{4}{3}}}{55a}-\frac{6\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\;b^{4}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\right.}{77\sqrt{ax+b\,x^{\frac{1}{3}}}\;a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] 2/5*x^2*(a*x+b*x^(1/3))^(1/2)+4/55*b*x^(4/3)*(a*x+b*x^(1/3))^(1/2)/a-36/385
*b^2*x^(2/3)*(a*x+b*x^(1/3))^(1/2)/a^2+12/77*b^3*(a*x+b*x^(1/3))^(1/2)/a^3-
6/77/a^4*b^4*(-a*b)^(1/2)*((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(
-2*(x^(1/3)-(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3
))^(1/2)/(a*x+b*x^(1/3))^(1/2)*EllipticF(((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(
1/2)*a)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{1}{3}}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{a\,x+b\,x^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^(1/3))^(1/2),x)
[Out] int(x*(a*x + b*x^(1/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{ax + b\sqrt[3]{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(x*sqrt(a*x + b*x**(1/3)), x)
```

3.134 $\int \sqrt{b\sqrt[3]{x}+ax}\,dx$

Optimal. Leaf size=323

$$-\frac{2b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan\right.}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] -4/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(3/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)
+a*x)^(1/2)+4/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a+2/3*x*(b*x^(1/3)+a*x)^(1
/2)+4/5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/co
s(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)
/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a
^(1/2)+b^(1/2))^2)^(1/2)/a^(7/4)/(b*x^(1/3)+a*x)^(1/2)-2/5*b^(9/4)*x^(1/6)*
(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*
(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)
/a^(7/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {2004, 2018, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{4b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{3/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{2b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{7/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-4*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(3/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a) + (2*x*Sqr
t[b*x^(1/3) + a*x])/3 + (4*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*
x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)
*x^(1/6))/b^(1/4)], 1/2])/(5*a^(7/4)*Sqrt[b*x^(1/3) + a*x]) - (2*b^(9/4)*(S
qrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^
2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(7/4)*
Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt{b\sqrt[3]{x}+ax}\,dx = \frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{9}(2b)\int \frac{\sqrt[3]{x}}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$$

$$= \frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{1}{3}(2b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{(2b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5a}$$

$$= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(2b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{5a\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(4b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5a\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}-\frac{\left(4b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5a^{3/2}\sqrt{b\sqrt[3]{x}+ax}} -$$

$$= -\frac{4b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{3/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{4b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a}+\frac{2}{3}x\sqrt{b\sqrt[3]{x}+ax}+\frac{4b^{9/4}\left(\sqrt{b}+\right.}{}$$

Mathematica [C] time = 0.06, size = 94, normalized size = 0.29

$$\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\left(\left(ax^{2/3}+b\right)\sqrt{\frac{ax^{2/3}}{b}+1}-b\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{3a\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))*Sqrt[1 + (a*x^(2/3))/b] -
 b*Hypergeometric2F1[-1/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(3*a*Sqrt[1 + (a*x
^(2/3))/b])
```

fricas [F] time = 8.10, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ax+bx^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3)), x)
```

maple [A] time = 0.07, size = 207, normalized size = 0.64

$$\frac{2\sqrt{ax+b\,x^{\frac{1}{3}}}\;x}{3}+\frac{4\sqrt{ax+b\,x^{\frac{1}{3}}}\;b\,x^{\frac{1}{3}}}{15a}-\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\mathrm{EllipticE}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{\cdots}}{\cdots}\right)\cdots}{\sqrt{-\cdots}}}\cdots\right)}{a}\cdots\right.}{5\sqrt{ax+b\,x^{\frac{1}{3}}}\;a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] 2/3*x*(a*x+b*x^(1/3))^(1/2)+4/15*b*x^(1/3)*(a*x+b*x^(1/3))^(1/2)/a-2/5*b^2/
a^2*(-a*b)^(1/2)*((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-2*(x^(1/
3)-(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)/
(a*x+b*x^(1/3))^(1/2)*(-2*(-a*b)^(1/2)/a*EllipticE(((x^(1/3)+(-a*b)^(1/2)/a
)/(-a*b)^(1/2)*a)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/a*EllipticF(((x^(1/3)+(-a
*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ax+bx^{\frac{1}{3}}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3)), x)
```

mupad [B] time = 5.18, size = 40, normalized size = 0.12

$$\frac{6\,x\,\sqrt{a\,x+b\,x^{1/3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{7}{4};\;\frac{11}{4};\;-\frac{a\,x^{2/3}}{b}\right)}{7\,\sqrt{\frac{a\,x^{2/3}}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] (6*x*(a*x + b*x^(1/3))^(1/2)*hypergeom([-1/2, 7/4], 11/4, -(a*x^(2/3))/b))/
(7*((a*x^(2/3))/b + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{ax+b\sqrt[3]{x}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3)), x)
```

3.135 $\int \frac{\sqrt{b \sqrt[3]{x}+ax}}{x}\,dx$

Optimal. Leaf size=123

$$\frac{2b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}}+2\sqrt{ax+b\sqrt[3]{x}}$$

```
[Out] 2*(b*x^(1/3)+a*x)^(1/2)+2*b^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x
^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(1/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 2021, 2011, 329, 220}

$$\frac{2b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\sqrt{ax+b\sqrt[3]{x}}}+2\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x,x]
```

```
[Out] 2*Sqrt[b*x^(1/3) + a*x] + (2*b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b +
a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/
4)*x^(1/6))/b^(1/4)], 1/2])/(a^(1/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x}\,dx &= 3\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x}\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{b\sqrt[3]{x}+ax} + (2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\
&= 2\sqrt{b\sqrt[3]{x}+ax} + \frac{\left(2b\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}\\
&= 2\sqrt{b\sqrt[3]{x}+ax} + \frac{\left(4b\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}\\
&= 2\sqrt{b\sqrt[3]{x}+ax} + \frac{2b^{3/4}\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)^2}}\,\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt[4]{a}\,\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 54, normalized size = 0.44

$$\frac{6\sqrt{ax+b\sqrt[3]{x}}\;{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)}{\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x,x]
[Out] (6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-1/2, 1/4, 5/4, -((a*x^(2/3))/b)
])/Sqrt[1 + (a*x^(2/3))/b]
```

fricas [F] time = 1.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="fricas")
[Out] integral(sqrt(a*x + b*x^(1/3))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

maple [A] time = 0.06, size = 132, normalized size = 1.07

$$\frac{2\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\;b\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{\sqrt{ax+b\,x^{\frac{1}{3}}}\;a}+2\sqrt{ax+b\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(1/2)/x,x)
[Out] 2*(a*x+b*x^(1/3))^(1/2)+2*b*(-a*b)^(1/2)/a*((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)
^(1/2)*a)^(1/2)*(-2*(x^(1/3)-(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-1/(-a*
b)^(1/2)*a*x^(1/3))^(1/2)/(a*x+b*x^(1/3))^(1/2)*EllipticF(((x^(1/3)+(-a*b)^
(1/2)/a)/(-a*b)^(1/2)*a)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2)/x,x)
[Out] int((a*x + b*x^(1/3))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x,x)
[Out] Integral(sqrt(a*x + b*x**(1/3))/x, x)
```

3.136 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx$

Optimal. Leaf size=325

$$\frac{6a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{12a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\right.}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 12/5*a^(3/2)*(b+a*x^(2/3))*x^(1/3)/b/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a
*x)^(1/2)-6/5*(b*x^(1/3)+a*x)^(1/2)/x-12/5*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3
)-12/5*a^(5/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)+6/5*a^(5/4)*x^(1/6)*(
cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6
)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(
x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/
b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{6a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{12a^{5/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\right.}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

```
[Out] (12*a^(3/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[
b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(5*x) - (12*a*Sqrt[b*x^(1/3)
+ a*x])/(5*b*x^(1/3)) - (12*a^(5/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a
*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4
)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (6*a^(5/4)*(
Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))
^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)
*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^4}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} + \frac{1}{5}(6a)\,\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{(6a^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5b}$$
$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{\left(6a^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{5b\sqrt{b\sqrt[3]{x}+ax}}$$
$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{\left(12a^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{5b\sqrt{b\sqrt[3]{x}+ax}}$$
$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} + \frac{\left(12a^{3/2}\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right.}{5\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}}$$
$$= \frac{12a^{3/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{12a^{5/4}\left(\sqrt{b}+\sqrt{a}\right.}{}$$

Mathematica [C] time = 0.07, size = 59, normalized size = 0.18

$$-\frac{6\sqrt{ax+b\sqrt[3]{x}}\,{}_2F_1\left(-\frac{5}{4},-\frac{1}{2};-\frac{1}{4};-\frac{ax^{2/3}}{b}\right)}{5x\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^2,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-5/4, -1/2, -1/4, -((a*x^(2/3))
/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x)
```

fricas [F] time = 7.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 3.28-6/5/x^(1/3)*sqrt(a/x^(1/3)+b/x)+integrate(24*a/20/
3/(x^(1/3)*x^(1/3)*sqrt(x)*sqrt(a*(x^(1/3))^2+b)*sign(x)),x)
```

maple [A] time = 0.10, size = 213, normalized size = 0.66

$$-\frac{12\left(a\,x^{\frac{2}{3}}+b\right)a}{5\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;b}+\frac{6\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\;\mathrm{EllipticE}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a}+\frac{\sqrt{-ab}\;\cdots}{\cdots}\right)}{5\sqrt{ax+b\,x^{\frac{1}{3}}}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(1/2)/x^2,x)
```

```
[Out] -6/5*(a*x+b*x^(1/3))^(1/2)/x-12/5*(b+a*x^(2/3))*a/b/(x^(1/3)*(b+a*x^(2/3)))
^(1/2)+6/5*a/b*(-a*b)^(1/2)*((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)
*(-2*(x^(1/3)-(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1
/3))^(1/2)/(a*x+b*x^(1/3))^(1/2)*(-2*(-a*b)^(1/2)/a*EllipticE(((x^(1/3)+(-a
*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/a*EllipticF(((
x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\sqrt{a\,x+b\,x^{1/3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^2,x)
```

```
[Out] int((a*x + b*x^(1/3))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax+b\sqrt[3]{x}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**2, x)
```

3.137 $\int \frac{\sqrt{b \sqrt[3]{x}+ax}}{x^3} dx$

Optimal. Leaf size=188

$$\frac{10a^{11/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{20a^2\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{77bx^{4/3}}-\frac{6\sqrt{ax}}{1}$$

```
[Out] -6/11*(b*x^(1/3)+a*x)^(1/2)/x^2-12/77*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+20/
77*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)+10/77*a^(11/4)*x^(1/6)*(cos(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
)*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^
(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b
*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$\frac{20a^2\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}+\frac{10a^{11/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{12a\sqrt{ax+b\sqrt[3]{x}}}{77bx^{4/3}}-\frac{6\sqrt{ax}}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(11*x^2) - (12*a*Sqrt[b*x^(1/3) + a*x])/(77*b*x^
(4/3)) + (20*a^2*Sqrt[b*x^(1/3) + a*x])/(77*b^2*x^(2/3)) + (10*a^(11/4)*(Sq
rt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2
]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*b^(9/4)*
Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} + \frac{1}{11}(6a)\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} - \frac{\left(30a^2\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{77b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{\left(10a^3\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{\ldots}\right.}{77b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{\left(10a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \ldots\right.}{77b^2\sqrt{b\sqrt[3]{x}+\ldots}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{\left(20a^3\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \ldots\right.}{77b^2\sqrt{b\sqrt[3]{x}+a\ldots}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{11x^2} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{77bx^{4/3}} + \frac{20a^2\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} + \frac{10a^{11/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax\ldots}{\left(\sqrt{b}+\sqrt{a}\ldots\right.}}}{77b^{9/4}\sqrt{b\ldots}}$$

Mathematica [C] time = 0.05, size = 59, normalized size = 0.31

$$-\frac{6\sqrt{ax+b\sqrt[3]{x}}\;{}_2F_1\left(-\frac{11}{4},-\frac{1}{2};-\frac{7}{4};-\frac{ax^{2/3}}{b}\right)}{11x^2\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^3,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-11/4, -1/2, -7/4, -((a*x^(2/3)
)/b)])/(11*Sqrt[1 + (a*x^(2/3))/b]*x^2)
```

fricas [F] time = 1.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 3.256*((-420*b^3/4620/b^3/x^(1/3)/x^(1/3)-120*b^2*a/462
0/b^3)/x^(1/3)/x^(1/3)+200*b*a^2/4620/b^3)*sqrt(a/x^(1/3)+b/x)+integrate(60
0*b*a^3/4620/b^3/3/((x^(1/6))^5*sqrt(a*(x^(1/3))^2+b)*sign(x)),x)
```

maple [A] time = 0.08, size = 179, normalized size = 0.95

$$\frac{10\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\;a^2\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{77\sqrt{ax+b\,x^{\frac{1}{3}}}\;b^2}+\frac{20\sqrt{ax+b\,x^{\frac{1}{3}}}\;a^2}{77b^2x^{\frac{2}{3}}}-\frac{12\sqrt{ax}}{77}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(1/2)/x^3,x)
```

```
[Out] -6/11*(a*x+b*x^(1/3))^(1/2)/x^2-12/77*a*(a*x+b*x^(1/3))^(1/2)/b/x^(4/3)+20/
77*a^2*(a*x+b*x^(1/3))^(1/2)/b^2/x^(2/3)+10/77*a^2/b^2*(-a*b)^(1/2)*((x^(1/
3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-2*(x^(1/3)-(-a*b)^(1/2)/a)/(-a*b
)^(1/2)*a)^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)/(a*x+b*x^(1/3))^(1/2)*El
lipticF(((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^{\frac{1}{3}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(1/3))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+b\sqrt[3]{x}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**3, x)
```

3.138 $\int \frac{\sqrt{b \sqrt[3]{x}+ax}}{x^4}\, dx$

Optimal. Leaf size=413

$$-\frac{154a^{17/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{308a^{17/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] -308/1105*a^(9/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^
(1/3)+a*x)^(1/2)-6/17*(b*x^(1/3)+a*x)^(1/2)/x^3-12/221*a*(b*x^(1/3)+a*x)^(1
/2)/b/x^(7/3)+44/663*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(5/3)-308/3315*a^3*(b*
x^(1/3)+a*x)^(1/2)/b^3/x+308/1105*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)+308
/1105*a^(17/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)-154/1105*a^(17/4)*x^
(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)
*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(
1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)
^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{308a^{9/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{44a^2\sqrt{ax+b\sqrt[3]{x}}}{663b^2x^{5/3}}-\frac{154a^{17/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan\right.}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

```
[Out] (-308*a^(9/2)*(b + a*x^(2/3))*x^(1/3))/(1105*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(17*x^3) - (12*a*Sqrt[
b*x^(1/3) + a*x])/(221*b*x^(7/3)) + (44*a^2*Sqrt[b*x^(1/3) + a*x])/(663*b^2
*x^(5/3)) - (308*a^3*Sqrt[b*x^(1/3) + a*x])/(3315*b^3*x) + (308*a^4*Sqrt[b*
x^(1/3) + a*x])/(1105*b^4*x^(1/3)) + (308*a^(17/4)*(Sqrt[b] + Sqrt[a]*x^(1/
3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2
*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a
*x]) - (154*a^(17/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt
[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1
/4)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} + \frac{1}{17}(6a)\,\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} - \frac{(66a^2)\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{221b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} + \frac{(154a^3)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,d\right.}{663b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} - \frac{(154a^4)}{}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{}}{110}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{}}{110}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{}}{110}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}+ax}}{663b^2x^{5/3}} - \frac{308a^3\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{308a^4\sqrt{}}{110}$$

$$= -\frac{308a^{9/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{17x^3} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{221bx^{7/3}} + \frac{44a^2\sqrt{b\sqrt[3]{x}}}{663b^2x}$$

Mathematica [C] time = 0.05, size = 59, normalized size = 0.14

$$-\frac{6\sqrt{ax+b\sqrt[3]{x}}\;{}_2F_1\left(-\frac{17}{4},-\frac{1}{2};-\frac{13}{4};-\frac{ax^{2/3}}{b}\right)}{17x^3\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^4,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-17/4, -1/2, -13/4, -((a*x^(2/3
))/b)])/(17*Sqrt[1 + (a*x^(2/3))/b]*x^3)
```

fricas [F] time = 8.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 3.256*(((-540540*b^6/9189180/b^6/x^(1/3)/x^(1/3)-83160*
b^5*a/9189180/b^6)/x^(1/3)/x^(1/3)+101640*b^4*a^2/9189180/b^6)/x^(1/3)/x^(1
/3)-142296*b^3*a^3/9189180/b^6)/x^(1/3)*sqrt(a/x^(1/3)+b/x)+integrate(-1280
664*b^3*a^4/9189180/b^6/3/(x^(1/3)*x^(1/3)*sqrt(x)*sqrt(a*(x^(1/3))^2+b)*si
gn(x)),x)
```

maple [A] time = 0.08, size = 281, normalized size = 0.68

$$\frac{308\left(a x^{\frac{2}{3}}+b\right)a^{4}}{1105\sqrt{\left(a x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\, b^{4}}-\frac{154\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{a x^{\frac{1}{3}}}{\sqrt{-ab}}}\left(-\frac{2\sqrt{-ab}\,\mathrm{EllipticE}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{a}+\cdots\right)}{1105\sqrt{ax+b x^{\frac{1}{3}}}\, b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(1/2)/x^4,x)
```

```
[Out] -6/17*(a*x+b*x^(1/3))^(1/2)/x^3-12/221*a*(a*x+b*x^(1/3))^(1/2)/b/x^(7/3)+44
/663*a^2*(a*x+b*x^(1/3))^(1/2)/b^2/x^(5/3)-308/3315*a^3*(a*x+b*x^(1/3))^(1/
2)/b^3/x+308/1105*(a*x^(2/3)+b)*a^4/b^4/((a*x^(2/3)+b)*x^(1/3))^(1/2)-154/1
105*a^4/b^4*(-a*b)^(1/2)*((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-
2*(x^(1/3)-(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3)
)^(1/2)/(a*x+b*x^(1/3))^(1/2)*(-2*(-a*b)^(1/2)/a*EllipticE(((x^(1/3)+(-a*b)
^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2),1/2*2^(1/2))+(-a*b)^(1/2)/a*EllipticF(((x^(
1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2),1/2*2^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^4,x)
```

```
[Out] int((a*x + b*x^(1/3))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**4, x)
```

3.139 $\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx$

Optimal. Leaf size=276

$$-\frac{1326a^{23/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{33649b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{2652a^5\sqrt{ax+b\sqrt[3]{x}}}{33649b^5x^{2/3}}+\frac{7956a^4\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}-$$

```
[Out] -6/23*(b*x^(1/3)+a*x)^(1/2)/x^4-12/437*a*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+6
8/2185*a^2*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)-884/24035*a^3*(b*x^(1/3)+a*x)^
(1/2)/b^3/x^2+7956/168245*a^4*(b*x^(1/3)+a*x)^(1/2)/b^4/x^(4/3)-2652/33649*
a^5*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-1326/33649*a^(23/4)*x^(1/6)*(cos(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4
)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*
a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(21/4)
/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$-\frac{2652a^5\sqrt{ax+b\sqrt[3]{x}}}{33649b^5x^{2/3}}+\frac{7956a^4\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}-\frac{884a^3\sqrt{ax+b\sqrt[3]{x}}}{24035b^3x^2}+\frac{68a^2\sqrt{ax+b\sqrt[3]{x}}}{2185b^2x^{8/3}}-\frac{1326a^{23/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right.}{33649}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(23*x^4) - (12*a*Sqrt[b*x^(1/3) + a*x])/(437*b*x
^(10/3)) + (68*a^2*Sqrt[b*x^(1/3) + a*x])/(2185*b^2*x^(8/3)) - (884*a^3*Sqr
t[b*x^(1/3) + a*x])/(24035*b^3*x^2) + (7956*a^4*Sqrt[b*x^(1/3) + a*x])/(168
245*b^4*x^(4/3)) - (2652*a^5*Sqrt[b*x^(1/3) + a*x])/(33649*b^5*x^(2/3)) - (
1326*a^(23/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + S
qrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1
/2])/(33649*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{b\sqrt[3]{x}+ax}}{x^5}\,dx = 3\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} + \frac{1}{23}(6a)\,\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} - \frac{(102a^2)\,\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{437b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} + \frac{(442a^3)\,\text{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x\right.}{2185b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} - (3978a^4)\,\text{S}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b}}{168245}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b}}{168245}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b}}{168245}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b}}{168245}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{23x^4} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{437bx^{10/3}} + \frac{68a^2\sqrt{b\sqrt[3]{x}+ax}}{2185b^2x^{8/3}} - \frac{884a^3\sqrt{b\sqrt[3]{x}+ax}}{24035b^3x^2} + \frac{7956a^4\sqrt{b}}{168245}$$

Mathematica [C] time = 0.06, size = 59, normalized size = 0.21

$$-\frac{6\sqrt{ax+b\sqrt[3]{x}}\;{}_2F_1\left(-\frac{23}{4},-\frac{1}{2};-\frac{19}{4};-\frac{ax^{2/3}}{b}\right)}{23x^4\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(1/3) + a*x]/x^5,x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-23/4, -1/2, -19/4, -((a*x^(2/3
))/b)])/(23*Sqrt[1 + (a*x^(2/3))/b]*x^4)
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*x + b*x^(1/3))/x^5, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 3.26*(((((-2618916300*b^9/60235074900/b^9/x^(1/3)/x^(1/
3)-275675400*b^8*a/60235074900/b^9)/x^(1/3)/x^(1/3)+312432120*b^7*a^2/60235
074900/b^9)/x^(1/3)/x^(1/3)-369237960*b^6*a^3/60235074900/b^9)/x^(1/3)/x^(1
/3)+474734520*b^5*a^4/60235074900/b^9)/x^(1/3)/x^(1/3)-791224200*b^4*a^5/60
235074900/b^9)*sqrt(a/x^(1/3)+b/x)+integrate(-2373672600*b^4*a^6/6023507490
0/b^9/3/((x^(1/6))^5*sqrt(a*(x^(1/3))^2+b)*sign(x)),x)
```

maple [A] time = 0.08, size = 245, normalized size = 0.89

$$-\frac{1326\sqrt{-ab}\,\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(x^{\frac{1}{3}}-\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\;a^{5}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x^{\frac{1}{3}}+\frac{\sqrt{-ab}}{a}\right)a}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{33649\sqrt{ax+b\,x^{\frac{1}{3}}}\;b^{5}}-\frac{2652\sqrt{ax+b\,x^{\frac{1}{3}}}\;a^{5}}{33649b^{5}x^{\frac{2}{3}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(1/2)/x^5,x)
```

```
[Out] -6/23*(a*x+b*x^(1/3))^(1/2)/x^4-12/437*a*(a*x+b*x^(1/3))^(1/2)/b/x^(10/3)+6
8/2185*a^2*(a*x+b*x^(1/3))^(1/2)/b^2/x^(8/3)-884/24035*a^3*(a*x+b*x^(1/3))^
(1/2)/b^3/x^2+7956/168245*a^4*(a*x+b*x^(1/3))^(1/2)/b^4/x^(4/3)-2652/33649*
a^5*(a*x+b*x^(1/3))^(1/2)/b^5/x^(2/3)-1326/33649*a^5/b^5*(-a*b)^(1/2)*((x^(
1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2)*(-2*(x^(1/3)-(-a*b)^(1/2)/a)/(-a
*b)^(1/2)*a)^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)/(a*x+b*x^(1/3))^(1/2)*
EllipticF(((x^(1/3)+(-a*b)^(1/2)/a)/(-a*b)^(1/2)*a)^(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{1}{3}}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(1/3))/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+b\,x^{1/3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(1/2)/x^5,x)
[Out] int((a*x + b*x^(1/3))^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + b\sqrt[3]{x}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(1/3))/x**5, x)
```

3.140 $\int x^2 \left(b\sqrt[3]{x} + ax\right)^{3/2} dx$

Optimal. Leaf size=298

$$-\frac{884b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{100947a^{21/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{1768b^6\sqrt{ax+b\sqrt[3]{x}}}{100947a^5}-\frac{1768b^5x^{2/3}\sqrt{ax+b}}{168245a^4}$$

```
[Out] 2/9*x^3*(b*x^(1/3)+a*x)^(3/2)+1768/100947*b^6*(b*x^(1/3)+a*x)^(1/2)/a^5-176
8/168245*b^5*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^4+1768/216315*b^4*x^(4/3)*(b*x
^(1/3)+a*x)^(1/2)/a^3-136/19665*b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^2+8/1311*b^
2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2)/a+4/69*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)-884
/100947*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/c
os(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6
)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*
a^(1/2)+b^(1/2))^2)^(1/2)/a^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2021, 2024, 2011, 329, 220}

$$-\frac{1768b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{168245a^4}+\frac{1768b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{216315a^3}-\frac{136b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{19665a^2}-\frac{884b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{a}{\left(\sqrt{a}\right.}}}{100947a^{21/4}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (1768*b^6*Sqrt[b*x^(1/3) + a*x])/(100947*a^5) - (1768*b^5*x^(2/3)*Sqrt[b*x^
(1/3) + a*x])/(168245*a^4) + (1768*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(2163
15*a^3) - (136*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(19665*a^2) + (8*b^2*x^(8/3)*
Sqrt[b*x^(1/3) + a*x])/(1311*a) + (4*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/69 +
 (2*x^3*(b*x^(1/3) + a*x)^(3/2))/9 - (884*b^(27/4)*(Sqrt[b] + Sqrt[a]*x^(1/
3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2
*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*a^(21/4)*Sqrt[b*x^(1/3) +
 a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}dx = 3\,\text{Subst}\left(\int x^8\left(bx+ax^3\right)^{3/2}dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{1}{3}(2b)\,\text{Subst}\left(\int x^9\sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{1}{69}\left(4b^2\right)\text{Subst}\left(\int \frac{x^{10}}{\sqrt{bx+ax^3}}dx, x,$$

$$= \frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a} + \frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{9}x^3\left(b\sqrt[3]{x}+ax\right)^{3/2} - \frac{\left(68b^3\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2} + \frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a} + \frac{4}{69}bx^{10/3}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{9}x^3\left(b\sqrt[3]{x}\right.$$

$$= \frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3} - \frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2} + \frac{8b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a} + \frac{4}{69}bx^{10/3}$$

$$= -\frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4} + \frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3} - \frac{136b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{19665a^2} + \frac{8b^2x}{}$$

$$= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5} - \frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4} + \frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3} - \frac{136b^3x}{1}$$

$$= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5} - \frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4} + \frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3} - \frac{136b^3x}{1}$$

$$= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5} - \frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4} + \frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3} - \frac{136b^3x}{1}$$

$$= \frac{1768b^6\sqrt{b\sqrt[3]{x}+ax}}{100947a^5} - \frac{1768b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{168245a^4} + \frac{1768b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{216315a^3} - \frac{136b^3x}{1}$$

Mathematica [C] time = 0.15, size = 142, normalized size = 0.48

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(\left(ax^{2/3}+b\right)^2\sqrt{\frac{ax^{2/3}}{b}+1}\left(24035a^4x^{8/3}-17765a^3bx^2+12155a^2b^2x^{4/3}-7293ab^3x^{2/3}+3315b^4\right)\right.}{216315a^5\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]*(3315*b
^4 - 7293*a*b^3*x^(2/3) + 12155*a^2*b^2*x^(4/3) - 17765*a^3*b*x^2 + 24035*a
^4*x^(8/3)) - 3315*b^6*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b)])
)/(216315*a^5*Sqrt[1 + (a*x^(2/3))/b])
```

fricas [F] time = 1.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ax^3+bx^{\frac{7}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x^3 + b*x^(7/3))*sqrt(a*x + b*x^(1/3)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

maple [A] time = 0.14, size = 196, normalized size = 0.66

$$\frac{\frac{2a^8x^5}{9} + \frac{104a^7 b x^{\frac{13}{3}}}{207} + \frac{1126a^6b^2x^{\frac{11}{3}}}{3933} - \frac{16a^5b^3x^3}{19665} + \frac{272a^4b^4x^{\frac{7}{3}}}{216315} - \frac{3536a^3b^5x^{\frac{5}{3}}}{1514205} + \frac{3536a^2b^6x}{504735} + \frac{1768a\,b^7x^{\frac{1}{3}}}{100947} - \frac{884\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{\cdots}}{\cdots}}{\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+b*x^(1/3))^(3/2),x)
```

```
[Out] 2/1514205*(216755*x^(11/3)*a^6*b^2+380380*x^(13/3)*a^7*b-616*a^5*b^3*x^3-17
68*x^(5/3)*a^3*b^5+952*x^(7/3)*a^4*b^4+168245*a^8*x^5-6630*b^7*(-a*b)^(1/2)
*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3
)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+5304*a^2*b^6*x+13260*x^(1/
3)*a*b^7)/a^6/((a*x^(2/3)+b)*x^(1/3))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a\,x + b\,x^{1/3}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] int(x^2*(a*x + b*x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2*(a*x + b*x**(1/3))**(3/2), x)
```

3.141 $\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=408

$$-\frac{44b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{88b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 2/7*x^2*(b*x^(1/3)+a*x)^(3/2)-88/1105*b^5*(b+a*x^(2/3))*x^(1/3)/a^(7/2)/(x^
(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+88/3315*b^4*x^(1/3)*(b*x^(1/3)
+a*x)^(1/2)/a^3-88/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^2+24/1547*b^2*x^(5/3)
*(b*x^(1/3)+a*x)^(1/2)/a+12/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)+88/1105*b^(
21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))
,1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(
1/2))^2)^(1/2)/a^(15/4)/(b*x^(1/3)+a*x)^(1/2)-44/1105*b^(21/4)*x^(1/6)*(cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b
^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(
1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(
15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {2018, 2021, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{88b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105a^{7/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{44b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+88\ldots$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-88*b^5*(b + a*x^(2/3))*x^(1/3))/(1105*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))
*Sqrt[b*x^(1/3) + a*x]) + (88*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(3315*a^3)
 - (88*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^2) + (24*b^2*x^(5/3)*Sqrt[b*x^(
1/3) + a*x])/(1547*a) + (12*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/119 + (2*x^2*(
b*x^(1/3) + a*x)^(3/2))/7 + (88*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(
b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a
^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) - (44
*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[
a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])
/(1105*a^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int x\left(b\sqrt[3]{x}+ax\right)^{3/2}dx=3\,\text{Subst}\left(\int x^5\left(bx+ax^3\right)^{3/2}dx,x,\sqrt[3]{x}\right)$$

$$=\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{7}(6b)\,\text{Subst}\left(\int x^6\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)$$

$$=\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{119}\left(12b^2\right)\text{Subst}\left(\int\frac{x^7}{\sqrt{bx+ax^3}}dx,x,\right.$$

$$=\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(132b^3\right)\text{Subst}}{}$$

$$=-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{7}x^2\left(b\sqrt[3]{x}+\right.$$

$$=\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{}}$$

$$=\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{}}$$

$$=\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{}}$$

$$=\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}+\frac{24b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a}+\frac{12}{119}bx^{7/3}\sqrt{b\sqrt[3]{}}$$

$$=-\frac{88b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}}+\frac{88b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{3315a^3}-\frac{88b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^2}$$

Mathematica [C] time = 0.11, size = 123, normalized size = 0.30

$$\frac{2\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\left(\left(ax^{2/3}+b\right)^2\sqrt{\frac{ax^{2/3}}{b}+1}\left(221a^2x^{4/3}-143abx^{2/3}+77b^2\right)-77b^4\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)\right)}{1547a^3\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*((b + a*x^(2/3))^2*Sqrt[1 + (a*x^(2/3))/b]
*(77*b^2 - 143*a*b*x^(2/3) + 221*a^2*x^(4/3)) - 77*b^4*Hypergeometric2F1[-3
/2, 3/4, 7/4, -((a*x^(2/3))/b)]))/(1547*a^3*Sqrt[1 + (a*x^(2/3))/b])
```

fricas [F] time = 7.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ax^2+bx^{\frac{4}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

[Out] integral((a*x^2 + b*x^(4/3))*sqrt(a*x + b*x^(1/3)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate((a*x + b*x^(1/3))^(3/2)*x, x)

maple [A] time = 0.08, size = 261, normalized size = 0.64

$$\frac{\frac{2a^6x^4}{7} + \frac{80a^5b\,x^{\frac{10}{3}}}{119} + \frac{622a^4b^2x^{\frac{8}{3}}}{1547} - \frac{16a^3b^3x^2}{4641} + \frac{176a^2b^4x^{\frac{4}{3}}}{23205} - \frac{88\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,b^6\,\mathrm{EllipticE}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{1105} +}{\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a*x+b*x^(1/3))^(3/2),x)

[Out] 2/23205/a^4*(4665*x^(8/3)*a^4*b^2+7800*x^(10/3)*a^5*b-40*x^2*a^3*b^3-924*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+3315*x^4*a^6+308*x^(2/3)*a*b^5+88*x^(4/3)*a^2*b^4)/((a*x^(2/3)+b)*x^(1/3))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate((a*x + b*x^(1/3))^(3/2)*x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a\,x + b\,x^{1/3}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a*x + b*x^(1/3))^(3/2),x)

[Out] int(x*(a*x + b*x^(1/3))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x**(1/3)+a*x)**(3/2),x)

[Out] Integral(x*(a*x + b*x**(1/3))**(3/2), x)

3.142 $\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=208

$$\frac{4b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a}+\frac{12}{55}b$$

```
[Out] 2/5*x*(b*x^(1/3)+a*x)^(3/2)-8/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^2+24/385*b^2*x
^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a+12/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)+4/77*b
^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)
)),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b
^(1/2))^2)^(1/2)/a^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2004, 2018, 2021, 2024, 2011, 329, 220}

$$\frac{4b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a}+\frac{12}{55}b$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-8*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^2) + (24*b^2*x^(2/3)*Sqrt[b*x^(1/3) +
a*x])/(385*a) + (12*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/55 + (2*x*(b*x^(1/3) +
 a*x)^(3/2))/5 + (4*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(77*a^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
```

```
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \left(b\sqrt[3]{x}+ax\right)^{3/2}\,dx = \frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{5}(2b)\int \sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}\,dx$$

$$= \frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{5}(6b)\,\text{Subst}\left(\int x^3\sqrt{bx+ax^3}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\frac{1}{55}\left(12b^2\right)\text{Subst}\left(\int\frac{x^4}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)$$

$$= \frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}-\frac{\left(12b^3\right)\text{Subst}\left(\int\frac{x^2}{\sqrt{bx+\cdots}}\right.}{77a}$$

$$= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\cdots$$

$$= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\cdots$$

$$= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\cdots$$

$$= -\frac{8b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^2}+\frac{24b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a}+\frac{12}{55}bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}+\frac{2}{5}x\left(b\sqrt[3]{x}+ax\right)^{3/2}+\cdots$$

Mathematica [C] time = 0.10, size = 106, normalized size = 0.51

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(5b^3\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)-\left(5b-11ax^{2/3}\right)\left(ax^{2/3}+b\right)^2\sqrt{\frac{ax^{2/3}}{b}+1}\right)}{55a^2\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-((5*b - 11*a*x^(2/3))*(b + a*x^(2/3))^2*Sqrt[1 +
 (a*x^(2/3))/b]) + 5*b^3*Hypergeometric2F1[-3/2, 1/4, 5/4, -((a*x^(2/3))/b)
]))/(55*a^2*Sqrt[1 + (a*x^(2/3))/b])
```

fricas [F] time = 1.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2), x)
```

maple [A] time = 0.11, size = 164, normalized size = 0.79

$$\frac{\frac{2a^5x^3}{5}+\frac{56a^4b\,x^{\frac{7}{3}}}{55}+\frac{262a^3b^2x^{\frac{5}{3}}}{385}-\frac{16a^2b^3x}{385}-\frac{8a\,b^4x^{\frac{1}{3}}}{77}+\frac{4\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,b^4\,\text{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\ldots\right.}{77}}{\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(3/2),x)
```

```
[Out] 2/385*(10*b^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^
(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(
1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(
1/2))+131*a^3*b^2*x^(5/3)+196*a^4*b*x^(7/3)-8*a^2*b^3*x+77*a^5*x^3-20*a*b^4
*x^(1/3))/a^3/((a*x^(2/3)+b)*x^(1/3))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2), x)
```

mupad [B] time = 5.19, size = 40, normalized size = 0.19

$$\frac{2\,x\,\left(a\,x+b\,x^{1/3}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{9}{4};\ \frac{13}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{3\,\left(\frac{a\,x^{2/3}}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] (2*x*(a*x + b*x^(1/3))^(3/2)*hypergeom([-3/2, 9/4], 13/4, -(a*x^(2/3))/b))/
(3*((a*x^(2/3))/b + 1)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(3/2), x)
```

3.143 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=319

$$\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan\right.}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 2/3*(b*x^(1/3)+a*x)^(3/2)+8/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(1/2)/(x^(1/3)*a^
(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+4/5*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)-8/
5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/
4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)
+b^(1/2))^2)^(1/2)/a^(3/4)/(b*x^(1/3)+a*x)^(1/2)+4/5*b^(9/4)*x^(1/6)*(cos(2
*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/
3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/
4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2021, 2004, 2032, 329, 305, 220, 1196}

$$\frac{4b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan\right.}{5a^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

```
[Out] (8*b^2*(b + a*x^(2/3))*x^(1/3))/(5*Sqrt[a]*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt
[b*x^(1/3) + a*x]) + (4*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/5 + (2*(b*x^(1/3)
+ a*x)^(3/2))/3 - (8*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b*x^(1/3) + a*x]) + (4*b^(9/4)*(Sqrt[b]
 + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(
1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(3/4)*Sqrt[b
*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x}\,dx = 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + (2b)\,\text{Subst}\left(\int \sqrt{bx+ax^3}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{1}{5}\left(4b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(4b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \ldots\right.}{5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(8b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \ldots\right.}{5\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} + \frac{\left(8b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \ldots\right.}{5\sqrt{a}\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{8b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5\sqrt{a}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{4}{5}b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax} + \frac{2}{3}\left(b\sqrt[3]{x}+ax\right)^{3/2} - \frac{8b^{9/4}\left(\sqrt{\ldots}\right.}{\ldots}$$

Mathematica [C] time = 0.05, size = 60, normalized size = 0.19

$$\frac{2b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}\;{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)}{\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x,x]
```

```
[Out] (2*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, 3/4, 7/4, -((a*x
^(2/3))/b)])/Sqrt[1 + (a*x^(2/3))/b]
```

fricas [F] time = 8.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

maple [A] time = 0.08, size = 228, normalized size = 0.71

$$\frac{\frac{2a^3x^2}{3}+\frac{32a^2b\,x^{\frac{4}{3}}}{15}+\frac{8\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,b^3\,\mathrm{EllipticE}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{5}-\frac{4\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\cdots}{5}}{\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(3/2)/x,x)
```

```
[Out] 2/15/a*(12*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)
-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*Ellipt
icE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-6*b^3*((a*x^
(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)
^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+11*a*b^2*x^(2/3)+16*a^2*b*x^(4/3)
+5*a^3*x^2)/((a*x^(2/3)+b)*x^(1/3))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x,x)
```

```
[Out] int((a*x + b*x^(1/3))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x,x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x, x)
```

3.144 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=144

$$\frac{4a^{3/4}b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x}+4a\sqrt{ax+b\sqrt[3]{x}}$$

```
[Out] -2*(b*x^(1/3)+a*x)^(3/2)/x+4*a*(b*x^(1/3)+a*x)^(1/2)+4*a^(3/4)*b^(3/4)*x^(1
/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/
2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(
1/2)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2020, 2021, 2011, 329, 220}

$$\frac{4a^{3/4}b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{\sqrt{ax+b\sqrt[3]{x}}}-\frac{2\left(ax+b\sqrt[3]{x}\right)^{3/2}}{x}+4a\sqrt{ax+b\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] 4*a*Sqrt[b*x^(1/3) + a*x] - (2*(b*x^(1/3) + a*x)^(3/2))/x + (4*a^(3/4)*b^(3
/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(
1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/Sqrt[
b*x^(1/3) + a*x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^2}\,dx = 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^4}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + (6a)\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x}\,dx, x, \sqrt[3]{x}\right)$$

$$= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + (4ab)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{\left(4ab\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}$$

$$= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{\left(8ab\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b\sqrt[3]{x}+ax}}$$

$$= 4a\sqrt{b\sqrt[3]{x}+ax} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x} + \frac{4a^{3/4}b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\,\sqrt[6]{x}\,F\left(2\tan^{-1}\right.}{\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] time = 0.06, size = 60, normalized size = 0.42

$$-\frac{2b\sqrt{ax+b\sqrt[3]{x}}\;{}_2F_1\left(-\frac{3}{2},-\frac{3}{4};\frac{1}{4};-\frac{ax^{2/3}}{b}\right)}{x^{2/3}\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-3/2, -3/4, 1/4, -((a*x^(2/3)
)/b)])/(Sqrt[1 + (a*x^(2/3))/b]*x^(2/3))
```

fricas [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

maple [A] time = 0.07, size = 130, normalized size = 0.90

$$\frac{2a^2x^{\frac{4}{3}}+4\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,b\,x^{\frac{1}{3}}\,\text{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)-2b^2}{\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(3/2)/x^2,x)
[Out] 2/x^(1/3)*(2*x^(1/3)*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^
(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*b+x^(4/3)*a^2-b^2)/((a*x^(2/3)+b)*x^(1/3))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^2,x)
```

```
[Out] int((a*x + b*x^(1/3))^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**2,x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**2, x)
```

3.145 $\int \frac{\left(b \sqrt[3]{x}+ax\right)^{3/2}}{x^3} \, dx$

Optimal. Leaf size=350

$$\frac{4a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan\right.}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] -2/3*(b*x^(1/3)+a*x)^(3/2)/x^2+8/5*a^(5/2)*(b+a*x^(2/3))*x^(1/3)/b/(x^(1/3)
*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-4/5*a*(b*x^(1/3)+a*x)^(1/2)/x-8/5*a
^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3)-8/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Ellipti
cE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(
1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+
a*x)^(1/2)+4/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(
1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^
(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$\frac{4a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{8a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan\right.}{5b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (8*a^(5/2)*(b + a*x^(2/3))*x^(1/3))/(5*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b
*x^(1/3) + a*x]) - (4*a*Sqrt[b*x^(1/3) + a*x])/(5*x) - (8*a^2*Sqrt[b*x^(1/3
) + a*x])/(5*b*x^(1/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(3*x^2) - (8*a^(9/4)*
(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3)
)^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4
)*Sqrt[b*x^(1/3) + a*x]) + (4*a^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b +
 a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1
/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^3}\,dx = 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^7}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + (2a)\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^4}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{1}{5}\left(4a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$
$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(4a^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, \right.}{5b}$$
$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(4a^3\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\right.}{5b\sqrt{b\sqrt[3]{x}+}}$$
$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(8a^3\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\right.}{5b\sqrt{b\sqrt[3]{x}+}}$$
$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{3x^2} + \frac{\left(8a^{5/2}\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subs}}{5\sqrt{b}\,\sqrt{b\sqrt[3]{x}}}$$
$$= \frac{8a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{4a\sqrt{b\sqrt[3]{x}+ax}}{5x} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{5b\sqrt[3]{x}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3}}{3x^2}$$

Mathematica [C] time = 0.06, size = 62, normalized size = 0.18

$$-\frac{2b\sqrt{ax+b\sqrt[3]{x}}\;{}_2F_1\left(-\frac{9}{4},-\frac{3}{2};-\frac{5}{4};-\frac{ax^{2/3}}{b}\right)}{3x^{5/3}\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^3,x]
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-9/4, -3/2, -5/4, -((a*x^(2/3
))/b)])/(3*Sqrt[1 + (a*x^(2/3))/b]*x^(5/3))
```

fricas [F] time = 7.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 8.232*(-60*b^3/180/b^2/x^(1/3)/x^(1/3)-132*b^2*a/180/b^
2)/x^(1/3)*sqrt(a/x^(1/3)+b/x)+integrate(144*b^2*a^2/180/b^2/3/(x^(1/3)*x^(
1/3)*sqrt(x)*sqrt(a*(x^(1/3))^2+b)*sign(x)),x)
```

maple [A] time = 0.08, size = 339, normalized size = 0.97

$$-\frac{2\left(-12\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a x^{\frac{1}{3}}}{\sqrt{-ab}}}\,\sqrt{\left(a x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\,a^{2} b\, x^{\frac{8}{3}}\,\mathrm{EllipticE}\left(\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+6\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{\cdots}}{\sqrt{-ab}}}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(3/2)/x^3,x)
```

```
[Out] -2/15*(-12*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/
3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*x^(8
/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b
)^(1/2))^(1/2),1/2*2^(1/2))+6*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))
^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*
x^(1/3))^(1/2)*x^(8/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+12*x^(10/3)*(a*x+b*x^(1/3))^
(1/2)*a^3+12*x^(8/3)*(a*x+b*x^(1/3))^(1/2)*a^2*b+16*x^2*((a*x^(2/3)+b)*x^(1
/3))^(1/2)*a*b^2+11*x^(8/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^2*b+5*x^(4/3)*(
(a*x^(2/3)+b)*x^(1/3))^(1/2)*b^3)/b/x^3/(a*x^(2/3)+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x + b\,x^{1/3}\right)^{3/2}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(1/3))^(3/2)/x^3, x)
```

sympy [**F**]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**3,x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**3, x)
```

3.146 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=213

$$\frac{4a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{8a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}-\frac{24a^2\sqrt{ax+b\sqrt[3]{x}}}{385bx^{4/3}}-\frac{2\left(ax+b\sqrt[3]{\right.}}{5x^3}$$

```
[Out] -2/5*(b*x^(1/3)+a*x)^(3/2)/x^3-12/55*a*(b*x^(1/3)+a*x)^(1/2)/x^2-24/385*a^2
*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+8/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)
+4/77*a^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$\frac{8a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^2x^{2/3}}+\frac{4a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{24a^2\sqrt{ax+b\sqrt[3]{x}}}{385bx^{4/3}}-\frac{12a\sqrt{ax+}}{55x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-12*a*Sqrt[b*x^(1/3) + a*x])/(55*x^2) - (24*a^2*Sqrt[b*x^(1/3) + a*x])/(38
5*b*x^(4/3)) + (8*a^3*Sqrt[b*x^(1/3) + a*x])/(77*b^2*x^(2/3)) - (2*(b*x^(1/
3) + a*x)^(3/2))/(5*x^3) + (4*a^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^4}\,dx = 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3} + \frac{1}{5}(6a)\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^7}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3} + \frac{1}{55}\left(12a^2\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3} - \frac{\left(12a^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\ldots\right.}{77b}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3} + \frac{\left(4a^4\right)\ldots}{\ldots}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3} + \frac{\left(4a^4\sqrt{\ldots}\right.}{\ldots}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3} + \frac{\left(8a^4\sqrt{\ldots}\right.}{\ldots}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{55x^2} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{385bx^{4/3}} + \frac{8a^3\sqrt{b\sqrt[3]{x}+ax}}{77b^2x^{2/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{5x^3} + \frac{4a^{15/4}\ldots}{\ldots}$$

Mathematica [C] time = 0.07, size = 62, normalized size = 0.29

$$-\frac{2b\sqrt{ax+b\sqrt[3]{x}}\;{}_2F_1\left(-\frac{15}{4},-\frac{3}{2};-\frac{11}{4};-\frac{ax^{2/3}}{b}\right)}{5x^{8/3}\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-15/4, -3/2, -11/4, -((a*x^(2
/3))/b)])/(5*Sqrt[1 + (a*x^(2/3))/b]*x^(8/3))
```

fricas [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 0.572*(((-41580*b^6/207900/b^5/x^(1/3)/x^(1/3)-64260*b^
5*a/207900/b^5)/x^(1/3)/x^(1/3)-6480*b^4*a^2/207900/b^5)/x^(1/3)/x^(1/3)+10
800*b^3*a^3/207900/b^5)*sqrt(a/x^(1/3)+b/x)+integrate(10800*b^3*a^4/207900/
b^5/3/((x^(1/6))^5*sqrt(a*(x^(1/3))^2+b)*sign(x)),x)
```

maple [A] time = 0.09, size = 168, normalized size = 0.79

$$\frac{\frac{8a^{4}x^{5}}{77}+\frac{4\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,a^{3}x^{\frac{14}{3}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)}{77}+\frac{16a^{3}b\,x^{\frac{13}{3}}}{385}-\frac{262a^{2}b^{2}x^{\frac{11}{3}}}{385}-\frac{56a\,b^{3}x^{3}}{55}-\frac{2b^{4}x}{5}}{\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;b^{2}x^{\frac{14}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(3/2)/x^4,x)
```

```
[Out] 2/385*(10*a^3*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-
2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^
(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
x^(14/3)-131*x^(11/3)*a^2*b^2+8*x^(13/3)*a^3*b-196*a*b^3*x^3+20*x^5*a^4-77*
x^(7/3)*b^4)/b^2/((a*x^(2/3)+b)*x^(1/3))^(1/2)/x^(14/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^4,x)
[Out] int((a*x + b*x^(1/3))^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**4,x)
[Out] Integral((a*x + b*x**(1/3))**(3/2)/x**4, x)
```

3.147 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=438

$$-\frac{44a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{88a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(\cdots\right.}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] -2/7*(b*x^(1/3)+a*x)^(3/2)/x^4-88/1105*a^(11/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(
x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-12/119*a*(b*x^(1/3)+a*x)^(1/
2)/x^3-24/1547*a^2*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+88/4641*a^3*(b*x^(1/3)+a
*x)^(1/2)/b^2/x^(5/3)-88/3315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x+88/1105*a^5*(
b*x^(1/3)+a*x)^(1/2)/b^4/x^(1/3)+88/1105*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(
1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Elli
pticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+
b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1
/3)+a*x)^(1/2)-44/1105*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/
4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(
2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2020, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{88a^{11/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{1105b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{88a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^2x^{5/3}}-\frac{44a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\cdots\right.\right.}{1105b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (-88*a^(11/2)*(b + a*x^(2/3))*x^(1/3))/(1105*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) - (12*a*Sqrt[b*x^(1/3) + a*x])/(119*x^3) - (24*a^2
*Sqrt[b*x^(1/3) + a*x])/(1547*b*x^(7/3)) + (88*a^3*Sqrt[b*x^(1/3) + a*x])/(
4641*b^2*x^(5/3)) - (88*a^4*Sqrt[b*x^(1/3) + a*x])/(3315*b^3*x) + (88*a^5*S
qrt[b*x^(1/3) + a*x])/(1105*b^4*x^(1/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(7*x
^4) + (88*a^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b
] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4
)], 1/2])/(1105*b^(15/4)*Sqrt[b*x^(1/3) + a*x]) - (44*a^(21/4)*(Sqrt[b] + S
qrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)
*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(1105*b^(15/4)*Sqrt[b
*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^5}\,dx = 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} + \frac{1}{7}(6a)\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{10}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} + \frac{1}{119}\left(12a^2\right)\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} - \frac{\left(132a^3\right)\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\right.}{1547b}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{7x^4} + \frac{\left(44a^4\right)}{}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} - \frac{2\left(b\sqrt[3]{x}\right.}{}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5}{110}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5}{110}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5}{110}$$

$$= -\frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b\sqrt[3]{x}+ax}}{4641b^2x^{5/3}} - \frac{88a^4\sqrt{b\sqrt[3]{x}+ax}}{3315b^3x} + \frac{88a^5}{110}$$

$$= -\frac{88a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{1105b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{12a\sqrt{b\sqrt[3]{x}+ax}}{119x^3} - \frac{24a^2\sqrt{b\sqrt[3]{x}+ax}}{1547bx^{7/3}} + \frac{88a^3\sqrt{b}}{4641b}$$

Mathematica [C] time = 0.06, size = 62, normalized size = 0.14

$$-\frac{2b\sqrt{ax+b\sqrt[3]{x}}\;{}_2F_1\left(-\frac{21}{4},-\frac{3}{2};-\frac{17}{4};-\frac{ax^{2/3}}{b}\right)}{7x^{11/3}\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-21/4, -3/2, -17/4, -((a*x^(2
/3))/b)])/(7*Sqrt[1 + (a*x^(2/3))/b]*x^(11/3))
```

fricas [F] time = 8.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 3.252*((((-137837700*b^9/964863900/b^8/x^(1/3)/x^(1/3)-
186486300*b^8*a/964863900/b^8)/x^(1/3)/x^(1/3)-7484400*b^7*a^2/964863900/b^
8)/x^(1/3)/x^(1/3)+9147600*b^6*a^3/964863900/b^8)/x^(1/3)/x^(1/3)-12806640*
b^5*a^4/964863900/b^8)/x^(1/3)*sqrt(a/x^(1/3)+b/x)+integrate(-38419920*b^5*
a^5/964863900/b^8/3/(x^(1/3)*x^(1/3)*sqrt(x)*sqrt(a*(x^(1/3))^2+b)*sign(x))
,x)
```

maple [A] time = 0.09, size = 411, normalized size = 0.94

$$-\frac{88\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a x^{\frac{1}{3}}}{\sqrt{-ab}}}\,\sqrt{\left(a x^{\frac{2}{3}}+b\right) x^{\frac{1}{3}}}\;a^5 b x^{\frac{20}{3}}\operatorname{EllipticE}\left(\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{1105}+\frac{44\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(3/2)/x^5,x)
```

```
[Out] 2/23205*(-924*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^
(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*x
^(20/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1
/2)*a*x^(1/3))^(1/2)*x^(20/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticF(((a*x
^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+924*(a*x+b*x^(1/3))^(
1/2)*x^(22/3)*a^6+924*(a*x+b*x^(1/3))^(1/2)*x^(20/3)*a^5*b-88*x^6*((a*x^(2/
3)+b)*x^(1/3))^(1/2)*a^4*b^2-308*x^(20/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^5
*b-4665*x^(14/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^2*b^4+40*x^(16/3)*((a*x^(2
/3)+b)*x^(1/3))^(1/2)*a^3*b^3-7800*x^4*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a*b^5-
3315*x^(10/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*b^6)/b^4/x^7/(a*x^(2/3)+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+b\,x^{1/3}\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^5,x)
```

```
[Out] int((a*x + b*x^(1/3))^(3/2)/x^5, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

3.148 $\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=301

$$-\frac{884a^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{100947b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{1768a^6\sqrt{ax+b\sqrt[3]{x}}}{100947b^5x^{2/3}}+\frac{1768a^5\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}$$

```
[Out] -2/9*(b*x^(1/3)+a*x)^(3/2)/x^5-4/69*a*(b*x^(1/3)+a*x)^(1/2)/x^4-8/1311*a^2*
(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+136/19665*a^3*(b*x^(1/3)+a*x)^(1/2)/b^2/x^
(8/3)-1768/216315*a^4*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2+1768/168245*a^5*(b*x^(1
/3)+a*x)^(1/2)/b^4/x^(4/3)-1768/100947*a^6*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3
)-884/100947*a^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(
1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2020, 2025, 2011, 329, 220}

$$-\frac{1768a^6\sqrt{ax+b\sqrt[3]{x}}}{100947b^5x^{2/3}}+\frac{1768a^5\sqrt{ax+b\sqrt[3]{x}}}{168245b^4x^{4/3}}-\frac{1768a^4\sqrt{ax+b\sqrt[3]{x}}}{216315b^3x^2}+\frac{136a^3\sqrt{ax+b\sqrt[3]{x}}}{19665b^2x^{8/3}}-\frac{884a^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\cdots\right.}{10\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-4*a*Sqrt[b*x^(1/3) + a*x])/(69*x^4) - (8*a^2*Sqrt[b*x^(1/3) + a*x])/(1311
*b*x^(10/3)) + (136*a^3*Sqrt[b*x^(1/3) + a*x])/(19665*b^2*x^(8/3)) - (1768*
a^4*Sqrt[b*x^(1/3) + a*x])/(216315*b^3*x^2) + (1768*a^5*Sqrt[b*x^(1/3) + a*
x])/(168245*b^4*x^(4/3)) - (1768*a^6*Sqrt[b*x^(1/3) + a*x])/(100947*b^5*x^(
2/3)) - (2*(b*x^(1/3) + a*x)^(3/2))/(9*x^5) - (884*a^(27/4)*(Sqrt[b] + Sqrt
[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*El
lipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(100947*b^(21/4)*Sqrt[b*
x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(b\sqrt[3]{x}+ax\right)^{3/2}}{x^6}\,dx = 3\,\text{Subst}\left(\int \frac{\left(bx+ax^3\right)^{3/2}}{x^{16}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} + \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{\sqrt{bx+ax^3}}{x^{13}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} + \frac{1}{69}\left(4a^2\right)\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} - \frac{\left(68a^3\right)\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,d\cdots\right.}{1311b}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{2\left(b\sqrt[3]{x}+ax\right)^{3/2}}{9x^5} + \frac{\left(884a\cdots\right.}{\cdots}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} - \frac{2\left(\cdots\right.}{\cdots}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{17\cdots}{\cdots}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{17\cdots}{\cdots}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{17\cdots}{\cdots}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{17\cdots}{\cdots}$$

$$= -\frac{4a\sqrt{b\sqrt[3]{x}+ax}}{69x^4} - \frac{8a^2\sqrt{b\sqrt[3]{x}+ax}}{1311bx^{10/3}} + \frac{136a^3\sqrt{b\sqrt[3]{x}+ax}}{19665b^2x^{8/3}} - \frac{1768a^4\sqrt{b\sqrt[3]{x}+ax}}{216315b^3x^2} + \frac{17\cdots}{\cdots}$$

Mathematica [C] time = 0.07, size = 62, normalized size = 0.21

$$-\frac{2b\sqrt{ax+b\sqrt[3]{x}}\;{}_2F_1\left(-\frac{27}{4},-\frac{3}{2};-\frac{23}{4};-\frac{ax^{2/3}}{b}\right)}{9x^{14/3}\sqrt{\frac{ax^{2/3}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-2*b*Sqrt[b*x^(1/3) + a*x]*Hypergeometric2F1[-27/4, -3/2, -23/4, -((a*x^(2
/3))/b)])/(9*Sqrt[1 + (a*x^(2/3))/b]*x^(14/3))
```

fricas [F] time = 1.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 3.242*((((((-1264936572900*b^12/11384429156100/b^11/x^(
1/3)/x^(1/3)-1594920026700*b^11*a/11384429156100/b^11)/x^(1/3)/x^(1/3)-3473
5100400*b^10*a^2/11384429156100/b^11)/x^(1/3)/x^(1/3)+39366447120*b^9*a^3/1
1384429156100/b^11)/x^(1/3)/x^(1/3)-46523982960*b^8*a^4/11384429156100/b^11
)/x^(1/3)/x^(1/3)+59816549520*b^7*a^5/11384429156100/b^11)/x^(1/3)/x^(1/3)-
99694249200*b^6*a^6/11384429156100/b^11)*sqrt(a/x^(1/3)+b/x)+integrate(-996
94249200*b^6*a^7/11384429156100/b^11/3/((x^(1/6))^5*sqrt(a*(x^(1/3))^2+b)*s
ign(x)),x)
```

maple [A] time = 0.09, size = 201, normalized size = 0.67

$$-\frac{2\left(13260a^{7}x^{9}+6630\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,a^{6}x^{\frac{26}{3}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+5304a^{6}\cdots\right.}{1514205\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(1/3))^(3/2)/x^6,x)
```

```
[Out] -2/1514205*(6630*a^6*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^
(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^
(1/2))*x^(26/3)-1768*x^(23/3)*a^5*b^2+5304*x^(25/3)*a^6*b+952*x^7*a^4*b^3+2
16755*x^(17/3)*a^2*b^5-616*x^(19/3)*a^3*b^4+380380*x^5*a*b^6+13260*x^9*a^7+
168245*x^(13/3)*b^7)/b^5/((a*x^(2/3)+b)*x^(1/3))^(1/2)/x^(26/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(1/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(1/3))^(3/2)/x^6,x)
```

```
[Out] int((a*x + b*x^(1/3))^(3/2)/x^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(1/3)+a*x)**(3/2)/x**6,x)
```

```
[Out] Timed out
```

3.149 $\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=304

$$-\frac{5525b^{27/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{14421a^{29/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{11050b^6\sqrt{ax+b\sqrt[3]{x}}}{14421a^7}-\frac{2210b^5x^{2/3}\sqrt{ax+b\cdots}}{4807a^6}$$

```
[Out] 11050/14421*b^6*(b*x^(1/3)+a*x)^(1/2)/a^7-2210/4807*b^5*x^(2/3)*(b*x^(1/3)+
a*x)^(1/2)/a^6+15470/43263*b^4*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-1190/3933*
b^3*x^2*(b*x^(1/3)+a*x)^(1/2)/a^4+350/1311*b^2*x^(8/3)*(b*x^(1/3)+a*x)^(1/2
)/a^3-50/207*b*x^(10/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/9*x^4*(b*x^(1/3)+a*x)^(
1/2)/a-5525/14421*b^(27/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))
/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(29/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 2024, 2011, 329, 220}

$$-\frac{2210b^5x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{4807a^6}+\frac{15470b^4x^{4/3}\sqrt{ax+b\sqrt[3]{x}}}{43263a^5}-\frac{1190b^3x^2\sqrt{ax+b\sqrt[3]{x}}}{3933a^4}+\frac{350b^2x^{8/3}\sqrt{ax+b\sqrt[3]{x}}}{1311a^3}-\frac{5525b^{27/4}\sqrt[6]{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (11050*b^6*Sqrt[b*x^(1/3) + a*x])/(14421*a^7) - (2210*b^5*x^(2/3)*Sqrt[b*x^
(1/3) + a*x])/(4807*a^6) + (15470*b^4*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(43263
*a^5) - (1190*b^3*x^2*Sqrt[b*x^(1/3) + a*x])/(3933*a^4) + (350*b^2*x^(8/3)*
Sqrt[b*x^(1/3) + a*x])/(1311*a^3) - (50*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])/(
207*a^2) + (2*x^4*Sqrt[b*x^(1/3) + a*x])/(9*a) - (5525*b^(27/4)*(Sqrt[b] +
Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6
)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(14421*a^(29/4)*Sqrt
[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
```

```
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\text{Subst}\left(\int \frac{x^{14}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} - \frac{(25b)\,\text{Subst}\left(\int \frac{x^{12}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{9a}$$

$$= -\frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} + \frac{\left(175b^2\right)\text{Subst}\left(\int \frac{x^{10}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{69a^2}$$

$$= \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+ax}}{9a} - \frac{\left(2975b^3\right)\text{Subst}\left(\int \right.}{131}$$

$$= -\frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} + \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}}{207a^2} + \frac{2x^4\sqrt{b\sqrt[3]{x}+}}{9a}$$

$$= \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} + \frac{350b^2x^{8/3}\sqrt{b\sqrt[3]{x}+ax}}{1311a^3} - \frac{50bx^{10/3}}{20}$$

$$= -\frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2\sqrt{b\sqrt[3]{x}+ax}}{3933a^4} + \frac{350b^2x}{}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2}{39}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2}{39}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2}{39}$$

$$= \frac{11050b^6\sqrt{b\sqrt[3]{x}+ax}}{14421a^7} - \frac{2210b^5x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{4807a^6} + \frac{15470b^4x^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{43263a^5} - \frac{1190b^3x^2}{39}$$

Mathematica [C] time = 0.10, size = 161, normalized size = 0.53

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(4807a^7x^{14/3}-418a^6bx^4+550a^5b^2x^{10/3}-770a^4b^3x^{8/3}+1190a^3b^4x^2-2210a^2b^5x^{4/3}-16575b^7\sqrt{\frac{a}{}}\right.}{43263a^7\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(16575*b^7 + 6630*a*b^6*x^(2/3) - 2210*a^2*b^5*x^(
4/3) + 1190*a^3*b^4*x^2 - 770*a^4*b^3*x^(8/3) + 550*a^5*b^2*x^(10/3) - 418*
a^6*b*x^4 + 4807*a^7*x^(14/3) - 16575*b^7*Sqrt[1 + (a*x^(2/3))/b]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(43263*a^7*(b + a*x^(2/3)))
```

fricas [F] time = 1.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^5-abx^{\frac{13}{3}}+b^2x^{\frac{11}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^2+b^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^5 - a*b*x^(13/3) + b^2*x^(11/3))*sqrt(a*x + b*x^(1/3))/(a^3
*x^2 + b^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

maple [A] time = 0.10, size = 196, normalized size = 0.64

$$-\frac{-9614a^8x^5+836a^7b\,x^{\frac{13}{3}}-1100a^6b^2x^{\frac{11}{3}}+1540a^5b^3x^3-2380a^4b^4x^{\frac{7}{3}}+4420a^3b^5x^{\frac{5}{3}}-13260a^2b^6x-33150a\,b^7x}{43263\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] -1/43263*(-1100*a^6*b^2*x^(11/3)+836*a^7*b*x^(13/3)+1540*a^5*b^3*x^3+4420*a
^3*b^5*x^(5/3)-2380*a^4*b^4*x^(7/3)-9614*a^8*x^5+16575*b^7*(-a*b)^(1/2)*((a
*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-13260*a^2*b^6*x-33150*a*b^7*x^
(1/3))/((a*x^(2/3)+b)*x^(1/3))^(1/2)/a^8
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{ax + bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a*x + b*x^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{a\,x + b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x^4/(a*x + b*x^(1/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(a*x + b*x**(1/3)), x)
```

3.150 $\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=414

$$-\frac{209b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{221a^{23/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{418b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E}{221a^{23/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] -418/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(11/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^
(1/3)+a*x)^(1/2)+418/663*b^4*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^5-2090/4641*b^
3*x*(b*x^(1/3)+a*x)^(1/2)/a^4+570/1547*b^2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^
3-38/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/7*x^3*(b*x^(1/3)+a*x)^(1/2)/
a+418/221*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(23/4)/(b*x^(1/3)+a*x)^(1/2)-209/221*b^(21/4)
*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*
2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))
^2)^(1/2)/a^(23/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 414, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2018, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{418b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{11/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{570b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^3}-\frac{209b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\right.}{221a^{23/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-418*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(11/2)*(Sqrt[b] + Sqrt[a]*x^(1/3)
)*Sqrt[b*x^(1/3) + a*x]) + (418*b^4*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^5
) - (2090*b^3*x*Sqrt[b*x^(1/3) + a*x])/(4641*a^4) + (570*b^2*x^(5/3)*Sqrt[b
*x^(1/3) + a*x])/(1547*a^3) - (38*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a^2
) + (2*x^3*Sqrt[b*x^(1/3) + a*x])/(7*a) + (418*b^(21/4)*(Sqrt[b] + Sqrt[a]*
x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Ellipt
icE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3)
 + a*x]) - (209*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(
Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/
b^(1/4)], 1/2])/(221*a^(23/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\text{Subst}\left(\int \frac{x^{11}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} - \frac{(19b)\,\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7a}$$

$$= -\frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} + \frac{(285b^2)\,\text{Subst}\left(\int \frac{x^7}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{119a^2}$$

$$= \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a} - \frac{(3135b^3)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx}}\right.}{1547a^3}$$

$$= -\frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^2} + \frac{2x^3\sqrt{b\sqrt[3]{x}+ax}}{7a}$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+}}{119a^2}$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+}}{119a^2}$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+}}{119a^2}$$

$$= \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} + \frac{570b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^3} - \frac{38bx^{7/3}\sqrt{b\sqrt[3]{x}+}}{119a^2}$$

$$= -\frac{418b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{11/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{418b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^5} - \frac{2090b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^4} +$$

Mathematica **[C]** time = 0.08, size = 143, normalized size = 0.35

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(663a^5x^{11/3} - 78a^4bx^3 + 114a^3b^2x^{7/3} - 190a^2b^3x^{5/3} - 1463b^5\sqrt[3]{x}\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right) + 4\right.}{4641a^5\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(1463*b^5*x^(1/3) + 418*a*b^4*x - 190*a^2*b^3*x^(5
/3) + 114*a^3*b^2*x^(7/3) - 78*a^4*b*x^3 + 663*a^5*x^(11/3) - 1463*b^5*Sqrt
[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/3))/
b)]))/(4641*a^5*(b + a*x^(2/3)))
```

fricas **[F]** time = 12.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^4 - abx^{\frac{10}{3}} + b^2x^{\frac{8}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^2+b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^4 - a*b*x^(10/3) + b^2*x^(8/3))*sqrt(a*x + b*x^(1/3))/(a^3*
x^2 + b^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{ax + bx^{\frac{1}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

maple [A] time = 0.09, size = 263, normalized size = 0.64

$$-\frac{-1326a^6x^4+156a^5b\,x^{\frac{10}{3}}-228a^4b^2x^{\frac{8}{3}}+380a^3b^3x^2-836a^2b^4x^{\frac{4}{3}}+8778\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\cdots}}{4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] -1/4641/a^6*(-228*a^4*b^2*x^(8/3)+156*a^5*b*x^(10/3)+380*a^3*b^3*x^2+8778*b
^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b
)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticE(((
a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-4389*b^6*((a*x^(1/
3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-1326*a^6*x^4-2926*a*b^5*x^(2/3
)-836*a^2*b^4*x^(4/3))/((a*x^(2/3)+b)*x^(1/3))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{ax + bx^{\frac{1}{3}}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(a*x + b*x^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a\,x+b\,x^{1/3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(1/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*x + b*x**(1/3)), x)
```

3.151 $\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=216

$$\frac{39b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{17/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{78b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^4}+\frac{234b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^3}-$$

```
[Out] -78/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^4+234/385*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1
/2)/a^3-26/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/5*x^2*(b*x^(1/3)+a*x)^(
1/2)/a+39/77*b^(15/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(
1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(17/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 2024, 2011, 329, 220}

$$\frac{234b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^3}+\frac{39b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{77a^{17/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{78b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (-78*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^4) + (234*b^2*x^(2/3)*Sqrt[b*x^(1/3)
+ a*x])/(385*a^3) - (26*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])/(55*a^2) + (2*x^2*
Sqrt[b*x^(1/3) + a*x])/(5*a) + (39*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(77*a^(17/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
```

```
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\text{Subst}\left(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} - \frac{(13b)\,\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5a}$$

$$= -\frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \frac{\left(117b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{55a^2}$$

$$= \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} - \frac{\left(117b^3\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+a}}\right.}{77a^3}$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \cdots$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \cdots$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \cdots$$

$$= -\frac{78b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^4} + \frac{234b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^3} - \frac{26bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^2} + \frac{2x^2\sqrt{b\sqrt[3]{x}+ax}}{5a} + \cdots$$

Mathematica [C] time = 0.07, size = 124, normalized size = 0.57

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(77a^4x^{8/3}-14a^3bx^2+26a^2b^2x^{4/3}+195b^4\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)-78ab^3x^{2/3}-195b^4\right)}{385a^4\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-195*b^4 - 78*a*b^3*x^(2/3) + 26*a^2*b^2*x^(4/3)
- 14*a^3*b*x^2 + 77*a^4*x^(8/3) + 195*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hypergeom
etric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^4*(b + a*x^(2/3)))
```

fricas [F] time = 3.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^{2}x^{3}-abx^{\frac{7}{3}}+b^{2}x^{\frac{5}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^{3}x^{2}+b^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^3 - a*b*x^(7/3) + b^2*x^(5/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^2 + b^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

maple [A] time = 0.05, size = 163, normalized size = 0.75

$$\frac{154a^{5}x^{3}-28a^{4}b\,x^{\frac{7}{3}}+52a^{3}b^{2}x^{\frac{5}{3}}-156a^{2}b^{3}x-390a\,b^{4}x^{\frac{1}{3}}+195\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}}{385\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] 1/385*(52*a^3*b^2*x^(5/3)-28*a^4*b*x^(7/3)+195*b^4*(-a*b)^(1/2)*((a*x^(1/3)
+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2
))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-156*a^2*b^3*x+154*a^5*x^3-390*a*b^4*x^
(1/3))/((a*x^(2/3)+b)*x^(1/3))^(1/2)/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{2}}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(1/3))^(1/2), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*x**(1/3)), x)
```

3.152 $\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=326

$$\frac{7b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{14b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\,\mathrm{t}\right.}{5a^{11/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 14/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(5/2)/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)
+a*x)^(1/2)-14/15*b*x^(1/3)*(b*x^(1/3)+a*x)^(1/2)/a^2+2/3*x*(b*x^(1/3)+a*x)
^(1/2)/a-14/5*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1
/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(
1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(11/4)/(b*x^(1/3)+a*x)^(1/2)+7/5*b^(9/4)*x
^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^
(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2
)^(1/2)/a^(11/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {2018, 2024, 2032, 329, 305, 220, 1196}

$$\frac{14b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{5/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{7b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{5a^{11/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{14b^{9/4}\sqrt[6]{x}\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (14*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(5/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) - (14*b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])/(15*a^2) + (2*x*
Sqrt[b*x^(1/3) + a*x])/(3*a) - (14*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt
[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[
(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*a^(11/4)*Sqrt[b*x^(1/3) + a*x]) + (7*b
^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5
*a^(11/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} - \frac{(7b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{3a}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{(7b^2)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{5a^2}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(7b^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{}\right.}{5a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(14b^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x,\right.}{5a^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} + \frac{\left(14b^{5/2}\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x\right.}{5a^{5/2}\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{14b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{5/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{14b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^2} + \frac{2x\sqrt{b\sqrt[3]{x}+ax}}{3a} - \frac{14b^{9/4}\left(\sqrt{\right.}}{}$$

Mathematica [C] time = 0.09, size = 106, normalized size = 0.33

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(5a^2x^{5/3}+7b^2\sqrt[3]{x}\sqrt{\frac{ax^{2/3}}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)-2abx-7b^2\sqrt[3]{x}\right)}{15a^2\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(-7*b^2*x^(1/3) - 2*a*b*x + 5*a^2*x^(5/3) + 7*b^2*
Sqrt[1 + (a*x^(2/3))/b]*x^(1/3)*Hypergeometric2F1[1/2, 3/4, 7/4, -((a*x^(2/
3))/b)]))/(15*a^2*(b + a*x^(2/3)))
```

fricas [F] time = 7.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2-abx^{\frac{4}{3}}+b^2x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^2+b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^2 + b^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

maple [A] time = 0.06, size = 228, normalized size = 0.70

$$-\frac{-10a^3x^2+4a^2b\,x^{\frac{4}{3}}-42\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,b^3\,\mathrm{EllipticE}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+21\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}}{15\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] -1/15/a^3*(-42*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(
1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*El
lipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+21*b^3*(
(a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+
(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+14*a*b^2*x^(2/3)+4*a^2*b*x^(
4/3)-10*a^3*x^2)/((a*x^(2/3)+b)*x^(1/3))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax+bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*x^(1/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^(1/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*x + b*x**(1/3)), x)
```

3.153 $\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=126

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{5/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 2*(b*x^(1/3)+a*x)^(1/2)/a-b^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(
1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x
^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(5/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2010, 2013, 2011, 329, 220}

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}}{a}-\frac{b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{5/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^(1/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(1/3) + a*x])/a - (b^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(5/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt
[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n -
 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{b\sqrt[3]{x}+ax}}\,dx = \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a} - \frac{b\int \frac{1}{x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}\,dx}{3a}$$

$$= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{a}$$

$$= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a} - \frac{\left(b\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{a\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a} - \frac{\left(2b\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{a\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{2\sqrt{b\sqrt[3]{x}+ax}}{a} - \frac{b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\,\sqrt[6]{x}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{a^{5/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] time = 0.04, size = 80, normalized size = 0.63

$$\frac{2\sqrt{ax+b\sqrt[3]{x}}\left(-b\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)+ax^{2/3}+b\right)}{a\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^(1/3) + a*x],x]
[Out] (2*Sqrt[b*x^(1/3) + a*x]*(b + a*x^(2/3) - b*Sqrt[1 + (a*x^(2/3))/b]*Hyperge
ometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a*(b + a*x^(2/3)))
```

fricas [F] time = 1.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2 - abx^{\frac{4}{3}} + b^2x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^3+b^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^3 + b^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

maple [A] time = 0.05, size = 127, normalized size = 1.01

$$\frac{2a^{2}x + 2ab\,x^{\frac{1}{3}} - \sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,b\,\mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)}{\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] (-b*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/
3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*Elli
pticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+2*a*b*x^(1
/3)+2*a^2*x)/((a*x^(2/3)+b)*x^(1/3))^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(1/3)), x)
```

mupad [B] time = 5.27, size = 40, normalized size = 0.32

$$\frac{2\,x\,\sqrt{\frac{b}{a\,x^{2/3}}+1}\;{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ -\frac{b}{a\,x^{2/3}}\right)}{\sqrt{a\,x+b\,x^{1/3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1/3))^(1/2),x)
```

```
[Out] (2*x*(b/(a*x^(2/3)) + 1)^(1/2)*hypergeom([-3/4, 1/2], 1/4, -b/(a*x^(2/3))))
/(a*x + b*x^(1/3))^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**(1/3)), x)
```

3.154 $\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=294

$$\frac{3\sqrt[4]{a}\,\sqrt[6]{x}\,\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{6\sqrt[4]{a}\,\sqrt[6]{x}\,\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\tan^{-1}\right.}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 6*(b+a*x^(2/3))*x^(1/3)*a^(1/2)/b/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)
^(1/2)-6*(b*x^(1/3)+a*x)^(1/2)/b/x^(1/3)-6*a^(1/4)*x^(1/6)*(cos(2*arctan(a^
(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*Ell
ipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)
+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1
/3)+a*x)^(1/2)+3*a^(1/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(
x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2018, 2025, 2032, 329, 305, 220, 1196}

$$\frac{3\sqrt[4]{a}\,\sqrt[6]{x}\,\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\,\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{6\sqrt[4]{a}\,\sqrt[6]{x}\,\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\,\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\tan^{-1}\right.}{b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (6*Sqrt[a]*(b + a*x^(2/3))*x^(1/3))/(b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x
^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(b*x^(1/3)) - (6*a^(1/4)*(Sqrt[b
] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^
(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*
x^(1/3) + a*x]) + (3*a^(1/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3
))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/
6))/b^(1/4)], 1/2])/(b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{(3a)\,\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(3a\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(6a\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} + \frac{\left(6\sqrt{a}\,\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{b}\,\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(6\sqrt{a}\,\sqrt{b+ax^{2\ldots}}\right.}{\ldots}$$

$$= \frac{6\sqrt{a}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{b\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{b\sqrt[3]{x}} - \frac{6\sqrt[4]{a}\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\,\sqrt[3]{x}\right)^2}}\ldots}{b^{3/4}\sqrt{b\sqrt[3]{x}+\ldots}}$$

Mathematica [C] time = 0.07, size = 54, normalized size = 0.18

$$-\frac{6\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-\frac{ax^{2/3}}{b}\right)}{\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-1/4, 1/2, 3/4, -((a*x^(2/3))
/b)])/Sqrt[b*x^(1/3) + a*x]
```

fricas [F] time = 6.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2-abx^{\frac{4}{3}}+b^2x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^4+b^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^4 + b^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

maple [A] time = 0.08, size = 253, normalized size = 0.86

$$-\frac{3\left(-2\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,b\,\mathrm{EllipticE}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}}{\left(a\,x^{\frac{2}{3}}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] -3*(-2*((a*x^(2/3)+b)*x^(1/3))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a
*x^(1/3))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2
*2^(1/2))*b+((a*x^(2/3)+b)*x^(1/3))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1
/2)*a*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
),1/2*2^(1/2))*b+2*x^(2/3)*(a*x+b*x^(1/3))^(1/2)*a+2*(a*x+b*x^(1/3))^(1/2)*
b)/x^(1/3)/(a*x^(2/3)+b)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(1/3))^(1/2)),x)
[Out] int(1/(x*(a*x + b*x^(1/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/3)+a*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(a*x + b*x**(1/3))), x)
```

3.155 $\int \frac{1}{x^2 \sqrt{b \sqrt[3]{x}+ax}} \, dx$

Optimal. Leaf size=163

$$\frac{5a^{7/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{10a\sqrt{ax+b\sqrt[3]{x}}}{7b^2x^{2/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}$$

```
[Out] -6/7*(b*x^(1/3)+a*x)^(1/2)/b/x^(4/3)+10/7*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/
3)+5/7*a^(7/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/
b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^
(1/2)+b^(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 2025, 2011, 329, 220}

$$\frac{5a^{7/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{7b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{10a\sqrt{ax+b\sqrt[3]{x}}}{7b^2x^{2/3}}-\frac{6\sqrt{ax+b\sqrt[3]{x}}}{7bx^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(7*b*x^(4/3)) + (10*a*Sqrt[b*x^(1/3) + a*x])/(7*
b^2*x^(2/3)) + (5*a^(7/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/
(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))
/b^(1/4)], 1/2])/(7*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
```

```
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{b\sqrt[3]{x}+ax}}\,dx &= 3\,\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)\\ &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}}-\frac{(15a)\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7b}\\ &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}}+\frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}}+\frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{7b^2}\\ &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}}+\frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}}+\frac{\left(5a^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\,\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{7b^2\sqrt{b\sqrt[3]{x}+ax}}\\ &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}}+\frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}}+\frac{\left(10a^2\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{7b^2\sqrt{b\sqrt[3]{x}+ax}}\\ &= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{7bx^{4/3}}+\frac{10a\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{2/3}}+\frac{5a^{7/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\,\sqrt[6]{x}\,F\left(2\tan\right.}{7b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 59, normalized size = 0.36

$$-\frac{6\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{2};-\frac{3}{4};-\frac{ax^{2/3}}{b}\right)}{7x\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-7/4, 1/2, -3/4, -((a*x^(2/3)
)/b)])/(7*x*Sqrt[b*x^(1/3) + a*x])
```

fricas [F] time = 1.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2-abx^{\frac{4}{3}}+b^2x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^5+b^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^5 + b^3*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 3.266*(-12*b/84/b^2/x^(1/3)/x^(1/3)+20*a/84/b^2)*sqrt(a
/x^(1/3)+b/x)+integrate(60*a^2/84/b^2/3/((x^(1/6))^5*sqrt(a*(x^(1/3))^2+b)*
sign(x)),x)
```

maple [A] time = 0.09, size = 142, normalized size = 0.87

$$\frac{10a^{2}x^{\frac{5}{3}}+5\sqrt{-ab}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,a\,x^{\frac{4}{3}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+4abx-6b^{2}x^{\frac{1}{3}}}{7\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;b^{2}x^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] 1/7*(5*a*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*
x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)
*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(4/
3)+4*a*b*x+10*x^(5/3)*a^2-6*b^2*x^(1/3))/b^2/((a*x^(2/3)+b)*x^(1/3))^(1/2)/
x^(4/3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^{\frac{1}{3}}}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2}\,\sqrt{a\,x+b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(1/3))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(1/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*x**(1/3))), x)
```

3.156 $\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx$

Optimal. Leaf size=388

$$-\frac{77a^{13/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{154a^{13/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E}{65b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] -154/65*a^(7/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1
/3)+a*x)^(1/2)-6/13*(b*x^(1/3)+a*x)^(1/2)/b/x^(7/3)+22/39*a*(b*x^(1/3)+a*x)
^(1/2)/b^2/x^(5/3)-154/195*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x+154/65*a^3*(b*x^
(1/3)+a*x)^(1/2)/b^4/x^(1/3)+154/65*a^(13/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*
x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE
(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/
2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a
*x)^(1/2)-77/65*a^(13/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)
^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4
)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(
x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {2018, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{154a^{7/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{65b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}-\frac{77a^{13/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{65b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}+154a^{13/4}\sqrt[6]{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-154*a^(7/2)*(b + a*x^(2/3))*x^(1/3))/(65*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3))*
Sqrt[b*x^(1/3) + a*x]) - (6*Sqrt[b*x^(1/3) + a*x])/(13*b*x^(7/3)) + (22*a*S
qrt[b*x^(1/3) + a*x])/(39*b^2*x^(5/3)) - (154*a^2*Sqrt[b*x^(1/3) + a*x])/(1
95*b^3*x) + (154*a^3*Sqrt[b*x^(1/3) + a*x])/(65*b^4*x^(1/3)) + (154*a^(13/4
)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/
3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(65*b^(
15/4)*Sqrt[b*x^(1/3) + a*x]) - (77*a^(13/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan
[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(65*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} - \frac{(33a)\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{13b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} + \frac{(77a^2)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{39b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} - \frac{(77a^3)\,\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx,\right.}{65b^3}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{(77a^4)}{}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{(77a^4\sqrt{}}{}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{(154a^4}{}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}+ax}}{195b^3x} + \frac{154a^3\sqrt{b\sqrt[3]{x}+ax}}{65b^4\sqrt[3]{x}} - \frac{(154a^{7/}}{}$$

$$= -\frac{154a^{7/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{65b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{6\sqrt{b\sqrt[3]{x}+ax}}{13bx^{7/3}} + \frac{22a\sqrt{b\sqrt[3]{x}+ax}}{39b^2x^{5/3}} - \frac{154a^2\sqrt{b\sqrt[3]{x}}}{195b^3x}$$

Mathematica [C] time = 0.06, size = 59, normalized size = 0.15

$$-\frac{6\sqrt{\frac{ax^{2/3}}{b}+1}\;{}_2F_1\left(-\frac{13}{4},\frac{1}{2};-\frac{9}{4};-\frac{ax^{2/3}}{b}\right)}{13x^2\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-13/4, 1/2, -9/4, -((a*x^(2/3
))/b)])/(13*x^2*Sqrt[b*x^(1/3) + a*x])
```

fricas [F] time = 7.39, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2 - abx^{\frac{4}{3}} + b^2x^{\frac{2}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^3x^6+b^3x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x
^6 + b^3*x^4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign
 by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sy
m(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Va
lueEvaluation time: 1.416*((-3780*b^4/49140/b^5/x^(1/3)/x^(1/3)+4620*b^3*a/
49140/b^5)/x^(1/3)/x^(1/3)-6468*b^2*a^2/49140/b^5)/x^(1/3)*sqrt(a/x^(1/3)+b
/x)+integrate(-58212*b^2*a^3/49140/b^5/3/(x^(1/3)*x^(1/3)*sqrt(x)*sqrt(a*(x
^(1/3))^2+b)*sign(x)),x)
```

maple [A] time = 0.09, size = 365, normalized size = 0.94

$$-\frac{462\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{2}\,\sqrt{\frac{-a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^{3} b\,x^{\frac{10}{3}}\,\mathrm{EllipticE}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)-231\sqrt{a\,x \cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x+b*x^(1/3))^(1/2),x)
```

```
[Out] -1/195*(462*a^3*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*((-
a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/
2)*x^(10/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2
))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-231*a^3*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a
*b)^(1/2))^(1/2)*2^(1/2)*((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1
/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*x^(10/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*Ellip
ticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-462*x^(10/3
)*(a*x+b*x^(1/3))^(1/2)*a^3*b+44*x^(8/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^2*
b^2+154*x^(10/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^3*b-20*x^2*((a*x^(2/3)+b)*
x^(1/3))^(1/2)*a*b^3-462*x^4*(a*x+b*x^(1/3))^(1/2)*a^4+90*x^(4/3)*((a*x^(2/
3)+b)*x^(1/3))^(1/2)*b^4)/x^(11/3)/(a*x^(2/3)+b)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}\, x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{3}\,\sqrt{a\,x+b\,x^{1/3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(1/3))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(1/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{ax+b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*x**(1/3))), x)
```

3.157 $\int \frac{1}{x^4 \sqrt{b \sqrt[3]{x}+ax}}\, dx$

Optimal. Leaf size=251

$$-\frac{663a^{19/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{1463b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{1326a^4\sqrt{ax+b\sqrt[3]{x}}}{1463b^5x^{2/3}}+\frac{3978a^3\sqrt{ax+b\sqrt[3]{x}}}{7315b^4x^{4/3}}$$

```
[Out] -6/19*(b*x^(1/3)+a*x)^(1/2)/b/x^(10/3)+34/95*a*(b*x^(1/3)+a*x)^(1/2)/b^2/x^
(8/3)-442/1045*a^2*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2+3978/7315*a^3*(b*x^(1/3)+a
*x)^(1/2)/b^4/x^(4/3)-1326/1463*a^4*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)-663/1
463*a^(19/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2
*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1
/2)+b^(1/2))^2)^(1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2018, 2025, 2011, 329, 220}

$$-\frac{1326a^4\sqrt{ax+b\sqrt[3]{x}}}{1463b^5x^{2/3}}+\frac{3978a^3\sqrt{ax+b\sqrt[3]{x}}}{7315b^4x^{4/3}}-\frac{442a^2\sqrt{ax+b\sqrt[3]{x}}}{1045b^3x^2}-\frac{663a^{19/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F}{1463b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

```
[Out] (-6*Sqrt[b*x^(1/3) + a*x])/(19*b*x^(10/3)) + (34*a*Sqrt[b*x^(1/3) + a*x])/(
95*b^2*x^(8/3)) - (442*a^2*Sqrt[b*x^(1/3) + a*x])/(1045*b^3*x^2) + (3978*a^
3*Sqrt[b*x^(1/3) + a*x])/(7315*b^4*x^(4/3)) - (1326*a^4*Sqrt[b*x^(1/3) + a*
x])/(1463*b^5*x^(2/3)) - (663*a^(19/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b
+ a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(
1/4)*x^(1/6))/b^(1/4)], 1/2])/(1463*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{b\sqrt[3]{x}+ax}}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x^{10}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} - \frac{(51a)\,\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{19b}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} + \frac{\left(221a^2\right)\text{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{95b^2}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} - \frac{\left(1989a^3\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{bx+ax^3}}\right.}{1045b^3}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} + \frac{(1989}{}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} - \frac{1326a}{1}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} - \frac{1326a}{1}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} - \frac{1326a}{1}$$

$$= -\frac{6\sqrt{b\sqrt[3]{x}+ax}}{19bx^{10/3}} + \frac{34a\sqrt{b\sqrt[3]{x}+ax}}{95b^2x^{8/3}} - \frac{442a^2\sqrt{b\sqrt[3]{x}+ax}}{1045b^3x^2} + \frac{3978a^3\sqrt{b\sqrt[3]{x}+ax}}{7315b^4x^{4/3}} - \frac{1326a}{1}$$

Mathematica [C] time = 0.09, size = 59, normalized size = 0.24

$$-\frac{6\sqrt{\frac{ax^{2/3}}{b}+1}\;{}_2F_1\left(-\frac{19}{4},\frac{1}{2};-\frac{15}{4};-\frac{ax^{2/3}}{b}\right)}{19x^3\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x^(1/3) + a*x]),x]
```

[Out] (-6*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-19/4, 1/2, -15/4, -((a*x^(2/3))/b)])/(19*x^3*Sqrt[b*x^(1/3) + a*x])

fricas [F] time = 1.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2x^2 - abx^{\frac{4}{3}} + b^2x^{\frac{2}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^3x^7 + b^3x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="fricas")

[Out] integral((a^2*x^2 - a*b*x^(4/3) + b^2*x^(2/3))*sqrt(a*x + b*x^(1/3))/(a^3*x^7 + b^3*x^5), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="giac")

[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:INPUT:sage2OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation time: 8.266*((((-8108100*b^7/154053900/b^8/x^(1/3)/x^(1/3)+9189180*b^6*a/154053900/b^8)/x^(1/3)/x^(1/3)-10859940*b^5*a^2/154053900/b^8)/x^(1/3)/x^(1/3)+13962780*b^4*a^3/154053900/b^8)/x^(1/3)/x^(1/3)-23271300*b^3*a^4/154053900/b^8)*sqrt(a/x^(1/3)+b/x)+integrate(-69813900*b^3*a^5/154053900/b^8/3/((x^(1/6))^5*sqrt(a*(x^(1/3))^2+b)*sign(x)),x)

maple [A] time = 0.10, size = 179, normalized size = 0.71

$$-\frac{6630a^5x^{\frac{17}{3}} + 3315\sqrt{-ab}\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\sqrt{-\frac{2\left(a x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\sqrt{-\frac{a x^{\frac{1}{3}}}{\sqrt{-ab}}}\, a^4x^{\frac{16}{3}}\,\mathrm{EllipticF}\left(\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right) + 2652a}{7315\sqrt{\left(a x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\, b^5x^{\frac{16}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(a*x+b*x^(1/3))^(1/2),x)

[Out] -1/7315*(3315*a^4*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(16/3)+2652*a^4*b*x^5+6630*x^(17/3)*a^5+476*x^(11/3)*a^2*b^3-884*x^(13/3)*a^3*b^2-308*a*b^4*x^3+2310*x^(7/3)*b^5)/b^5/((a*x^(2/3)+b)*x^(1/3))^(1/2)/x^(16/3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^{\frac{1}{3}}}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(1/3))*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \sqrt{a\,x + b\,x^{1/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(1/3))^(1/2)),x)
```

```
[Out] int(1/(x^4*(a*x + b*x^(1/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \sqrt{ax + b\sqrt[3]{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a*x + b*x**(1/3))), x)
```

3.158 $\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=437

$$-\frac{4807b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{442a^{27/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{4807b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+}{\left(\sqrt{a}\sqrt[3]{x}+\right.}}}{221a^{27/4}\sqrt{ax+b}}$$

```
[Out] -3*x^4/a/(b*x^(1/3)+a*x)^(1/2)-4807/221*b^5*(b+a*x^(2/3))*x^(1/3)/a^(13/2)/
(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+4807/663*b^4*x^(1/3)*(b*x^(
1/3)+a*x)^(1/2)/a^6-24035/4641*b^3*x*(b*x^(1/3)+a*x)^(1/2)/a^5+6555/1547*b^
2*x^(5/3)*(b*x^(1/3)+a*x)^(1/2)/a^4-437/119*b*x^(7/3)*(b*x^(1/3)+a*x)^(1/2)
/a^3+23/7*x^3*(b*x^(1/3)+a*x)^(1/2)/a^2+4807/221*b^(21/4)*x^(1/6)*(cos(2*ar
ctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4
)))*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*
a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(27/4)
/(b*x^(1/3)+a*x)^(1/2)-4807/442*b^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin
(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*
((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(27/4)/(b*x^(1/3)+a*x)^
(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2022, 2024, 2032, 329, 305, 220, 1196}

$$-\frac{4807b^5\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221a^{13/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{6555b^2x^{5/3}\sqrt{ax+b\sqrt[3]{x}}}{1547a^4}-\frac{4807b^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}}{442a^{27/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-4807*b^5*(b + a*x^(2/3))*x^(1/3))/(221*a^(13/2)*(Sqrt[b] + Sqrt[a]*x^(1/3
))*Sqrt[b*x^(1/3) + a*x]) - (3*x^4)/(a*Sqrt[b*x^(1/3) + a*x]) + (4807*b^4*x
^(1/3)*Sqrt[b*x^(1/3) + a*x])/(663*a^6) - (24035*b^3*x*Sqrt[b*x^(1/3) + a*x
])/(4641*a^5) + (6555*b^2*x^(5/3)*Sqrt[b*x^(1/3) + a*x])/(1547*a^4) - (437*
b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])/(119*a^3) + (23*x^3*Sqrt[b*x^(1/3) + a*x])
/(7*a^2) + (4807*b^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/
(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))
/b^(1/4)], 1/2])/(221*a^(27/4)*Sqrt[b*x^(1/3) + a*x]) - (4807*b^(21/4)*(Sqr
t[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]
*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(442*a^(27/4)
*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^4}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^{14}}{\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{69\,\text{Subst}\left(\int \frac{x^{11}}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} - \frac{(437b)\,\text{Subst}\left(\int \frac{x^9}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{14a^2}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x}+ax}}{7a^2} + \frac{\left(6555b^2\right)\text{Subst}\left(\int \frac{}{\sqrt{bx}}\right.}{238a^3}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}}{119a^3} + \frac{23x^3\sqrt{b\sqrt[3]{x}+a}}{7a^2}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b\sqrt[3]{x}+ax}}{1547a^4} - \frac{437bx^{7/3}\sqrt{b\sqrt[3]{x}}}{119a^3}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b}}{1547a^4}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b}}{1547a^4}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b}}{1547a^4}$$

$$= -\frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} - \frac{24035b^3x\sqrt{b\sqrt[3]{x}+ax}}{4641a^5} + \frac{6555b^2x^{5/3}\sqrt{b}}{1547a^4}$$

$$= -\frac{4807b^5\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221a^{13/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{3x^4}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{4807b^4\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{663a^6} -$$

Mathematica [C] time = 0.11, size = 131, normalized size = 0.30

$$\frac{2x^{2/3}\left(663a^5x^{10/3} - 897a^4bx^{8/3} + 1311a^3b^2x^2 - 2185a^2b^3x^{4/3} + 33649b^5\sqrt{\frac{ax^{2/3}}{b}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^{2/3}}{b}\right) + 4807a\right.}{4641a^6\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(2/3)*(-33649*b^5 + 4807*a*b^4*x^(2/3) - 2185*a^2*b^3*x^(4/3) + 1311*a
^3*b^2*x^2 - 897*a^4*b*x^(8/3) + 663*a^5*x^(10/3) + 33649*b^5*Sqrt[1 + (a*x
```

```
^(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(4641*a^6*S
qrt[b*x^(1/3) + a*x])
```

fricas [F] time = 8.99, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^4x^6+3\,a^2b^2x^{\frac{14}{3}}-2\,ab^3x^4-\left(2\,a^3bx^5-b^4x^3\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^6x^4+2\,a^3b^3x^2+b^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^6 + 3*a^2*b^2*x^(14/3) - 2*a*b^3*x^4 - (2*a^3*b*x^5 - b^4*x
^3)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

maple [A] time = 0.12, size = 384, normalized size = 0.88

$$-\frac{-2652\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^6x^4+3588\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^5b\,x^{\frac{10}{3}}-5244\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^4b^2x^{\frac{8}{3}}+8740\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x+b*x^(1/3))^(3/2),x)
```

```
[Out] -1/9282/a^7*(-5244*x^(8/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^4*b^2+3588*x^(10
/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^5*b+8740*x^2*((a*x^(2/3)+b)*x^(1/3))^(1
/2)*a^3*b^3+201894*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a
*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2
)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^
(1/2))^(1/2),1/2*2^(1/2))-100947*b^6*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2)
)^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a
*x^(1/3))^(1/2)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^
(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))-2652*((a*x^(2/3)+b)*x^(1/3))^(1/2)*
x^4*a^6-39452*x^(2/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a*b^5-27846*x^(2/3)*(a*
x+b*x^(1/3))^(1/2)*a*b^5-19228*x^(4/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^2*b^
4)/x^(1/3)/(a*x^(2/3)+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] int(x^4/(a*x + b*x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**4/(a*x + b*x**(1/3))**(3/2), x)
```

3.159 $\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=239

$$\frac{663b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154a^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{663b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^5}+\frac{1989b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^4}-$$

```
[Out] -3*x^3/a/(b*x^(1/3)+a*x)^(1/2)-663/77*b^3*(b*x^(1/3)+a*x)^(1/2)/a^5+1989/38
5*b^2*x^(2/3)*(b*x^(1/3)+a*x)^(1/2)/a^4-221/55*b*x^(4/3)*(b*x^(1/3)+a*x)^(1
/2)/a^3+17/5*x^2*(b*x^(1/3)+a*x)^(1/2)/a^2+663/154*b^(15/4)*x^(1/6)*(cos(2*
arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1
/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3
)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(21/
4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2022, 2024, 2011, 329, 220}

$$\frac{1989b^2x^{2/3}\sqrt{ax+b\sqrt[3]{x}}}{385a^4}+\frac{663b^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154a^{21/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{663b^3\sqrt{ax+b\sqrt[3]{x}}}{77a^5}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-3*x^3)/(a*Sqrt[b*x^(1/3) + a*x]) - (663*b^3*Sqrt[b*x^(1/3) + a*x])/(77*a^
5) + (1989*b^2*x^(2/3)*Sqrt[b*x^(1/3) + a*x])/(385*a^4) - (221*b*x^(4/3)*Sq
rt[b*x^(1/3) + a*x])/(55*a^3) + (17*x^2*Sqrt[b*x^(1/3) + a*x])/(5*a^2) + (6
63*b^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqr
t[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2
])/(154*a^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^{11}}{\left(bx+ax^3\right)^{3/2}}\,dx,x,\sqrt[3]{x}\right)$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{51\,\text{Subst}\left(\int \frac{x^8}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2}-\frac{(221b)\,\text{Subst}\left(\int \frac{x^6}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{10a^2}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3}+\frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2}+\frac{\left(1989b^2\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{bx+ax}}\right.}{110a^3}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4}-\frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3}+\frac{17x^2\sqrt{b\sqrt[3]{x}+ax}}{5a^2}-$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5}+\frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4}-\frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5}+\frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4}-\frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5}+\frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4}-\frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3}$$

$$= -\frac{3x^3}{a\sqrt{b\sqrt[3]{x}+ax}}-\frac{663b^3\sqrt{b\sqrt[3]{x}+ax}}{77a^5}+\frac{1989b^2x^{2/3}\sqrt{b\sqrt[3]{x}+ax}}{385a^4}-\frac{221bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}}{55a^3}$$

Mathematica [C] time = 0.13, size = 124, normalized size = 0.52

$$\frac{\sqrt{ax+b\sqrt[3]{x}}\left(154a^4x^{8/3}-238a^3bx^2+442a^2b^2x^{4/3}+3315b^4\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)-1326ab^3x^{2/3}-3315b\right.}{385a^5\left(ax^{2/3}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*x^(1/3) + a*x]*(-3315*b^4 - 1326*a*b^3*x^(2/3) + 442*a^2*b^2*x^(4/3
) - 238*a^3*b*x^2 + 154*a^4*x^(8/3) + 3315*b^4*Sqrt[1 + (a*x^(2/3))/b]*Hype
rgeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(385*a^5*(b + a*x^(2/3)))
```

fricas [F] time = 1.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^5+3\,a^2b^2x^{\frac{11}{3}}-2\,ab^3x^3-\left(2\,a^3bx^4-b^4x^2\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^6x^4+2\,a^3b^3x^2+b^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^5 + 3*a^2*b^2*x^(11/3) - 2*a*b^3*x^3 - (2*a^3*b*x^4 - b^4*x
^2)*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

maple [A] time = 0.09, size = 260, normalized size = 1.09

$$-\frac{-308\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^5x^3+476\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^4b\,x^{\frac{7}{3}}-884\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^3b^2x^{\frac{5}{3}}+2652\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+b*x^(1/3))^(3/2),x)
```

```
[Out] -1/770*(-884*x^(5/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^3*b^2+476*x^(7/3)*((a*
x^(2/3)+b)*x^(1/3))^(1/2)*a^4*b-3315*((a*x^(2/3)+b)*x^(1/3))^(1/2)*Elliptic
F(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*(-a*b)^(1/2)*(
(a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(
-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*b^4+2652*x*((a*x^(2/3)
+b)*x^(1/3))^(1/2)*a^2*b^3-308*((a*x^(2/3)+b)*x^(1/3))^(1/2)*x^3*a^5+4320*x
^(1/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a*b^4+2310*x^(1/3)*(a*x+b*x^(1/3))^(1/
2)*a*b^4)/x^(1/3)/(a*x^(2/3)+b)/a^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(1/3)+a*x)**(3/2),x)
[Out] Integral(x**3/(a*x + b*x**(1/3))**(3/2), x)
```

3.160 $\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=349

$$\frac{77b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{77b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\right.}{5a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] -3*x^2/a/(b*x^(1/3)+a*x)^(1/2)+77/5*b^2*(b+a*x^(2/3))*x^(1/3)/a^(7/2)/(x^(1
/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-77/15*b*x^(1/3)*(b*x^(1/3)+a*x)^
(1/2)/a^3+11/3*x*(b*x^(1/3)+a*x)^(1/2)/a^2-77/5*b^(9/4)*x^(1/6)*(cos(2*arct
an(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
)*EllipticE(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^
(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(15/4)/(
b*x^(1/3)+a*x)^(1/2)+77/10*b^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arc
tan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*
x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2022, 2024, 2032, 329, 305, 220, 1196}

$$\frac{77b^2\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5a^{7/2}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{77b^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10a^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{77b^{9/4}\sqrt[6]{x}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (77*b^2*(b + a*x^(2/3))*x^(1/3))/(5*a^(7/2)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqr
t[b*x^(1/3) + a*x]) - (3*x^2)/(a*Sqrt[b*x^(1/3) + a*x]) - (77*b*x^(1/3)*Sqr
t[b*x^(1/3) + a*x])/(15*a^3) + (11*x*Sqrt[b*x^(1/3) + a*x])/(3*a^2) - (77*b
^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5
*a^(15/4)*Sqrt[b*x^(1/3) + a*x]) + (77*b^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*
Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*Arc
Tan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(10*a^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
```

```
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^2}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{x^8}{\left(bx+ax^3\right)^{3/2}}\,dx,x,\sqrt[3]{x}\right)$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{33\,\text{Subst}\left(\int \frac{x^5}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{2a}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} - \frac{(77b)\,\text{Subst}\left(\int \frac{x^3}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{6a^2}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^2\right)\text{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Sul}}{10a^3\sqrt{b\sqrt[3]{}}}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^2\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Sul}}{5a^3\sqrt{b\sqrt[3]{}}}$$

$$= -\frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{x}+ax}}{3a^2} + \frac{\left(77b^{5/2}\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{S}}{5a^{7/2}\sqrt{b}}$$

$$= \frac{77b^2\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5a^{7/2}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{3x^2}{a\sqrt{b\sqrt[3]{x}+ax}} - \frac{77b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}{15a^3} + \frac{11x\sqrt{b\sqrt[3]{}}}{3a^2}$$

Mathematica [C] time = 0.09, size = 94, normalized size = 0.27

$$\frac{2x^{2/3}\left(5a^2x^{4/3} - 77b^2\sqrt{\frac{ax^{2/3}}{b}+1}\;{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^{2/3}}{b}\right) - 11abx^{2/3} + 77b^2\right)}{15a^3\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(2/3)*(77*b^2 - 11*a*b*x^(2/3) + 5*a^2*x^(4/3) - 77*b^2*Sqrt[1 + (a*x^
(2/3))/b]*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^(2/3))/b)]))/(15*a^3*Sqrt
[b*x^(1/3) + a*x])
```

fricas [F] time = 8.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^4 + 3\,a^2b^2x^{\frac{8}{3}} - 2\,ab^3x^2 - \left(2\,a^3bx^3 - b^4x\right)x^{\frac{1}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^6x^4 + 2\,a^3b^3x^2 + b^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^4 + 3*a^2*b^2*x^(8/3) - 2*a*b^3*x^2 - (2*a^3*b*x^3 - b^4*x)
*x^(1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

maple [A] time = 0.07, size = 312, normalized size = 0.89

$$-\frac{-20\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\,a^3x^2+44\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\,a^2b\,x^{\frac{4}{3}}-462\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,\sqrt{\left(a\,x^{\frac{2}{3}}+b\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+b*x^(1/3))^(3/2),x)
```

```
[Out] -1/30/a^4*(-462*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^
(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*(
(a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2),1/2*2^(1/2))+231*b^3*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2
)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/
3))^(1/2)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+64*x^(2/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*
a*b^2+90*x^(2/3)*(a*x+b*x^(1/3))^(1/2)*a*b^2+44*x^(4/3)*((a*x^(2/3)+b)*x^(1
/3))^(1/2)*a^2*b-20*((a*x^(2/3)+b)*x^(1/3))^(1/2)*x^2*a^3)/x^(1/3)/(a*x^(2/
3)+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a*x + b*x**(1/3))**(3/2), x)
```

3.161 $\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{5b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{5\sqrt{ax+b\sqrt[3]{x}}}{a^2}-\frac{3x}{a\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] -3*x/a/(b*x^(1/3)+a*x)^(1/2)+5*(b*x^(1/3)+a*x)^(1/2)/a^2-5/2*b^(3/4)*x^(1/6
)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(
1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2)
)*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/
2)/a^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2018, 2022, 2024, 2011, 329, 220}

$$-\frac{5b^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{9/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{5\sqrt{ax+b\sqrt[3]{x}}}{a^2}-\frac{3x}{a\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (-3*x)/(a*Sqrt[b*x^(1/3) + a*x]) + (5*Sqrt[b*x^(1/3) + a*x])/a^2 - (5*b^(3/
4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1
/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2*a^(
9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2022

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(n - j)*
(p + 1)), x] - Dist[(c^n*(m + j*p - n + j + 1))/(b*(n - j)*(p + 1)), Int[(c
*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !Int
egerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &&
 GtQ[m + j*p + 1, n - j]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx &= 3\,\text{Subst}\left(\int \frac{x^5}{\left(bx+ax^3\right)^{3/2}}\,dx,x,\sqrt[3]{x}\right)\\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{15\,\text{Subst}\left(\int \frac{x^2}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{2a}\\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2}-\frac{(5b)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx,x,\sqrt[3]{x}\right)}{2a^2}\\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2}-\frac{\left(5b\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx,x,\sqrt[3]{x}\right)}{2a^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2}-\frac{\left(5b\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx,x,\sqrt[6]{x}\right)}{a^2\sqrt{b\sqrt[3]{x}+ax}}\\
&= -\frac{3x}{a\sqrt{b\sqrt[3]{x}+ax}}+\frac{5\sqrt{b\sqrt[3]{x}+ax}}{a^2}-\frac{5b^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\,\sqrt[6]{x}\,F\left(2\tan^{-1}\right.}{2a^{9/4}\sqrt{b\sqrt[3]{x}+ax}}
\end{aligned}
$$

Mathematica [C] time = 0.09, size = 82, normalized size = 0.55

$$
\frac{\sqrt{ax+b\sqrt[3]{x}}\left(-5b\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{ax^{2/3}}{b}\right)+2ax^{2/3}+5b\right)}{a^2\left(ax^{2/3}+b\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^(1/3) + a*x)^(3/2),x]
```

```
[Out] (Sqrt[b*x^(1/3) + a*x]*(5*b + 2*a*x^(2/3) - 5*b*Sqrt[1 + (a*x^(2/3))/b]*Hyp
ergeometric2F1[1/4, 1/2, 5/4, -((a*x^(2/3))/b)]))/(a^2*(b + a*x^(2/3)))
```

fricas [F] time = 1.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^4x^3+3\,a^2b^2x^{\frac{5}{3}}-2\,ab^3x-\left(2\,a^3bx^2-b^4\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^6x^4+2\,a^3b^3x^2+b^6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^4 + 2*a^3*b^3*x^2 + b^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

maple [A] time = 0.06, size = 184, normalized size = 1.23

$$\frac{4\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^2x+4\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;ab\,x^{\frac{1}{3}}+6\sqrt{ax+b\,x^{\frac{1}{3}}}\;ab\,x^{\frac{1}{3}}-5\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;\sqrt{-ab}\;\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\;\sqrt{\frac{a\,x^{\frac{1}{3}}+\cdots}{\sqrt{\cdots}}}\cdots}{2\left(a\,x^{\frac{2}{3}}+b\right)a^3x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+b*x^(1/3))^(3/2),x)
```

```
[Out] 1/2*(-5*((a*x^(2/3)+b)*x^(1/3))^(1/2)*(-a*b)^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/
3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/
2))*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/
2))/(-a*b)^(1/2))^(1/2)*b+4*x^(1/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a*b+6*x^(
1/3)*(a*x+b*x^(1/3))^(1/2)*a*b+4*((a*x^(2/3)+b)*x^(1/3))^(1/2)*x*a^2)/x^(1/
3)/(a*x^(2/3)+b)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(a*x + b*x^(1/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a\,x+b\,x^{1/3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] int(x/(a*x + b*x^(1/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x/(a*x + b*x**(1/3))**(3/2), x)
```

3.162 $\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=296

$$-\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\ldots}{\ldots}\right.\right.}{a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 3*x^(2/3)/b/(b*x^(1/3)+a*x)^(1/2)-3*(b+a*x^(2/3))*x^(1/3)/b/a^(1/2)/(x^(1/3
)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)+3*x^(1/6)*(cos(2*arctan(a^(1/4)*x^
(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(s
in(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2)
)*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^(1
/3)+a*x)^(1/2)-3/2*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)
/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1
/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3
)*a^(1/2)+b^(1/2))^2)^(1/2)/a^(3/4)/b^(3/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2006, 2018, 2032, 329, 305, 220, 1196}

$$-\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{3\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}E\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\ldots}{\ldots}\right.\right.}{a^{3/4}b^{3/4}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(1/3) + a*x)^(-3/2),x]
```

```
[Out] (-3*(b + a*x^(2/3))*x^(1/3))/(Sqrt[a]*b*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*
x^(1/3) + a*x]) + (3*x^(2/3))/(b*Sqrt[b*x^(1/3) + a*x]) + (3*(Sqrt[b] + Sqr
t[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*E
llipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(a^(3/4)*b^(3/4)*Sqrt[b
*x^(1/3) + a*x]) - (3*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqr
t[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(
1/4)], 1/2])/(2*a^(3/4)*b^(3/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\int \frac{1}{\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}}\,dx}{2b}$$

$$= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{3\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{\sqrt{x}}{\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{x}\right)}{2b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{b\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} - \frac{\left(3\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{a}\sqrt{b}\sqrt{b\sqrt[3]{x}+ax}} + \frac{\left(3\sqrt{b+ax^{2/3}}\,\sqrt[6]{x}\right)\mathrm{S}}{\sqrt{a}\sqrt{b}}$$

$$= -\frac{3\left(b+ax^{2/3}\right)\sqrt[3]{x}}{\sqrt{a}\,b\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} + \frac{3x^{2/3}}{b\sqrt{b\sqrt[3]{x}+ax}} + \frac{3\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)}}}{a^{3/4}b^{3/4}\sqrt{b}}$$

Mathematica [C] time = 0.04, size = 62, normalized size = 0.21

$$\frac{2x^{2/3}\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{ax^{2/3}}{b}\right)}{b\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(1/3) + a*x)^(-3/2),x]
```

```
[Out] (2*Sqrt[1 + (a*x^(2/3))/b]*x^(2/3)*Hypergeometric2F1[3/4, 3/2, 7/4, -((a*x^
(2/3))/b)])/(b*Sqrt[b*x^(1/3) + a*x])
```

fricas [F] time = 7.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^3+3\,a^2b^2x^{\frac{5}{3}}-2\,ab^3x-\left(2\,a^3bx^2-b^4\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^6x^5+2\,a^3b^3x^3+b^6x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^5 + 2*a^3*b^3*x^3 + b^6*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^(1/3))^(-3/2), x)
```

maple [A] time = 0.06, size = 242, normalized size = 0.82

$$\frac{-3\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\;b\,\mathrm{EllipticE}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}},\frac{\sqrt{2}}{2}\right)+\frac{3\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\,\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{\cdots}}{\sqrt{-ab}}}\cdots}{\cdots}}{\left(a\,x^{\frac{2}{3}}+b\right)ab\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(1/3))^(3/2),x)
```

```
[Out] 3/2/a*(-2*((a*x^(2/3)+b)*x^(1/3))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1
/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2
)*a*x^(1/3))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),
1/2*2^(1/2))*b+((a*x^(2/3)+b)*x^(1/3))^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)
^(1/2)*a*x^(1/3))^(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(
1/2),1/2*2^(1/2))*b+2*(a*x+b*x^(1/3))^(1/2)*a*x^(2/3))/x^(1/3)/(a*x^(2/3)+b
)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(1/3))^(-3/2), x)
```

mupad [B] time = 5.35, size = 40, normalized size = 0.14

$$\frac{2\,x\left(\frac{a\,x^{2/3}}{b}+1\right)^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\ \frac{7}{4};\ -\frac{a\,x^{2/3}}{b}\right)}{\left(a\,x+b\,x^{1/3}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1/3))^(3/2),x)
```

```
[Out] (2*x*((a*x^(2/3))/b + 1)^(3/2)*hypergeom([3/4, 3/2], 7/4, -(a*x^(2/3))/b))/
(a*x + b*x^(1/3))^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(1/3))**(-3/2), x)
```

3.163 $\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{5a^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{5\sqrt{ax+b\sqrt[3]{x}}}{b^2x^{2/3}}+\frac{3}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 3/b/x^(1/3)/(b*x^(1/3)+a*x)^(1/2)-5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(2/3)-5/2*a
^(3/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arcta
n(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))
),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^
(1/2))^2)^(1/2)/b^(9/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2018, 2023, 2025, 2011, 329, 220}

$$-\frac{5a^{3/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{2b^{9/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{5\sqrt{ax+b\sqrt[3]{x}}}{b^2x^{2/3}}+\frac{3}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x]) - (5*Sqrt[b*x^(1/3) + a*x])/(b^2*x^(2/3
)) - (5*a^(3/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] +
 Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)],
 1/2])/(2*b^(9/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
```

```
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} + \frac{15\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} - \frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}} - \frac{(5a)\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b^2}$$

$$= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} - \frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}} - \frac{\left(5a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\sqrt{b+ax^2}}\,dx, x, \sqrt[3]{\right.}}{2b^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} - \frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}} - \frac{\left(5a\sqrt{b+ax^{2/3}}\sqrt[6]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{b+ax^4}}\,dx, x, \sqrt[6]{x}\right.}{b^2\sqrt{b\sqrt[3]{x}+ax}}$$

$$= \frac{3}{b\sqrt[3]{x}\sqrt{b\sqrt[3]{x}+ax}} - \frac{5\sqrt{b\sqrt[3]{x}+ax}}{b^2x^{2/3}} - \frac{5a^{3/4}\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{\frac{b+ax^{2/3}}{\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)^2}}\sqrt[6]{x}\,F\left(2\,\tan\right.}{2b^{9/4}\sqrt{b\sqrt[3]{x}+ax}}$$

Mathematica [C] time = 0.07, size = 62, normalized size = 0.39

$$-\frac{2\sqrt{\frac{ax^{2/3}}{b}+1}\;{}_2F_1\left(-\frac{3}{4},\frac{3}{2};\frac{1}{4};-\frac{ax^{2/3}}{b}\right)}{b\sqrt[3]{x}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^(1/3) + a*x)^(3/2)),x]
```

[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-3/4, 3/2, 1/4, -((a*x^(2/3))
/b)])/(b*x^(1/3)*Sqrt[b*x^(1/3) + a*x])

fricas [F] time = 1.69, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^{4}x^{3}+3\,a^{2}b^{2}x^{\frac{5}{3}}-2\,ab^{3}x-\left(2\,a^{3}bx^{2}-b^{4}\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^{6}x^{6}+2\,a^{3}b^{3}x^{4}+b^{6}x^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")

[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^6 + 2*a^3*b^3*x^4 + b^6*x^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")

[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)

maple [A] time = 0.07, size = 181, normalized size = 1.15

$$-\frac{6\sqrt{ax+b\,x^{\frac{1}{3}}}\;ax+4\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;ax+5\sqrt{-ab}\;\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\;\sqrt{2}\;\sqrt{\frac{-a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\;\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\;\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;x^{\frac{2}{3}}\,\mathrm{E}}{2\left(a\,x^{\frac{2}{3}}+b\right)b^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a*x+b*x^(1/3))^(3/2),x)

[Out] -1/2*(5*(-a*b)^(1/2)*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*2^(1/2)*
((-a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^
(1/2)*EllipticF(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*
x^(2/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)+6*(a*x+b*x^(1/3))^(1/2)*x*a+4*x^(1/3)
*((a*x^(2/3)+b)*x^(1/3))^(1/2)*b+4*((a*x^(2/3)+b)*x^(1/3))^(1/2)*x*a)/b^2/x
/(a*x^(2/3)+b)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+bx^{\frac{1}{3}}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(1/3))^(3/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(1/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*x**(1/3))**(3/2)), x)
```

3.164 $\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=383

$$\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,E\left(2\tan\cdots\right.}{5b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 3/b/x^(4/3)/(b*x^(1/3)+a*x)^(1/2)+77/5*a^(5/2)*(b+a*x^(2/3))*x^(1/3)/b^4/(x
^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-11/3*(b*x^(1/3)+a*x)^(1/2)/b^
2/x^(5/3)+77/15*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x-77/5*a^2*(b*x^(1/3)+a*x)^(1/2
)/b^4/x^(1/3)-77/5*a^(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^
2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arctan(a^(1
/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))
/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)+77/10*a^
(9/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan
(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))
,1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(
1/2))^2)^(1/2)/b^(15/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2023, 2025, 2032, 329, 305, 220, 1196}

$$\frac{77a^{5/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{5b^4\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{10b^{15/4}\sqrt{ax+b\sqrt[3]{x}}}-\frac{77a^{9/4}\sqrt[6]{x}\left(\sqrt{\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(4/3)*Sqrt[b*x^(1/3) + a*x]) + (77*a^(5/2)*(b + a*x^(2/3))*x^(1/3))/
(5*b^4*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (11*Sqrt[b*x^(1
/3) + a*x])/(3*b^2*x^(5/3)) + (77*a*Sqrt[b*x^(1/3) + a*x])/(15*b^3*x) - (77
*a^2*Sqrt[b*x^(1/3) + a*x])/(5*b^4*x^(1/3)) - (77*a^(9/4)*(Sqrt[b] + Sqrt[a
]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*Elli
pticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(5*b^(15/4)*Sqrt[b*x^(1/3)
 + a*x]) + (77*a^(9/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sq
rt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^
(1/4)], 1/2])/(10*b^(15/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
 b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^2\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x^4\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{33\,\text{Subst}\left(\int \frac{1}{x^5\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} - \frac{(77a)\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{6b^2}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} + \frac{\left(77a^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{bx+ax^3}}\,dx\right.}{10b^3}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^3\right)}{}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^3\sqrt{\right.}}{}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^3\sqrt{\right.}}{}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+ax}}{15b^3x} - \frac{77a^2\sqrt{b\sqrt[3]{x}+ax}}{5b^4\sqrt[3]{x}} + \frac{\left(77a^{5/2}\right.}{}$$

$$= \frac{3}{bx^{4/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{77a^{5/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{5b^4\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{11\sqrt{b\sqrt[3]{x}+ax}}{3b^2x^{5/3}} + \frac{77a\sqrt{b\sqrt[3]{x}+}}{15b^3x}$$

Mathematica [C] time = 0.08, size = 64, normalized size = 0.17

$$-\frac{2\sqrt{\frac{ax^{2/3}}{b}+1}\;{}_2F_1\left(-\frac{9}{4},\frac{3}{2};-\frac{5}{4};-\frac{ax^{2/3}}{b}\right)}{3bx^{4/3}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-9/4, 3/2, -5/4, -((a*x^(2/3)
)/b)])/(3*b*x^(4/3)*Sqrt[b*x^(1/3) + a*x])
```

fricas [F] time = 7.17, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^3+3\,a^2b^2x^{\frac{5}{3}}-2\,ab^3x-\left(2\,a^3bx^2-b^4\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^6x^7+2\,a^3b^3x^5+b^6x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^7 + 2*a^3*b^3*x^5 + b^6*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

maple [A] time = 0.07, size = 339, normalized size = 0.89

$$-\frac{-462\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\, \sqrt{-\frac{2\left(a x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\, \sqrt{-\frac{a x^{\frac{1}{3}}}{\sqrt{-ab}}}\, \sqrt{\left(a x^{\frac{2}{3}}+b\right) x^{\frac{1}{3}}}\; a^2 b\, x^{\frac{8}{3}}\, \mathrm{EllipticE}\left(\sqrt{\frac{a x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right)+231\sqrt{\frac{a x}{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+b*x^(1/3))^(3/2),x)
```

```
[Out] -1/30*(-462*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1
/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*x^(
8/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))/(-a*
b)^(1/2))^(1/2),1/2*2^(1/2))+231*a^2*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/
2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)
*a*x^(1/3))^(1/2)*x^(8/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticF(((a*x^(1/
3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+462*(a*x+b*x^(1/3))^(1/2)
*a^3*x^(10/3)+372*(a*x+b*x^(1/3))^(1/2)*a^2*b*x^(8/3)-44*((a*x^(2/3)+b)*x^(
1/3))^(1/2)*a*b^2*x^2-64*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^2*b*x^(8/3)+20*((a
*x^(2/3)+b)*x^(1/3))^(1/2)*b^3*x^(4/3))/x^3/(a*x^(2/3)+b)/b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2 \left(a\,x + b\,x^{1/3}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(1/3))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(1/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(ax + b\sqrt[3]{x}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*x + b*x**(1/3))**(3/2)), x)
```

3.165 $\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=246

$$\frac{663a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{663a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^5x^{2/3}}-\frac{1989a^2\sqrt{ax+b\sqrt[3]{x}}}{385b^4x^{4/3}}+$$

```
[Out] 3/b/x^(7/3)/(b*x^(1/3)+a*x)^(1/2)-17/5*(b*x^(1/3)+a*x)^(1/2)/b^2/x^(8/3)+22
1/55*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^2-1989/385*a^2*(b*x^(1/3)+a*x)^(1/2)/b^4
/x^(4/3)+663/77*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(2/3)+663/154*a^(15/4)*x^(1
/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x
^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/
2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(
1/2)/b^(21/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {2018, 2023, 2025, 2011, 329, 220}

$$\frac{663a^3\sqrt{ax+b\sqrt[3]{x}}}{77b^5x^{2/3}}-\frac{1989a^2\sqrt{ax+b\sqrt[3]{x}}}{385b^4x^{4/3}}+\frac{663a^{15/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{154b^{21/4}\sqrt{ax+b\sqrt[3]{x}}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(7/3)*Sqrt[b*x^(1/3) + a*x]) - (17*Sqrt[b*x^(1/3) + a*x])/(5*b^2*x^(
8/3)) + (221*a*Sqrt[b*x^(1/3) + a*x])/(55*b^3*x^2) - (1989*a^2*Sqrt[b*x^(1/
3) + a*x])/(385*b^4*x^(4/3)) + (663*a^3*Sqrt[b*x^(1/3) + a*x])/(77*b^5*x^(2
/3)) + (663*a^(15/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt
[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1
/4)], 1/2])/(154*b^(21/4)*Sqrt[b*x^(1/3) + a*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^3\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x^7\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{51\,\text{Subst}\left(\int \frac{1}{x^8\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} - \frac{(221a)\,\text{Subst}\left(\int \frac{1}{x^6\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{10b^2}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} + \frac{\left(1989a^2\right)\text{Subst}\left(\int \frac{}{x^4\sqrt{b}}\right.}{110b^3}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} -$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} + \frac{6}{}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} + \frac{6}{}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} + \frac{6}{}$$

$$= \frac{3}{bx^{7/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{17\sqrt{b\sqrt[3]{x}+ax}}{5b^2x^{8/3}} + \frac{221a\sqrt{b\sqrt[3]{x}+ax}}{55b^3x^2} - \frac{1989a^2\sqrt{b\sqrt[3]{x}+ax}}{385b^4x^{4/3}} + \frac{6}{}$$

Mathematica [C] time = 0.07, size = 64, normalized size = 0.26

$$-\frac{2\sqrt{\frac{ax^{2/3}}{b}+1}\,{}_2F_1\left(-\frac{15}{4},\frac{3}{2};-\frac{11}{4};-\frac{ax^{2/3}}{b}\right)}{5bx^{7/3}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-15/4, 3/2, -11/4, -((a*x^(2/
3))/b])/(5*b*x^(7/3)*Sqrt[b*x^(1/3) + a*x])
```

fricas [F] time = 1.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^3+3\,a^2b^2x^{\frac{5}{3}}-2\,ab^3x-\left(2\,a^3bx^2-b^4\right)x^{\frac{1}{3}}\right)\sqrt{ax+bx^{\frac{1}{3}}}}{a^6x^8+2\,a^3b^3x^6+b^6x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^8 + 2*a^3*b^3*x^6 + b^6*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

maple [A] time = 0.08, size = 261, normalized size = 1.06

$$\frac{2310\sqrt{ax + b\,x^{\frac{1}{3}}}\;a^4x^5 + 4320\sqrt{\left(a\,x^{\frac{2}{3}} + b\right)x^{\frac{1}{3}}}\;a^4x^5 + 3315\sqrt{\frac{a\,x^{\frac{1}{3}} + \sqrt{-ab}}{\sqrt{-ab}}}\;\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}} - \sqrt{-ab}\right)}{\sqrt{-ab}}}\;\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\;\sqrt{\left(a\,x^{\frac{2}{3}} + b\right)x^{\frac{1}{3}}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x+b*x^(1/3))^(3/2),x)
```

```
[Out] 1/770*(3315*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-
a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)*EllipticF
(((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))*x^(14/3)*((a*x^
(2/3)+b)*x^(1/3))^(1/2)*(-a*b)^(1/2)*a^3-884*x^(11/3)*((a*x^(2/3)+b)*x^(1/3
))^(1/2)*a^2*b^2+2652*x^(13/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^3*b+476*x^3*
((a*x^(2/3)+b)*x^(1/3))^(1/2)*a*b^3+2310*(a*x+b*x^(1/3))^(1/2)*x^5*a^4+4320
*((a*x^(2/3)+b)*x^(1/3))^(1/2)*x^5*a^4-308*x^(7/3)*((a*x^(2/3)+b)*x^(1/3))^
(1/2)*b^4)/b^5/x^5/(a*x^(2/3)+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\left(a\,x + b\,x^{1/3}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(1/3))^(3/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(1/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a*x + b*x**(1/3))**(3/2)), x)
```

3.166 $\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=471

$$-\frac{4807a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt[6]{x}}{\sqrt[4]{b}}\right)|\frac{1}{2}\right)}{442b^{27/4}\sqrt{ax+b\sqrt[3]{x}}}+\frac{4807a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{\frac{ax^{2/3}+b}{\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)}}}{221b^{27/4}\sqrt{ax+b\sqrt[3]{x}}}$$

```
[Out] 3/b/x^(10/3)/(b*x^(1/3)+a*x)^(1/2)-4807/221*a^(11/2)*(b+a*x^(2/3))*x^(1/3)/
b^7/(x^(1/3)*a^(1/2)+b^(1/2))/(b*x^(1/3)+a*x)^(1/2)-23/7*(b*x^(1/3)+a*x)^(1
/2)/b^2/x^(11/3)+437/119*a*(b*x^(1/3)+a*x)^(1/2)/b^3/x^3-6555/1547*a^2*(b*x
^(1/3)+a*x)^(1/2)/b^4/x^(7/3)+24035/4641*a^3*(b*x^(1/3)+a*x)^(1/2)/b^5/x^(5
/3)-4807/663*a^4*(b*x^(1/3)+a*x)^(1/2)/b^6/x+4807/221*a^5*(b*x^(1/3)+a*x)^(
1/2)/b^7/x^(1/3)+4807/221*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^
(1/4)))^2)^(1/2)/cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticE(sin(2*arc
tan(a^(1/4)*x^(1/6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*
x^(2/3))/(x^(1/3)*a^(1/2)+b^(1/2))^2)^(1/2)/b^(27/4)/(b*x^(1/3)+a*x)^(1/2)-
4807/442*a^(21/4)*x^(1/6)*(cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))^2)^(1/2)/
cos(2*arctan(a^(1/4)*x^(1/6)/b^(1/4)))*EllipticF(sin(2*arctan(a^(1/4)*x^(1/
6)/b^(1/4))),1/2*2^(1/2))*(x^(1/3)*a^(1/2)+b^(1/2))*((b+a*x^(2/3))/(x^(1/3)
*a^(1/2)+b^(1/2))^2)^(1/2)/b^(27/4)/(b*x^(1/3)+a*x)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {2018, 2023, 2025, 2032, 329, 305, 220, 1196}

$$-\frac{4807a^{11/2}\sqrt[3]{x}\left(ax^{2/3}+b\right)}{221b^7\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)\sqrt{ax+b\sqrt[3]{x}}}+\frac{24035a^3\sqrt{ax+b\sqrt[3]{x}}}{4641b^5x^{5/3}}-\frac{6555a^2\sqrt{ax+b\sqrt[3]{x}}}{1547b^4x^{7/3}}-\frac{4807a^{21/4}\sqrt[6]{x}\left(\sqrt{a}\sqrt[3]{x}+\sqrt{b}\right)}{442b^{27}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] 3/(b*x^(10/3)*Sqrt[b*x^(1/3) + a*x]) - (4807*a^(11/2)*(b + a*x^(2/3))*x^(1/
3))/(221*b^7*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[b*x^(1/3) + a*x]) - (23*Sqrt[
b*x^(1/3) + a*x])/(7*b^2*x^(11/3)) + (437*a*Sqrt[b*x^(1/3) + a*x])/(119*b^3
*x^3) - (6555*a^2*Sqrt[b*x^(1/3) + a*x])/(1547*b^4*x^(7/3)) + (24035*a^3*Sq
rt[b*x^(1/3) + a*x])/(4641*b^5*x^(5/3)) - (4807*a^4*Sqrt[b*x^(1/3) + a*x])/
(663*b^6*x) + (4807*a^5*Sqrt[b*x^(1/3) + a*x])/(221*b^7*x^(1/3)) + (4807*a^
(21/4)*(Sqrt[b] + Sqrt[a]*x^(1/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*
x^(1/3))^2]*x^(1/6)*EllipticE[2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(2
21*b^(27/4)*Sqrt[b*x^(1/3) + a*x]) - (4807*a^(21/4)*(Sqrt[b] + Sqrt[a]*x^(1
/3))*Sqrt[(b + a*x^(2/3))/(Sqrt[b] + Sqrt[a]*x^(1/3))^2]*x^(1/6)*EllipticF[
2*ArcTan[(a^(1/4)*x^(1/6))/b^(1/4)], 1/2])/(442*b^(27/4)*Sqrt[b*x^(1/3) + a
*x])
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 305

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, D
ist[1/q, Int[1/Sqrt[a + b*x^4], x], x] - Dist[1/q, Int[(1 - q*x^2)/Sqrt[a +
```

```
b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1196

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + c*x^4])/(a*(1 + q^2*x^2)), x] + Simp[(d*(
1 + q^2*x^2)*Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticE[2*ArcTan[q*x],
 1/2])/(q*Sqrt[a + c*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e},
 x] && PosQ[c/a]
```

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j]
 && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^4\left(b\sqrt[3]{x}+ax\right)^{3/2}}\,dx = 3\,\text{Subst}\left(\int \frac{1}{x^{10}\left(bx+ax^3\right)^{3/2}}\,dx, x, \sqrt[3]{x}\right)$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} + \frac{69\,\text{Subst}\left(\int \frac{1}{x^{11}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{2b}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} - \frac{(437a)\,\text{Subst}\left(\int \frac{1}{x^{9}\sqrt{bx+ax^3}}\,dx, x, \sqrt[3]{x}\right)}{14b^2}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} + \frac{\left(6555a^2\right)\text{Subst}\left(\int \frac{1}{x^7\sqrt{bx+\cdots}}\right.}{238b^3}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} - \frac{(72\cdots}{}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24\cdots}{}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24\cdots}{}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24\cdots}{}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24\cdots}{}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24\cdots}{}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\sqrt[3]{x}+ax}}{119b^3x^3} - \frac{6555a^2\sqrt{b\sqrt[3]{x}+ax}}{1547b^4x^{7/3}} + \frac{24\cdots}{}$$

$$= \frac{3}{bx^{10/3}\sqrt{b\sqrt[3]{x}+ax}} - \frac{4807a^{11/2}\left(b+ax^{2/3}\right)\sqrt[3]{x}}{221b^7\left(\sqrt{b}+\sqrt{a}\sqrt[3]{x}\right)\sqrt{b\sqrt[3]{x}+ax}} - \frac{23\sqrt{b\sqrt[3]{x}+ax}}{7b^2x^{11/3}} + \frac{437a\sqrt{b\cdots}}{119b\cdots}$$

Mathematica [C] time = 0.08, size = 64, normalized size = 0.14

$$-\frac{2\sqrt{\frac{ax^{2/3}}{b}+1}\;{}_2F_1\left(-\frac{21}{4},\frac{3}{2};-\frac{17}{4};-\frac{ax^{2/3}}{b}\right)}{7bx^{10/3}\sqrt{ax+b\sqrt[3]{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^(1/3) + a*x)^(3/2)),x]
```

```
[Out] (-2*Sqrt[1 + (a*x^(2/3))/b]*Hypergeometric2F1[-21/4, 3/2, -17/4, -((a*x^(2/
3))/b)])/(7*b*x^(10/3)*Sqrt[b*x^(1/3) + a*x])
```

fricas [F] time = 7.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^4x^3 + 3\,a^2b^2x^{\frac{5}{3}} - 2\,ab^3x - \left(2\,a^3bx^2 - b^4\right)x^{\frac{1}{3}}\right)\sqrt{ax + bx^{\frac{1}{3}}}}{a^6x^9 + 2\,a^3b^3x^7 + b^6x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^4*x^3 + 3*a^2*b^2*x^(5/3) - 2*a*b^3*x - (2*a^3*b*x^2 - b^4)*x^(
1/3))*sqrt(a*x + b*x^(1/3))/(a^6*x^9 + 2*a^3*b^3*x^7 + b^6*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

maple [A] time = 0.07, size = 411, normalized size = 0.87

$$\frac{-201894\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}\,\sqrt{-\frac{2\left(a\,x^{\frac{1}{3}}-\sqrt{-ab}\right)}{\sqrt{-ab}}}\,\sqrt{-\frac{a\,x^{\frac{1}{3}}}{\sqrt{-ab}}}\,\sqrt{\left(a\,x^{\frac{2}{3}}+b\right)x^{\frac{1}{3}}}\;a^5b\,x^{\frac{20}{3}}\,\text{EllipticE}\left(\sqrt{\frac{a\,x^{\frac{1}{3}}+\sqrt{-ab}}{\sqrt{-ab}}}, \frac{\sqrt{2}}{2}\right) + 10094}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*x+b*x^(1/3))^(3/2),x)
```

```
[Out] 1/9282*(-201894*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-2*(a*
x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*b)^(1/2)*a*x^(1/3))^(1/2)
*x^(20/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticE(((a*x^(1/3)+(-a*b)^(1/2))
/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+100947*a^5*b*((a*x^(1/3)+(-a*b)^(1/2))/(-
a*b)^(1/2))^(1/2)*(-2*(a*x^(1/3)-(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2)*(-1/(-a*
b)^(1/2)*a*x^(1/3))^(1/2)*x^(20/3)*((a*x^(2/3)+b)*x^(1/3))^(1/2)*EllipticF(
((a*x^(1/3)+(-a*b)^(1/2))/(-a*b)^(1/2))^(1/2),1/2*2^(1/2))+201894*(a*x+b*x^
(1/3))^(1/2)*a^6*x^(22/3)+174048*(a*x+b*x^(1/3))^(1/2)*a^5*b*x^(20/3)-19228
*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^4*b^2*x^6-39452*((a*x^(2/3)+b)*x^(1/3))^(1
/2)*a^5*b*x^(20/3)-5244*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^2*b^4*x^(14/3)+8740
*((a*x^(2/3)+b)*x^(1/3))^(1/2)*a^3*b^3*x^(16/3)+3588*((a*x^(2/3)+b)*x^(1/3)
)^(1/2)*a*b^5*x^4-2652*((a*x^(2/3)+b)*x^(1/3))^(1/2)*b^6*x^(10/3))/x^7/(a*x
^(2/3)+b)/b^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{1}{3}}\right)^{\frac{3}{2}} x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(1/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(1/3))^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(a\,x+b\,x^{1/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(1/3))^(3/2)),x)
```

```
[Out] int(1/(x^4*(a*x + b*x^(1/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(ax+b\sqrt[3]{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(1/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a*x + b*x**(1/3))**(3/2)), x)
```

3.167 $\int x^3\sqrt{bx^{2/3}+ax}\,dx$

Optimal. Leaf size=371

$$\frac{8388608b^{12}\left(ax+bx^{2/3}\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(ax+bx^{2/3}\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(ax+bx^{2/3}\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}-\frac{524288b^{9}\left(ax+bx^{2/3}\right)^{3/}}{4345965a^{10}}$$

```
[Out] -524288/4345965*b^9*(b*x^(2/3)+a*x)^(3/2)/a^10+8388608/152108775*b^12*(b*x^
(2/3)+a*x)^(3/2)/a^13/x-4194304/50702925*b^11*(b*x^(2/3)+a*x)^(3/2)/a^12/x^
(2/3)+1048576/10140585*b^10*(b*x^(2/3)+a*x)^(3/2)/a^11/x^(1/3)+65536/482885
*b^8*x^(1/3)*(b*x^(2/3)+a*x)^(3/2)/a^9-360448/2414425*b^7*x^(2/3)*(b*x^(2/3
)+a*x)^(3/2)/a^8+90112/557175*b^6*x*(b*x^(2/3)+a*x)^(3/2)/a^7-45056/260015*
b^5*x^(4/3)*(b*x^(2/3)+a*x)^(3/2)/a^6+2816/15295*b^4*x^(5/3)*(b*x^(2/3)+a*x
)^(3/2)/a^5-1408/7245*b^3*x^2*(b*x^(2/3)+a*x)^(3/2)/a^4+352/1725*b^2*x^(7/3
)*(b*x^(2/3)+a*x)^(3/2)/a^3-16/75*b*x^(8/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/9*x
^3*(b*x^(2/3)+a*x)^(3/2)/a
```

Rubi [A] time = 0.63, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{8388608b^{12}\left(ax+bx^{2/3}\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(ax+bx^{2/3}\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(ax+bx^{2/3}\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}-\frac{524288b^{9}\left(ax+bx^{2/3}\right)^{3/}}{4345965a^{10}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-524288*b^9*(b*x^(2/3) + a*x)^(3/2))/(4345965*a^10) + (8388608*b^12*(b*x^(
2/3) + a*x)^(3/2))/(152108775*a^13*x) - (4194304*b^11*(b*x^(2/3) + a*x)^(3/
2))/(50702925*a^12*x^(2/3)) + (1048576*b^10*(b*x^(2/3) + a*x)^(3/2))/(10140
585*a^11*x^(1/3)) + (65536*b^8*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(482885*a^9
) - (360448*b^7*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))/(2414425*a^8) + (90112*b^6
*x*(b*x^(2/3) + a*x)^(3/2))/(557175*a^7) - (45056*b^5*x^(4/3)*(b*x^(2/3) +
a*x)^(3/2))/(260015*a^6) + (2816*b^4*x^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(1529
5*a^5) - (1408*b^3*x^2*(b*x^(2/3) + a*x)^(3/2))/(7245*a^4) + (352*b^2*x^(7/
3)*(b*x^(2/3) + a*x)^(3/2))/(1725*a^3) - (16*b*x^(8/3)*(b*x^(2/3) + a*x)^(3
/2))/(75*a^2) + (2*x^3*(b*x^(2/3) + a*x)^(3/2))/(9*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{bx^{2/3}+ax}\,dx &= \frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}-\frac{(8b)\int x^{8/3}\sqrt{bx^{2/3}+ax}\,dx}{9a}\\
&= -\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}+\frac{\left(176b^2\right)\int x^{7/3}\sqrt{bx^{2/3}+ax}\,dx}{225a^2}\\
&= \frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx^{2/3}+ax\right)^{3/2}}{9a}-\frac{\left(704b^3\right)\int x^2\sqrt{\cdots}}{1035\cdots}\\
&= -\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}+\frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}-\frac{16bx^{8/3}\left(bx^{2/3}+ax\right)^{3/2}}{75a^2}+\frac{2x^3\left(bx\cdots\right.}{\cdots}\\
&= \frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5}-\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}+\frac{352b^2x^{7/3}\left(bx^{2/3}+ax\right)^{3/2}}{1725a^3}-\frac{16bx^{\cdots}}{\cdots}\\
&= -\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6}+\frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5}-\frac{1408b^3x^2\left(bx^{2/3}+ax\right)^{3/2}}{7245a^4}+\frac{3\cdots}{\cdots}\\
&= \frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7}-\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6}+\frac{2816b^4x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{15295a^5}-\frac{14\cdots}{\cdots}\\
&= -\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8}+\frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7}-\frac{45056b^5x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{260015a^6}+\cdots\\
&= \frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9}-\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8}+\frac{90112b^6x\left(bx^{2/3}+ax\right)^{3/2}}{557175a^7}-\cdots\\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}+\frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9}-\frac{360448b^7x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{2414425a^8}-\cdots\\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}+\frac{1048576b^{10}\left(bx^{2/3}+ax\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}+\frac{65536b^8\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{482885a^9}-\cdots\\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}-\frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}}+\frac{1048576b^{10}\left(bx^{2/3}+ax\right)^{3/2}}{10140585a^{11}\sqrt[3]{x}}+\cdots\\
&= -\frac{524288b^9\left(bx^{2/3}+ax\right)^{3/2}}{4345965a^{10}}+\frac{8388608b^{12}\left(bx^{2/3}+ax\right)^{3/2}}{152108775a^{13}x}-\frac{4194304b^{11}\left(bx^{2/3}+ax\right)^{3/2}}{50702925a^{12}x^{2/3}}+\cdots
\end{aligned}$$

Mathematica [A] time = 0.18, size = 181, normalized size = 0.49

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(16900975a^{12}x^4-16224936a^{11}bx^{11/3}+15519504a^{10}b^2x^{10/3}-14780480a^9b^3x^3+14002\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(4194304*b^12 - 6291456*a*b^11*x^(
1/3) + 7864320*a^2*b^10*x^(2/3) - 9175040*a^3*b^9*x + 10321920*a^4*b^8*x^(4
/3) - 11354112*a^5*b^7*x^(5/3) + 12300288*a^6*b^6*x^2 - 13178880*a^7*b^5*x^
(7/3) + 14002560*a^8*b^4*x^(8/3) - 14780480*a^9*b^3*x^3 + 15519504*a^10*b^2
```

```
*x^(10/3) - 16224936*a^11*b*x^(11/3) + 16900975*a^12*x^4))/(152108775*a^13*
x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 396, normalized size = 1.07

$$-\frac{8388608\, b^{\frac{27}{2}}}{152108775\, a^{13}} + \frac{2\left(\frac{27\left(676039\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}} - 8817900\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}} b + 53117350\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}} b^2 - 195695500\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}} b^3 + 492116625\left(ax^{\frac{1}{3}}+ \cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -8388608/152108775*b^(27/2)/a^13 + 2/152108775*(27*(676039*(a*x^(1/3) + b)^
(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/2)
*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^(17
/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3) + b)
^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3)
+ b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x^(1/3)
 + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sqrt(a*x^
(1/3) + b)*b^12)*b/a^12 + 13*(1300075*(a*x^(1/3) + b)^(27/2) - 18253053*(a*
x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478056150*(a
*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4 - 26574297
75*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2)*b^6 - 463
3467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)^(11/2)*b^8
- 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3) + b)^(7/2)*b
^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3) + b)^(3/2
)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^12)/a
```

maple [A] time = 0.07, size = 156, normalized size = 0.42

$$-\frac{2\sqrt{ax+b\,x^{\frac{2}{3}}}\,\left(a\,x^{\frac{1}{3}}+b\right)\left(-16900975a^{12}x^4+16224936a^{11}b\,x^{\frac{11}{3}}-15519504a^{10}b^2x^{\frac{10}{3}}+14780480a^9b^3x^3-14\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^(2/3)+a*x)^(1/2),x)
```

```
[Out] -2/152108775*(b*x^(2/3)+a*x)^(1/2)*(a*x^(1/3)+b)*(16224936*x^(11/3)*a^11*b-
15519504*x^(10/3)*a^10*b^2-14002560*x^(8/3)*a^8*b^4+13178880*x^(7/3)*a^7*b^
5+11354112*x^(5/3)*a^5*b^7-10321920*x^(4/3)*a^4*b^8-16900975*x^4*a^12+14780
480*x^3*a^9*b^3-7864320*x^(2/3)*a^2*b^10-12300288*x^2*a^6*b^6+6291456*x^(1/
3)*a*b^11+9175040*x*a^3*b^9-4194304*b^12)/x^(1/3)/a^13
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + bx^{\frac{2}{3}}}\, x^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x^3, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{a\,x+b\,x^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^3*(a*x + b*x^(2/3))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \sqrt{ax + bx^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*x + b*x**(2/3)), x)
```

3.168 $\int x^2\sqrt{bx^{2/3}+ax}\,dx$

Optimal. Leaf size=283

$$-\frac{131072b^9\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(ax+bx^{2/3}\right)^{3/2}}{323323a^8\sqrt[3]{x}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{3/2}}{46189a^7}-\frac{9216b}{}$$

```
[Out] 8192/46189*b^6*(b*x^(2/3)+a*x)^(3/2)/a^7-131072/1616615*b^9*(b*x^(2/3)+a*x)
^(3/2)/a^10/x+196608/1616615*b^8*(b*x^(2/3)+a*x)^(3/2)/a^9/x^(2/3)-49152/32
3323*b^7*(b*x^(2/3)+a*x)^(3/2)/a^8/x^(1/3)-9216/46189*b^5*x^(1/3)*(b*x^(2/3
)+a*x)^(3/2)/a^6+4608/20995*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(3/2)/a^5-384/1615*
b^3*x*(b*x^(2/3)+a*x)^(3/2)/a^4+576/2261*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(3/2)/
a^3-36/133*b*x^(5/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/7*x^2*(b*x^(2/3)+a*x)^(3/2
)/a
```

Rubi [A] time = 0.44, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{131072b^9\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^{10}x}+\frac{196608b^8\left(ax+bx^{2/3}\right)^{3/2}}{1616615a^9x^{2/3}}-\frac{49152b^7\left(ax+bx^{2/3}\right)^{3/2}}{323323a^8\sqrt[3]{x}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{3/2}}{46189a^7}-\frac{9216b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (8192*b^6*(b*x^(2/3) + a*x)^(3/2))/(46189*a^7) - (131072*b^9*(b*x^(2/3) + a
*x)^(3/2))/(1616615*a^10*x) + (196608*b^8*(b*x^(2/3) + a*x)^(3/2))/(1616615
*a^9*x^(2/3)) - (49152*b^7*(b*x^(2/3) + a*x)^(3/2))/(323323*a^8*x^(1/3)) -
(9216*b^5*x^(1/3)*(b*x^(2/3) + a*x)^(3/2))/(46189*a^6) + (4608*b^4*x^(2/3)*
(b*x^(2/3) + a*x)^(3/2))/(20995*a^5) - (384*b^3*x*(b*x^(2/3) + a*x)^(3/2))/
(1615*a^4) + (576*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(3/2))/(2261*a^3) - (36*b*x
^(5/3)*(b*x^(2/3) + a*x)^(3/2))/(133*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(3/2))
/(7*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^2\sqrt{bx^{2/3}+ax}\,dx = \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} - \frac{(6b)\int x^{5/3}\sqrt{bx^{2/3}+ax}\,dx}{7a}$$
$$= -\frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} + \frac{\left(96b^2\right)\int x^{4/3}\sqrt{bx^{2/3}+ax}\,dx}{133a^2}$$
$$= \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(bx^{2/3}+ax\right)^{3/2}}{7a} - \frac{\left(192b^3\right)\int x\sqrt{b}}{323}$$
$$= -\frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx^{5/3}\left(bx^{2/3}+ax\right)^{3/2}}{133a^2} + \frac{2x^2\left(bx^{2/3}\right.}{7}$$
$$= \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{576b^2x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}}{2261a^3} - \frac{36bx^{5/3}}{}$$
$$= -\frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384b^3x\left(bx^{2/3}+ax\right)^{3/2}}{1615a^4} + \frac{576b}{}$$
$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b^4x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{20995a^5} - \frac{384b^3}{}$$
$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}} - \frac{9216b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{46189a^6} + \frac{4608b^4x^2}{}$$
$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} + \frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}} - \frac{49152b^7\left(bx^{2/3}+ax\right)^{3/2}}{323323a^8\sqrt[3]{x}} - \frac{9216b^5\sqrt[3]{x}}{4}$$
$$= \frac{8192b^6\left(bx^{2/3}+ax\right)^{3/2}}{46189a^7} - \frac{131072b^9\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^{10}x} + \frac{196608b^8\left(bx^{2/3}+ax\right)^{3/2}}{1616615a^9x^{2/3}} - \frac{49152b^7}{323}$$

Mathematica [A] time = 0.13, size = 144, normalized size = 0.51

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(230945a^9x^3-218790a^8bx^{8/3}+205920a^7b^2x^{7/3}-192192a^6b^3x^2+177408a^5b^4x^{5/3}-16\right.}{1616615a^{10}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(-65536*b^9 + 98304*a*b^8*x^(1/3)
- 122880*a^2*b^7*x^(2/3) + 143360*a^3*b^6*x - 161280*a^4*b^5*x^(4/3) + 1774
08*a^5*b^4*x^(5/3) - 192192*a^6*b^3*x^2 + 205920*a^7*b^2*x^(7/3) - 218790*a
^8*b*x^(8/3) + 230945*a^9*x^3))/(1616615*a^10*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 312, normalized size = 1.10

$$\frac{131072\, b^{\frac{21}{2}}}{1616615\, a^{10}} + \frac{2\left(\frac{21\left(12155\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}} - 122265\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}} b + 554268\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} b^2 - 1492260\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}} b^3 + 2645370\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}} b^4 - 32332\right.}{a^9}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 131072/1616615*b^(21/2)/a^10 + 2/1616615*(21*(12155*(a*x^(1/3) + b)^(19/2)
- 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)^(15/2)*b^2 - 149
2260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)*b^4 - 3233
230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^6 - 1662804
*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 - 230945*sqrt
(a*x^(1/3) + b)*b^9)*b/a^9 + 5*(46189*(a*x^(1/3) + b)^(21/2) - 510510*(a*x^
(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752*(a*x^(1/
3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 22221108*(a*x^(1
/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 16628040*(a*x^(1
/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(a*x^(1/3)
 + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^9)/a
```

maple [A] time = 0.05, size = 123, normalized size = 0.43

$$-\frac{2\sqrt{ax+b\,x^{\frac{2}{3}}}\,\left(a\,x^{\frac{1}{3}}+b\right)\left(-230945a^9x^3+218790a^8b\,x^{\frac{8}{3}}-205920a^7b^2x^{\frac{7}{3}}+192192a^6b^3x^2-177408a^5b^4x^{\frac{5}{3}}+\right.}{1616615a^{10}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] -2/1616615*(a*x+b*x^(2/3))^(1/2)*(a*x^(1/3)+b)*(218790*x^(8/3)*a^8*b-205920
*x^(7/3)*a^7*b^2-177408*x^(5/3)*a^5*b^4+161280*x^(4/3)*a^4*b^5-230945*x^3*a
^9+122880*x^(2/3)*a^2*b^7+192192*x^2*a^6*b^3-98304*x^(1/3)*a*b^8-143360*x*a
^3*b^6+65536*b^9)/x^(1/3)/a^10
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{2}{3}}}\,x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^2*(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*x + b*x**(2/3)), x)
```

3.169 $\int x\sqrt{bx^{2/3}+ax}\,dx$

Optimal. Leaf size=195

$$\frac{2048b^6\left(ax+bx^{2/3}\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(ax+bx^{2/3}\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{3/2}}{1001a^5\sqrt[3]{x}}-\frac{128b^3\left(ax+bx^{2/3}\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\left(ax+b\right.}{143a^3}$$

```
[Out] -128/429*b^3*(b*x^(2/3)+a*x)^(3/2)/a^4+2048/15015*b^6*(b*x^(2/3)+a*x)^(3/2)
/a^7/x-1024/5005*b^5*(b*x^(2/3)+a*x)^(3/2)/a^6/x^(2/3)+256/1001*b^4*(b*x^(2
/3)+a*x)^(3/2)/a^5/x^(1/3)+48/143*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(3/2)/a^3-24/
65*b*x^(2/3)*(b*x^(2/3)+a*x)^(3/2)/a^2+2/5*x*(b*x^(2/3)+a*x)^(3/2)/a
```

Rubi [A] time = 0.27, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{2048b^6\left(ax+bx^{2/3}\right)^{3/2}}{15015a^7x}-\frac{1024b^5\left(ax+bx^{2/3}\right)^{3/2}}{5005a^6x^{2/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{3/2}}{1001a^5\sqrt[3]{x}}-\frac{128b^3\left(ax+bx^{2/3}\right)^{3/2}}{429a^4}+\frac{48b^2\sqrt[3]{x}\left(ax+b\right.}{143a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-128*b^3*(b*x^(2/3) + a*x)^(3/2))/(429*a^4) + (2048*b^6*(b*x^(2/3) + a*x)^
(3/2))/(15015*a^7*x) - (1024*b^5*(b*x^(2/3) + a*x)^(3/2))/(5005*a^6*x^(2/3)
) + (256*b^4*(b*x^(2/3) + a*x)^(3/2))/(1001*a^5*x^(1/3)) + (48*b^2*x^(1/3)*
(b*x^(2/3) + a*x)^(3/2))/(143*a^3) - (24*b*x^(2/3)*(b*x^(2/3) + a*x)^(3/2))
/(65*a^2) + (2*x*(b*x^(2/3) + a*x)^(3/2))/(5*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x\sqrt{bx^{2/3}+ax}\,dx = \frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a} - \frac{(4b)\int x^{2/3}\sqrt{bx^{2/3}+ax}\,dx}{5a}$$
$$= -\frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2} + \frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a} + \frac{\left(8b^2\right)\int \sqrt[3]{x}\sqrt{bx^{2/3}+ax}\,dx}{13a^2}$$
$$= \frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3} - \frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2} + \frac{2x\left(bx^{2/3}+ax\right)^{3/2}}{5a} - \frac{\left(64b^3\right)\int \sqrt{bx^{2/3}+}}{143a^3}$$
$$= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4} + \frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3} - \frac{24bx^{2/3}\left(bx^{2/3}+ax\right)^{3/2}}{65a^2} + \frac{2x\left(bx^{2/3}+a\right.}{5a}$$
$$= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4} + \frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}} + \frac{48b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}}{143a^3} - \frac{24bx^{2/3}\left(bx^{2/3}\right.}{65a^2}$$
$$= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4} - \frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}} + \frac{256b^4\left(bx^{2/3}+ax\right)^{3/2}}{1001a^5\sqrt[3]{x}} + \frac{48b^2\sqrt[3]{x}\left(bx^{2/3}\right.}{143a^3}$$
$$= -\frac{128b^3\left(bx^{2/3}+ax\right)^{3/2}}{429a^4} + \frac{2048b^6\left(bx^{2/3}+ax\right)^{3/2}}{15015a^7x} - \frac{1024b^5\left(bx^{2/3}+ax\right)^{3/2}}{5005a^6x^{2/3}} + \frac{256b^4\left(bx^{2/3}+\right.}{1001a^5\sqrt[3]{}}$$

Mathematica [**A**] time = 0.09, size = 107, normalized size = 0.55

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(3003a^6x^2-2772a^5bx^{5/3}+2520a^4b^2x^{4/3}-2240a^3b^3x+1920a^2b^4x^{2/3}-1536ab^5\sqrt[3]{x}+\right.}{15015a^7\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(1024*b^6 - 1536*a*b^5*x^(1/3) + 1
920*a^2*b^4*x^(2/3) - 2240*a^3*b^3*x + 2520*a^4*b^2*x^(4/3) - 2772*a^5*b*x^
(5/3) + 3003*a^6*x^2))/(15015*a^7*x^(1/3))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.19, size = 228, normalized size = 1.17

$$-\frac{2048\,b^{\frac{15}{2}}}{15015\,a^7}+\frac{2\left(\frac{15\left(231\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}-1638\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b+5005\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^2-8580\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^3+9009\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^4-6006\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^5+3003\sqrt{ax}\right.}{a^6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -2048/15015*b^(15/2)/a^7 + 2/15015*(15*(231*(a*x^(1/3) + b)^(13/2) - 1638*(
a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3)
+ b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2
)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*x^(1/3) + b)^(15/2)
 - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025
*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(
1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3)
 + b)*b^7)/a^6)/a
```

maple [A] time = 0.05, size = 90, normalized size = 0.46

$$-\frac{2\sqrt{ax+b\,x^{\frac{2}{3}}}\,\left(a\,x^{\frac{1}{3}}+b\right)\left(-3003a^{6}x^{2}+2772a^{5}b\,x^{\frac{5}{3}}-2520a^{4}b^{2}x^{\frac{4}{3}}+2240a^{3}b^{3}x-1920a^{2}b^{4}x^{\frac{2}{3}}+1536a\,b^{5}x^{\frac{1}{3}}\right.}{15015a^{7}x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] -2/15015*(a*x+b*x^(2/3))^(1/2)*(a*x^(1/3)+b)*(2772*x^(5/3)*a^5*b-2520*x^(4/
3)*a^4*b^2-1920*x^(2/3)*a^2*b^4-3003*x^2*a^6+1536*x^(1/3)*a*b^5+2240*x*a^3*
b^3-1024*b^6)/x^(1/3)/a^7
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax+bx^{\frac{2}{3}}}\,x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{a\,x+b\,x^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x*(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{ax+bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*x + b*x**(2/3)), x)
```

3.170 $\int \sqrt{bx^{2/3} + ax}\, dx$

Optimal. Leaf size=109

$$-\frac{32b^3\left(ax+bx^{2/3}\right)^{3/2}}{105a^4x}+\frac{16b^2\left(ax+bx^{2/3}\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{7a^2\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}$$

```
[Out] 2/3*(b*x^(2/3)+a*x)^(3/2)/a-32/105*b^3*(b*x^(2/3)+a*x)^(3/2)/a^4/x+16/35*b^
2*(b*x^(2/3)+a*x)^(3/2)/a^3/x^(2/3)-4/7*b*(b*x^(2/3)+a*x)^(3/2)/a^2/x^(1/3)
```

Rubi [A] time = 0.14, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2002, 2016, 2014}

$$-\frac{32b^3\left(ax+bx^{2/3}\right)^{3/2}}{105a^4x}+\frac{16b^2\left(ax+bx^{2/3}\right)^{3/2}}{35a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{3/2}}{7a^2\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(3*a) - (32*b^3*(b*x^(2/3) + a*x)^(3/2))/(105*a
^4*x) + (16*b^2*(b*x^(2/3) + a*x)^(3/2))/(35*a^3*x^(2/3)) - (4*b*(b*x^(2/3)
 + a*x)^(3/2))/(7*a^2*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{bx^{2/3}+ax}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a} - \frac{(2b)\int \frac{\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}\,dx}{3a} \\ &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a} - \frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}} + \frac{\left(8b^2\right)\int \frac{\sqrt{bx^{2/3}+ax}}{x^{2/3}}\,dx}{21a^2} \\ &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a} + \frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}} - \frac{\left(16b^3\right)\int \frac{\sqrt{bx^{2/3}+ax}}{x}\,dx}{105a^3} \\ &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{3a} - \frac{32b^3\left(bx^{2/3}+ax\right)^{3/2}}{105a^4x} + \frac{16b^2\left(bx^{2/3}+ax\right)^{3/2}}{35a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{3/2}}{7a^2\sqrt[3]{x}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 70, normalized size = 0.64

$$\frac{2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\left(35a^3x-30a^2bx^{2/3}+24ab^2\sqrt[3]{x}-16b^3\right)}{105a^4\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x],x]
[Out] (2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*(-16*b^3 + 24*a*b^2*x^(1/3) - 30*a
^2*b*x^(2/3) + 35*a^3*x))/(105*a^4*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.18, size = 143, normalized size = 1.31

$$\frac{32\,b^{\frac{9}{2}}}{105\,a^4}+\frac{2\left(\frac{9\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+b}\,b^3\right)b}{a^3}+\frac{35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^2-420\left(ax^{\frac{1}{3}}\right.}{a^3}\right.}{105\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
[Out] 32/105*b^(9/2)/a^4 + 2/105*(9*(5*(a*x^(1/3) + b)^(7/2) - 21*(a*x^(1/3) + b)
^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(1/3) + b)*b^3)*b/a^3
 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/2)*b + 378*(a*x^(1/3)
 + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*sqrt(a*x^(1/3) + b)*b
^4)/a^3)/a
```

maple [A] time = 0.04, size = 57, normalized size = 0.52

$$-\frac{2\sqrt{ax+b\,x^{\frac{2}{3}}}\,\left(a\,x^{\frac{1}{3}}+b\right)\left(-35a^3x+30a^2b\,x^{\frac{2}{3}}-24a\,b^2x^{\frac{1}{3}}+16b^3\right)}{105a^4x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] -2/105*(a*x+b*x^(2/3))^(1/2)*(a*x^(1/3)+b)*(30*a^2*b*x^(2/3)-24*a*b^2*x^(1/
3)-35*a^3*x+16*b^3)/x^(1/3)/a^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3)), x)
```

mupad [B] time = 5.19, size = 40, normalized size = 0.37

$$\frac{3\,x\,\sqrt{a\,x+b\,x^{2/3}}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},4;\;5;\;-\frac{a\,x^{1/3}}{b}\right)}{4\,\sqrt{\frac{a\,x^{1/3}}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] (3*x*(a*x + b*x^(2/3))^(1/2)*hypergeom([-1/2, 4], 5, -(a*x^(1/3))/b))/(4*((
a*x^(1/3))/b + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + bx^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3)), x)
```

3.171 $\int \frac{\sqrt{bx^{2/3}+ax}}{x} \, dx$

Optimal. Leaf size=23

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

[Out] 2*(b*x^(2/3)+a*x)^(3/2)/a/x

Rubi [A] time = 0.04, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^(2/3) + a*x]/x,x]

[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x} \, dx = \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{ax}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{ax}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x,x]

[Out] (2*(b*x^(2/3) + a*x)^(3/2))/(a*x)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.21, size = 23, normalized size = 1.00

$$\frac{2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}}{a}-\frac{2\,b^{\frac{3}{2}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="giac")
[Out] 2*(a*x^(1/3) + b)^(3/2)/a - 2*b^(3/2)/a
```

maple [A] time = 0.04, size = 27, normalized size = 1.17

$$\frac{2\sqrt{ax+b\,x^{\frac{2}{3}}}\,\left(a\,x^{\frac{1}{3}}+b\right)}{a\,x^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2)/x,x)
[Out] 2*(a*x+b*x^(2/3))^(1/2)/x^(1/3)*(a*x^(1/3)+b)/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^(2/3))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x,x)
[Out] int((a*x + b*x^(2/3))^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x,x)
[Out] Integral(sqrt(a*x + b*x**(2/3))/x, x)
```

3.172 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx$

Optimal. Leaf size=90

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4bx^{2/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

```
[Out] 3/4*a^2*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)-3/2*(b*x^(2/
3)+a*x)^(1/2)/x-3/4*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(2/3)
```

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{4b^{3/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4bx^{2/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^2,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(2*x) - (3*a*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(2/3)
) + (3*a^2*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(4*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}+\frac{1}{4}a\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}-\frac{a^2\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{8b}\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b}\\ &= -\frac{3\sqrt{bx^{2/3}+ax}}{2x}-\frac{3a\sqrt{bx^{2/3}+ax}}{4bx^{2/3}}+\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{4b^{3/2}}\end{aligned}$$

Mathematica [C] time = 0.06, size = 57, normalized size = 0.63

$$-\frac{2a^2\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\;{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^3\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^2,x]
[Out] (-2*a^2*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 3, 5/2
, 1 + (a*x^(1/3))/b])/(b^3*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.28, size = 72, normalized size = 0.80

$$-\frac{3\left(\frac{a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3+\sqrt{ax^{\frac{1}{3}}+b}\,a^3b}{a^2bx^{\frac{2}{3}}}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="giac")
[Out] -3/4*(a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + ((a*x^(1/3) +
 b)^(3/2)*a^3 + sqrt(a*x^(1/3) + b)*a^3*b)/(a^2*b*x^(2/3)))/a
```

maple [A] time = 0.05, size = 80, normalized size = 0.89

$$\frac{3\sqrt{ax+b\,x^{\frac{2}{3}}}\left(a^2b\,x^{\frac{2}{3}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{5}{2}}-\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{3}{2}}\right)}{4\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2)/x^2,x)
```

```
[Out] 3/4*(a*x+b*x^(2/3))^(1/2)*(arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b*x^(2/3)*a
^2-(a*x^(1/3)+b)^(3/2)*b^(3/2)-(a*x^(1/3)+b)^(1/2)*b^(5/2))/x/(a*x^(1/3)+b)
^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x + b\,x^{2/3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^2,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^{\frac{2}{3}}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**2, x)
```

3.173 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$

Optimal. Leaf size=178

$$-\frac{21a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}}+\frac{21a^4\sqrt{ax+bx^{2/3}}}{128b^4x^{2/3}}-\frac{7a^3\sqrt{ax+bx^{2/3}}}{64b^3x}+\frac{7a^2\sqrt{ax+bx^{2/3}}}{80b^2x^{4/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{40bx^{5/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

```
[Out] -21/128*a^5*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)-3/5*(b*x
^(2/3)+a*x)^(1/2)/x^2-3/40*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(5/3)+7/80*a^2*(b*x^
(2/3)+a*x)^(1/2)/b^2/x^(4/3)-7/64*a^3*(b*x^(2/3)+a*x)^(1/2)/b^3/x+21/128*a^
4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A] time = 0.30, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{21a^4\sqrt{ax+bx^{2/3}}}{128b^4x^{2/3}}-\frac{7a^3\sqrt{ax+bx^{2/3}}}{64b^3x}+\frac{7a^2\sqrt{ax+bx^{2/3}}}{80b^2x^{4/3}}-\frac{21a^5\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{128b^{9/2}}-\frac{3a\sqrt{ax+bx^{2/3}}}{40bx^{5/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(5*x^2) - (3*a*Sqrt[b*x^(2/3) + a*x])/(40*b*x^(5
/3)) + (7*a^2*Sqrt[b*x^(2/3) + a*x])/(80*b^2*x^(4/3)) - (7*a^3*Sqrt[b*x^(2/
3) + a*x])/(64*b^3*x) + (21*a^4*Sqrt[b*x^(2/3) + a*x])/(128*b^4*x^(2/3)) -
(21*a^5*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(128*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} + \frac{1}{10}a\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} - \frac{(7a^2)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{80b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} + \frac{(7a^3)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{96b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} - \frac{(7a^4)\int \frac{1}{x\sqrt{bx^{2/}}}}{128b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+}}{128b^4x^{2/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+}}{128b^4x^{2/3}}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{5x^2} - \frac{3a\sqrt{bx^{2/3}+ax}}{40bx^{5/3}} + \frac{7a^2\sqrt{bx^{2/3}+ax}}{80b^2x^{4/3}} - \frac{7a^3\sqrt{bx^{2/3}+ax}}{64b^3x} + \frac{21a^4\sqrt{bx^{2/3}+}}{128b^4x^{2/3}}$$

Mathematica [C] time = 0.07, size = 57, normalized size = 0.32

$$\frac{2a^5\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\;{}_2F_1\left(\frac{3}{2},6;\frac{5}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^6\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^3,x]
```

```
[Out] (2*a^5*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 6, 5/2,
 1 + (a*x^(1/3))/b])/(b^6*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.28, size = 126, normalized size = 0.71

$$\frac{\frac{105\,a^6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^6 - 490\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^6b + 896\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^6b^2 - 790\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^6b^3 - 105\sqrt{ax^{\frac{1}{3}}+b}\,a^6b^4}{a^5b^4x^{\frac{5}{3}}}}{640\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/640*(105*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(
a*x^(1/3) + b)^(9/2)*a^6 - 490*(a*x^(1/3) + b)^(7/2)*a^6*b + 896*(a*x^(1/3)
 + b)^(5/2)*a^6*b^2 - 790*(a*x^(1/3) + b)^(3/2)*a^6*b^3 - 105*sqrt(a*x^(1/3
) + b)*a^6*b^4)/(a^5*b^4*x^(5/3)))/a
```

maple [A] time = 0.06, size = 125, normalized size = 0.70

$$-\frac{\sqrt{ax+b\,x^{\frac{2}{3}}}\left(105a^5b^4x^{\frac{5}{3}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+105\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{17}{2}}+790\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{15}{2}}-896\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{\frac{13}{2}}+\right.}{640\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{17}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2)/x^3,x)
```

```
[Out] -1/640*(a*x+b*x^(2/3))^(1/2)*(-105*(a*x^(1/3)+b)^(9/2)*b^(9/2)+490*(a*x^(1/
3)+b)^(7/2)*b^(11/2)-896*(a*x^(1/3)+b)^(5/2)*b^(13/2)+105*arctanh((a*x^(1/3
)+b)^(1/2)/b^(1/2))*b^4*x^(5/3)*a^5+790*(a*x^(1/3)+b)^(3/2)*b^(15/2)+105*(a
*x^(1/3)+b)^(1/2)*b^(17/2))/x^2/(a*x^(1/3)+b)^(1/2)/b^(17/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**3, x)
```

3.174 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx$

Optimal. Leaf size=266

$$\frac{1287a^8\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{16384b^{15/2}}-\frac{1287a^7\sqrt{ax+bx^{2/3}}}{16384b^7x^{2/3}}+\frac{429a^6\sqrt{ax+bx^{2/3}}}{8192b^6x}-\frac{429a^5\sqrt{ax+bx^{2/3}}}{10240b^5x^{4/3}}+\frac{1287a^4\sqrt{ax+bx^{2/3}}}{35840b^4x^{5/3}}$$

```
[Out] 1287/16384*a^8*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)-3/8*
(b*x^(2/3)+a*x)^(1/2)/x^3-3/112*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/448*a^
2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-143/4480*a^3*(b*x^(2/3)+a*x)^(1/2)/b^3/
x^2+1287/35840*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(5/3)-429/10240*a^5*(b*x^(2/
3)+a*x)^(1/2)/b^5/x^(4/3)+429/8192*a^6*(b*x^(2/3)+a*x)^(1/2)/b^6/x-1287/163
84*a^7*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

Rubi [A] time = 0.48, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$-\frac{1287a^7\sqrt{ax+bx^{2/3}}}{16384b^7x^{2/3}}+\frac{429a^6\sqrt{ax+bx^{2/3}}}{8192b^6x}-\frac{429a^5\sqrt{ax+bx^{2/3}}}{10240b^5x^{4/3}}+\frac{1287a^4\sqrt{ax+bx^{2/3}}}{35840b^4x^{5/3}}-\frac{143a^3\sqrt{ax+bx^{2/3}}}{4480b^3x^2}+\frac{13a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(8*x^3) - (3*a*Sqrt[b*x^(2/3) + a*x])/(112*b*x^(
8/3)) + (13*a^2*Sqrt[b*x^(2/3) + a*x])/(448*b^2*x^(7/3)) - (143*a^3*Sqrt[b*
x^(2/3) + a*x])/(4480*b^3*x^2) + (1287*a^4*Sqrt[b*x^(2/3) + a*x])/(35840*b^
4*x^(5/3)) - (429*a^5*Sqrt[b*x^(2/3) + a*x])/(10240*b^5*x^(4/3)) + (429*a^6
*Sqrt[b*x^(2/3) + a*x])/(8192*b^6*x) - (1287*a^7*Sqrt[b*x^(2/3) + a*x])/(16
384*b^7*x^(2/3)) + (1287*a^8*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x
]])/(16384*b^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} + \frac{1}{16}a\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} - \frac{\left(13a^2\right)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{224b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} + \frac{\left(143a^3\right)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{2688b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} - \frac{\left(429a^4\right)\int \frac{}{x^{2}}}{8960}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/}}}{35840b^4x}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/}}}{35840b^4x}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/}}}{35840b^4x}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/}}}{35840b^4x}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/}}}{35840b^4x}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{8x^3} - \frac{3a\sqrt{bx^{2/3}+ax}}{112bx^{8/3}} + \frac{13a^2\sqrt{bx^{2/3}+ax}}{448b^2x^{7/3}} - \frac{143a^3\sqrt{bx^{2/3}+ax}}{4480b^3x^2} + \frac{1287a^4\sqrt{bx^{2/}}}{35840b^4x}$$

Mathematica [C] time = 0.05, size = 57, normalized size = 0.21

$$-\frac{2a^8\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\;{}_2F_1\left(\frac{3}{2},9;\frac{5}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^9\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^4,x]
```

```
[Out] (-2*a^8*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 9, 5/2
, 1 + (a*x^(1/3))/b])/(b^9*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.35, size = 177, normalized size = 0.67

$$-\frac{\frac{45045\,a^9\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^7}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^9-345345\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^9b+1150149\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^9b^2-2167737\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^9b^3+2518087\left(ax^{\frac{1}{3}}\right.}{a^8b^7x^{\frac{8}{3}}}}{573440\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/573440*(45045*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) +
(45045*(a*x^(1/3) + b)^(15/2)*a^9 - 345345*(a*x^(1/3) + b)^(13/2)*a^9*b + 1
150149*(a*x^(1/3) + b)^(11/2)*a^9*b^2 - 2167737*(a*x^(1/3) + b)^(9/2)*a^9*b
^3 + 2518087*(a*x^(1/3) + b)^(7/2)*a^9*b^4 - 1831739*(a*x^(1/3) + b)^(5/2)*
a^9*b^5 + 801535*(a*x^(1/3) + b)^(3/2)*a^9*b^6 + 45045*sqrt(a*x^(1/3) + b)*
a^9*b^7)/(a^8*b^7*x^(8/3)))/a
```

maple [A] time = 0.06, size = 167, normalized size = 0.63

$$-\frac{\sqrt{ax+b\,x^{\frac{2}{3}}}\left(-45045a^8b^7x^{\frac{8}{3}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+45045\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{29}{2}}+801535\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{27}{2}}-1831739\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2)/x^4,x)
```

```
[Out] -1/573440*(a*x+b*x^(2/3))^(1/2)*(45045*b^(15/2)*(a*x^(1/3)+b)^(15/2)-345345
*b^(17/2)*(a*x^(1/3)+b)^(13/2)+1150149*b^(19/2)*(a*x^(1/3)+b)^(11/2)-216773
7*b^(21/2)*(a*x^(1/3)+b)^(9/2)+2518087*b^(23/2)*(a*x^(1/3)+b)^(7/2)-1831739
*b^(25/2)*(a*x^(1/3)+b)^(5/2)+801535*b^(27/2)*(a*x^(1/3)+b)^(3/2)-45045*arc
tanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b^7*x^(8/3)*a^8+45045*b^(29/2)*(a*x^(1/3)
+b)^(1/2))/x^3/(a*x^(1/3)+b)^(1/2)/b^(29/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\sqrt{a\,x+b\,x^{2/3}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^4,x)
```

```
[Out] int((a*x + b*x^(2/3))^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**4, x)
```

3.175 $\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx$

Optimal. Leaf size=354

$$-\frac{12597a^{11}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{262144b^{21/2}}+\frac{12597a^{10}\sqrt{ax+bx^{2/3}}}{262144b^{10}x^{2/3}}-\frac{4199a^9\sqrt{ax+bx^{2/3}}}{131072b^9x}+\frac{4199a^8\sqrt{ax+bx^{2/3}}}{163840b^8x^{4/3}}-\frac{12597a^7\sqrt{a}}{573440}$$

```
[Out] -12597/262144*a^11*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)-
3/11*(b*x^(2/3)+a*x)^(1/2)/x^4-3/220*a*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/
1320*a^2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)-323/21120*a^3*(b*x^(2/3)+a*x)^(
1/2)/b^3/x^3+323/19712*a^4*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(8/3)-4199/236544*a^
5*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+4199/215040*a^6*(b*x^(2/3)+a*x)^(1/2)/b
^6/x^2-12597/573440*a^7*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(5/3)+4199/163840*a^8*(
b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-4199/131072*a^9*(b*x^(2/3)+a*x)^(1/2)/b^9/
x+12597/262144*a^10*(b*x^(2/3)+a*x)^(1/2)/b^10/x^(2/3)
```

Rubi [A] time = 0.66, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{12597a^{10}\sqrt{ax+bx^{2/3}}}{262144b^{10}x^{2/3}}-\frac{4199a^9\sqrt{ax+bx^{2/3}}}{131072b^9x}+\frac{4199a^8\sqrt{ax+bx^{2/3}}}{163840b^8x^{4/3}}-\frac{12597a^7\sqrt{ax+bx^{2/3}}}{573440b^7x^{5/3}}+\frac{4199a^6\sqrt{ax+bx^{2/3}}}{215040b^6x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(11*x^4) - (3*a*Sqrt[b*x^(2/3) + a*x])/(220*b*x^
(11/3)) + (19*a^2*Sqrt[b*x^(2/3) + a*x])/(1320*b^2*x^(10/3)) - (323*a^3*Sqr
t[b*x^(2/3) + a*x])/(21120*b^3*x^3) + (323*a^4*Sqrt[b*x^(2/3) + a*x])/(1971
2*b^4*x^(8/3)) - (4199*a^5*Sqrt[b*x^(2/3) + a*x])/(236544*b^5*x^(7/3)) + (4
199*a^6*Sqrt[b*x^(2/3) + a*x])/(215040*b^6*x^2) - (12597*a^7*Sqrt[b*x^(2/3)
 + a*x])/(573440*b^7*x^(5/3)) + (4199*a^8*Sqrt[b*x^(2/3) + a*x])/(163840*b^
8*x^(4/3)) - (4199*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^9*x) + (12597*a^10*
Sqrt[b*x^(2/3) + a*x])/(262144*b^10*x^(2/3)) - (12597*a^11*ArcTanh[(Sqrt[b]
*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(262144*b^(21/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
```

```
+ j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}+\frac{1}{22}a\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}-\frac{\left(19a^2\right)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{440b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}+\frac{\left(323a^3\right)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{7920b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}-\frac{\left(323a^4\right)\int \frac{1}{x^{3}\cdots}}{8448\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}\cdots}}{19712b^4x\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}\cdots}}{19712b^4x\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}\cdots}}{19712b^4x\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}\cdots}}{19712b^4x\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}\cdots}}{19712b^4x\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}\cdots}}{19712b^4x\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}\cdots}}{19712b^4x\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}\cdots}}{19712b^4x\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}\cdots}}{19712b^4x\cdots}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{11x^4}-\frac{3a\sqrt{bx^{2/3}+ax}}{220bx^{11/3}}+\frac{19a^2\sqrt{bx^{2/3}+ax}}{1320b^2x^{10/3}}-\frac{323a^3\sqrt{bx^{2/3}+ax}}{21120b^3x^3}+\frac{323a^4\sqrt{bx^{2/3}\cdots}}{19712b^4x\cdots}$$

Mathematica [C] time = 0.05, size = 57, normalized size = 0.16

$$\frac{2a^{11}\left(a\sqrt[3]{x}+b\right)\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{3}{2},12;\frac{5}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^{12}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*x^(2/3) + a*x]/x^5,x]
```

```
[Out] (2*a^11*(b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[3/2, 12, 5/
2, 1 + (a*x^(1/3))/b])/(b^12*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.43, size = 228, normalized size = 0.64

$$\frac{\frac{14549535\,a^{12}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{12}-155195040\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{12}b+749786037\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{12}b^{2}-2163862272\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/302776320*(14549535*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b
^10) + (14549535*(a*x^(1/3) + b)^(21/2)*a^12 - 155195040*(a*x^(1/3) + b)^(1
9/2)*a^12*b + 749786037*(a*x^(1/3) + b)^(17/2)*a^12*b^2 - 2163862272*(a*x^(
1/3) + b)^(15/2)*a^12*b^3 + 4139920070*(a*x^(1/3) + b)^(13/2)*a^12*b^4 - 55
03713280*(a*x^(1/3) + b)^(11/2)*a^12*b^5 + 5174056250*(a*x^(1/3) + b)^(9/2)
*a^12*b^6 - 3424523520*(a*x^(1/3) + b)^(7/2)*a^12*b^7 + 1551313995*(a*x^(1/
3) + b)^(5/2)*a^12*b^8 - 450357600*(a*x^(1/3) + b)^(3/2)*a^12*b^9 - 1454953
5*sqrt(a*x^(1/3) + b)*a^12*b^10)/(a^11*b^10*x^(11/3)))/a
```

maple [A] time = 0.06, size = 209, normalized size = 0.59

$$-\frac{\sqrt{ax+b\,x^{\frac{2}{3}}}\left(14549535a^{11}b^{10}x^{\frac{11}{3}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+14549535\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{41}{2}}+450357600\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{3}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(1/2)/x^5,x)
```

```
[Out] -1/302776320*(a*x+b*x^(2/3))^(1/2)*(-14549535*(a*x^(1/3)+b)^(21/2)*b^(21/2)
+155195040*(a*x^(1/3)+b)^(19/2)*b^(23/2)-749786037*(a*x^(1/3)+b)^(17/2)*b^(
25/2)+2163862272*(a*x^(1/3)+b)^(15/2)*b^(27/2)-4139920070*(a*x^(1/3)+b)^(13
/2)*b^(29/2)+5503713280*(a*x^(1/3)+b)^(11/2)*b^(31/2)-5174056250*(a*x^(1/3)
+b)^(9/2)*b^(33/2)+3424523520*(a*x^(1/3)+b)^(7/2)*b^(35/2)-1551313995*(a*x^
(1/3)+b)^(5/2)*b^(37/2)+14549535*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b^10*
x^(11/3)*a^11+450357600*(a*x^(1/3)+b)^(3/2)*b^(39/2)+14549535*(a*x^(1/3)+b)
^(1/2)*b^(41/2))/x^4/(a*x^(1/3)+b)^(1/2)/b^(41/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x + b*x^(2/3))/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a\,x+b\,x^{2/3}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(1/2)/x^5,x)
[Out] int((a*x + b*x^(2/3))^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^{\frac{2}{3}}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(1/2)/x**5,x)
[Out] Integral(sqrt(a*x + b*x**(2/3))/x**5, x)
```

3.176 $\int x^2 \left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=343

$$-\frac{1048576b^{11}\left(ax+bx^{2/3}\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(ax+bx^{2/3}\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^9\left(ax+bx^{2/3}\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^9x^{2/3}}-$$

```
[Out] 45056/557175*b^6*(b*x^(2/3)+a*x)^(5/2)/a^7-1048576/152108775*b^11*(b*x^(2/3
)+a*x)^(5/2)/a^12/x^(5/3)+524288/30421755*b^10*(b*x^(2/3)+a*x)^(5/2)/a^11/x
^(4/3)-131072/4345965*b^9*(b*x^(2/3)+a*x)^(5/2)/a^10/x+65536/1448655*b^8*(b
*x^(2/3)+a*x)^(5/2)/a^9/x^(2/3)-90112/1448655*b^7*(b*x^(2/3)+a*x)^(5/2)/a^8
/x^(1/3)-11264/111435*b^5*x^(1/3)*(b*x^(2/3)+a*x)^(5/2)/a^6+5632/45885*b^4*
x^(2/3)*(b*x^(2/3)+a*x)^(5/2)/a^5-352/2415*b^3*x*(b*x^(2/3)+a*x)^(5/2)/a^4+
176/1035*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(5/2)/a^3-44/225*b*x^(5/3)*(b*x^(2/3)+
a*x)^(5/2)/a^2+2/9*x^2*(b*x^(2/3)+a*x)^(5/2)/a
```

Rubi [A] time = 0.62, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{1048576b^{11}\left(ax+bx^{2/3}\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(ax+bx^{2/3}\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^9\left(ax+bx^{2/3}\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{1448655a^9x^{2/3}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (45056*b^6*(b*x^(2/3) + a*x)^(5/2))/(557175*a^7) - (1048576*b^11*(b*x^(2/3)
 + a*x)^(5/2))/(152108775*a^12*x^(5/3)) + (524288*b^10*(b*x^(2/3) + a*x)^(5
/2))/(30421755*a^11*x^(4/3)) - (131072*b^9*(b*x^(2/3) + a*x)^(5/2))/(434596
5*a^10*x) + (65536*b^8*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^9*x^(2/3)) - (90
112*b^7*(b*x^(2/3) + a*x)^(5/2))/(1448655*a^8*x^(1/3)) - (11264*b^5*x^(1/3)
*(b*x^(2/3) + a*x)^(5/2))/(111435*a^6) + (5632*b^4*x^(2/3)*(b*x^(2/3) + a*x
)^(5/2))/(45885*a^5) - (352*b^3*x*(b*x^(2/3) + a*x)^(5/2))/(2415*a^4) + (17
6*b^2*x^(4/3)*(b*x^(2/3) + a*x)^(5/2))/(1035*a^3) - (44*b*x^(5/3)*(b*x^(2/3
) + a*x)^(5/2))/(225*a^2) + (2*x^2*(b*x^(2/3) + a*x)^(5/2))/(9*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^2\left(bx^{2/3}+ax\right)^{3/2}dx=\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}-\frac{(22b)\int x^{5/3}\left(bx^{2/3}+ax\right)^{3/2}dx}{27a}$$
$$=-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}+\frac{\left(88b^2\right)\int x^{4/3}\left(bx^{2/3}+ax\right)^{3/2}dx}{135a^2}$$
$$=\frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}+\frac{2x^2\left(bx^{2/3}+ax\right)^{5/2}}{9a}-\frac{\left(176b^3\right)\int x \ldots}{\ldots}$$
$$=-\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}-\frac{44bx^{5/3}\left(bx^{2/3}+ax\right)^{5/2}}{225a^2}+\frac{2x^2\left(b\ldots\right.}{\ldots}$$
$$=\frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5}-\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4}+\frac{176b^2x^{4/3}\left(bx^{2/3}+ax\right)^{5/2}}{1035a^3}-\frac{44bx\ldots}{\ldots}$$
$$=-\frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6}+\frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5}-\frac{352b^3x\left(bx^{2/3}+ax\right)^{5/2}}{2415a^4}+\ldots$$
$$=\frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6}+\frac{5632b^4x^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{45885a^5}-\ldots$$
$$=\frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}}-\frac{11264b^5\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{111435a^6}+\ldots$$
$$=\frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}}-\frac{90112b^7\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^8\sqrt[3]{x}}-\ldots$$
$$=\frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{1448655a^9x^{2/3}}-\ldots$$
$$=\frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}+\frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}}-\frac{131072b^9\left(bx^{2/3}+ax\right)^{5/2}}{4345965a^{10}x}+\ldots$$
$$=\frac{45056b^6\left(bx^{2/3}+ax\right)^{5/2}}{557175a^7}-\frac{1048576b^{11}\left(bx^{2/3}+ax\right)^{5/2}}{152108775a^{12}x^{5/3}}+\frac{524288b^{10}\left(bx^{2/3}+ax\right)^{5/2}}{30421755a^{11}x^{4/3}}-\ldots$$

Mathematica [A] time = 0.16, size = 172, normalized size = 0.50

$$2\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\left(16900975a^{11}x^{11/3}-14872858a^{10}bx^{10/3}+12932920a^9b^2x^3-11085360a^8b^3x^{8/3}+933\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(-524288*b^11 + 1310720*a*b^10*x
^(1/3) - 2293760*a^2*b^9*x^(2/3) + 3440640*a^3*b^8*x - 4730880*a^4*b^7*x^(4
/3) + 6150144*a^5*b^6*x^(5/3) - 7687680*a^6*b^5*x^2 + 9335040*a^7*b^4*x^(7/
3) - 11085360*a^8*b^3*x^(8/3) + 12932920*a^9*b^2*x^3 - 14872858*a^10*b*x^(1
0/3) + 16900975*a^11*x^(11/3)))/(152108775*a^12*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.31, size = 770, normalized size = 2.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/16900975*b*(524288*b^(25/2)/a^12 + (25*(88179*(a*x^(1/3) + b)^(23/2) - 10
62347*(a*x^(1/3) + b)^(21/2)*b + 5870865*(a*x^(1/3) + b)^(19/2)*b^2 - 19684
665*(a*x^(1/3) + b)^(17/2)*b^3 + 44618574*(a*x^(1/3) + b)^(15/2)*b^4 - 7207
6158*(a*x^(1/3) + b)^(13/2)*b^5 + 85180914*(a*x^(1/3) + b)^(11/2)*b^6 - 743
64290*(a*x^(1/3) + b)^(9/2)*b^7 + 47805615*(a*x^(1/3) + b)^(7/2)*b^8 - 2230
9287*(a*x^(1/3) + b)^(5/2)*b^9 + 7436429*(a*x^(1/3) + b)^(3/2)*b^10 - 20281
17*sqrt(a*x^(1/3) + b)*b^11)*b/a^11 + 3*(676039*(a*x^(1/3) + b)^(25/2) - 88
17900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) + b)^(21/2)*b^2 - 1956
95500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3) + b)^(17/2)*b^4 - 8
92371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(1/3) + b)^(13/2)*b^6
 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a*x^(1/3) + b)^(9/2)*
b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(a*x^(1/3) + b)^(5/2)
*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*sqrt(a*x^(1/3) + b)*
b^12)/a^11)/a) - 2/152108775*a*(4194304*b^(27/2)/a^13 - (27*(676039*(a*x^(1
/3) + b)^(25/2) - 8817900*(a*x^(1/3) + b)^(23/2)*b + 53117350*(a*x^(1/3) +
b)^(21/2)*b^2 - 195695500*(a*x^(1/3) + b)^(19/2)*b^3 + 492116625*(a*x^(1/3)
 + b)^(17/2)*b^4 - 892371480*(a*x^(1/3) + b)^(15/2)*b^5 + 1201269300*(a*x^(
1/3) + b)^(13/2)*b^6 - 1216870200*(a*x^(1/3) + b)^(11/2)*b^7 + 929553625*(a
*x^(1/3) + b)^(9/2)*b^8 - 531173500*(a*x^(1/3) + b)^(7/2)*b^9 + 223092870*(
a*x^(1/3) + b)^(5/2)*b^10 - 67603900*(a*x^(1/3) + b)^(3/2)*b^11 + 16900975*
sqrt(a*x^(1/3) + b)*b^12)*b/a^12 + 13*(1300075*(a*x^(1/3) + b)^(27/2) - 182
53053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^2 - 478
056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2)*b^4 -
 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^(15/2)*
b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3) + b)^(1
1/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1/3) + b
)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x^(1/3)
+ b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^12)/a)
```

maple [A] time = 0.05, size = 145, normalized size = 0.42

$$\frac{2\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(a\,x^{\frac{1}{3}}+b\right)\left(16900975a^{11}x^{\frac{11}{3}}-14872858a^{10}b\,x^{\frac{10}{3}}+12932920a^{9}b^{2}x^{3}-11085360a^{8}b^{3}x^{\frac{8}{3}}+9335\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/152108775*(a*x+b*x^(2/3))^(3/2)*(a*x^(1/3)+b)*(16900975*x^(11/3)*a^11-148
72858*x^(10/3)*a^10*b+12932920*x^3*a^9*b^2-11085360*x^(8/3)*a^8*b^3+9335040
*x^(7/3)*a^7*b^4-7687680*x^2*a^6*b^5+6150144*x^(5/3)*a^5*b^6-4730880*x^(4/3
)*a^4*b^7+3440640*x*a^3*b^8-2293760*x^(2/3)*a^2*b^9+1310720*x^(1/3)*a*b^10-
524288*b^11)/x/a^12
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((a*x + b*x^(2/3))^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a\,x+b\,x^{2/3}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x + b*x^(2/3))^(3/2),x)
[Out] int(x^2*(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x**2*(a*x + b*x**(2/3))**(3/2), x)
```

3.177 $\int x\left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=255

$$\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(ax+bx^{2/3}\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(ax+bx^{2/3}\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(ax\right.}{4199a}$$

```
[Out] -256/1615*b^3*(b*x^(2/3)+a*x)^(5/2)/a^4+65536/4849845*b^8*(b*x^(2/3)+a*x)^(
5/2)/a^9/x^(5/3)-32768/969969*b^7*(b*x^(2/3)+a*x)^(5/2)/a^8/x^(4/3)+8192/13
8567*b^6*(b*x^(2/3)+a*x)^(5/2)/a^7/x-4096/46189*b^5*(b*x^(2/3)+a*x)^(5/2)/a
^6/x^(2/3)+512/4199*b^4*(b*x^(2/3)+a*x)^(5/2)/a^5/x^(1/3)+64/323*b^2*x^(1/3
)*(b*x^(2/3)+a*x)^(5/2)/a^3-32/133*b*x^(2/3)*(b*x^(2/3)+a*x)^(5/2)/a^2+2/7*
x*(b*x^(2/3)+a*x)^(5/2)/a
```

Rubi [A] time = 0.42, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{65536b^8\left(ax+bx^{2/3}\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(ax+bx^{2/3}\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(ax+bx^{2/3}\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(ax+bx^{2/3}\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(ax\right.}{4199a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-256*b^3*(b*x^(2/3) + a*x)^(5/2))/(1615*a^4) + (65536*b^8*(b*x^(2/3) + a*x
)^(5/2))/(4849845*a^9*x^(5/3)) - (32768*b^7*(b*x^(2/3) + a*x)^(5/2))/(96996
9*a^8*x^(4/3)) + (8192*b^6*(b*x^(2/3) + a*x)^(5/2))/(138567*a^7*x) - (4096*
b^5*(b*x^(2/3) + a*x)^(5/2))/(46189*a^6*x^(2/3)) + (512*b^4*(b*x^(2/3) + a*
x)^(5/2))/(4199*a^5*x^(1/3)) + (64*b^2*x^(1/3)*(b*x^(2/3) + a*x)^(5/2))/(32
3*a^3) - (32*b*x^(2/3)*(b*x^(2/3) + a*x)^(5/2))/(133*a^2) + (2*x*(b*x^(2/3)
 + a*x)^(5/2))/(7*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x\left(bx^{2/3}+ax\right)^{3/2}dx=\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{(16b)\int x^{2/3}\left(bx^{2/3}+ax\right)^{3/2}dx}{21a}$$
$$=-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}+\frac{\left(32b^2\right)\int\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{3/2}dx}{57a^2}$$
$$=\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}+ax\right)^{5/2}}{7a}-\frac{\left(128b^3\right)\int\left(bx^{2/\ldots}\right.}{323\ldots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx^{2/3}+ax\right)^{5/2}}{133a^2}+\frac{2x\left(bx^{2/3}\ldots\right.}{7a}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx^{2/3}+ax\right)^{5/2}}{323a^3}-\frac{32bx^{2/3}\left(bx\ldots\right.}{133\ldots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/3}+ax\right)^{5/2}}{4199a^5\sqrt[3]{x}}+\frac{64b^2\sqrt[3]{x}\left(bx\ldots\right.}{323\ldots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(bx^{2/3}+ax\right)^{5/2}}{46189a^6x^{2/3}}+\frac{512b^4\left(bx^{2/\ldots}\right.}{4199a\ldots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(bx^{2/3}+ax\right)^{5/2}}{138567a^7x}-\frac{4096b^5\left(b\ldots\right.}{4618\ldots}$$
$$=-\frac{256b^3\left(bx^{2/3}+ax\right)^{5/2}}{1615a^4}+\frac{65536b^8\left(bx^{2/3}+ax\right)^{5/2}}{4849845a^9x^{5/3}}-\frac{32768b^7\left(bx^{2/3}+ax\right)^{5/2}}{969969a^8x^{4/3}}+\frac{8192b^6\left(\ldots\right.}{13\ldots}$$

Mathematica [A] time = 0.11, size = 135, normalized size = 0.53

$$\frac{2\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\left(692835a^8x^{8/3}-583440a^7bx^{7/3}+480480a^6b^2x^2-384384a^5b^3x^{5/3}+295680a^4b^4x^{4/3}\ldots\right.}{4849845a^9\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(32768*b^8 - 81920*a*b^7*x^(1/3)
 + 143360*a^2*b^6*x^(2/3) - 215040*a^3*b^5*x + 295680*a^4*b^4*x^(4/3) - 384
384*a^5*b^3*x^(5/3) + 480480*a^6*b^2*x^2 - 583440*a^7*b*x^(7/3) + 692835*a^
8*x^(8/3)))/(4849845*a^9*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.30, size = 602, normalized size = 2.36

$$-\frac{2}{692835}b\left(\frac{32768\,b^{\frac{19}{2}}}{a^9}-\frac{\frac{19\left(6435\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}-58344\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b+235620\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b^2-556920\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^3+850850\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^4-\cdots\right.}{a^8}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -2/692835*b*(32768*b^(19/2)/a^9 - (19*(6435*(a*x^(1/3) + b)^(17/2) - 58344*
(a*x^(1/3) + b)^(15/2)*b + 235620*(a*x^(1/3) + b)^(13/2)*b^2 - 556920*(a*x^
(1/3) + b)^(11/2)*b^3 + 850850*(a*x^(1/3) + b)^(9/2)*b^4 - 875160*(a*x^(1/3
) + b)^(7/2)*b^5 + 612612*(a*x^(1/3) + b)^(5/2)*b^6 - 291720*(a*x^(1/3) + b
)^(3/2)*b^7 + 109395*sqrt(a*x^(1/3) + b)*b^8)*b/a^8 + 9*(12155*(a*x^(1/3) +
 b)^(19/2) - 122265*(a*x^(1/3) + b)^(17/2)*b + 554268*(a*x^(1/3) + b)^(15/2
)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)*b^3 + 2645370*(a*x^(1/3) + b)^(11/2)
*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340*(a*x^(1/3) + b)^(7/2)*b^
6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*x^(1/3) + b)^(3/2)*b^8 -
230945*sqrt(a*x^(1/3) + b)*b^9)/a^8)/a) + 2/1616615*a*(65536*b^(21/2)/a^10
+ (21*(12155*(a*x^(1/3) + b)^(19/2) - 122265*(a*x^(1/3) + b)^(17/2)*b + 554
268*(a*x^(1/3) + b)^(15/2)*b^2 - 1492260*(a*x^(1/3) + b)^(13/2)*b^3 + 26453
70*(a*x^(1/3) + b)^(11/2)*b^4 - 3233230*(a*x^(1/3) + b)^(9/2)*b^5 + 2771340
*(a*x^(1/3) + b)^(7/2)*b^6 - 1662804*(a*x^(1/3) + b)^(5/2)*b^7 + 692835*(a*
x^(1/3) + b)^(3/2)*b^8 - 230945*sqrt(a*x^(1/3) + b)*b^9)*b/a^9 + 5*(46189*(
a*x^(1/3) + b)^(21/2) - 510510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3
) + b)^(17/2)*b^2 - 7759752*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3
) + b)^(13/2)*b^4 - 22221108*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/
3) + b)^(9/2)*b^6 - 16628040*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3)
 + b)^(5/2)*b^8 - 3233230*(a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3)
 + b)*b^10)/a^9)/a)
```

maple [A] time = 0.05, size = 112, normalized size = 0.44

$$\frac{2\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(a\,x^{\frac{1}{3}}+b\right)\left(692835a^8x^{\frac{8}{3}}-583440a^7b\,x^{\frac{7}{3}}+480480a^6b^2x^2-384384a^5b^3x^{\frac{5}{3}}+295680a^4b^4x^{\frac{4}{3}}-\cdots\right.}{4849845a^9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/4849845*(a*x+b*x^(2/3))^(3/2)*(a*x^(1/3)+b)*(692835*x^(8/3)*a^8-583440*x^
(7/3)*a^7*b+480480*a^6*b^2*x^2-384384*x^(5/3)*a^5*b^3+295680*x^(4/3)*a^4*b^
4-215040*a^3*b^5*x+143360*x^(2/3)*a^2*b^6-81920*x^(1/3)*a*b^7+32768*b^8)/x/
a^9
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(a\,x+b\,x^{2/3}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x + b*x^(2/3))^(3/2),x)
[Out] int(x*(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x*(a*x + b*x**(2/3))**(3/2), x)
```

3.178 $\int \left(bx^{2/3}+ax\right)^{3/2}\,dx$

Optimal. Leaf size=169

$$-\frac{512b^5\left(ax+bx^{2/3}\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(ax+bx^{2/3}\right)^{5/2}}{429a^4x}+\frac{32b^2\left(ax+bx^{2/3}\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{5/2}}{13a^2\sqrt[3]{x}}+$$

```
[Out] 2/5*(b*x^(2/3)+a*x)^(5/2)/a-512/15015*b^5*(b*x^(2/3)+a*x)^(5/2)/a^6/x^(5/3)
+256/3003*b^4*(b*x^(2/3)+a*x)^(5/2)/a^5/x^(4/3)-64/429*b^3*(b*x^(2/3)+a*x)^
(5/2)/a^4/x+32/143*b^2*(b*x^(2/3)+a*x)^(5/2)/a^3/x^(2/3)-4/13*b*(b*x^(2/3)+
a*x)^(5/2)/a^2/x^(1/3)
```

Rubi [A] time = 0.25, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2002, 2016, 2014}

$$-\frac{512b^5\left(ax+bx^{2/3}\right)^{5/2}}{15015a^6x^{5/3}}+\frac{256b^4\left(ax+bx^{2/3}\right)^{5/2}}{3003a^5x^{4/3}}-\frac{64b^3\left(ax+bx^{2/3}\right)^{5/2}}{429a^4x}+\frac{32b^2\left(ax+bx^{2/3}\right)^{5/2}}{143a^3x^{2/3}}-\frac{4b\left(ax+bx^{2/3}\right)^{5/2}}{13a^2\sqrt[3]{x}}+$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b*x^(2/3) + a*x)^(5/2))/(5*a) - (512*b^5*(b*x^(2/3) + a*x)^(5/2))/(1501
5*a^6*x^(5/3)) + (256*b^4*(b*x^(2/3) + a*x)^(5/2))/(3003*a^5*x^(4/3)) - (64
*b^3*(b*x^(2/3) + a*x)^(5/2))/(429*a^4*x) + (32*b^2*(b*x^(2/3) + a*x)^(5/2)
)/(143*a^3*x^(2/3)) - (4*b*(b*x^(2/3) + a*x)^(5/2))/(13*a^2*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \left(bx^{2/3}+ax\right)^{3/2} dx = \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{(2b)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{\sqrt[3]{x}}dx}{3a}$$
$$= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}} + \frac{\left(16b^2\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{2/3}}dx}{39a^2}$$
$$= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} + \frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}} - \frac{\left(32b^3\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}dx}{143a^3}$$
$$= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x} + \frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}} - \frac{4b\left(bx^{2/3}+ax\right)^{5/2}}{13a^2\sqrt[3]{x}} + \frac{(12}{}$$
$$= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} + \frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x} + \frac{32b^2\left(bx^{2/3}+ax\right)^{5/2}}{143a^3x^{2/3}} -$$
$$= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{5a} - \frac{512b^5\left(bx^{2/3}+ax\right)^{5/2}}{15015a^6x^{5/3}} + \frac{256b^4\left(bx^{2/3}+ax\right)^{5/2}}{3003a^5x^{4/3}} - \frac{64b^3\left(bx^{2/3}+ax\right)^{5/2}}{429a^4x} +$$

Mathematica [A] time = 0.07, size = 98, normalized size = 0.58

$$\frac{2\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\left(3003a^5x^{5/3}-2310a^4bx^{4/3}+1680a^3b^2x-1120a^2b^3x^{2/3}+640ab^4\sqrt[3]{x}-256b^5\right)}{15015a^6\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*(-256*b^5 + 640*a*b^4*x^(1/3) -
1120*a^2*b^3*x^(2/3) + 1680*a^3*b^2*x - 2310*a^4*b*x^(4/3) + 3003*a^5*x^(5/
3)))/(15015*a^6*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.23, size = 434, normalized size = 2.57

$$\frac{2}{3003}b\left(\frac{256\,b^{\frac{13}{2}}}{a^6} + \frac{\frac{13\left(63\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}-385\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b+990\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b^2-1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^3+1155\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^4-693\sqrt{ax^{\frac{1}{3}}+b}\,b^5\right)b}{a^5} + \frac{3\left(231\left(\right.\right.}{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

[Out] 2/3003*b*(256*b^(13/2)/a^6 + (13*(63*(a*x^(1/3) + b)^(11/2) - 385*(a*x^(1/3) + b)^(9/2)*b + 990*(a*x^(1/3) + b)^(7/2)*b^2 - 1386*(a*x^(1/3) + b)^(5/2)*b^3 + 1155*(a*x^(1/3) + b)^(3/2)*b^4 - 693*sqrt(a*x^(1/3) + b)*b^5)*b/a^5 + 3*(231*(a*x^(1/3) + b)^(13/2) - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)/a^5)/a) - 2/15015*a*(1024*b^(15/2)/a^7 - (15*(231*(a*x^(1/3) + b)^(13/2) - 1638*(a*x^(1/3) + b)^(11/2)*b + 5005*(a*x^(1/3) + b)^(9/2)*b^2 - 8580*(a*x^(1/3) + b)^(7/2)*b^3 + 9009*(a*x^(1/3) + b)^(5/2)*b^4 - 6006*(a*x^(1/3) + b)^(3/2)*b^5 + 3003*sqrt(a*x^(1/3) + b)*b^6)*b/a^6 + 7*(429*(a*x^(1/3) + b)^(15/2) - 3465*(a*x^(1/3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025*(a*x^(1/3) + b)^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(1/3) + b)^(5/2)*b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a^6)/a)

maple [A] time = 0.05, size = 79, normalized size = 0.47

$$\frac{2\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(a\,x^{\frac{1}{3}}+b\right)\left(3003a^{5}x^{\frac{5}{3}}-2310a^{4}b\,x^{\frac{4}{3}}+1680a^{3}b^{2}x-1120a^{2}b^{3}x^{\frac{2}{3}}+640a\,b^{4}x^{\frac{1}{3}}-256b^{5}\right)}{15015a^{6}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+b*x^(2/3))^(3/2),x)

[Out] 2/15015*(a*x+b*x^(2/3))^(3/2)*(a*x^(1/3)+b)*(3003*x^(5/3)*a^5-2310*a^4*b*x^(4/3)+1680*a^3*b^2*x-1120*x^(2/3)*a^2*b^3+640*a*b^4*x^(1/3)-256*b^5)/x/a^6

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")

[Out] integrate((a*x + b*x^(2/3))^(3/2), x)

mupad [B] time = 5.14, size = 40, normalized size = 0.24

$$\frac{x\left(a\,x+b\,x^{2/3}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},6;\ 7;\ -\frac{a\,x^{1/3}}{b}\right)}{2\left(\frac{a\,x^{1/3}}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + b*x^(2/3))^(3/2),x)

[Out] (x*(a*x + b*x^(2/3))^(3/2)*hypergeom([-3/2, 6], 7, -(a*x^(1/3))/b))/(2*((a*x^(1/3))/b + 1)^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**(2/3)+a*x)**(3/2),x)

[Out] Integral((a*x + b*x**(2/3))**(3/2), x)

3.179 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=84

$$\frac{16b^2\left(ax+bx^{2/3}\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(ax+bx^{2/3}\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}$$

```
[Out] 16/105*b^2*(b*x^(2/3)+a*x)^(5/2)/a^3/x^(5/3)-8/21*b*(b*x^(2/3)+a*x)^(5/2)/a
^2/x^(4/3)+2/3*(b*x^(2/3)+a*x)^(5/2)/a/x
```

Rubi [A] time = 0.14, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16b^2\left(ax+bx^{2/3}\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(ax+bx^{2/3}\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(ax+bx^{2/3}\right)^{5/2}}{3ax}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

```
[Out] (16*b^2*(b*x^(2/3) + a*x)^(5/2))/(105*a^3*x^(5/3)) - (8*b*(b*x^(2/3) + a*x)
^(5/2))/(21*a^2*x^(4/3)) + (2*(b*x^(2/3) + a*x)^(5/2))/(3*a*x)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}-\frac{(4b)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{4/3}}\,dx}{9a}\\
&= -\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}+\frac{\left(8b^2\right)\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^{5/3}}\,dx}{63a^2}\\
&= \frac{16b^2\left(bx^{2/3}+ax\right)^{5/2}}{105a^3x^{5/3}}-\frac{8b\left(bx^{2/3}+ax\right)^{5/2}}{21a^2x^{4/3}}+\frac{2\left(bx^{2/3}+ax\right)^{5/2}}{3ax}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 63, normalized size = 0.75

$$\frac{2\left(a\sqrt[3]{x}+b\right)^2\left(35a^2x^{2/3}-20ab\sqrt[3]{x}+8b^2\right)\sqrt{ax+bx^{2/3}}}{105a^3\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x,x]
```

```
[Out] (2*(b + a*x^(1/3))^2*(8*b^2 - 20*a*b*x^(1/3) + 35*a^2*x^(2/3))*Sqrt[b*x^(2/
3) + a*x])/(105*a^3*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 0.21, size = 265, normalized size = 3.15

$$-\frac{2}{35}\,b\left(\frac{8\,b^{\frac{7}{2}}}{a^3}-\frac{\frac{7\left(3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}-10\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b+15\sqrt{ax^{\frac{1}{3}}+b}\,b^2\right)b}{a^2}+\frac{3\left(5\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}-21\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b+35\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^2-35\sqrt{ax^{\frac{1}{3}}+b}\,b^3\right)}{a^2}}{a}\right)+\frac{2}{10\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -2/35*b*(8*b^(7/2)/a^3 - (7*(3*(a*x^(1/3) + b)^(5/2) - 10*(a*x^(1/3) + b)^(
3/2)*b + 15*sqrt(a*x^(1/3) + b)*b^2)*b/a^2 + 3*(5*(a*x^(1/3) + b)^(7/2) - 2
1*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^(1/3
) + b)*b^3)/a^2)/a) + 2/105*a*(16*b^(9/2)/a^4 + (9*(5*(a*x^(1/3) + b)^(7/2)
 - 21*(a*x^(1/3) + b)^(5/2)*b + 35*(a*x^(1/3) + b)^(3/2)*b^2 - 35*sqrt(a*x^
(1/3) + b)*b^3)*b/a^3 + (35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b)^(7/
2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 + 315*
sqrt(a*x^(1/3) + b)*b^4)/a^3)/a)
```

maple [A] time = 0.05, size = 48, normalized size = 0.57

$$\frac{2\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(a\,x^{\frac{1}{3}}+b\right)\left(35a^2x^{\frac{2}{3}}-20ab\,x^{\frac{1}{3}}+8b^2\right)}{105a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x,x)
```

```
[Out] 2/105*(a*x+b*x^(2/3))^(3/2)*(a*x^(1/3)+b)*(35*a^2*x^(2/3)-20*a*b*x^(1/3)+8*
b^2)/x/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x,x)
[Out] int((a*x + b*x^(2/3))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x,x)
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x, x)
```

3.180 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=78

$$-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)+\frac{6b\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

```
[Out] 2*(b*x^(2/3)+a*x)^(3/2)/x-6*b^(3/2)*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)
^(1/2))+6*b*(b*x^(2/3)+a*x)^(1/2)/x^(1/3)
```

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2029, 206}

$$-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)+\frac{6b\sqrt{ax+bx^{2/3}}}{\sqrt[3]{x}}+\frac{2\left(ax+bx^{2/3}\right)^{3/2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (6*b*Sqrt[b*x^(2/3) + a*x])/x^(1/3) + (2*(b*x^(2/3) + a*x)^(3/2))/x - 6*b^(
3/2)*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}\,dx &= \frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}+b\int \frac{\sqrt{bx^{2/3}+ax}}{x^{4/3}}\,dx\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}+b^2\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-\left(6b^2\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)\\
&= \frac{6b\sqrt{bx^{2/3}+ax}}{\sqrt[3]{x}}+\frac{2\left(bx^{2/3}+ax\right)^{3/2}}{x}-6b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)
\end{aligned}$$

Mathematica [A] time = 0.10, size = 88, normalized size = 1.13

$$\frac{2\sqrt{ax+bx^{2/3}}\left(\sqrt{a\sqrt[3]{x}+b}\left(a\sqrt[3]{x}+4b\right)-3b^{3/2}\tanh^{-1}\left(\frac{\sqrt{a\sqrt[3]{x}+b}}{\sqrt{b}}\right)\right)}{\sqrt[3]{x}\sqrt{a\sqrt[3]{x}+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^2,x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(Sqrt[b + a*x^(1/3)]*(4*b + a*x^(1/3)) - 3*b^(3/2)
*ArcTanh[Sqrt[b + a*x^(1/3)]/Sqrt[b]]))/(Sqrt[b + a*x^(1/3)]*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.23, size = 83, normalized size = 1.06

$$\frac{6\,b^{2}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}+2\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}+6\sqrt{ax^{\frac{1}{3}}+b}\,b-\frac{2\left(3\,b^{2}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+4\sqrt{-b}\,b^{\frac{3}{2}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 6*b^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/sqrt(-b) + 2*(a*x^(1/3) + b)^(3/
2) + 6*sqrt(a*x^(1/3) + b)*b - 2*(3*b^2*arctan(sqrt(b)/sqrt(-b)) + 4*sqrt(-
b)*b^(3/2))/sqrt(-b)
```

maple [A] time = 0.05, size = 69, normalized size = 0.88

$$-\frac{2\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(3b^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-3\sqrt{a\,x^{\frac{1}{3}}+b}\;b-\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}\right)}{\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x^2,x)
```

```
[Out] -2*(a*x+b*x^(2/3))^(3/2)*(3*b^(3/2)*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))-(a
*x^(1/3)+b)^(3/2)-3*(a*x^(1/3)+b)^(1/2)*b)/x/(a*x^(1/3)+b)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^2,x)
[Out] int((a*x + b*x^(2/3))^(3/2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**2,x)
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**2, x)
```

3.181 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=113

$$\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{3/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{8bx^{2/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4x}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

```
[Out] -(b*x^(2/3)+a*x)^(3/2)/x^2+3/8*a^3*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^
(1/2))/b^(3/2)-3/4*a*(b*x^(2/3)+a*x)^(1/2)/x-3/8*a^2*(b*x^(2/3)+a*x)^(1/2)/
b/x^(2/3)
```

Rubi [A] time = 0.18, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{3/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{8bx^{2/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{4x}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(4*x) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(8*b*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/x^2 + (3*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt
[b*x^(2/3) + a*x]])/(8*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^3}\,dx = -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{1}{2}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^2}\,dx$$
$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{1}{8}a^2\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx$$
$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}-\frac{a^3\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{16b}$$
$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{\left(3a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{}{\sqrt{b}}\right.}{8b}$$
$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{4x}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{8bx^{2/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^2}+\frac{3a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{8b^{3/2}}$$

Mathematica [C] time = 0.07, size = 61, normalized size = 0.54

$$\frac{6a^3\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{5b^4\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^3,x]
```

```
[Out] (6*a^3*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 4, 7/
2, 1 + (a*x^(1/3))/b])/(5*b^4*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.26, size = 92, normalized size = 0.81

$$-\frac{\frac{3\,a^4\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{3\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^4+8\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^4b-3\sqrt{ax^{\frac{1}{3}}+b}\,a^4b^2}{a^3bx}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*a^4*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + (3*(a*x^(1/
3) + b)^(5/2)*a^4 + 8*(a*x^(1/3) + b)^(3/2)*a^4*b - 3*sqrt(a*x^(1/3) + b)*a
^4*b^2)/(a^3*b*x))/a
```

maple [A] time = 0.06, size = 93, normalized size = 0.82

$$-\frac{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(-3a^3bx\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-3\sqrt{a\,x^{\frac{1}{3}}+b}\,b^{\frac{7}{2}}+8\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{5}{2}}+3\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^{\frac{3}{2}}\right)}{8\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x^3,x)
```

```
[Out] -1/8*(a*x+b*x^(2/3))^(3/2)*(3*(a*x^(1/3)+b)^(5/2)*b^(3/2)+8*(a*x^(1/3)+b)^(
3/2)*b^(5/2)-3*(a*x^(1/3)+b)^(1/2)*b^(7/2)-3*arctanh((a*x^(1/3)+b)^(1/2)/b^
(1/2))*x*a^3*b)/x^2/(a*x^(1/3)+b)^(3/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x + b\,x^{2/3}\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^3,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**3,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**3, x)
```

3.182 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=203

$$-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{9/2}}+\frac{21a^5\sqrt{ax+bx^{2/3}}}{512b^4x^{2/3}}-\frac{7a^4\sqrt{ax+bx^{2/3}}}{256b^3x}+\frac{7a^3\sqrt{ax+bx^{2/3}}}{320b^2x^{4/3}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{160bx^{5/3}}-\frac{\left(ax+bx^{2/3}\right)^{3/2}}{2x^3}$$

```
[Out] -1/2*(b*x^(2/3)+a*x)^(3/2)/x^3-21/512*a^6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3
)+a*x)^(1/2))/b^(9/2)-3/20*a*(b*x^(2/3)+a*x)^(1/2)/x^2-3/160*a^2*(b*x^(2/3)
+a*x)^(1/2)/b/x^(5/3)+7/320*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-7/256*a^4
*(b*x^(2/3)+a*x)^(1/2)/b^3/x+21/512*a^5*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A] time = 0.34, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2029, 206}

$$\frac{21a^5\sqrt{ax+bx^{2/3}}}{512b^4x^{2/3}}-\frac{7a^4\sqrt{ax+bx^{2/3}}}{256b^3x}+\frac{7a^3\sqrt{ax+bx^{2/3}}}{320b^2x^{4/3}}-\frac{21a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{9/2}}-\frac{3a^2\sqrt{ax+bx^{2/3}}}{160bx^{5/3}}-\frac{3a\sqrt{ax+bx^{2/3}}}{20x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(20*x^2) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(160*
b*x^(5/3)) + (7*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^2*x^(4/3)) - (7*a^4*Sqrt[
b*x^(2/3) + a*x])/(256*b^3*x) + (21*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^4*x^(
2/3)) - (b*x^(2/3) + a*x)^(3/2)/(2*x^3) - (21*a^6*ArcTanh[(Sqrt[b]*x^(1/3))
/Sqrt[b*x^(2/3) + a*x]])/(512*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^4}\,dx = -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+\frac{1}{4}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^3}\,dx$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+\frac{1}{40}a^2\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}-\frac{\left(7a^3\right)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{320b}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{2x^3}+\frac{\left(7a^4\right)\int \frac{1}{x^{4/3}\sqrt{b}}}{384b}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}-\frac{\left(bx^{2/3}+ax\right)}{2x^3}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+\frac{21a^5\sqrt{bx^{2/3}}}{512b^4x^{2/}}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+\frac{21a^5\sqrt{bx^{2/3}}}{512b^4x^{2/}}$$

$$= -\frac{3a\sqrt{bx^{2/3}+ax}}{20x^2}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{160bx^{5/3}}+\frac{7a^3\sqrt{bx^{2/3}+ax}}{320b^2x^{4/3}}-\frac{7a^4\sqrt{bx^{2/3}+ax}}{256b^3x}+\frac{21a^5\sqrt{bx^{2/3}}}{512b^4x^{2/}}$$

Mathematica [C] time = 0.05, size = 61, normalized size = 0.30

$$-\frac{6a^6\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{5}{2},7;\frac{7}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{5b^7\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^4,x]
```

```
[Out] (-6*a^6*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 7, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^7*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.37, size = 143, normalized size = 0.70

$$\frac{\frac{105\,a^7\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^4}+\frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^7-595\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^7b+1386\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^7b^2-1686\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^7b^3-595\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^7b^4+105\sqrt{ax^{\frac{1}{3}}+b}\,a^7b}{a^6b^4x^2}}{2560\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 1/2560*(105*a^7*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*
(a*x^(1/3) + b)^(11/2)*a^7 - 595*(a*x^(1/3) + b)^(9/2)*a^7*b + 1386*(a*x^(1
/3) + b)^(7/2)*a^7*b^2 - 1686*(a*x^(1/3) + b)^(5/2)*a^7*b^3 - 595*(a*x^(1/3
) + b)^(3/2)*a^7*b^4 + 105*sqrt(a*x^(1/3) + b)*a^7*b^5)/(a^6*b^4*x^2))/a
```

maple [A] time = 0.06, size = 139, normalized size = 0.68

$$\frac{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(-105a^6b^4x^2\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+105\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{19}{2}}-595\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{17}{2}}-1686\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b\right.}{2560\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{17}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x^4,x)
```

```
[Out] 1/2560*(a*x+b*x^(2/3))^(3/2)*(105*b^(9/2)*(a*x^(1/3)+b)^(11/2)-595*b^(11/2)
*(a*x^(1/3)+b)^(9/2)+1386*b^(13/2)*(a*x^(1/3)+b)^(7/2)-1686*b^(15/2)*(a*x^(
1/3)+b)^(5/2)-595*b^(17/2)*(a*x^(1/3)+b)^(3/2)+105*b^(19/2)*(a*x^(1/3)+b)^(
1/2)-105*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b^4*x^2*a^6)/x^3/(a*x^(1/3)+b
)^(3/2)/b^(17/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^4,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**4,x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(3/2)/x**4, x)
```

3.183 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=291

$$\frac{429a^9\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{32768b^{15/2}}-\frac{429a^8\sqrt{ax+bx^{2/3}}}{32768b^7x^{2/3}}+\frac{143a^7\sqrt{ax+bx^{2/3}}}{16384b^6x}-\frac{143a^6\sqrt{ax+bx^{2/3}}}{20480b^5x^{4/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{71680b^4x^{5/3}}-\frac{143a}{2}$$

```
[Out] -1/3*(b*x^(2/3)+a*x)^(3/2)/x^4+429/32768*a^9*arctanh(x^(1/3)*b^(1/2)/(b*x^(
2/3)+a*x)^(1/2))/b^(15/2)-1/16*a*(b*x^(2/3)+a*x)^(1/2)/x^3-1/224*a^2*(b*x^(
2/3)+a*x)^(1/2)/b/x^(8/3)+13/2688*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-143
/26880*a^4*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2+429/71680*a^5*(b*x^(2/3)+a*x)^(1/2
)/b^4/x^(5/3)-143/20480*a^6*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+143/16384*a^7
*(b*x^(2/3)+a*x)^(1/2)/b^6/x-429/32768*a^8*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3
)
```

Rubi [A] time = 0.52, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2020, 2025, 2029, 206}

$$-\frac{429a^8\sqrt{ax+bx^{2/3}}}{32768b^7x^{2/3}}+\frac{143a^7\sqrt{ax+bx^{2/3}}}{16384b^6x}-\frac{143a^6\sqrt{ax+bx^{2/3}}}{20480b^5x^{4/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{71680b^4x^{5/3}}-\frac{143a^4\sqrt{ax+bx^{2/3}}}{26880b^3x^2}+\frac{13a^3\sqrt{ax}}{2688b}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] -(a*Sqrt[b*x^(2/3) + a*x])/(16*x^3) - (a^2*Sqrt[b*x^(2/3) + a*x])/(224*b*x^
(8/3)) + (13*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^2*x^(7/3)) - (143*a^4*Sqrt[
b*x^(2/3) + a*x])/(26880*b^3*x^2) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(71680*
b^4*x^(5/3)) - (143*a^6*Sqrt[b*x^(2/3) + a*x])/(20480*b^5*x^(4/3)) + (143*a
^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^6*x) - (429*a^8*Sqrt[b*x^(2/3) + a*x])/(
32768*b^7*x^(2/3)) - (b*x^(2/3) + a*x)^(3/2)/(3*x^4) + (429*a^9*ArcTanh[(Sq
rt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^5}\,dx &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{1}{6}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^4}\,dx \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{1}{96}a^2\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}-\frac{\left(13a^3\right)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{1344b} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{3x^4}+\frac{\left(143a^4\right)\int \ldots}{161\ldots} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}-\frac{\left(bx^{2/3}+\ldots\right.}{3x^{\ldots}} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{\ldots}}{7168\ldots} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{\ldots}}{7168\ldots} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{\ldots}}{7168\ldots} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{\ldots}}{7168\ldots} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{\ldots}}{7168\ldots} \\
&= -\frac{a\sqrt{bx^{2/3}+ax}}{16x^3}-\frac{a^2\sqrt{bx^{2/3}+ax}}{224bx^{8/3}}+\frac{13a^3\sqrt{bx^{2/3}+ax}}{2688b^2x^{7/3}}-\frac{143a^4\sqrt{bx^{2/3}+ax}}{26880b^3x^2}+\frac{429a^5\sqrt{\ldots}}{7168\ldots}
\end{aligned}
$$

Mathematica [C] time = 0.06, size = 61, normalized size = 0.21

$$\frac{6a^9\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{5}{2},10;\frac{7}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{5b^{10}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^5,x]
```

```
[Out] (6*a^9*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 10, 7
/2, 1 + (a*x^(1/3))/b])/(5*b^10*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="fricas")
```

[Out] Timed out

giac [A] time = 0.38, size = 194, normalized size = 0.67

$$-\frac{\frac{45045\,a^{10}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{7}}+\frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{10}-390390\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{10}b+1495494\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^{10}b^{2}-3317886\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^{10}b^{3}+4685824\left(a\right.}{a^{9}b}}{3440640\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/3440640*(45045*a^10*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7)
+ (45045*(a*x^(1/3) + b)^(17/2)*a^10 - 390390*(a*x^(1/3) + b)^(15/2)*a^10*b
 + 1495494*(a*x^(1/3) + b)^(13/2)*a^10*b^2 - 3317886*(a*x^(1/3) + b)^(11/2)
*a^10*b^3 + 4685824*(a*x^(1/3) + b)^(9/2)*a^10*b^4 - 4349826*(a*x^(1/3) + b
)^(7/2)*a^10*b^5 + 2633274*(a*x^(1/3) + b)^(5/2)*a^10*b^6 + 390390*(a*x^(1/
3) + b)^(3/2)*a^10*b^7 - 45045*sqrt(a*x^(1/3) + b)*a^10*b^8)/(a^9*b^7*x^3))
/a
```

maple [A] time = 0.06, size = 181, normalized size = 0.62

$$-\frac{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(-45045a^{9}b^{7}x^{3}\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-45045\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{31}{2}}+390390\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{29}{2}}+2633274\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x^5,x)
```

```
[Out] -1/3440640*(a*x+b*x^(2/3))^(3/2)*(45045*(a*x^(1/3)+b)^(17/2)*b^(15/2)-39039
0*(a*x^(1/3)+b)^(15/2)*b^(17/2)+1495494*(a*x^(1/3)+b)^(13/2)*b^(19/2)-33178
86*(a*x^(1/3)+b)^(11/2)*b^(21/2)+4685824*(a*x^(1/3)+b)^(9/2)*b^(23/2)-43498
26*(a*x^(1/3)+b)^(7/2)*b^(25/2)+2633274*(a*x^(1/3)+b)^(5/2)*b^(27/2)+390390
*(a*x^(1/3)+b)^(3/2)*b^(29/2)-45045*(a*x^(1/3)+b)^(1/2)*b^(31/2)-45045*arct
anh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b^7*x^3*a^9)/x^4/(a*x^(1/3)+b)^(3/2)/b^(29
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^5,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^5, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**5,x)
```

```
[Out] Timed out
```

3.184 $\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx$

Optimal. Leaf size=379

$$-\frac{12597a^{12}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{2097152b^{21/2}}+\frac{12597a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{10}x^{2/3}}-\frac{4199a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^9x}+\frac{4199a^9\sqrt{ax+bx^{2/3}}}{1310720b^8x^{4/3}}-\frac{12597a^8\sqrt{ax}}{4587520b}$$

```
[Out] -1/4*(b*x^(2/3)+a*x)^(3/2)/x^5-12597/2097152*a^12*arctanh(x^(1/3)*b^(1/2)/(
b*x^(2/3)+a*x)^(1/2))/b^(21/2)-3/88*a*(b*x^(2/3)+a*x)^(1/2)/x^4-3/1760*a^2*
(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/10560*a^3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(
10/3)-323/168960*a^4*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3+323/157696*a^5*(b*x^(2/3
)+a*x)^(1/2)/b^4/x^(8/3)-4199/1892352*a^6*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)
+4199/1720320*a^7*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2-12597/4587520*a^8*(b*x^(2/3
)+a*x)^(1/2)/b^7/x^(5/3)+4199/1310720*a^9*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)
-4199/1048576*a^10*(b*x^(2/3)+a*x)^(1/2)/b^9/x+12597/2097152*a^11*(b*x^(2/3
)+a*x)^(1/2)/b^10/x^(2/3)
```

Rubi [A] time = 0.72, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2020, 2025, 2029, 206}

$$\frac{12597a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{10}x^{2/3}}-\frac{4199a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^9x}+\frac{4199a^9\sqrt{ax+bx^{2/3}}}{1310720b^8x^{4/3}}-\frac{12597a^8\sqrt{ax+bx^{2/3}}}{4587520b^7x^{5/3}}+\frac{4199a^7\sqrt{ax+bx^{2/3}}}{1720320b^6x^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-3*a*Sqrt[b*x^(2/3) + a*x])/(88*x^4) - (3*a^2*Sqrt[b*x^(2/3) + a*x])/(1760
*b*x^(11/3)) + (19*a^3*Sqrt[b*x^(2/3) + a*x])/(10560*b^2*x^(10/3)) - (323*a
^4*Sqrt[b*x^(2/3) + a*x])/(168960*b^3*x^3) + (323*a^5*Sqrt[b*x^(2/3) + a*x]
)/(157696*b^4*x^(8/3)) - (4199*a^6*Sqrt[b*x^(2/3) + a*x])/(1892352*b^5*x^(7
/3)) + (4199*a^7*Sqrt[b*x^(2/3) + a*x])/(1720320*b^6*x^2) - (12597*a^8*Sqrt
[b*x^(2/3) + a*x])/(4587520*b^7*x^(5/3)) + (4199*a^9*Sqrt[b*x^(2/3) + a*x])
/(1310720*b^8*x^(4/3)) - (4199*a^10*Sqrt[b*x^(2/3) + a*x])/(1048576*b^9*x)
+ (12597*a^11*Sqrt[b*x^(2/3) + a*x])/(2097152*b^10*x^(2/3)) - (b*x^(2/3) +
a*x)^(3/2)/(4*x^5) - (12597*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) +
 a*x]])/(2097152*b^(21/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
```

```
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(bx^{2/3}+ax\right)^{3/2}}{x^6}\,dx &= -\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{1}{8}a\int \frac{\sqrt{bx^{2/3}+ax}}{x^5}\,dx \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{1}{176}a^2\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}-\frac{\left(19a^3\right)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{3520b} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{\left(bx^{2/3}+ax\right)^{3/2}}{4x^5}+\frac{\left(323a^4\right)\int \cdots}{6\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}-\frac{\left(bx^{2/3}\cdots\right.}{\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\cdots}{157\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\cdots}{157\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\cdots}{157\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\cdots}{157\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\cdots}{157\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\cdots}{157\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\cdots}{157\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\cdots}{157\cdots} \\
&= -\frac{3a\sqrt{bx^{2/3}+ax}}{88x^4}-\frac{3a^2\sqrt{bx^{2/3}+ax}}{1760bx^{11/3}}+\frac{19a^3\sqrt{bx^{2/3}+ax}}{10560b^2x^{10/3}}-\frac{323a^4\sqrt{bx^{2/3}+ax}}{168960b^3x^3}+\frac{323a^5\cdots}{157\cdots}
\end{aligned}
$$

Mathematica [C] time = 0.10, size = 61, normalized size = 0.16

$$
-\frac{6a^{12}\left(a\sqrt[3]{x}+b\right)^2\sqrt{ax+bx^{2/3}}\;{}_2F_1\left(\frac{5}{2},13;\frac{7}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{5b^{13}\sqrt[3]{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(3/2)/x^6,x]
```

```
[Out] (-6*a^12*(b + a*x^(1/3))^2*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[5/2, 13,
 7/2, 1 + (a*x^(1/3))/b])/(5*b^13*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.47, size = 245, normalized size = 0.65

$$\frac{\frac{14549535\,a^{13}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}a^{13}-169744575\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{13}b+904981077\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{13}b^{2}-2913648309\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{13}b}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/2422210560*(14549535*a^13*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*
b^10) + (14549535*(a*x^(1/3) + b)^(23/2)*a^13 - 169744575*(a*x^(1/3) + b)^(
21/2)*a^13*b + 904981077*(a*x^(1/3) + b)^(19/2)*a^13*b^2 - 2913648309*(a*x^
(1/3) + b)^(17/2)*a^13*b^3 + 6303782342*(a*x^(1/3) + b)^(15/2)*a^13*b^4 - 9
643633350*(a*x^(1/3) + b)^(13/2)*a^13*b^5 + 10677769530*(a*x^(1/3) + b)^(11
/2)*a^13*b^6 - 8598579770*(a*x^(1/3) + b)^(9/2)*a^13*b^7 + 4975837515*(a*x^
(1/3) + b)^(7/2)*a^13*b^8 - 2001671595*(a*x^(1/3) + b)^(5/2)*a^13*b^9 - 169
744575*(a*x^(1/3) + b)^(3/2)*a^13*b^10 + 14549535*sqrt(a*x^(1/3) + b)*a^13*
b^11)/(a^12*b^10*x^4))/a
```

maple [A] time = 0.07, size = 223, normalized size = 0.59

$$\frac{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}\left(-14549535a^{12}b^{10}x^{4}\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+14549535\sqrt{a\,x^{\frac{1}{3}}+b}\;b^{\frac{43}{2}}-169744575\left(a\,x^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^{\frac{41}{2}}-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^(2/3))^(3/2)/x^6,x)
```

```
[Out] 1/2422210560*(a*x+b*x^(2/3))^(3/2)*(14549535*(a*x^(1/3)+b)^(23/2)*b^(21/2)-
169744575*(a*x^(1/3)+b)^(21/2)*b^(23/2)+904981077*(a*x^(1/3)+b)^(19/2)*b^(2
5/2)-2913648309*(a*x^(1/3)+b)^(17/2)*b^(27/2)+6303782342*(a*x^(1/3)+b)^(15/
2)*b^(29/2)-9643633350*(a*x^(1/3)+b)^(13/2)*b^(31/2)+10677769530*(a*x^(1/3)
+b)^(11/2)*b^(33/2)-8598579770*(a*x^(1/3)+b)^(9/2)*b^(35/2)+4975837515*(a*x
^(1/3)+b)^(7/2)*b^(37/2)-2001671595*(a*x^(1/3)+b)^(5/2)*b^(39/2)-169744575*
(a*x^(1/3)+b)^(3/2)*b^(41/2)+14549535*(a*x^(1/3)+b)^(1/2)*b^(43/2)-14549535
*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b^10*x^4*a^12)/x^5/(a*x^(1/3)+b)^(3/2
)/b^(41/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^(2/3)+a*x)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a\,x+b\,x^{2/3}\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^(2/3))^(3/2)/x^6,x)
```

```
[Out] int((a*x + b*x^(2/3))^(3/2)/x^6, x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**(2/3)+a*x)**(3/2)/x**6,x)
```

```
[Out] Timed out
```

3.185 $\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=401

$$-\frac{16777216b^{13}\sqrt{ax+bx^{2/3}}}{11700675a^{14}\sqrt[3]{x}}+\frac{8388608b^{12}\sqrt{ax+bx^{2/3}}}{11700675a^{13}}-\frac{2097152b^{11}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{ax+bx^{2/3}}}{2340135a^{11}}$$

```
[Out] 8388608/11700675*b^12*(b*x^(2/3)+a*x)^(1/2)/a^13-16777216/11700675*b^13*(b*
x^(2/3)+a*x)^(1/2)/a^14/x^(1/3)-2097152/3900225*b^11*x^(1/3)*(b*x^(2/3)+a*x
)^(1/2)/a^12+1048576/2340135*b^10*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^11-131072
/334305*b^9*x*(b*x^(2/3)+a*x)^(1/2)/a^10+65536/185725*b^8*x^(4/3)*(b*x^(2/3
)+a*x)^(1/2)/a^9-180224/557175*b^7*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^8+117145
6/3900225*b^6*x^2*(b*x^(2/3)+a*x)^(1/2)/a^7-73216/260015*b^5*x^(7/3)*(b*x^(
2/3)+a*x)^(1/2)/a^6+36608/137655*b^4*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-9152
/36225*b^3*x^3*(b*x^(2/3)+a*x)^(1/2)/a^4+416/1725*b^2*x^(10/3)*(b*x^(2/3)+a
*x)^(1/2)/a^3-52/225*b*x^(11/3)*(b*x^(2/3)+a*x)^(1/2)/a^2+2/9*x^4*(b*x^(2/3
)+a*x)^(1/2)/a
```

Rubi [A] time = 0.73, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2016, 2002, 2014}

$$-\frac{16777216b^{13}\sqrt{ax+bx^{2/3}}}{11700675a^{14}\sqrt[3]{x}}+\frac{8388608b^{12}\sqrt{ax+bx^{2/3}}}{11700675a^{13}}-\frac{2097152b^{11}\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{3900225a^{12}}+\frac{1048576b^{10}x^{2/3}\sqrt{ax+bx^{2/3}}}{2340135a^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (8388608*b^12*Sqrt[b*x^(2/3) + a*x])/(11700675*a^13) - (16777216*b^13*Sqrt[
b*x^(2/3) + a*x])/(11700675*a^14*x^(1/3)) - (2097152*b^11*x^(1/3)*Sqrt[b*x^
(2/3) + a*x])/(3900225*a^12) + (1048576*b^10*x^(2/3)*Sqrt[b*x^(2/3) + a*x])
/(2340135*a^11) - (131072*b^9*x*Sqrt[b*x^(2/3) + a*x])/(334305*a^10) + (655
36*b^8*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(185725*a^9) - (180224*b^7*x^(5/3)*Sq
rt[b*x^(2/3) + a*x])/(557175*a^8) + (1171456*b^6*x^2*Sqrt[b*x^(2/3) + a*x])
/(3900225*a^7) - (73216*b^5*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(260015*a^6) + (
36608*b^4*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(137655*a^5) - (9152*b^3*x^3*Sqrt[
b*x^(2/3) + a*x])/(36225*a^4) + (416*b^2*x^(10/3)*Sqrt[b*x^(2/3) + a*x])/(1
725*a^3) - (52*b*x^(11/3)*Sqrt[b*x^(2/3) + a*x])/(225*a^2) + (2*x^4*Sqrt[b*
x^(2/3) + a*x])/(9*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{(26b)\int \frac{x^{11/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{27a}$$

$$= -\frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} + \frac{\left(208b^2\right)\int \frac{x^{10/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{225a^2}$$

$$= \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+ax}}{9a} - \frac{\left(4576b^3\right)\int \frac{x^3}{\sqrt{bx^{2/3}+a}}}{5175a^3}$$

$$= -\frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52bx^{11/3}\sqrt{bx^{2/3}+ax}}{225a^2} + \frac{2x^4\sqrt{bx^{2/3}+}}{9a}$$

$$= \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2x^{10/3}\sqrt{bx^{2/3}+ax}}{1725a^3} - \frac{52bx^{11/3}}{22}$$

$$= -\frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152b^3x^3\sqrt{bx^{2/3}+ax}}{36225a^4} + \frac{416b^2}{}$$

$$= \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} + \frac{36608b^4x^{8/3}\sqrt{bx^{2/3}+ax}}{137655a^5} - \frac{9152}{}$$

$$= -\frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{73216b^5x^{7/3}\sqrt{bx^{2/3}+ax}}{260015a^6} + \frac{36}{}$$

$$= \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{1171456b^6x^2\sqrt{bx^{2/3}+ax}}{3900225a^7} - \frac{732}{}$$

$$= -\frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180224b^7x^{5/3}\sqrt{bx^{2/3}+ax}}{557175a^8} + \frac{117}{}$$

$$= \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} + \frac{65536b^8x^{4/3}\sqrt{bx^{2/3}+ax}}{185725a^9} - \frac{180}{}$$

$$= -\frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} - \frac{131072b^9x\sqrt{bx^{2/3}+ax}}{334305a^{10}} -$$

$$= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1048576b^{10}x^{2/3}\sqrt{bx^{2/3}+ax}}{2340135a^{11}} -$$

$$= \frac{8388608b^{12}\sqrt{bx^{2/3}+ax}}{11700675a^{13}} - \frac{16777216b^{13}\sqrt{bx^{2/3}+ax}}{11700675a^{14}\sqrt[3]{x}} - \frac{2097152b^{11}\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{3900225a^{12}} + \frac{1}{}$$

Mathematica [A] time = 0.23, size = 185, normalized size = 0.46

$$2\sqrt{ax+bx^{2/3}}\left(1300075a^{13}x^{13/3} - 1352078a^{12}bx^4 + 1410864a^{11}b^2x^{11/3} - 1478048a^{10}b^3x^{10/3} + 1555840a^9b^4x^3\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-8388608*b^13 + 4194304*a*b^12*x^(1/3) - 3145728*
a^2*b^11*x^(2/3) + 2621440*a^3*b^10*x - 2293760*a^4*b^9*x^(4/3) + 2064384*a
^5*b^8*x^(5/3) - 1892352*a^6*b^7*x^2 + 1757184*a^7*b^6*x^(7/3) - 1647360*a^
8*b^5*x^(8/3) + 1555840*a^9*b^4*x^3 - 1478048*a^10*b^3*x^(10/3) + 1410864*a
^11*b^2*x^(11/3) - 1352078*a^12*b*x^4 + 1300075*a^13*x^(13/3)))/(11700675*a
^14*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.30, size = 206, normalized size = 0.51

$$\frac{16777216\,b^{\frac{27}{2}}}{11700675\,a^{14}}+\frac{2\left(1300075\left(ax^{\frac{1}{3}}+b\right)^{\frac{27}{2}}-18253053\left(ax^{\frac{1}{3}}+b\right)^{\frac{25}{2}}b+119041650\left(ax^{\frac{1}{3}}+b\right)^{\frac{23}{2}}b^2-478056150\left(ax^{\frac{1}{3}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 16777216/11700675*b^(27/2)/a^14 + 2/11700675*(1300075*(a*x^(1/3) + b)^(27/2
) - 18253053*(a*x^(1/3) + b)^(25/2)*b + 119041650*(a*x^(1/3) + b)^(23/2)*b^
2 - 478056150*(a*x^(1/3) + b)^(21/2)*b^3 + 1320944625*(a*x^(1/3) + b)^(19/2
)*b^4 - 2657429775*(a*x^(1/3) + b)^(17/2)*b^5 + 4015671660*(a*x^(1/3) + b)^
(15/2)*b^6 - 4633467300*(a*x^(1/3) + b)^(13/2)*b^7 + 4106936925*(a*x^(1/3)
+ b)^(11/2)*b^8 - 2788660875*(a*x^(1/3) + b)^(9/2)*b^9 + 1434168450*(a*x^(1
/3) + b)^(7/2)*b^10 - 547591590*(a*x^(1/3) + b)^(5/2)*b^11 + 152108775*(a*x
^(1/3) + b)^(3/2)*b^12 - 35102025*sqrt(a*x^(1/3) + b)*b^13)/a^14
```

maple [A] time = 0.05, size = 167, normalized size = 0.42

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(1300075a^{13}x^{\frac{13}{3}}-1352078a^{12}b\,x^4+1410864a^{11}b^2x^{\frac{11}{3}}-1478048a^{10}b^3x^{\frac{10}{3}}+1555840a^9b^4x^3-16473\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 2/11700675*x^(1/3)*(a*x^(1/3)+b)*(1300075*x^(13/3)*a^13-1352078*x^4*a^12*b+
1410864*x^(11/3)*a^11*b^2-1478048*x^(10/3)*a^10*b^3+1555840*x^3*a^9*b^4-164
7360*x^(8/3)*a^8*b^5+1757184*x^(7/3)*a^7*b^6-1892352*x^2*a^6*b^7+2064384*x^
(5/3)*a^5*b^8-2293760*x^(4/3)*a^4*b^9+2621440*x*a^3*b^10-3145728*x^(2/3)*a^
2*b^11+4194304*x^(1/3)*a*b^12-8388608*b^13)/(a*x+b*x^(2/3))^(1/2)/a^14
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(a*x + b*x^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^(2/3))^(1/2),x)
[Out] int(x^4/(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(x**4/sqrt(a*x + b*x**(2/3)), x)
```

3.186 $\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=313

$$\frac{524288b^{10}\sqrt{ax+bx^{2/3}}}{323323a^{11}\sqrt[3]{x}}-\frac{262144b^9\sqrt{ax+bx^{2/3}}}{323323a^{10}}+\frac{196608b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{323323a^9}-\frac{163840b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{323323a^8}+\frac{20480b^6}{4}$$

```
[Out] -262144/323323*b^9*(b*x^(2/3)+a*x)^(1/2)/a^10+524288/323323*b^10*(b*x^(2/3)
+a*x)^(1/2)/a^11/x^(1/3)+196608/323323*b^8*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^
9-163840/323323*b^7*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^8+20480/46189*b^6*x*(b*
x^(2/3)+a*x)^(1/2)/a^7-18432/46189*b^5*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a^6+15
36/4199*b^4*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-768/2261*b^3*x^2*(b*x^(2/3)+a
*x)^(1/2)/a^4+720/2261*b^2*x^(7/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-40/133*b*x^(8/
3)*(b*x^(2/3)+a*x)^(1/2)/a^2+2/7*x^3*(b*x^(2/3)+a*x)^(1/2)/a
```

Rubi [A] time = 0.53, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$\frac{524288b^{10}\sqrt{ax+bx^{2/3}}}{323323a^{11}\sqrt[3]{x}}-\frac{262144b^9\sqrt{ax+bx^{2/3}}}{323323a^{10}}+\frac{196608b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{323323a^9}-\frac{163840b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{323323a^8}+\frac{20480b^6}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-262144*b^9*Sqrt[b*x^(2/3) + a*x])/(323323*a^10) + (524288*b^10*Sqrt[b*x^(
2/3) + a*x])/(323323*a^11*x^(1/3)) + (196608*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a
*x])/(323323*a^9) - (163840*b^7*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(323323*a^8)
 + (20480*b^6*x*Sqrt[b*x^(2/3) + a*x])/(46189*a^7) - (18432*b^5*x^(4/3)*Sqr
t[b*x^(2/3) + a*x])/(46189*a^6) + (1536*b^4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/
(4199*a^5) - (768*b^3*x^2*Sqrt[b*x^(2/3) + a*x])/(2261*a^4) + (720*b^2*x^(7
/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^3) - (40*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]
)/(133*a^2) + (2*x^3*Sqrt[b*x^(2/3) + a*x])/(7*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} - \frac{(20b)\int \frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a}$$
$$= -\frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} + \frac{\left(120b^2\right)\int \frac{x^{7/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{133a^2}$$
$$= \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+ax}}{7a} - \frac{\left(1920b^3\right)\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}$$
$$= -\frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^2} + \frac{2x^3\sqrt{bx^{2/3}+a}}{7a}$$
$$= \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^3} - \frac{40bx^{8/3}\sqrt{bx^{2/}}}{133a^2}$$
$$= -\frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^4} + \frac{720b^2x^{7/}}{2}$$
$$= \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + \frac{1536b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^5} - \frac{768b^3x^2}{22}$$
$$= -\frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - \frac{18432b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{46189a^6} + 1536$$
$$= \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + \frac{20480b^6x\sqrt{bx^{2/3}+ax}}{46189a^7} - 1843$$
$$= -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - \frac{163840b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{323323a^8} + 204$$
$$= -\frac{262144b^9\sqrt{bx^{2/3}+ax}}{323323a^{10}} + \frac{524288b^{10}\sqrt{bx^{2/3}+ax}}{323323a^{11}\sqrt[3]{x}} + \frac{196608b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{323323a^9} - 16384$$

Mathematica [A] time = 0.14, size = 148, normalized size = 0.47

$$\frac{2\sqrt{ax+bx^{2/3}}\left(46189a^{10}x^{10/3} - 48620a^9bx^3 + 51480a^8b^2x^{8/3} - 54912a^7b^3x^{7/3} + 59136a^6b^4x^2 - 64512a^5b^5x^{5/3}\right.}{323323a^{11}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(262144*b^10 - 131072*a*b^9*x^(1/3) + 98304*a^2*b^
8*x^(2/3) - 81920*a^3*b^7*x + 71680*a^4*b^6*x^(4/3) - 64512*a^5*b^5*x^(5/3)
 + 59136*a^6*b^4*x^2 - 54912*a^7*b^3*x^(7/3) + 51480*a^8*b^2*x^(8/3) - 4862
0*a^9*b*x^3 + 46189*a^10*x^(10/3)))/(323323*a^11*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.19, size = 164, normalized size = 0.52

$$-\frac{524288\,b^{\frac{21}{2}}}{323323\,a^{11}}+\frac{2\left(46189\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}-510510\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}b+2567565\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}b^{2}-7759752\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}b^{3}-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -524288/323323*b^(21/2)/a^11 + 2/323323*(46189*(a*x^(1/3) + b)^(21/2) - 510
510*(a*x^(1/3) + b)^(19/2)*b + 2567565*(a*x^(1/3) + b)^(17/2)*b^2 - 7759752
*(a*x^(1/3) + b)^(15/2)*b^3 + 15668730*(a*x^(1/3) + b)^(13/2)*b^4 - 2222110
8*(a*x^(1/3) + b)^(11/2)*b^5 + 22632610*(a*x^(1/3) + b)^(9/2)*b^6 - 1662804
0*(a*x^(1/3) + b)^(7/2)*b^7 + 8729721*(a*x^(1/3) + b)^(5/2)*b^8 - 3233230*(
a*x^(1/3) + b)^(3/2)*b^9 + 969969*sqrt(a*x^(1/3) + b)*b^10)/a^11
```

maple [A] time = 0.04, size = 134, normalized size = 0.43

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(46189a^{10}x^{\frac{10}{3}}-48620a^{9}b\,x^{3}+51480a^{8}b^{2}x^{\frac{8}{3}}-54912a^{7}b^{3}x^{\frac{7}{3}}+59136a^{6}b^{4}x^{2}-64512a^{5}b^{5}x^{\frac{5}{3}}+71680\right.}{323323\sqrt{ax+b\,x^{\frac{2}{3}}}\;a^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 2/323323*x^(1/3)*(a*x^(1/3)+b)*(46189*x^(10/3)*a^10-48620*x^3*a^9*b+51480*a
^8*b^2*x^(8/3)-54912*x^(7/3)*a^7*b^3+59136*a^6*b^4*x^2-64512*a^5*b^5*x^(5/3
)+71680*x^(4/3)*a^4*b^6-81920*x*a^3*b^7+98304*a^2*b^8*x^(2/3)-131072*x^(1/3
)*a*b^9+262144*b^10)/(a*x+b*x^(2/3))^(1/2)/a^11
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(a*x + b*x^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3}}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*x + b*x**(2/3)), x)
```

3.187 $\int \frac{x^2}{\sqrt{bx^{2/3}+ax}} dx$

Optimal. Leaf size=225

$$-\frac{4096b^7\sqrt{ax+bx^{2/3}}}{2145a^8\sqrt[3]{x}}+\frac{2048b^6\sqrt{ax+bx^{2/3}}}{2145a^7}-\frac{512b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^6}+\frac{256b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^5}-\frac{224b^3x\sqrt{ax+bx^{2/3}}}{429a^4}$$

```
[Out] 2048/2145*b^6*(b*x^(2/3)+a*x)^(1/2)/a^7-4096/2145*b^7*(b*x^(2/3)+a*x)^(1/2)
/a^8/x^(1/3)-512/715*b^5*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^6+256/429*b^4*x^(2
/3)*(b*x^(2/3)+a*x)^(1/2)/a^5-224/429*b^3*x*(b*x^(2/3)+a*x)^(1/2)/a^4+336/7
15*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-28/65*b*x^(5/3)*(b*x^(2/3)+a*x)^(1
/2)/a^2+2/5*x^2*(b*x^(2/3)+a*x)^(1/2)/a
```

Rubi [A] time = 0.35, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{4096b^7\sqrt{ax+bx^{2/3}}}{2145a^8\sqrt[3]{x}}+\frac{2048b^6\sqrt{ax+bx^{2/3}}}{2145a^7}-\frac{512b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^6}+\frac{256b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^5}-\frac{224b^3x\sqrt{ax+bx^{2/3}}}{429a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2048*b^6*Sqrt[b*x^(2/3) + a*x])/(2145*a^7) - (4096*b^7*Sqrt[b*x^(2/3) + a*
x])/(2145*a^8*x^(1/3)) - (512*b^5*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^6)
+ (256*b^4*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(429*a^5) - (224*b^3*x*Sqrt[b*x^(
2/3) + a*x])/(429*a^4) + (336*b^2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^3)
- (28*b*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(65*a^2) + (2*x^2*Sqrt[b*x^(2/3) + a
*x])/(5*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{(14b)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{15a}$$

$$= -\frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} + \frac{\left(56b^2\right)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{65a^2}$$

$$= \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} - \frac{\left(112b^3\right)\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{143a^3}$$

$$= -\frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^2} + \frac{2x^2\sqrt{bx^{2/3}+ax}}{5a} +$$

$$= \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^3} - \frac{28bx^{5/3}\sqrt{bx^{2/3}+a}}{65a^2}$$

$$= -\frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+ax}}{429a^4} + \frac{336b^2x^{4/3}\sqrt{bx^{2/3}}}{715a^3}$$

$$= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^5} - \frac{224b^3x\sqrt{bx^{2/3}+a}}{429a^4}$$

$$= \frac{2048b^6\sqrt{bx^{2/3}+ax}}{2145a^7} - \frac{4096b^7\sqrt{bx^{2/3}+ax}}{2145a^8\sqrt[3]{x}} - \frac{512b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^6} + \frac{256b^4x^{2/3}\sqrt{bx^{2/3}+a}}{429a^5}$$

Mathematica [A] time = 0.13, size = 111, normalized size = 0.49

$$\frac{2\sqrt{ax+bx^{2/3}}\left(429a^7x^{7/3} - 462a^6bx^2 + 504a^5b^2x^{5/3} - 560a^4b^3x^{4/3} + 640a^3b^4x - 768a^2b^5x^{2/3} + 1024ab^6\sqrt[3]{x} - 204\right.}{2145a^8\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(-2048*b^7 + 1024*a*b^6*x^(1/3) - 768*a^2*b^5*x^(2
/3) + 640*a^3*b^4*x - 560*a^4*b^3*x^(4/3) + 504*a^5*b^2*x^(5/3) - 462*a^6*b
*x^2 + 429*a^7*x^(7/3)))/(2145*a^8*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.27, size = 122, normalized size = 0.54

$$\frac{4096\,b^{\frac{15}{2}}}{2145\,a^8} + \frac{2\left(429\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}} - 3465\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}b + 12285\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}b^2 - 25025\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}b^3 + 32175\left(ax^{\frac{1}{3}}+\right.\right.}{2145\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4096/2145*b^(15/2)/a^8 + 2/2145*(429*(a*x^(1/3) + b)^(15/2) - 3465*(a*x^(1/
3) + b)^(13/2)*b + 12285*(a*x^(1/3) + b)^(11/2)*b^2 - 25025*(a*x^(1/3) + b)
^(9/2)*b^3 + 32175*(a*x^(1/3) + b)^(7/2)*b^4 - 27027*(a*x^(1/3) + b)^(5/2)*
b^5 + 15015*(a*x^(1/3) + b)^(3/2)*b^6 - 6435*sqrt(a*x^(1/3) + b)*b^7)/a^8
```

maple [A] time = 0.05, size = 101, normalized size = 0.45

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(429a^{7}x^{\frac{7}{3}}-462a^{6}b\,x^{2}+504a^{5}b^{2}x^{\frac{5}{3}}-560a^{4}b^{3}x^{\frac{4}{3}}+640a^{3}b^{4}x-768a^{2}b^{5}x^{\frac{2}{3}}+1024a\,b^{6}x^{\frac{1}{3}}-2048b\right.}{2145\sqrt{ax+b\,x^{\frac{2}{3}}}\;a^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 2/2145*x^(1/3)*(a*x^(1/3)+b)*(429*x^(7/3)*a^7-462*x^2*a^6*b+504*x^(5/3)*a^5
*b^2-560*a^4*x^(4/3)*b^3+640*x*a^3*b^4-768*x^(2/3)*a^2*b^5+1024*x^(1/3)*a*b
^6-2048*b^7)/(a*x+b*x^(2/3))^(1/2)/a^8
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(a*x + b*x^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*x + b*x**(2/3)), x)
```

3.188 $\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=137

$$\frac{256b^4\sqrt{ax+bx^{2/3}}}{105a^5\sqrt[3]{x}}-\frac{128b^3\sqrt{ax+bx^{2/3}}}{105a^4}+\frac{32b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{35a^3}-\frac{16bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^2}+\frac{2x\sqrt{ax+bx^{2/3}}}{3a}$$

```
[Out] -128/105*b^3*(b*x^(2/3)+a*x)^(1/2)/a^4+256/105*b^4*(b*x^(2/3)+a*x)^(1/2)/a^
5/x^(1/3)+32/35*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(1/2)/a^3-16/21*b*x^(2/3)*(b*x^
(2/3)+a*x)^(1/2)/a^2+2/3*x*(b*x^(2/3)+a*x)^(1/2)/a
```

Rubi [A] time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{256b^4\sqrt{ax+bx^{2/3}}}{105a^5\sqrt[3]{x}}-\frac{128b^3\sqrt{ax+bx^{2/3}}}{105a^4}+\frac{32b^2\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{35a^3}-\frac{16bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^2}+\frac{2x\sqrt{ax+bx^{2/3}}}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (-128*b^3*Sqrt[b*x^(2/3) + a*x])/(105*a^4) + (256*b^4*Sqrt[b*x^(2/3) + a*x]
)/(105*a^5*x^(1/3)) + (32*b^2*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(35*a^3) - (16
*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^2) + (2*x*Sqrt[b*x^(2/3) + a*x])/(3
*a)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx = \frac{2x\sqrt{bx^{2/3}+ax}}{3a} - \frac{(8b)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{9a}$$
$$= -\frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} + \frac{\left(16b^2\right)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^2}$$
$$= \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} - \frac{\left(64b^3\right)\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{105a^3}$$
$$= -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4} + \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2} + \frac{2x\sqrt{bx^{2/3}+ax}}{3a} + \cdot$$
$$= -\frac{128b^3\sqrt{bx^{2/3}+ax}}{105a^4} + \frac{256b^4\sqrt{bx^{2/3}+ax}}{105a^5\sqrt[3]{x}} + \frac{32b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{35a^3} - \frac{16bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^2}$$

Mathematica [A] time = 0.08, size = 74, normalized size = 0.54

$$\frac{2\sqrt{ax+bx^{2/3}}\left(35a^4x^{4/3}-40a^3bx+48a^2b^2x^{2/3}-64ab^3\sqrt[3]{x}+128b^4\right)}{105a^5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x]*(128*b^4 - 64*a*b^3*x^(1/3) + 48*a^2*b^2*x^(2/3) -
 40*a^3*b*x + 35*a^4*x^(4/3)))/(105*a^5*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.18, size = 80, normalized size = 0.58

$$-\frac{256\,b^{\frac{9}{2}}}{105\,a^5}+\frac{2\left(35\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}-180\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}b+378\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}b^2-420\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}b^3+315\sqrt{ax^{\frac{1}{3}}+b}\,b^4\right)}{105\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -256/105*b^(9/2)/a^5 + 2/105*(35*(a*x^(1/3) + b)^(9/2) - 180*(a*x^(1/3) + b
)^(7/2)*b + 378*(a*x^(1/3) + b)^(5/2)*b^2 - 420*(a*x^(1/3) + b)^(3/2)*b^3 +
 315*sqrt(a*x^(1/3) + b)*b^4)/a^5
```

maple [A] time = 0.05, size = 68, normalized size = 0.50

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(35a^4x^{\frac{4}{3}}-40a^3bx+48a^2b^2x^{\frac{2}{3}}-64a\,b^3x^{\frac{1}{3}}+128b^4\right)x^{\frac{1}{3}}}{105\sqrt{ax+b\,x^{\frac{2}{3}}}\;a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 2/105*x^(1/3)*(a*x^(1/3)+b)*(35*x^(4/3)*a^4-40*x*a^3*b+48*x^(2/3)*a^2*b^2-6
4*x^(1/3)*a*b^3+128*b^4)/(a*x+b*x^(2/3))^(1/2)/a^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(a*x + b*x^(2/3)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] int(x/(a*x + b*x^(2/3))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*x + b*x**(2/3)), x)
```

3.189 $\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

```
[Out] 2*(b*x^(2/3)+a*x)^(1/2)/a-4*b*(b*x^(2/3)+a*x)^(1/2)/a^2/x^(1/3)
```

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{2\sqrt{ax+bx^{2/3}}}{a}-\frac{4b\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*Sqrt[b*x^(2/3) + a*x])/a - (4*b*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx &= \frac{2\sqrt{bx^{2/3}+ax}}{a}-\frac{(2b)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{3a}\\ &= \frac{2\sqrt{bx^{2/3}+ax}}{a}-\frac{4b\sqrt{bx^{2/3}+ax}}{a^2\sqrt[3]{x}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 36, normalized size = 0.77

$$\frac{2\left(a\sqrt[3]{x}-2b\right)\sqrt{ax+bx^{2/3}}}{a^2\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*x^(2/3) + a*x],x]
```

```
[Out] (2*(-2*b + a*x^(1/3))*Sqrt[b*x^(2/3) + a*x])/(a^2*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.49, size = 36, normalized size = 0.77

$$\frac{4\,b^{\frac{3}{2}}}{a^2}+\frac{2\left(\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}-3\,\sqrt{ax^{\frac{1}{3}}+b}\,b\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 4*b^(3/2)/a^2 + 2*((a*x^(1/3) + b)^(3/2) - 3*sqrt(a*x^(1/3) + b)*b)/a^2
```

maple [A] time = 0.04, size = 36, normalized size = 0.77

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(a\,x^{\frac{1}{3}}-2b\right)x^{\frac{1}{3}}}{\sqrt{ax+b\,x^{\frac{2}{3}}}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 2*x^(1/3)*(a*x^(1/3)+b)*(a*x^(1/3)-2*b)/(a*x+b*x^(2/3))^(1/2)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x + b*x^(2/3)), x)
```

mupad [B] time = 5.22, size = 40, normalized size = 0.85

$$\frac{3\,x\,\sqrt{\frac{a\,x^{1/3}}{b}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},2;\;3;\;-\frac{a\,x^{1/3}}{b}\right)}{2\,\sqrt{a\,x+b\,x^{2/3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(2/3))^(1/2),x)
```

```
[Out] (3*x*((a*x^(1/3))/b + 1)^(1/2)*hypergeom([1/2, 2], 3, -(a*x^(1/3))/b))/(2*(
a*x + b*x^(2/3))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**(2/3)), x)
```

3.190 $\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=61

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

```
[Out] 3*a*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)-3*(b*x^(2/3)+a*x
)^(1/2)/b/x^(2/3)
```

Rubi [A] time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}-\frac{3\sqrt{ax+bx^{2/3}}}{bx^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(b*x^(2/3)) + (3*a*ArcTanh[(Sqrt[b]*x^(1/3))/Sqr
t[b*x^(2/3) + a*x]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx &= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}-\frac{a\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{2b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}+\frac{(3a)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b}\\
&= -\frac{3\sqrt{bx^{2/3}+ax}}{bx^{2/3}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 90, normalized size = 1.48

$$\frac{6a\sqrt[3]{x}\left(a\sqrt[3]{x}+b\right)\left(\frac{\tanh^{-1}\left(\sqrt{\frac{a\sqrt[3]{x}}{b}+1}\right)}{2\sqrt{\frac{a\sqrt[3]{x}}{b}+1}}-\frac{b}{2a\sqrt[3]{x}}\right)}{b^2\sqrt{x^{2/3}\left(a\sqrt[3]{x}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (6*a*(b + a*x^(1/3))*x^(1/3)*(-1/2*b/(a*x^(1/3)) + ArcTanh[Sqrt[1 + (a*x^(1
/3))/b]]/(2*Sqrt[1 + (a*x^(1/3))/b])))/(b^2*Sqrt[(b + a*x^(1/3))*x^(2/3)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.22, size = 51, normalized size = 0.84

$$-\frac{3\left(\frac{a^2\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}+\frac{\sqrt{ax^{\frac{1}{3}}+b}\,a}{bx^{\frac{1}{3}}}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -3*(a^2*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) + sqrt(a*x^(1/3)
+ b)*a/(b*x^(1/3)))/a
```

maple [A] time = 0.05, size = 61, normalized size = 1.00

$$\frac{3\sqrt{a\,x^{\frac{1}{3}}+b}\,\left(ab\,x^{\frac{1}{3}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-\sqrt{a\,x^{\frac{1}{3}}+b}\,b^{\frac{3}{2}}\right)}{\sqrt{ax+b\,x^{\frac{2}{3}}}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 3*(a*x^(1/3)+b)^(1/2)*(arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b*x^(1/3)*a-(a*
x^(1/3)+b)^(1/2)*b^(3/2))/(a*x+b*x^(2/3))^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(2/3))^(1/2)),x)
[Out] int(1/(x*(a*x + b*x^(2/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(2/3)+a*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(a*x + b*x**(2/3))), x)
```

3.191 $\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=153

$$-\frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{64b^{9/2}}+\frac{105a^3\sqrt{ax+bx^{2/3}}}{64b^4x^{2/3}}-\frac{35a^2\sqrt{ax+bx^{2/3}}}{32b^3x}+\frac{7a\sqrt{ax+bx^{2/3}}}{8b^2x^{4/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

```
[Out] -105/64*a^4*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)-3/4*(b*x
^(2/3)+a*x)^(1/2)/b/x^(5/3)+7/8*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)-35/32*a
^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x+105/64*a^3*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A] time = 0.24, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$\frac{105a^3\sqrt{ax+bx^{2/3}}}{64b^4x^{2/3}}-\frac{35a^2\sqrt{ax+bx^{2/3}}}{32b^3x}-\frac{105a^4\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{64b^{9/2}}+\frac{7a\sqrt{ax+bx^{2/3}}}{8b^2x^{4/3}}-\frac{3\sqrt{ax+bx^{2/3}}}{4bx^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(4*b*x^(5/3)) + (7*a*Sqrt[b*x^(2/3) + a*x])/(8*b
^2*x^(4/3)) - (35*a^2*Sqrt[b*x^(2/3) + a*x])/(32*b^3*x) + (105*a^3*Sqrt[b*x
^(2/3) + a*x])/(64*b^4*x^(2/3)) - (105*a^4*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/(64*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} - \frac{(7a)\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{8b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} + \frac{\left(35a^2\right)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{48b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} - \frac{\left(35a^3\right)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{64b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} + \frac{\left(35a^4\right)\int \frac{}{x}}{1}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} - \frac{\left(105a^4\right)\mathrm{Su}}{}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{4bx^{5/3}} + \frac{7a\sqrt{bx^{2/3}+ax}}{8b^2x^{4/3}} - \frac{35a^2\sqrt{bx^{2/3}+ax}}{32b^3x} + \frac{105a^3\sqrt{bx^{2/3}+ax}}{64b^4x^{2/3}} - \frac{105a^4\tanh}{6}$$

Mathematica [C] time = 0.06, size = 48, normalized size = 0.31

$$-\frac{6a^4\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^5\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-6*a^4*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 5, 3/2, 1 + (a*x^(1/3)
)/b])/(b^5*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.26, size = 109, normalized size = 0.71

$$\frac{\frac{105\,a^5\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^4} + \frac{105\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^5 - 385\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^5b + 511\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^5b^2 - 279\sqrt{ax^{\frac{1}{3}}+b}\,a^5b^3}{a^4b^4x^{\frac{4}{3}}}}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(105*a^5*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) + (105*(a
*x^(1/3) + b)^(7/2)*a^5 - 385*(a*x^(1/3) + b)^(5/2)*a^5*b + 511*(a*x^(1/3)
+ b)^(3/2)*a^5*b^2 - 279*sqrt(a*x^(1/3) + b)*a^5*b^3)/(a^4*b^4*x^(4/3)))/a
```

maple [A] time = 0.05, size = 126, normalized size = 0.82

$$-\frac{\sqrt{a\,x^{\frac{1}{3}}+b}\,\left(105a^{4}b\,x^{\frac{7}{3}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-105\sqrt{a\,x^{\frac{1}{3}}+b}\;a^{3}b^{\frac{3}{2}}x^{2}+70\sqrt{a\,x^{\frac{1}{3}}+b}\;a^{2}b^{\frac{5}{2}}x^{\frac{5}{3}}-56\sqrt{a\,x^{\frac{1}{3}}+b}\;a\,b\right.}{64\sqrt{ax+b\,x^{\frac{2}{3}}}\;b^{\frac{11}{2}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] -1/64*(a*x^(1/3)+b)^(1/2)*(105*x^(7/3)*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))
*a^4*b+70*x^(5/3)*(a*x^(1/3)+b)^(1/2)*b^(5/2)*a^2-56*x^(4/3)*(a*x^(1/3)+b)^
(1/2)*b^(7/2)*a+48*(a*x^(1/3)+b)^(1/2)*b^(9/2)*x-105*x^2*(a*x^(1/3)+b)^(1/2
)*b^(3/2)*a^3)/x^2/(a*x+b*x^(2/3))^(1/2)/b^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{2}\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(2/3))^(1/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(2/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*x + b*x**(2/3))), x)
```

3.192 $\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx$

Optimal. Leaf size=241

$$\frac{1287a^7\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{1024b^{15/2}}-\frac{1287a^6\sqrt{ax+bx^{2/3}}}{1024b^7x^{2/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{512b^6x}-\frac{429a^4\sqrt{ax+bx^{2/3}}}{640b^5x^{4/3}}+\frac{1287a^3\sqrt{ax+bx^{2/3}}}{2240b^4x^{5/3}}-1$$

```
[Out] 1287/1024*a^7*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)-3/7*(
b*x^(2/3)+a*x)^(1/2)/b/x^(8/3)+13/28*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)-14
3/280*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2+1287/2240*a^3*(b*x^(2/3)+a*x)^(1/2)
/b^4/x^(5/3)-429/640*a^4*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)+429/512*a^5*(b*x
^(2/3)+a*x)^(1/2)/b^6/x-1287/1024*a^6*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

Rubi [A] time = 0.41, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$-\frac{1287a^6\sqrt{ax+bx^{2/3}}}{1024b^7x^{2/3}}+\frac{429a^5\sqrt{ax+bx^{2/3}}}{512b^6x}-\frac{429a^4\sqrt{ax+bx^{2/3}}}{640b^5x^{4/3}}+\frac{1287a^3\sqrt{ax+bx^{2/3}}}{2240b^4x^{5/3}}-\frac{143a^2\sqrt{ax+bx^{2/3}}}{280b^3x^2}+\frac{1287a^7}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(7*b*x^(8/3)) + (13*a*Sqrt[b*x^(2/3) + a*x])/(28
*b^2*x^(7/3)) - (143*a^2*Sqrt[b*x^(2/3) + a*x])/(280*b^3*x^2) + (1287*a^3*S
qrt[b*x^(2/3) + a*x])/(2240*b^4*x^(5/3)) - (429*a^4*Sqrt[b*x^(2/3) + a*x])/
(640*b^5*x^(4/3)) + (429*a^5*Sqrt[b*x^(2/3) + a*x])/(512*b^6*x) - (1287*a^6
*Sqrt[b*x^(2/3) + a*x])/(1024*b^7*x^(2/3)) + (1287*a^7*ArcTanh[(Sqrt[b]*x^(
1/3))/Sqrt[b*x^(2/3) + a*x]])/(1024*b^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} - \frac{(13a)\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{14b}$$
$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} + \frac{\left(143a^2\right)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{168b^2}$$
$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} - \frac{\left(429a^3\right)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{560b^3}$$
$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} + \frac{(429\cdots}{\cdots}$$
$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429\cdots}{\cdots}$$
$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429\cdots}{\cdots}$$
$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429\cdots}{\cdots}$$
$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429\cdots}{\cdots}$$
$$= -\frac{3\sqrt{bx^{2/3}+ax}}{7bx^{8/3}} + \frac{13a\sqrt{bx^{2/3}+ax}}{28b^2x^{7/3}} - \frac{143a^2\sqrt{bx^{2/3}+ax}}{280b^3x^2} + \frac{1287a^3\sqrt{bx^{2/3}+ax}}{2240b^4x^{5/3}} - \frac{429\cdots}{\cdots}$$

Mathematica [C] time = 0.08, size = 48, normalized size = 0.20

$$\frac{6a^7\sqrt{ax+bx^{2/3}}\,{}_2F_1\left(\frac{1}{2},8;\frac{3}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^8\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[b*x^(2/3) + a*x]),x]
[Out] (6*a^7*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 8, 3/2, 1 + (a*x^(1/3))
/b])/(b^8*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.31, size = 160, normalized size = 0.66

$$-\frac{\frac{45045\,a^8\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^7} + \frac{45045\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}a^8 - 300300\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^8b + 849849\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^8b^2 - 1317888\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^8b^3 + 1200199\left(ax^{\frac{1}{3}}+\cdots\right.}{a^7b^7x^{\frac{7}{3}}}}{35840\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/35840*(45045*a^8*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + (
45045*(a*x^(1/3) + b)^(13/2)*a^8 - 300300*(a*x^(1/3) + b)^(11/2)*a^8*b + 84
9849*(a*x^(1/3) + b)^(9/2)*a^8*b^2 - 1317888*(a*x^(1/3) + b)^(7/2)*a^8*b^3
+ 1200199*(a*x^(1/3) + b)^(5/2)*a^8*b^4 - 631540*(a*x^(1/3) + b)^(3/2)*a^8*
b^5 + 169995*sqrt(a*x^(1/3) + b)*a^8*b^6)/(a^7*b^7*x^(7/3)))/a
```

maple [A] time = 0.05, size = 188, normalized size = 0.78

$$\frac{\sqrt{a\,x^{\frac{1}{3}}+b}\,\left(45045 a^{7} b\,x^{\frac{13}{3}}\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-45045\sqrt{a\,x^{\frac{1}{3}}+b}\;a^{6} b^{\frac{3}{2}} x^{4}+30030\sqrt{a\,x^{\frac{1}{3}}+b}\;a^{5} b^{\frac{5}{2}} x^{\frac{11}{3}}-24024\sqrt{a}}{35840}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] 1/35840*(a*x^(1/3)+b)^(1/2)*(45045*x^(13/3)*arctanh((a*x^(1/3)+b)^(1/2)/b^(
1/2))*a^7*b+30030*x^(11/3)*(a*x^(1/3)+b)^(1/2)*b^(5/2)*a^5-24024*x^(10/3)*(
a*x^(1/3)+b)^(1/2)*b^(7/2)*a^4-18304*x^(8/3)*(a*x^(1/3)+b)^(1/2)*b^(11/2)*a
^2+16640*x^(7/3)*(a*x^(1/3)+b)^(1/2)*b^(13/2)*a-15360*(a*x^(1/3)+b)^(1/2)*b
^(15/2)*x^2+20592*x^3*(a*x^(1/3)+b)^(1/2)*b^(9/2)*a^3-45045*x^4*(a*x^(1/3)+
b)^(1/2)*b^(3/2)*a^6)/x^4/(a*x+b*x^(2/3))^(1/2)/b^(17/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^{\frac{2}{3}}}\,x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\sqrt{a\,x+b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(2/3))^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x + b*x^(2/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3}\sqrt{ax+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(a*x + b*x**(2/3))), x)
```

3.193 $\int \frac{1}{x^4 \sqrt{bx^{2/3}+ax}}\, dx$

Optimal. Leaf size=329

$$-\frac{138567a^{10}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{131072b^{21/2}}+\frac{138567a^9\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{2/3}}-\frac{46189a^8\sqrt{ax+bx^{2/3}}}{65536b^9x}+\frac{46189a^7\sqrt{ax+bx^{2/3}}}{81920b^8x^{4/3}}-\frac{138567}{28}$$

```
[Out] -138567/131072*a^10*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)
-3/10*(b*x^(2/3)+a*x)^(1/2)/b/x^(11/3)+19/60*a*(b*x^(2/3)+a*x)^(1/2)/b^2/x^
(10/3)-323/960*a^2*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3+323/896*a^3*(b*x^(2/3)+a*x
)^(1/2)/b^4/x^(8/3)-4199/10752*a^4*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)+46189/
107520*a^5*(b*x^(2/3)+a*x)^(1/2)/b^6/x^2-138567/286720*a^6*(b*x^(2/3)+a*x)^
(1/2)/b^7/x^(5/3)+46189/81920*a^7*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)-46189/6
5536*a^8*(b*x^(2/3)+a*x)^(1/2)/b^9/x+138567/131072*a^9*(b*x^(2/3)+a*x)^(1/2
)/b^10/x^(2/3)
```

Rubi [A] time = 0.58, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2029, 206}

$$\frac{138567a^9\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{2/3}}-\frac{46189a^8\sqrt{ax+bx^{2/3}}}{65536b^9x}+\frac{46189a^7\sqrt{ax+bx^{2/3}}}{81920b^8x^{4/3}}-\frac{138567a^6\sqrt{ax+bx^{2/3}}}{286720b^7x^{5/3}}+\frac{46189a^5\sqrt{ax+b}}{107520b^6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-3*Sqrt[b*x^(2/3) + a*x])/(10*b*x^(11/3)) + (19*a*Sqrt[b*x^(2/3) + a*x])/(
60*b^2*x^(10/3)) - (323*a^2*Sqrt[b*x^(2/3) + a*x])/(960*b^3*x^3) + (323*a^3
*Sqrt[b*x^(2/3) + a*x])/(896*b^4*x^(8/3)) - (4199*a^4*Sqrt[b*x^(2/3) + a*x]
)/(10752*b^5*x^(7/3)) + (46189*a^5*Sqrt[b*x^(2/3) + a*x])/(107520*b^6*x^2)
- (138567*a^6*Sqrt[b*x^(2/3) + a*x])/(286720*b^7*x^(5/3)) + (46189*a^7*Sqrt
[b*x^(2/3) + a*x])/(81920*b^8*x^(4/3)) - (46189*a^8*Sqrt[b*x^(2/3) + a*x])/
(65536*b^9*x) + (138567*a^9*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(2/3)) -
(138567*a^10*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(131072*b^(2
1/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}\,dx = -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} - \frac{(19a)\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{20b}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} + \frac{\left(323a^2\right)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{360b^2}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} - \frac{\left(323a^3\right)\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx}{384b^3}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} + \frac{\left(4199a^4\right.}{}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{}}{1075}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{}}{1075}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{}}{1075}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{}}{1075}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{}}{1075}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{}}{1075}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{}}{1075}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{}}{1075}$$

$$= -\frac{3\sqrt{bx^{2/3}+ax}}{10bx^{11/3}} + \frac{19a\sqrt{bx^{2/3}+ax}}{60b^2x^{10/3}} - \frac{323a^2\sqrt{bx^{2/3}+ax}}{960b^3x^3} + \frac{323a^3\sqrt{bx^{2/3}+ax}}{896b^4x^{8/3}} - \frac{4199a^4\sqrt{}}{1075}$$

Mathematica [C] time = 0.07, size = 48, normalized size = 0.15

$$-\frac{6a^{10}\sqrt{ax+bx^{2/3}}\;{}_2F_1\left(\frac{1}{2},11;\frac{3}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^{11}\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[b*x^(2/3) + a*x]),x]
```

```
[Out] (-6*a^10*Sqrt[b*x^(2/3) + a*x]*Hypergeometric2F1[1/2, 11, 3/2, 1 + (a*x^(1/
3))/b])/(b^11*x^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.36, size = 211, normalized size = 0.64

$$\frac{\frac{14549535\,a^{11}\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b^{10}}+\frac{14549535\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{11}-140645505\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{11}b+609140532\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^{11}b^{2}-1554721740\left(ax^{\frac{1}{3}}+b\right)^{\frac{13}{2}}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/13762560*(14549535*a^11*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^
10) + (14549535*(a*x^(1/3) + b)^(19/2)*a^11 - 140645505*(a*x^(1/3) + b)^(17
/2)*a^11*b + 609140532*(a*x^(1/3) + b)^(15/2)*a^11*b^2 - 1554721740*(a*x^(1
/3) + b)^(13/2)*a^11*b^3 + 2585198330*(a*x^(1/3) + b)^(11/2)*a^11*b^4 - 291
8514950*(a*x^(1/3) + b)^(9/2)*a^11*b^5 + 2255541300*(a*x^(1/3) + b)^(7/2)*a
^11*b^6 - 1168982220*(a*x^(1/3) + b)^(5/2)*a^11*b^7 + 382331775*(a*x^(1/3)
+ b)^(3/2)*a^11*b^8 - 68025825*sqrt(a*x^(1/3) + b)*a^11*b^9)/(a^10*b^10*x^(
10/3)))/a
```

maple [A] time = 0.05, size = 248, normalized size = 0.75

$$-\frac{\sqrt{a\,x^{\frac{1}{3}}+b}\,\left(14549535 a^{10} b\, x^{\frac{19}{3}}\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-14549535\sqrt{a\,x^{\frac{1}{3}}+b}\;a^{9} b^{\frac{3}{2}} x^{6}+9699690\sqrt{a\,x^{\frac{1}{3}}+b}\;a^{8} b^{\frac{5}{2}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*x+b*x^(2/3))^(1/2),x)
```

```
[Out] -1/13762560*(a*x^(1/3)+b)^(1/2)*(14549535*x^(19/3)*arctanh((a*x^(1/3)+b)^(1
/2)/b^(1/2))*a^10*b+9699690*x^(17/3)*(a*x^(1/3)+b)^(1/2)*b^(5/2)*a^8-775975
2*x^(16/3)*(a*x^(1/3)+b)^(1/2)*b^(7/2)*a^7-5912192*x^(14/3)*(a*x^(1/3)+b)^(
1/2)*b^(11/2)*a^5+5374720*x^(13/3)*(a*x^(1/3)+b)^(1/2)*b^(13/2)*a^4+4630528
*x^(11/3)*(a*x^(1/3)+b)^(1/2)*b^(17/2)*a^2-4358144*x^(10/3)*(a*x^(1/3)+b)^(
1/2)*b^(19/2)*a+4128768*(a*x^(1/3)+b)^(1/2)*b^(21/2)*x^3-4961280*x^4*(a*x^(
1/3)+b)^(1/2)*b^(15/2)*a^3+6651216*x^5*(a*x^(1/3)+b)^(1/2)*b^(9/2)*a^6-1454
9535*x^6*(a*x^(1/3)+b)^(1/2)*b^(3/2)*a^9)/x^6/(a*x+b*x^(2/3))^(1/2)/b^(23/2
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax + bx^{\frac{2}{3}}}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^(2/3))*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{a\,x + b\,x^{2/3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(2/3))^(1/2)),x)
```

```
[Out] int(1/(x^4*(a*x + b*x^(2/3))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4}\sqrt{ax + bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(a*x + b*x**(2/3))), x)
```

3.194 $\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=336

$$\frac{1048576b^{10}\sqrt{ax+bx^{2/3}}}{29393a^{12}\sqrt[3]{x}}-\frac{524288b^9\sqrt{ax+bx^{2/3}}}{29393a^{11}}+\frac{393216b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{29393a^9}+409$$

```
[Out] -6*x^4/a/(b*x^(2/3)+a*x)^(1/2)-524288/29393*b^9*(b*x^(2/3)+a*x)^(1/2)/a^11+
1048576/29393*b^10*(b*x^(2/3)+a*x)^(1/2)/a^12/x^(1/3)+393216/29393*b^8*x^(1
/3)*(b*x^(2/3)+a*x)^(1/2)/a^10-327680/29393*b^7*x^(2/3)*(b*x^(2/3)+a*x)^(1/
2)/a^9+40960/4199*b^6*x*(b*x^(2/3)+a*x)^(1/2)/a^8-36864/4199*b^5*x^(4/3)*(b
*x^(2/3)+a*x)^(1/2)/a^7+33792/4199*b^4*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^6-16
896/2261*b^3*x^2*(b*x^(2/3)+a*x)^(1/2)/a^5+15840/2261*b^2*x^(7/3)*(b*x^(2/3
)+a*x)^(1/2)/a^4-880/133*b*x^(8/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+44/7*x^3*(b*x^
(2/3)+a*x)^(1/2)/a^2
```

Rubi [A] time = 0.60, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2015, 2016, 2002, 2014}

$$\frac{1048576b^{10}\sqrt{ax+bx^{2/3}}}{29393a^{12}\sqrt[3]{x}}-\frac{524288b^9\sqrt{ax+bx^{2/3}}}{29393a^{11}}+\frac{393216b^8\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{29393a^{10}}-\frac{327680b^7x^{2/3}\sqrt{ax+bx^{2/3}}}{29393a^9}+409$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^4)/(a*Sqrt[b*x^(2/3) + a*x]) - (524288*b^9*Sqrt[b*x^(2/3) + a*x])/(29
393*a^11) + (1048576*b^10*Sqrt[b*x^(2/3) + a*x])/(29393*a^12*x^(1/3)) + (39
3216*b^8*x^(1/3)*Sqrt[b*x^(2/3) + a*x])/(29393*a^10) - (327680*b^7*x^(2/3)*
Sqrt[b*x^(2/3) + a*x])/(29393*a^9) + (40960*b^6*x*Sqrt[b*x^(2/3) + a*x])/(4
199*a^8) - (36864*b^5*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^7) + (33792*b^
4*x^(5/3)*Sqrt[b*x^(2/3) + a*x])/(4199*a^6) - (16896*b^3*x^2*Sqrt[b*x^(2/3)
 + a*x])/(2261*a^5) + (15840*b^2*x^(7/3)*Sqrt[b*x^(2/3) + a*x])/(2261*a^4)
- (880*b*x^(8/3)*Sqrt[b*x^(2/3) + a*x])/(133*a^3) + (44*x^3*Sqrt[b*x^(2/3)
+ a*x])/(7*a^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
```

```
] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{22\int \frac{x^3}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} - \frac{(440b)\int \frac{x^{8/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^2}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} + \frac{\left(2640b^2\right)\int \frac{x^{7/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{133a^3}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+ax}}{133a^3} + \frac{44x^3\sqrt{bx^{2/3}+ax}}{7a^2} -$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{2/3}+ax}}{2261a^4} - \frac{880bx^{8/3}\sqrt{bx^{2/3}+}}{133a^3}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{2/3}+ax}}{2261a^5} + \frac{15840b^2x^{7/3}\sqrt{bx^{}}}{2261a^4}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2/3}+ax}}{4199a^6} - \frac{16896b^3x^2\sqrt{bx^{}}}{2261a^5}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx^{2/3}+ax}}{4199a^7} + \frac{33792b^4x^{5/3}\sqrt{bx^{2}}}{4199a^6}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9} + \frac{40960b^6x\sqrt{bx^{2/3}+ax}}{4199a^8} - \frac{36864b^5x^{4/3}\sqrt{bx}}{4199a^7}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{bx^{2/3}+ax}}{29393a^9} + \frac{40960b^6x\sqrt{b}}{4199a}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{29393a^{10}} - \frac{327680b^7x^{2/3}\sqrt{b}}{29393a^9}$$

$$= -\frac{6x^4}{a\sqrt{bx^{2/3}+ax}} - \frac{524288b^9\sqrt{bx^{2/3}+ax}}{29393a^{11}} + \frac{1048576b^{10}\sqrt{bx^{2/3}+ax}}{29393a^{12}\sqrt[3]{x}} + \frac{393216b^8\sqrt[3]{x}\sqrt{bx^{2}}}{29393a^{10}}$$

Mathematica [A] time = 0.19, size = 161, normalized size = 0.48

$$\frac{2\sqrt[3]{x}\left(4199a^{11}x^{11/3} - 4862a^{10}bx^{10/3} + 5720a^9b^2x^3 - 6864a^8b^3x^{8/3} + 8448a^7b^4x^{7/3} - 10752a^6b^5x^2 + 14336a^5b^6x^{5/}\right.}{29393a^{12}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*x^(1/3)*(524288*b^11 + 262144*a*b^10*x^(1/3) - 65536*a^2*b^9*x^(2/3) + 3
2768*a^3*b^8*x - 20480*a^4*b^7*x^(4/3) + 14336*a^5*b^6*x^(5/3) - 10752*a^6*
b^5*x^2 + 8448*a^7*b^4*x^(7/3) - 6864*a^8*b^3*x^(8/3) + 5720*a^9*b^2*x^3 -
4862*a^10*b*x^(10/3) + 4199*a^11*x^(11/3)))/(29393*a^12*Sqrt[b*x^(2/3) + a*
x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.25, size = 214, normalized size = 0.64

$$-\frac{1048576\,b^{\frac{21}{2}}}{29393\,a^{12}}+\frac{6\,b^{11}}{\sqrt{ax^{\frac{1}{3}}+b}\,a^{12}}+\frac{2\left(4199\left(ax^{\frac{1}{3}}+b\right)^{\frac{21}{2}}a^{240}-51051\left(ax^{\frac{1}{3}}+b\right)^{\frac{19}{2}}a^{240}b+285285\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^{240}b^{2}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1048576/29393*b^(21/2)/a^12 + 6*b^11/(sqrt(a*x^(1/3) + b)*a^12) + 2/29393*
(4199*(a*x^(1/3) + b)^(21/2)*a^240 - 51051*(a*x^(1/3) + b)^(19/2)*a^240*b +
 285285*(a*x^(1/3) + b)^(17/2)*a^240*b^2 - 969969*(a*x^(1/3) + b)^(15/2)*a^
240*b^3 + 2238390*(a*x^(1/3) + b)^(13/2)*a^240*b^4 - 3703518*(a*x^(1/3) + b
)^(11/2)*a^240*b^5 + 4526522*(a*x^(1/3) + b)^(9/2)*a^240*b^6 - 4157010*(a*x
^(1/3) + b)^(7/2)*a^240*b^7 + 2909907*(a*x^(1/3) + b)^(5/2)*a^240*b^8 - 161
6615*(a*x^(1/3) + b)^(3/2)*a^240*b^9 + 969969*sqrt(a*x^(1/3) + b)*a^240*b^1
0)/a^252
```

maple [A] time = 0.05, size = 143, normalized size = 0.43

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(4199a^{11}x^{\frac{11}{3}}-4862a^{10}b\,x^{\frac{10}{3}}+5720a^{9}b^{2}x^{3}-6864a^{8}b^{3}x^{\frac{8}{3}}+8448a^{7}b^{4}x^{\frac{7}{3}}-10752a^{6}b^{5}x^{2}+14336a\cdots\right.}{29393\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/29393*x*(a*x^(1/3)+b)*(4199*a^11*x^(11/3)-4862*a^10*b*x^(10/3)+5720*a^9*b
^2*x^3-6864*a^8*b^3*x^(8/3)+8448*a^7*b^4*x^(7/3)-10752*a^6*b^5*x^2+14336*a^
5*b^6*x^(5/3)-20480*a^4*b^7*x^(4/3)+32768*a^3*b^8*x-65536*a^2*b^9*x^(2/3)+2
62144*a*b^10*x^(1/3)+524288*b^11)/(a*x+b*x^(2/3))^(3/2)/a^12
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x + b*x^(2/3))^(3/2),x)
[Out] int(x^4/(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x**4/(a*x + b*x**(2/3))**(3/2), x)
```

3.195 $\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=248

$$-\frac{65536b^7\sqrt{ax+bx^{2/3}}}{2145a^9\sqrt[3]{x}}+\frac{32768b^6\sqrt{ax+bx^{2/3}}}{2145a^8}-\frac{8192b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^6}-\frac{3584b^3x\sqrt{a}}{429a}$$

```
[Out] -6*x^3/a/(b*x^(2/3)+a*x)^(1/2)+32768/2145*b^6*(b*x^(2/3)+a*x)^(1/2)/a^8-655
36/2145*b^7*(b*x^(2/3)+a*x)^(1/2)/a^9/x^(1/3)-8192/715*b^5*x^(1/3)*(b*x^(2/
3)+a*x)^(1/2)/a^7+4096/429*b^4*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^6-3584/429*b
^3*x*(b*x^(2/3)+a*x)^(1/2)/a^5+5376/715*b^2*x^(4/3)*(b*x^(2/3)+a*x)^(1/2)/a
^4-448/65*b*x^(5/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+32/5*x^2*(b*x^(2/3)+a*x)^(1/2
)/a^2
```

Rubi [A] time = 0.41, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2015, 2016, 2002, 2014}

$$-\frac{65536b^7\sqrt{ax+bx^{2/3}}}{2145a^9\sqrt[3]{x}}+\frac{32768b^6\sqrt{ax+bx^{2/3}}}{2145a^8}-\frac{8192b^5\sqrt[3]{x}\sqrt{ax+bx^{2/3}}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{ax+bx^{2/3}}}{429a^6}-\frac{3584b^3x\sqrt{a}}{429a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^3)/(a*Sqrt[b*x^(2/3) + a*x]) + (32768*b^6*Sqrt[b*x^(2/3) + a*x])/(214
5*a^8) - (65536*b^7*Sqrt[b*x^(2/3) + a*x])/(2145*a^9*x^(1/3)) - (8192*b^5*x
^(1/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^7) + (4096*b^4*x^(2/3)*Sqrt[b*x^(2/3)
+ a*x])/(429*a^6) - (3584*b^3*x*Sqrt[b*x^(2/3) + a*x])/(429*a^5) + (5376*b^
2*x^(4/3)*Sqrt[b*x^(2/3) + a*x])/(715*a^4) - (448*b*x^(5/3)*Sqrt[b*x^(2/3)
+ a*x])/(65*a^3) + (32*x^2*Sqrt[b*x^(2/3) + a*x])/(5*a^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{16\int \frac{x^2}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}-\frac{(224b)\int \frac{x^{5/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{15a^2}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}-\frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3}+\frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}+\frac{\left(896b^2\right)\int \frac{x^{4/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{65a^3}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4}-\frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3}+\frac{32x^2\sqrt{bx^{2/3}+ax}}{5a^2}-$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}-\frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5}+\frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+ax}}{715a^4}-\frac{448bx^{5/3}\sqrt{bx^{2/3}+ax}}{65a^3}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6}-\frac{3584b^3x\sqrt{bx^{2/3}+ax}}{429a^5}+\frac{5376b^2x^{4/3}\sqrt{bx^{2/3}+}}{715a^4}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}-\frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+ax}}{429a^6}-\frac{3584b^3x\sqrt{bx^{2/3}+}}{429a^5}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8}-\frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7}+\frac{4096b^4x^{2/3}\sqrt{bx^{2/3}+}}{429a^6}$$

$$= -\frac{6x^3}{a\sqrt{bx^{2/3}+ax}}+\frac{32768b^6\sqrt{bx^{2/3}+ax}}{2145a^8}-\frac{65536b^7\sqrt{bx^{2/3}+ax}}{2145a^9\sqrt[3]{x}}-\frac{8192b^5\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{715a^7}$$

Mathematica [A] time = 0.13, size = 122, normalized size = 0.49

$$\frac{2\left(429a^8x^3-528a^7bx^{8/3}+672a^6b^2x^{7/3}-896a^5b^3x^2+1280a^4b^4x^{5/3}-2048a^3b^5x^{4/3}+4096a^2b^6x-16384ab^7x^{2/3}\right.}{2145a^9\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(-32768*b^8*x^(1/3) - 16384*a*b^7*x^(2/3) + 4096*a^2*b^6*x - 2048*a^3*b^
5*x^(4/3) + 1280*a^4*b^4*x^(5/3) - 896*a^5*b^3*x^2 + 672*a^6*b^2*x^(7/3) -
528*a^7*b*x^(8/3) + 429*a^8*x^3))/(2145*a^9*Sqrt[b*x^(2/3) + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 163, normalized size = 0.66

$$\frac{65536\,b^{\frac{15}{2}}}{2145\,a^{9}} - \frac{6\,b^{8}}{\sqrt{ax^{\frac{1}{3}} + b}\,a^{9}} + \frac{2\left(429\left(ax^{\frac{1}{3}} + b\right)^{\frac{15}{2}} a^{126} - 3960\left(ax^{\frac{1}{3}} + b\right)^{\frac{13}{2}} a^{126} b + 16380\left(ax^{\frac{1}{3}} + b\right)^{\frac{11}{2}} a^{126} b^{2} - 4004 \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 65536/2145*b^(15/2)/a^9 - 6*b^8/(sqrt(a*x^(1/3) + b)*a^9) + 2/2145*(429*(a*
x^(1/3) + b)^(15/2)*a^126 - 3960*(a*x^(1/3) + b)^(13/2)*a^126*b + 16380*(a*
x^(1/3) + b)^(11/2)*a^126*b^2 - 40040*(a*x^(1/3) + b)^(9/2)*a^126*b^3 + 643
50*(a*x^(1/3) + b)^(7/2)*a^126*b^4 - 72072*(a*x^(1/3) + b)^(5/2)*a^126*b^5
+ 60060*(a*x^(1/3) + b)^(3/2)*a^126*b^6 - 51480*sqrt(a*x^(1/3) + b)*a^126*b
^7)/a^135
```

maple [A] time = 0.05, size = 110, normalized size = 0.44

$$\frac{2\left(a\,x^{\frac{1}{3}} + b\right)\left(429 a^{8} x^{\frac{8}{3}} - 528 a^{7} b\,x^{\frac{7}{3}} + 672 a^{6} b^{2} x^{2} - 896 a^{5} b^{3} x^{\frac{5}{3}} + 1280 a^{4} b^{4} x^{\frac{4}{3}} - 2048 a^{3} b^{5} x + 4096 a^{2} b^{6} x^{\frac{2}{3}} - 163 \cdots\right.}{2145\left(ax + b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}} a^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/2145*x*(a*x^(1/3)+b)*(429*a^8*x^(8/3)-528*a^7*b*x^(7/3)+672*a^6*b^2*x^2-8
96*a^5*b^3*x^(5/3)+1280*x^(4/3)*a^4*b^4-2048*a^3*b^5*x+4096*x^(2/3)*a^2*b^6
-16384*x^(1/3)*a*b^7-32768*b^8)/(a*x+b*x^(2/3))^(3/2)/a^9
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{3}}{{\left(a\,x + b\,x^{2/3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^3/(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**3/(a*x + b*x**(2/3))**(3/2), x)
```

3.196 $\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{512b^4\sqrt{ax+bx^{2/3}}}{21a^6\sqrt[3]{x}}-\frac{256b^3\sqrt{ax+bx^{2/3}}}{21a^5}+\frac{64b^2\sqrt[3]{x}\,\sqrt{ax+bx^{2/3}}}{7a^4}-\frac{160bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^3}+\frac{20x\sqrt{ax+bx^{2/3}}}{3a^2}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

```
[Out] -6*x^2/a/(b*x^(2/3)+a*x)^(1/2)-256/21*b^3*(b*x^(2/3)+a*x)^(1/2)/a^5+512/21*
b^4*(b*x^(2/3)+a*x)^(1/2)/a^6/x^(1/3)+64/7*b^2*x^(1/3)*(b*x^(2/3)+a*x)^(1/2
)/a^4-160/21*b*x^(2/3)*(b*x^(2/3)+a*x)^(1/2)/a^3+20/3*x*(b*x^(2/3)+a*x)^(1/
2)/a^2
```

Rubi [A] time = 0.24, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2015, 2016, 2002, 2014}

$$\frac{512b^4\sqrt{ax+bx^{2/3}}}{21a^6\sqrt[3]{x}}-\frac{256b^3\sqrt{ax+bx^{2/3}}}{21a^5}+\frac{64b^2\sqrt[3]{x}\,\sqrt{ax+bx^{2/3}}}{7a^4}-\frac{160bx^{2/3}\sqrt{ax+bx^{2/3}}}{21a^3}+\frac{20x\sqrt{ax+bx^{2/3}}}{3a^2}-\frac{6x^2}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x^2)/(a*Sqrt[b*x^(2/3) + a*x]) - (256*b^3*Sqrt[b*x^(2/3) + a*x])/(21*a^
5) + (512*b^4*Sqrt[b*x^(2/3) + a*x])/(21*a^6*x^(1/3)) + (64*b^2*x^(1/3)*Sqr
t[b*x^(2/3) + a*x])/(7*a^4) - (160*b*x^(2/3)*Sqrt[b*x^(2/3) + a*x])/(21*a^3
) + (20*x*Sqrt[b*x^(2/3) + a*x])/(3*a^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
```

```
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{10\int \frac{x}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$
$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2} - \frac{(80b)\int \frac{x^{2/3}}{\sqrt{bx^{2/3}+ax}}\,dx}{9a^2}$$
$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2} + \frac{\left(160b^2\right)\int \frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\,dx}{21a^3}$$
$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} + \frac{20x\sqrt{bx^{2/3}+ax}}{3a^2} - (12$$
$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4} - \frac{160bx^{2/3}\sqrt{bx^{2/3}+ax}}{21a^3} + 2$$
$$= -\frac{6x^2}{a\sqrt{bx^{2/3}+ax}} - \frac{256b^3\sqrt{bx^{2/3}+ax}}{21a^5} + \frac{512b^4\sqrt{bx^{2/3}+ax}}{21a^6\sqrt[3]{x}} + \frac{64b^2\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}{7a^4} - 160$$

Mathematica [A] time = 0.10, size = 85, normalized size = 0.53

$$\frac{14a^5x^2-20a^4bx^{5/3}+32a^3b^2x^{4/3}-64a^2b^3x+256ab^4x^{2/3}+512b^5\sqrt[3]{x}}{21a^6\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (512*b^5*x^(1/3) + 256*a*b^4*x^(2/3) - 64*a^2*b^3*x + 32*a^3*b^2*x^(4/3) -
20*a^4*b*x^(5/3) + 14*a^5*x^2)/(21*a^6*Sqrt[b*x^(2/3) + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.23, size = 112, normalized size = 0.70

$$-\frac{512\,b^{\frac{9}{2}}}{21\,a^6}+\frac{6\,b^5}{\sqrt{ax^{\frac{1}{3}}+b}\,a^6}+\frac{2\left(7\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^{48}-45\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}a^{48}b+126\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^{48}b^2-210\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^{48}b^3+\right.}{21\,a^{54}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -512/21*b^(9/2)/a^6 + 6*b^5/(sqrt(a*x^(1/3) + b)*a^6) + 2/21*(7*(a*x^(1/3)
+ b)^(9/2)*a^48 - 45*(a*x^(1/3) + b)^(7/2)*a^48*b + 126*(a*x^(1/3) + b)^(5/
```

```
2)*a^48*b^2 - 210*(a*x^(1/3) + b)^(3/2)*a^48*b^3 + 315*sqrt(a*x^(1/3) + b)*
a^48*b^4)/a^54
```

maple [A] time = 0.05, size = 77, normalized size = 0.48

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(7a^{5}x^{\frac{5}{3}}-10a^{4}b\,x^{\frac{4}{3}}+16a^{3}b^{2}x-32a^{2}b^{3}x^{\frac{2}{3}}+128a\,b^{4}x^{\frac{1}{3}}+256b^{5}\right)x}{21\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2/21*x*(a*x^(1/3)+b)*(7*a^5*x^(5/3)-10*a^4*b*x^(4/3)+16*a^3*b^2*x-32*a^2*b^
3*x^(2/3)+128*a*b^4*x^(1/3)+256*b^5)/(a*x+b*x^(2/3))^(3/2)/a^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{2}}{\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] int(x^2/(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(x**2/(a*x + b*x**(2/3))**(3/2), x)
```

3.197 $\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{16b\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}}+\frac{8\sqrt{ax+bx^{2/3}}}{a^2}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

```
[Out] -6*x/a/(b*x^(2/3)+a*x)^(1/2)+8*(b*x^(2/3)+a*x)^(1/2)/a^2-16*b*(b*x^(2/3)+a*
x)^(1/2)/a^3/x^(1/3)
```

Rubi [A] time = 0.08, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2015, 2002, 2014}

$$-\frac{16b\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}}+\frac{8\sqrt{ax+bx^{2/3}}}{a^2}-\frac{6x}{a\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (-6*x)/(a*Sqrt[b*x^(2/3) + a*x]) + (8*Sqrt[b*x^(2/3) + a*x])/a^2 - (16*b*Sq
rt[b*x^(2/3) + a*x])/(a^3*x^(1/3))
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx = -\frac{6x}{a\sqrt{bx^{2/3}+ax}} + \frac{4\int \frac{1}{\sqrt{bx^{2/3}+ax}}\,dx}{a}$$

$$= -\frac{6x}{a\sqrt{bx^{2/3}+ax}} + \frac{8\sqrt{bx^{2/3}+ax}}{a^2} - \frac{(8b)\int \frac{1}{\sqrt[3]{x}\sqrt{bx^{2/3}+ax}}\,dx}{3a^2}$$

$$= -\frac{6x}{a\sqrt{bx^{2/3}+ax}} + \frac{8\sqrt{bx^{2/3}+ax}}{a^2} - \frac{16b\sqrt{bx^{2/3}+ax}}{a^3\sqrt[3]{x}}$$

Mathematica [**A**] time = 0.06, size = 60, normalized size = 0.88

$$\frac{2\left(a^2x^{2/3}-4ab\sqrt[3]{x}-8b^2\right)\sqrt{ax+bx^{2/3}}}{a^3\sqrt[3]{x}\left(a\sqrt[3]{x}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(b*x^(2/3) + a*x)^(3/2),x]
```

```
[Out] (2*(-8*b^2 - 4*a*b*x^(1/3) + a^2*x^(2/3))*Sqrt[b*x^(2/3) + a*x])/(a^3*(b +
a*x^(1/3))*x^(1/3))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 0.22, size = 60, normalized size = 0.88

$$\frac{16\,b^{\frac{3}{2}}}{a^3} - \frac{6\,b^2}{\sqrt{ax^{\frac{1}{3}}+b}\,a^3} + \frac{2\left(\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^6 - 6\sqrt{ax^{\frac{1}{3}}+b}\,a^6b\right)}{a^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 16*b^(3/2)/a^3 - 6*b^2/(sqrt(a*x^(1/3) + b)*a^3) + 2*((a*x^(1/3) + b)^(3/2)
*a^6 - 6*sqrt(a*x^(1/3) + b)*a^6*b)/a^9
```

maple [**A**] time = 0.05, size = 45, normalized size = 0.66

$$\frac{2\left(a\,x^{\frac{1}{3}}+b\right)\left(a^2x^{\frac{2}{3}}-4ab\,x^{\frac{1}{3}}-8b^2\right)x}{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 2*x*(a*x^(1/3)+b)*(a^2*x^(2/3)-4*a*b*x^(1/3)-8*b^2)/(a*x+b*x^(2/3))^(3/2)/a
^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(x/(a*x + b*x^(2/3))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(a\,x + b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x + b*x^(2/3))^(3/2),x)
[Out] int(x/(a*x + b*x^(2/3))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(x/(a*x + b*x**(2/3))**(3/2), x)
```

3.198 $\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

```
[Out] -6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(3/2)+6*x^(1/3)/b/(b*x^
(2/3)+a*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2006, 2029, 206}

$$\frac{6\sqrt[3]{x}}{b\sqrt{ax+bx^{2/3}}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*x^(1/3))/(b*Sqrt[b*x^(2/3) + a*x]) - (6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b
*x^(2/3) + a*x]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(bx^{2/3}+ax\right)^{3/2}}\,dx &= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}+\frac{\int \frac{1}{x^{2/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}\\
&= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b}\\
&= \frac{6\sqrt[3]{x}}{b\sqrt{bx^{2/3}+ax}}-\frac{6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{bx^{2/3}+ax}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 45, normalized size = 0.75

$$\frac{6\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*x^(2/3) + a*x)^(-3/2),x]
```

```
[Out] (6*x^(1/3)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (a*x^(1/3))/b])/(b*Sqrt[b*x^
(2/3) + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.21, size = 71, normalized size = 1.18

$$\frac{6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}-\frac{6\left(\sqrt{b}\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)+\sqrt{-b}\right)}{\sqrt{-b}\,b^{\frac{3}{2}}}+\frac{6}{\sqrt{ax^{\frac{1}{3}}+b}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b) - 6*(sqrt(b)*arctan(sqr
t(b)/sqrt(-b)) + sqrt(-b))/(sqrt(-b)*b^(3/2)) + 6/(sqrt(a*x^(1/3) + b)*b)
```

maple [A] time = 0.05, size = 56, normalized size = 0.93

$$-\frac{6\left(a\,x^{\frac{1}{3}}+b\right)\left(\sqrt{a\,x^{\frac{1}{3}}+b}\;b\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-b^{\frac{3}{2}}\right)x}{\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] -6*x*(a*x^(1/3)+b)*(arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*b*(a*x^(1/3)+b)^(1
/2)-b^(3/2))/(a*x+b*x^(2/3))^(3/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^(2/3))^(-3/2), x)
```

mupad [B] time = 5.36, size = 40, normalized size = 0.67

$$-\frac{2\,x\,{\left(\frac{b}{a\,x^{1/3}}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{3}{2};\ \frac{5}{2};\ -\frac{b}{a\,x^{1/3}}\right)}{{\left(a\,x+b\,x^{2/3}\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(2/3))^(3/2),x)
```

```
[Out] -(2*x*(b/(a*x^(1/3)) + 1)^(3/2)*hypergeom([3/2, 3/2], 5/2, -b/(a*x^(1/3))))
/(a*x + b*x^(2/3))^(3/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral((a*x + b*x**(2/3))**(-3/2), x)
```

3.199 $\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{105a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{9/2}}-\frac{105a^2\sqrt{ax+bx^{2/3}}}{8b^4x^{2/3}}+\frac{35a\sqrt{ax+bx^{2/3}}}{4b^3x}-\frac{7\sqrt{ax+bx^{2/3}}}{b^2x^{4/3}}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

```
[Out] 105/8*a^3*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(9/2)+6/b/x^(2/3
)/(b*x^(2/3)+a*x)^(1/2)-7*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(4/3)+35/4*a*(b*x^(2/
3)+a*x)^(1/2)/b^3/x-105/8*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(2/3)
```

Rubi [A] time = 0.24, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2029, 206}

$$-\frac{105a^2\sqrt{ax+bx^{2/3}}}{8b^4x^{2/3}}+\frac{105a^3\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{8b^{9/2}}+\frac{35a\sqrt{ax+bx^{2/3}}}{4b^3x}-\frac{7\sqrt{ax+bx^{2/3}}}{b^2x^{4/3}}+\frac{6}{bx^{2/3}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(2/3)*Sqrt[b*x^(2/3) + a*x]) - (7*Sqrt[b*x^(2/3) + a*x])/(b^2*x^(4/3
)) + (35*a*Sqrt[b*x^(2/3) + a*x])/(4*b^3*x) - (105*a^2*Sqrt[b*x^(2/3) + a*x
])/(8*b^4*x^(2/3)) + (105*a^3*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*
x]])/(8*b^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} + \frac{7\int \frac{1}{x^{5/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} - \frac{(35a)\int \frac{1}{x^{4/3}\sqrt{bx^{2/3}+ax}}\,dx}{6b^2}$$

$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} + \frac{\left(35a^2\right)\int \frac{1}{x\sqrt{bx^{2/3}+ax}}\,dx}{8b^3}$$

$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} - \frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}} - \frac{\left(35a^3\right)}{}$$

$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} - \frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}} + \frac{\left(105a^{3}\right.}{}$$

$$= \frac{6}{bx^{2/3}\sqrt{bx^{2/3}+ax}} - \frac{7\sqrt{bx^{2/3}+ax}}{b^2x^{4/3}} + \frac{35a\sqrt{bx^{2/3}+ax}}{4b^3x} - \frac{105a^2\sqrt{bx^{2/3}+ax}}{8b^4x^{2/3}} + \frac{105a^3}{}$$

Mathematica [C] time = 0.09, size = 48, normalized size = 0.33

$$-\frac{6a^3\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{\sqrt[3]{x}a}{b}+1\right)}{b^4\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (-6*a^3*x^(1/3)*Hypergeometric2F1[-1/2, 4, 1/2, 1 + (a*x^(1/3))/b])/(b^4*Sq
rt[b*x^(2/3) + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.26, size = 105, normalized size = 0.72

$$-\frac{105\,a^3\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{8\sqrt{-b}\,b^4} - \frac{6\,a^3}{\sqrt{ax^{\frac{1}{3}}+b}\,b^4} - \frac{57\left(ax^{\frac{1}{3}}+b\right)^{\frac{5}{2}}a^3 - 136\left(ax^{\frac{1}{3}}+b\right)^{\frac{3}{2}}a^3b + 87\sqrt{ax^{\frac{1}{3}}+b}\,a^3b^2}{8\,a^3b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -105/8*a^3*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^4) - 6*a^3/(sqr
t(a*x^(1/3) + b)*b^4) - 1/8*(57*(a*x^(1/3) + b)^(5/2)*a^3 - 136*(a*x^(1/3)
+ b)^(3/2)*a^3*b + 87*sqrt(a*x^(1/3) + b)*a^3*b^2)/(a^3*b^4*x)
```

maple [A] time = 0.06, size = 88, normalized size = 0.60

$$-\frac{\left(a\,x^{\frac{1}{3}}+b\right)\left(-105\sqrt{a\,x^{\frac{1}{3}}+b}\;a^{3}x\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)+105a^{3}\sqrt{b}\;x+35a^{2}b^{\frac{3}{2}}x^{\frac{2}{3}}-14a\,b^{\frac{5}{2}}x^{\frac{1}{3}}+8b^{\frac{7}{2}}\right)}{8\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] -1/8*(a*x^(1/3)+b)*(-14*b^(5/2)*x^(1/3)*a+35*b^(3/2)*x^(2/3)*a^2+105*x*a^3*
b^(1/2)+8*b^(7/2)-105*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*(a*x^(1/3)+b)^(1
/2)*x*a^3)/(a*x+b*x^(2/3))^(3/2)/b^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x*(a*x + b*x^(2/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x*(a*x + b*x**(2/3))**(3/2)), x)
```

3.200 $\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{9009a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{512b^{15/2}}+\frac{9009a^5\sqrt{ax+bx^{2/3}}}{512b^7x^{2/3}}-\frac{3003a^4\sqrt{ax+bx^{2/3}}}{256b^6x}+\frac{3003a^3\sqrt{ax+bx^{2/3}}}{320b^5x^{4/3}}-\frac{1287a^2\sqrt{ax+b}}{160b^4x^{5/3}}$$

```
[Out] -9009/512*a^6*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(15/2)+6/b/x
^(5/3)/(b*x^(2/3)+a*x)^(1/2)-13/2*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(7/3)+143/20*
a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^2-1287/160*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/x^(5
/3)+3003/320*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(4/3)-3003/256*a^4*(b*x^(2/3)+
a*x)^(1/2)/b^6/x+9009/512*a^5*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(2/3)
```

Rubi [A] time = 0.41, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {2023, 2025, 2029, 206}

$$\frac{9009a^5\sqrt{ax+bx^{2/3}}}{512b^7x^{2/3}}-\frac{3003a^4\sqrt{ax+bx^{2/3}}}{256b^6x}+\frac{3003a^3\sqrt{ax+bx^{2/3}}}{320b^5x^{4/3}}-\frac{1287a^2\sqrt{ax+bx^{2/3}}}{160b^4x^{5/3}}-\frac{9009a^6\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^2}}\right.}{512b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(5/3)*Sqrt[b*x^(2/3) + a*x]) - (13*Sqrt[b*x^(2/3) + a*x])/(2*b^2*x^(
7/3)) + (143*a*Sqrt[b*x^(2/3) + a*x])/(20*b^3*x^2) - (1287*a^2*Sqrt[b*x^(2/
3) + a*x])/(160*b^4*x^(5/3)) + (3003*a^3*Sqrt[b*x^(2/3) + a*x])/(320*b^5*x^
(4/3)) - (3003*a^4*Sqrt[b*x^(2/3) + a*x])/(256*b^6*x) + (9009*a^5*Sqrt[b*x^
(2/3) + a*x])/(512*b^7*x^(2/3)) - (9009*a^6*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[
b*x^(2/3) + a*x]])/(512*b^(15/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^2\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}+\frac{13\int \frac{1}{x^{8/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}-\frac{(143a)\int \frac{1}{x^{7/3}\sqrt{bx^{2/3}+ax}}\,dx}{12b^2}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}+\frac{\left(429a^2\right)\int \frac{1}{x^2\sqrt{bx^{2/3}+ax}}\,dx}{40b^3}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}-\frac{(300\ldots}{}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003\ldots}{}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003\ldots}{}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003\ldots}{}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003\ldots}{}$$

$$= \frac{6}{bx^{5/3}\sqrt{bx^{2/3}+ax}}-\frac{13\sqrt{bx^{2/3}+ax}}{2b^2x^{7/3}}+\frac{143a\sqrt{bx^{2/3}+ax}}{20b^3x^2}-\frac{1287a^2\sqrt{bx^{2/3}+ax}}{160b^4x^{5/3}}+\frac{3003\ldots}{}$$

Mathematica [C] time = 0.08, size = 48, normalized size = 0.20

$$\frac{6a^6\sqrt[3]{x}\ {}_2F_1\left(-\frac{1}{2},7;\frac{1}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^7\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (6*a^6*x^(1/3)*Hypergeometric2F1[-1/2, 7, 1/2, 1 + (a*x^(1/3))/b])/(b^7*Sqr
t[b*x^(2/3) + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.39, size = 156, normalized size = 0.66

$$\frac{9009\,a^6\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{512\,\sqrt{-b}\,b^7}+\frac{6\,a^6}{\sqrt{ax^{\frac{1}{3}}+b}\,b^7}+\frac{29685\left(ax^{\frac{1}{3}}+b\right)^{\frac{11}{2}}a^6-163095\left(ax^{\frac{1}{3}}+b\right)^{\frac{9}{2}}a^6b+364194\left(ax^{\frac{1}{3}}+b\right)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 9009/512*a^6*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^7) + 6*a^6/(s
qrt(a*x^(1/3) + b)*b^7) + 1/2560*(29685*(a*x^(1/3) + b)^(11/2)*a^6 - 163095
*(a*x^(1/3) + b)^(9/2)*a^6*b + 364194*(a*x^(1/3) + b)^(7/2)*a^6*b^2 - 41609
4*(a*x^(1/3) + b)^(5/2)*a^6*b^3 + 246505*(a*x^(1/3) + b)^(3/2)*a^6*b^4 - 62
475*sqrt(a*x^(1/3) + b)*a^6*b^5)/(a^6*b^7*x^2)
```

maple [A] time = 0.06, size = 126, normalized size = 0.53

$$-\frac{\left(a\,x^{\frac{1}{3}}+b\right)\left(45045\sqrt{a\,x^{\frac{1}{3}}+b}\;a^6x^2\,\mathrm{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-45045a^6\sqrt{b}\;x^2-15015a^5b^{\frac{3}{2}}x^{\frac{5}{3}}+6006a^4b^{\frac{5}{2}}x^{\frac{4}{3}}-3432}{2560\left(ax+b\,x^{\frac{2}{3}}\right)^{\frac{3}{2}}b^{\frac{15}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] -1/2560*(a*x^(1/3)+b)*(1280*b^(13/2)+45045*(a*x^(1/3)+b)^(1/2)*arctanh((a*x
^(1/3)+b)^(1/2)/b^(1/2))*x^2*a^6+6006*x^(4/3)*b^(5/2)*a^4-3432*x*b^(7/2)*a^
3+2288*x^(2/3)*b^(9/2)*a^2-1664*x^(1/3)*b^(11/2)*a-45045*x^2*a^6*b^(1/2)-15
015*x^(5/3)*b^(3/2)*a^5)/x/(a*x+b*x^(2/3))^(3/2)/b^(15/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{1}{x^2\left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a*x + b*x^(2/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*x + b*x**(2/3))**(3/2)), x)
```

3.201 $\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=324

$$\frac{692835a^9\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{32768b^{21/2}}-\frac{692835a^8\sqrt{ax+bx^{2/3}}}{32768b^{10}x^{2/3}}+\frac{230945a^7\sqrt{ax+bx^{2/3}}}{16384b^9x}-\frac{46189a^6\sqrt{ax+bx^{2/3}}}{4096b^8x^{4/3}}+\frac{138567}{14}$$

```
[Out] 692835/32768*a^9*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(21/2)+6/
b/x^(8/3)/(b*x^(2/3)+a*x)^(1/2)-19/3*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(10/3)+323
/48*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^3-1615/224*a^2*(b*x^(2/3)+a*x)^(1/2)/b^4/
x^(8/3)+20995/2688*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(7/3)-46189/5376*a^4*(b*
x^(2/3)+a*x)^(1/2)/b^6/x^2+138567/14336*a^5*(b*x^(2/3)+a*x)^(1/2)/b^7/x^(5/
3)-46189/4096*a^6*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(4/3)+230945/16384*a^7*(b*x^(
2/3)+a*x)^(1/2)/b^9/x-692835/32768*a^8*(b*x^(2/3)+a*x)^(1/2)/b^10/x^(2/3)
```

Rubi [A] time = 0.60, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2029, 206}

$$-\frac{692835a^8\sqrt{ax+bx^{2/3}}}{32768b^{10}x^{2/3}}+\frac{230945a^7\sqrt{ax+bx^{2/3}}}{16384b^9x}-\frac{46189a^6\sqrt{ax+bx^{2/3}}}{4096b^8x^{4/3}}+\frac{138567a^5\sqrt{ax+bx^{2/3}}}{14336b^7x^{5/3}}-\frac{46189a^4\sqrt{ax}}{5376b^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(8/3)*Sqrt[b*x^(2/3) + a*x]) - (19*Sqrt[b*x^(2/3) + a*x])/(3*b^2*x^(
10/3)) + (323*a*Sqrt[b*x^(2/3) + a*x])/(48*b^3*x^3) - (1615*a^2*Sqrt[b*x^(2
/3) + a*x])/(224*b^4*x^(8/3)) + (20995*a^3*Sqrt[b*x^(2/3) + a*x])/(2688*b^5
*x^(7/3)) - (46189*a^4*Sqrt[b*x^(2/3) + a*x])/(5376*b^6*x^2) + (138567*a^5*
Sqrt[b*x^(2/3) + a*x])/(14336*b^7*x^(5/3)) - (46189*a^6*Sqrt[b*x^(2/3) + a*
x])/(4096*b^8*x^(4/3)) + (230945*a^7*Sqrt[b*x^(2/3) + a*x])/(16384*b^9*x) -
 (692835*a^8*Sqrt[b*x^(2/3) + a*x])/(32768*b^10*x^(2/3)) + (692835*a^9*ArcT
anh[(Sqrt[b]*x^(1/3))/Sqrt[b*x^(2/3) + a*x]])/(32768*b^(21/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
```

```
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^3\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}+\frac{19\int \frac{1}{x^{11/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}-\frac{(323a)\int \frac{1}{x^{10/3}\sqrt{bx^{2/3}+ax}}\,dx}{18b^2}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}+\frac{\left(1615a^2\right)\int \frac{1}{x^3\sqrt{bx^{2/3}+ax}}\,dx}{96b^3}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}-\frac{(209\ldots}{}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{2099\ldots}{}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{2099\ldots}{}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{2099\ldots}{}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{2099\ldots}{}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{2099\ldots}{}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{2099\ldots}{}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{2099\ldots}{}$$

$$= \frac{6}{bx^{8/3}\sqrt{bx^{2/3}+ax}}-\frac{19\sqrt{bx^{2/3}+ax}}{3b^2x^{10/3}}+\frac{323a\sqrt{bx^{2/3}+ax}}{48b^3x^3}-\frac{1615a^2\sqrt{bx^{2/3}+ax}}{224b^4x^{8/3}}+\frac{2099\ldots}{}$$

Mathematica [C] time = 0.07, size = 48, normalized size = 0.15

$$-\frac{6a^9\sqrt[3]{x}\,{}_2F_1\left(-\frac{1}{2},10;\frac{1}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^{10}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (-6*a^9*x^(1/3)*Hypergeometric2F1[-1/2, 10, 1/2, 1 + (a*x^(1/3))/b])/(b^10*
Sqrt[b*x^(2/3) + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.38, size = 207, normalized size = 0.64

$$-\frac{692835\,a^9\arctan\left(\frac{\sqrt{ax^{\frac{1}{3}}+b}}{\sqrt{-b}}\right)}{32768\,\sqrt{-b}\,b^{10}}-\frac{6\,a^9}{\sqrt{ax^{\frac{1}{3}}+b}\,b^{10}}-\frac{10420767\left(ax^{\frac{1}{3}}+b\right)^{\frac{17}{2}}a^9-88937058\left(ax^{\frac{1}{3}}+b\right)^{\frac{15}{2}}a^9b+334408\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] -692835/32768*a^9*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^10) - 6*
a^9/(sqrt(a*x^(1/3) + b)*b^10) - 1/688128*(10420767*(a*x^(1/3) + b)^(17/2)*
a^9 - 88937058*(a*x^(1/3) + b)^(15/2)*a^9*b + 334408914*(a*x^(1/3) + b)^(13
/2)*a^9*b^2 - 724860666*(a*x^(1/3) + b)^(11/2)*a^9*b^3 + 993296384*(a*x^(1/
3) + b)^(9/2)*a^9*b^4 - 884769030*(a*x^(1/3) + b)^(7/2)*a^9*b^5 + 503730990
*(a*x^(1/3) + b)^(5/2)*a^9*b^6 - 169799070*(a*x^(1/3) + b)^(3/2)*a^9*b^7 +
26738145*sqrt(a*x^(1/3) + b)*a^9*b^8)/(a^9*b^10*x^3)
```

maple [A] time = 0.07, size = 159, normalized size = 0.49

$$\frac{\left(a\,x^{\frac{1}{3}}+b\right)\left(14549535\sqrt{a\,x^{\frac{1}{3}}+b}\;a^9x^3\operatorname{arctanh}\left(\frac{\sqrt{a\,x^{\frac{1}{3}}+b}}{\sqrt{b}}\right)-14549535a^9\sqrt{b}\;x^3-4849845a^8b^{\frac{3}{2}}x^{\frac{8}{3}}+1939938a^{7}\cdots\right)}{68812\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] 1/688128*(a*x^(1/3)+b)*(14549535*(a*x^(1/3)+b)^(1/2)*arctanh((a*x^(1/3)+b)^
(1/2)/b^(1/2))*x^3*a^9-229376*b^(19/2)-537472*b^(11/2)*x^(4/3)*a^4-4849845*
b^(3/2)*x^(8/3)*a^8+272384*b^(17/2)*x^(1/3)*a+413440*b^(13/2)*x*a^3-330752*
b^(15/2)*x^(2/3)*a^2+739024*b^(9/2)*x^(5/3)*a^5-14549535*x^3*a^9*b^(1/2)+19
39938*b^(5/2)*x^(7/3)*a^7-1108536*b^(7/2)*x^2*a^6)/x^2/(a*x+b*x^(2/3))^(3/2
)/b^(21/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax+bx^{\frac{2}{3}}\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,{\left(a\,x+b\,x^{2/3}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x + b*x^(2/3))^(3/2)),x)
[Out] int(1/(x^3*(a*x + b*x^(2/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**(2/3)+a*x)**(3/2),x)
[Out] Integral(1/(x**3*(a*x + b*x**(2/3))**(3/2)), x)
```

3.202 $\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx$

Optimal. Leaf size=412

$$-\frac{50702925a^{12}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt[3]{x}}{\sqrt{ax+bx^{2/3}}}\right)}{2097152b^{27/2}}+\frac{50702925a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{13}x^{2/3}}-\frac{16900975a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^{12}x}+\frac{3380195a^9\sqrt{ax+b}}{262144b^{11}x^{4/3}}$$

```
[Out] -50702925/2097152*a^12*arctanh(x^(1/3)*b^(1/2)/(b*x^(2/3)+a*x)^(1/2))/b^(27
/2)+6/b/x^(11/3)/(b*x^(2/3)+a*x)^(1/2)-25/4*(b*x^(2/3)+a*x)^(1/2)/b^2/x^(13
/3)+575/88*a*(b*x^(2/3)+a*x)^(1/2)/b^3/x^4-2415/352*a^2*(b*x^(2/3)+a*x)^(1/
2)/b^4/x^(11/3)+15295/2112*a^3*(b*x^(2/3)+a*x)^(1/2)/b^5/x^(10/3)-260015/33
792*a^4*(b*x^(2/3)+a*x)^(1/2)/b^6/x^3+185725/22528*a^5*(b*x^(2/3)+a*x)^(1/2
)/b^7/x^(8/3)-2414425/270336*a^6*(b*x^(2/3)+a*x)^(1/2)/b^8/x^(7/3)+482885/4
9152*a^7*(b*x^(2/3)+a*x)^(1/2)/b^9/x^2-1448655/131072*a^8*(b*x^(2/3)+a*x)^(
1/2)/b^10/x^(5/3)+3380195/262144*a^9*(b*x^(2/3)+a*x)^(1/2)/b^11/x^(4/3)-169
00975/1048576*a^10*(b*x^(2/3)+a*x)^(1/2)/b^12/x+50702925/2097152*a^11*(b*x^
(2/3)+a*x)^(1/2)/b^13/x^(2/3)
```

Rubi [A] time = 0.84, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2029, 206}

$$\frac{50702925a^{11}\sqrt{ax+bx^{2/3}}}{2097152b^{13}x^{2/3}}-\frac{16900975a^{10}\sqrt{ax+bx^{2/3}}}{1048576b^{12}x}+\frac{3380195a^9\sqrt{ax+bx^{2/3}}}{262144b^{11}x^{4/3}}-\frac{1448655a^8\sqrt{ax+bx^{2/3}}}{131072b^{10}x^{5/3}}+482$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] 6/(b*x^(11/3)*Sqrt[b*x^(2/3) + a*x]) - (25*Sqrt[b*x^(2/3) + a*x])/(4*b^2*x^
(13/3)) + (575*a*Sqrt[b*x^(2/3) + a*x])/(88*b^3*x^4) - (2415*a^2*Sqrt[b*x^(
2/3) + a*x])/(352*b^4*x^(11/3)) + (15295*a^3*Sqrt[b*x^(2/3) + a*x])/(2112*b
^5*x^(10/3)) - (260015*a^4*Sqrt[b*x^(2/3) + a*x])/(33792*b^6*x^3) + (185725
*a^5*Sqrt[b*x^(2/3) + a*x])/(22528*b^7*x^(8/3)) - (2414425*a^6*Sqrt[b*x^(2/
3) + a*x])/(270336*b^8*x^(7/3)) + (482885*a^7*Sqrt[b*x^(2/3) + a*x])/(49152
*b^9*x^2) - (1448655*a^8*Sqrt[b*x^(2/3) + a*x])/(131072*b^10*x^(5/3)) + (33
80195*a^9*Sqrt[b*x^(2/3) + a*x])/(262144*b^11*x^(4/3)) - (16900975*a^10*Sqr
t[b*x^(2/3) + a*x])/(1048576*b^12*x) + (50702925*a^11*Sqrt[b*x^(2/3) + a*x]
)/(2097152*b^13*x^(2/3)) - (50702925*a^12*ArcTanh[(Sqrt[b]*x^(1/3))/Sqrt[b*
x^(2/3) + a*x]])/(2097152*b^(27/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{1}{x^4\left(bx^{2/3}+ax\right)^{3/2}}\,dx = \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} + \frac{25\int \frac{1}{x^{14/3}\sqrt{bx^{2/3}+ax}}\,dx}{b}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} - \frac{(575a)\int \frac{1}{x^{13/3}\sqrt{bx^{2/3}+ax}}\,dx}{24b^2}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} + \frac{\left(4025a^2\right)\int \frac{1}{x^4\sqrt{bx^{2/3}+ax}}}{176b^3}$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} -$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

$$= \frac{6}{bx^{11/3}\sqrt{bx^{2/3}+ax}} - \frac{25\sqrt{bx^{2/3}+ax}}{4b^2x^{13/3}} + \frac{575a\sqrt{bx^{2/3}+ax}}{88b^3x^4} - \frac{2415a^2\sqrt{bx^{2/3}+ax}}{352b^4x^{11/3}} +$$

Mathematica [C] time = 0.09, size = 48, normalized size = 0.12

$$\frac{6a^{12}\sqrt[3]{x}\;{}_2F_1\left(-\frac{1}{2},13;\frac{1}{2};\frac{\sqrt[3]{x}\,a}{b}+1\right)}{b^{13}\sqrt{ax+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(b*x^(2/3) + a*x)^(3/2)),x]
```

```
[Out] (6*a^12*x^(1/3)*Hypergeometric2F1[-1/2, 13, 1/2, 1 + (a*x^(1/3))/b])/(b^13*
Sqrt[b*x^(2/3) + a*x])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.45, size = 258, normalized size = 0.63

$$\frac{50702925\, a^{12} \arctan\left(\frac{\sqrt{a x^{\frac{1}{3}} + b}}{\sqrt{-b}}\right)}{2097152 \sqrt{-b}\, b^{13}} + \frac{6\, a^{12}}{\sqrt{a x^{\frac{1}{3}} + b}\, b^{13}} + \frac{1257960429 \left(a x^{\frac{1}{3}} + b\right)^{\frac{23}{2}} a^{12} - 14537792973 \left(a x^{\frac{1}{3}} + b\right)^{\frac{21}{2}} a^{12} b + 7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 50702925/2097152*a^12*arctan(sqrt(a*x^(1/3) + b)/sqrt(-b))/(sqrt(-b)*b^13)
+ 6*a^12/(sqrt(a*x^(1/3) + b)*b^13) + 1/69206016*(1257960429*(a*x^(1/3) + b
)^(23/2)*a^12 - 14537792973*(a*x^(1/3) + b)^(21/2)*a^12*b + 76667241519*(a*
x^(1/3) + b)^(19/2)*a^12*b^2 - 243717614415*(a*x^(1/3) + b)^(17/2)*a^12*b^3
 + 519393101810*(a*x^(1/3) + b)^(15/2)*a^12*b^4 - 780150847218*(a*x^(1/3) +
 b)^(13/2)*a^12*b^5 + 844265343246*(a*x^(1/3) + b)^(11/2)*a^12*b^6 - 659969
685518*(a*x^(1/3) + b)^(9/2)*a^12*b^7 + 366679446705*(a*x^(1/3) + b)^(7/2)*
a^12*b^8 - 138840292305*(a*x^(1/3) + b)^(5/2)*a^12*b^9 + 32660709939*(a*x^(
1/3) + b)^(3/2)*a^12*b^10 - 3724872723*sqrt(a*x^(1/3) + b)*a^12*b^11)/(a^12
*b^13*x^4)
```

maple [A] time = 0.08, size = 192, normalized size = 0.47

$$-\frac{\left(a x^{\frac{1}{3}} + b\right) \left(1673196525 \sqrt{a x^{\frac{1}{3}} + b}\; a^{12} x^{4} \operatorname{arctanh}\left(\frac{\sqrt{a x^{\frac{1}{3}} + b}}{\sqrt{b}}\right) - 1673196525 a^{12} \sqrt{b}\, x^{4} - 557732175 a^{11} b^{\frac{3}{2}} x^{\frac{11}{3}} + 22}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a*x+b*x^(2/3))^(3/2),x)
```

```
[Out] -1/69206016*(a*x^(1/3)+b)*(17301504*b^(25/2)-1673196525*x^4*a^12*b^(1/2)-19
660800*b^(23/2)*x^(1/3)*a+22609920*b^(21/2)*x^(2/3)*a^2+1673196525*(a*x^(1/
3)+b)^(1/2)*arctanh((a*x^(1/3)+b)^(1/2)/b^(1/2))*x^4*a^12-26378240*b^(19/2)
*x*a^3+31324160*b^(17/2)*x^(4/3)*a^4-38036480*b^(15/2)*x^(5/3)*a^5+47545600
*b^(13/2)*x^2*a^6-61809280*b^(11/2)*x^(7/3)*a^7+84987760*b^(9/2)*x^(8/3)*a^
8-127481640*b^(7/2)*x^3*a^9+223092870*b^(5/2)*x^(10/3)*a^10-557732175*b^(3/
2)*x^(11/3)*a^11)/x^3/(a*x+b*x^(2/3))^(3/2)/b^(27/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a x + b x^{\frac{2}{3}}\right)^{\frac{3}{2}} x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^(2/3)+a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*x + b*x^(2/3))^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(a\,x+b\,x^{2/3}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x + b*x^(2/3))^(3/2)),x)
```

```
[Out] int(1/(x^4*(a*x + b*x^(2/3))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{4} \left(ax + bx^{\frac{2}{3}}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**(2/3)+a*x)**(3/2),x)
```

```
[Out] Integral(1/(x**4*(a*x + b*x**(2/3))**(3/2)), x)
```

3.203 $\int x^2 \left(ax^2+bx^3\right) dx$

Optimal. Leaf size=17

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

```
[Out] 1/5*a*x^5+1/6*b*x^6
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^5)/5 + (b*x^6)/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(ax^2+bx^3\right) dx &= \int \left(ax^4+bx^5\right) dx\\ &= \frac{ax^5}{5}+\frac{bx^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^5}{5}+\frac{bx^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3),x]
```

```
[Out] (a*x^5)/5 + (b*x^6)/6
```

fricas [A] time = 0.50, size = 13, normalized size = 0.76

$$\frac{1}{6}x^6b+\frac{1}{5}x^5a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*x^6*b + 1/5*x^5*a
```

giac [A] time = 0.19, size = 13, normalized size = 0.76

$$\frac{1}{6}bx^6+\frac{1}{5}ax^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{6}b\,x^{6}+\frac{1}{5}a\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2),x)
```

```
[Out] 1/5*a*x^5+1/6*b*x^6
```

maxima [A] time = 1.33, size = 13, normalized size = 0.76

$$\frac{1}{6}\,bx^{6}+\frac{1}{5}\,ax^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/6*b*x^6 + 1/5*a*x^5
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^{5}\,\left(6\,a+5\,b\,x\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3),x)
```

```
[Out] (x^5*(6*a + 5*b*x))/30
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^{5}}{5}+\frac{bx^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2),x)
```

```
[Out] a*x**5/5 + b*x**6/6
```

3.204 $\int x \left(ax^2+bx^3\right) \, dx$

Optimal. Leaf size=17

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

[Out] 1/4*a*x^4+1/5*b*x^5

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

[In] Int[x*(a*x^2 + b*x^3),x]

[Out] (a*x^4)/4 + (b*x^5)/5

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int x \left(ax^2+bx^3\right) \, dx &= \int \left(ax^3+bx^4\right) \, dx\\ &= \frac{ax^4}{4}+\frac{bx^5}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^4}{4}+\frac{bx^5}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a*x^2 + b*x^3),x]

[Out] (a*x^4)/4 + (b*x^5)/5

fricas [A] time = 0.42, size = 13, normalized size = 0.76

$$\frac{1}{5}x^5b+\frac{1}{4}x^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^3+a*x^2),x, algorithm="fricas")

[Out] 1/5*x^5*b + 1/4*x^4*a

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^5+\frac{1}{4}\,ax^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{5}b\,x^{5}+\frac{1}{4}a\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2),x)
```

```
[Out] 1/4*a*x^4+1/5*b*x^5
```

maxima [A] time = 1.34, size = 13, normalized size = 0.76

$$\frac{1}{5}\,bx^{5}+\frac{1}{4}\,ax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] 1/5*b*x^5 + 1/4*a*x^4
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^{4}\,\left(5\,a+4\,b\,x\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3),x)
```

```
[Out] (x^4*(5*a + 4*b*x))/20
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{ax^{4}}{4}+\frac{bx^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2),x)
```

```
[Out] a*x**4/4 + b*x**5/5
```

3.205 $\int \left(ax^2+bx^3\right)\,dx$

Optimal. Leaf size=17

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

```
[Out] 1/3*a*x^3+1/4*b*x^4
```

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[a*x^2 + b*x^3,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

Rubi steps

$$\int \left(ax^2+bx^3\right)\,dx=\frac{ax^3}{3}+\frac{bx^4}{4}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^3}{3}+\frac{bx^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*x^2 + b*x^3,x]
```

```
[Out] (a*x^3)/3 + (b*x^4)/4
```

fricas [A] time = 0.58, size = 13, normalized size = 0.76

$$\frac{1}{4}x^4b+\frac{1}{3}x^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*b + 1/3*x^3*a
```

giac [A] time = 0.16, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="giac")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

maple [A] time = 0.04, size = 14, normalized size = 0.82

$$\frac{1}{4}b\,x^4+\frac{1}{3}a\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b*x^3+a*x^2,x)
```

```
[Out] 1/3*a*x^3+1/4*b*x^4
```

maxima [A] time = 1.34, size = 13, normalized size = 0.76

$$\frac{1}{4}\,bx^4 + \frac{1}{3}\,ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x^3+a*x^2,x, algorithm="maxima")
```

```
[Out] 1/4*b*x^4 + 1/3*a*x^3
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^3\,\left(4\,a+3\,b\,x\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*x^2 + b*x^3,x)
```

```
[Out] (x^3*(4*a + 3*b*x))/12
```

sympy [A] time = 0.07, size = 12, normalized size = 0.71

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b*x**3+a*x**2,x)
```

```
[Out] a*x**3/3 + b*x**4/4
```

3.206 $\int \frac{ax^2+bx^3}{x} \, dx$

Optimal. Leaf size=17

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

```
[Out] 1/2*a*x^2+1/3*b*x^3
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{ax^2+bx^3}{x} \, dx &= \int \left(ax+bx^2\right) \, dx \\ &= \frac{ax^2}{2} + \frac{bx^3}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{ax^2}{2} + \frac{bx^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/x,x]
```

```
[Out] (a*x^2)/2 + (b*x^3)/3
```

fricas [A] time = 0.51, size = 13, normalized size = 0.76

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="fricas")
```

```
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

giac [A] time = 0.15, size = 13, normalized size = 0.76

$$\frac{1}{3}bx^3 + \frac{1}{2}ax^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="giac")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

maple [A] time = 0.06, size = 14, normalized size = 0.82

$$\frac{1}{3} b\,x^{3}+\frac{1}{2} a\,x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/x,x)
[Out] 1/2*a*x^2+1/3*b*x^3
```

maxima [A] time = 1.36, size = 13, normalized size = 0.76

$$\frac{1}{3} \, bx^{3} + \frac{1}{2} \, ax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x,x, algorithm="maxima")
[Out] 1/3*b*x^3 + 1/2*a*x^2
```

mupad [B] time = 0.02, size = 13, normalized size = 0.76

$$\frac{x^2\,\left(3\,a+2\,b\,x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)/x,x)
[Out] (x^2*(3*a + 2*b*x))/6
```

sympy [A] time = 0.06, size = 12, normalized size = 0.71

$$\frac{a x^{2}}{2} + \frac{b x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/x,x)
[Out] a*x**2/2 + b*x**3/3
```

3.207 $\int \frac{ax^2+bx^3}{x^2}\,dx$

Optimal. Leaf size=12

$$ax + \frac{bx^2}{2}$$

```
[Out] a*x+1/2*b*x^2
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{ax^2 + bx^3}{x^2}\,dx &= \int (a + bx)\,dx\\ &= ax + \frac{bx^2}{2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$ax + \frac{bx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)/x^2,x]
```

```
[Out] a*x + (b*x^2)/2
```

fricas [A] time = 0.52, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*b*x^2 + a*x
```

giac [A] time = 0.15, size = 10, normalized size = 0.83

$$\frac{1}{2}\,bx^2 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*b*x^2 + a*x
```

maple [A] time = 0.05, size = 11, normalized size = 0.92

$$\frac{1}{2}b\,x^{2}+a x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)/x^2,x)
```

```
[Out] a*x+1/2*b*x^2
```

maxima [A] time = 1.34, size = 10, normalized size = 0.83

$$\frac{1}{2}\,b x^{2}+a x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*x^2 + a*x
```

mupad [B] time = 0.02, size = 10, normalized size = 0.83

$$\frac{b\,x^2}{2}+a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)/x^2,x)
```

```
[Out] a*x + (b*x^2)/2
```

sympy [A] time = 0.06, size = 8, normalized size = 0.67

$$a x+\frac{b x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)/x**2,x)
```

```
[Out] a*x + b*x**2/2
```

3.208 $\int x^2 \left(ax^2+bx^3\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*b^2*x^9
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3)^2,x]
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^2\left(ax^2+bx^3\right)^2\,dx &= \int x^6(a+bx)^2\,dx\\ &= \int \left(a^2x^6+2abx^7+b^2x^8\right)dx\\ &= \frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^7}{7}+\frac{1}{4}abx^8+\frac{b^2x^9}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^2,x]
[Out] (a^2*x^7)/7 + (a*b*x^8)/4 + (b^2*x^9)/9
```

fricas [A] time = 0.45, size = 24, normalized size = 0.80

$$\frac{1}{9}x^9b^2+\frac{1}{4}x^8ba+\frac{1}{7}x^7a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*b^2 + 1/4*x^8*b*a + 1/7*x^7*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{9} b^2 x^9 + \frac{1}{4} abx^8 + \frac{1}{7} a^2 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{9}b^2x^9 + \frac{1}{4}ab\,x^8 + \frac{1}{7}a^2x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^2,x)
```

```
[Out] 1/7*a^2*x^7+1/4*a*b*x^8+1/9*b^2*x^9
```

maxima [A] time = 1.35, size = 24, normalized size = 0.80

$$\frac{1}{9} b^2 x^9 + \frac{1}{4} abx^8 + \frac{1}{7} a^2 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b^2*x^9 + 1/4*a*b*x^8 + 1/7*a^2*x^7
```

mupad [B] time = 0.04, size = 24, normalized size = 0.80

$$\frac{a^2\,x^7}{7} + \frac{a\,b\,x^8}{4} + \frac{b^2\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3)^2,x)
```

```
[Out] (a^2*x^7)/7 + (b^2*x^9)/9 + (a*b*x^8)/4
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^7}{7} + \frac{abx^8}{4} + \frac{b^2x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**2,x)
```

```
[Out] a**2*x**7/7 + a*b*x**8/4 + b**2*x**9/9
```

3.209 $\int x \left(ax^2+bx^3\right)^2 \, dx$

Optimal. Leaf size=30

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

```
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*b^2*x^8
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 43}

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x \left(ax^2+bx^3\right)^2 \, dx &= \int x^5(a+bx)^2 \, dx\\ &= \int \left(a^2x^5+2abx^6+b^2x^7\right) dx\\ &= \frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^6}{6}+\frac{2}{7}abx^7+\frac{b^2x^8}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^6)/6 + (2*a*b*x^7)/7 + (b^2*x^8)/8
```

fricas [A] time = 0.52, size = 24, normalized size = 0.80

$$\frac{1}{8}x^8b^2+\frac{2}{7}x^7ba+\frac{1}{6}x^6a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] 1/8*x^8*b^2 + 2/7*x^7*b*a + 1/6*x^6*a^2
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

maple [A] time = 0.06, size = 25, normalized size = 0.83

$$\frac{1}{8}b^2x^8+\frac{2}{7}ab\,x^7+\frac{1}{6}a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^2,x)
[Out] 1/6*a^2*x^6+2/7*a*b*x^7+1/8*b^2*x^8
```

maxima [A] time = 1.20, size = 24, normalized size = 0.80

$$\frac{1}{8}\,b^2x^8+\frac{2}{7}\,abx^7+\frac{1}{6}\,a^2x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/8*b^2*x^8 + 2/7*a*b*x^7 + 1/6*a^2*x^6
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^6}{6}+\frac{2\,a\,b\,x^7}{7}+\frac{b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3)^2,x)
[Out] (a^2*x^6)/6 + (b^2*x^8)/8 + (2*a*b*x^7)/7
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^6}{6}+\frac{2abx^7}{7}+\frac{b^2x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**2,x)
[Out] a**2*x**6/6 + 2*a*b*x**7/7 + b**2*x**8/8
```

3.210 $\int \left(ax^2+bx^3\right)^2 dx$

Optimal. Leaf size=30

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

```
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 43}

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^2+bx^3\right)^2 dx &= \int x^4(a+bx)^2\,dx\\ &= \int \left(a^2x^4+2abx^5+b^2x^6\right) dx\\ &= \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

fricas [A] time = 0.47, size = 24, normalized size = 0.80

$$\frac{1}{7}x^7b^2+\frac{1}{3}x^6ba+\frac{1}{5}x^5a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] 1/7*x^7*b^2 + 1/3*x^6*b*a + 1/5*x^5*a^2
```

giac [A] time = 0.16, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{1}{3}\,abx^6 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{7}b^2x^7 + \frac{1}{3}ab\,x^6 + \frac{1}{5}a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2,x)
[Out] 1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7
```

maxima [A] time = 1.33, size = 24, normalized size = 0.80

$$\frac{1}{7}\,b^2x^7 + \frac{1}{3}\,abx^6 + \frac{1}{5}\,a^2x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^5}{5} + \frac{a\,b\,x^6}{3} + \frac{b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^2,x)
[Out] (a^2*x^5)/5 + (b^2*x^7)/7 + (a*b*x^6)/3
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^5}{5} + \frac{abx^6}{3} + \frac{b^2x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2,x)
[Out] a**2*x**5/5 + a*b*x**6/3 + b**2*x**7/7
```

3.211 $\int \frac{\left(ax^2+bx^3\right)^2}{x}\,dx$

Optimal. Leaf size=30

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

```
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3\right)^2}{x}\,dx &= \int x^3(a+bx)^2\,dx\\ &= \int \left(a^2x^3+2abx^4+b^2x^5\right)\,dx\\ &= \frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^4}{4}+\frac{2}{5}abx^5+\frac{b^2x^6}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2/x,x]
```

```
[Out] (a^2*x^4)/4 + (2*a*b*x^5)/5 + (b^2*x^6)/6
```

fricas [A] time = 0.56, size = 24, normalized size = 0.80

$$\frac{1}{6}b^2x^6+\frac{2}{5}abx^5+\frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="fricas")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

giac [A] time = 0.17, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="giac")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

maple [A] time = 0.04, size = 25, normalized size = 0.83

$$\frac{1}{6}b^2x^6 + \frac{2}{5}ab\,x^5 + \frac{1}{4}a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2/x,x)
[Out] 1/4*a^2*x^4+2/5*a*b*x^5+1/6*b^2*x^6
```

maxima [A] time = 1.27, size = 24, normalized size = 0.80

$$\frac{1}{6}\,b^2x^6 + \frac{2}{5}\,abx^5 + \frac{1}{4}\,a^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x,x, algorithm="maxima")
[Out] 1/6*b^2*x^6 + 2/5*a*b*x^5 + 1/4*a^2*x^4
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^4}{4} + \frac{2\,a\,b\,x^5}{5} + \frac{b^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^2/x,x)
[Out] (a^2*x^4)/4 + (b^2*x^6)/6 + (2*a*b*x^5)/5
```

sympy [A] time = 0.07, size = 26, normalized size = 0.87

$$\frac{a^2x^4}{4} + \frac{2abx^5}{5} + \frac{b^2x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2/x,x)
[Out] a**2*x**4/4 + 2*a*b*x**5/5 + b**2*x**6/6
```

3.212 $\int \frac{(ax^2+bx^3)^2}{x^2} dx$

Optimal. Leaf size=30

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

```
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3\right)^2}{x^2} dx &= \int x^2(a+bx)^2 \, dx\\ &= \int \left(a^2x^2+2abx^3+b^2x^4\right) dx\\ &= \frac{a^2x^3}{3}+\frac{1}{2}abx^4+\frac{b^2x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.00, size = 30, normalized size = 1.00

$$\frac{a^2x^3}{3} + \frac{1}{2}abx^4 + \frac{b^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^2/x^2,x]
```

```
[Out] (a^2*x^3)/3 + (a*b*x^4)/2 + (b^2*x^5)/5
```

fricas [A] time = 0.65, size = 24, normalized size = 0.80

$$\frac{1}{5}b^2x^5 + \frac{1}{2}abx^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="fricas")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

giac [A] time = 0.15, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="giac")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

maple [A] time = 0.05, size = 25, normalized size = 0.83

$$\frac{1}{5}b^2x^5 + \frac{1}{2}ab\,x^4 + \frac{1}{3}a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^2/x^2,x)
[Out] 1/3*a^2*x^3+1/2*a*b*x^4+1/5*b^2*x^5
```

maxima [A] time = 1.33, size = 24, normalized size = 0.80

$$\frac{1}{5}\,b^2x^5 + \frac{1}{2}\,abx^4 + \frac{1}{3}\,a^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^2/x^2,x, algorithm="maxima")
[Out] 1/5*b^2*x^5 + 1/2*a*b*x^4 + 1/3*a^2*x^3
```

mupad [B] time = 0.03, size = 24, normalized size = 0.80

$$\frac{a^2\,x^3}{3} + \frac{a\,b\,x^4}{2} + \frac{b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^2/x^2,x)
[Out] (a^2*x^3)/3 + (b^2*x^5)/5 + (a*b*x^4)/2
```

sympy [A] time = 0.07, size = 24, normalized size = 0.80

$$\frac{a^2x^3}{3} + \frac{abx^4}{2} + \frac{b^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**2/x**2,x)
[Out] a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5
```

3.213 $\int \frac{x^6}{ax^2+bx^3} dx$

Optimal. Leaf size=57

$$\frac{a^4 \log(a+bx)}{b^5} - \frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

```
[Out] -a^3*x/b^4+1/2*a^2*x^2/b^3-1/3*a*x^3/b^2+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2 x^2}{2b^3} - \frac{a^3 x}{b^4} + \frac{a^4 \log(a+bx)}{b^5} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{ax^2+bx^3}\,dx &= \int \frac{x^4}{a+bx}\,dx \\
&= \int \left(-\frac{a^3}{b^4} + \frac{a^2 x}{b^3} - \frac{a x^2}{b^2} + \frac{x^3}{b} + \frac{a^4}{b^4(a+bx)}\right) dx \\
&= -\frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b} + \frac{a^4 \log(a+bx)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 57, normalized size = 1.00

$$\frac{a^4 \log(a+bx)}{b^5} - \frac{a^3 x}{b^4} + \frac{a^2 x^2}{2b^3} - \frac{a x^3}{3b^2} + \frac{x^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3),x]
```

```
[Out] -((a^3*x)/b^4) + (a^2*x^2)/(2*b^3) - (a*x^3)/(3*b^2) + x^4/(4*b) + (a^4*Log
[a + b*x])/b^5
```

fricas [A] time = 0.54, size = 52, normalized size = 0.91

$$\frac{3\,b^4x^4 - 4\,ab^3x^3 + 6\,a^2b^2x^2 - 12\,a^3bx + 12\,a^4\log\left(bx + a\right)}{12\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^4*x^4 - 4*a*b^3*x^3 + 6*a^2*b^2*x^2 - 12*a^3*b*x + 12*a^4*log(b*x
 + a))/b^5
```

giac [A] time = 0.15, size = 53, normalized size = 0.93

$$\frac{a^4\log\left(|bx + a|\right)}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] a^4*log(abs(b*x + a))/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 1
2*a^3*x)/b^4
```

maple [A] time = 0.04, size = 52, normalized size = 0.91

$$\frac{x^4}{4b} - \frac{a\,x^3}{3b^2} + \frac{a^2x^2}{2b^3} + \frac{a^4\ln\left(bx + a\right)}{b^5} - \frac{a^3x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2),x)
```

```
[Out] -a^3/b^4*x+1/2*a^2/b^3*x^2-1/3*a/b^2*x^3+1/4*x^4/b+a^4*ln(b*x+a)/b^5
```

maxima [A] time = 1.27, size = 52, normalized size = 0.91

$$\frac{a^4\log\left(bx + a\right)}{b^5} + \frac{3\,b^3x^4 - 4\,ab^2x^3 + 6\,a^2bx^2 - 12\,a^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] a^4*log(b*x + a)/b^5 + 1/12*(3*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3
*x)/b^4
```

mupad [B] time = 5.09, size = 51, normalized size = 0.89

$$\frac{x^4}{4\,b} + \frac{a^4\,\ln\left(a + b\,x\right)}{b^5} - \frac{a\,x^3}{3\,b^2} - \frac{a^3\,x}{b^4} + \frac{a^2\,x^2}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3),x)
```

```
[Out] x^4/(4*b) + (a^4*log(a + b*x))/b^5 - (a*x^3)/(3*b^2) - (a^3*x)/b^4 + (a^2*x
^2)/(2*b^3)
```

sympy [A] time = 0.14, size = 49, normalized size = 0.86

$$\frac{a^4\log\left(a + bx\right)}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{2b^3} - \frac{ax^3}{3b^2} + \frac{x^4}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2),x)
```

```
[Out] a**4*log(a + b*x)/b**5 - a**3*x/b**4 + a**2*x**2/(2*b**3) - a*x**3/(3*b**2)
 + x**4/(4*b)
```

3.214 $\int \frac{x^5}{ax^2+bx^3} \, dx$

Optimal. Leaf size=44

$$-\frac{a^3 \log(a+bx)}{b^4}+\frac{a^2 x}{b^3}-\frac{a x^2}{2 b^2}+\frac{x^3}{3 b}$$

```
[Out] a^2*x/b^3-1/2*a*x^2/b^2+1/3*x^3/b-a^3*ln(b*x+a)/b^4
```

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2 x}{b^3}-\frac{a^3 \log(a+bx)}{b^4}-\frac{a x^2}{2 b^2}+\frac{x^3}{3 b}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{ax^2+bx^3} \, dx &= \int \frac{x^3}{a+bx} \, dx \\
&= \int \left(\frac{a^2}{b^3}-\frac{a x}{b^2}+\frac{x^2}{b}-\frac{a^3}{b^3 (a+bx)}\right) \, dx \\
&= \frac{a^2 x}{b^3}-\frac{a x^2}{2 b^2}+\frac{x^3}{3 b}-\frac{a^3 \log(a+bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 44, normalized size = 1.00

$$-\frac{a^3 \log(a+bx)}{b^4}+\frac{a^2 x}{b^3}-\frac{a x^2}{2 b^2}+\frac{x^3}{3 b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3),x]
```

```
[Out] (a^2*x)/b^3 - (a*x^2)/(2*b^2) + x^3/(3*b) - (a^3*Log[a + b*x])/b^4
```

fricas [A] time = 0.55, size = 41, normalized size = 0.93

$$\frac{2 b^3 x^3 - 3 a b^2 x^2 + 6 a^2 b x - 6 a^3 \log\left(bx+a\right)}{6 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b^3*x^3 - 3*a*b^2*x^2 + 6*a^2*b*x - 6*a^3*log(b*x + a))/b^4
```

giac [A] time = 0.14, size = 43, normalized size = 0.98

$$-\frac{a^{3}\log\left(|bx+a|\right)}{b^{4}}+\frac{2\,b^{2}x^{3}-3\,abx^{2}+6\,a^{2}x}{6\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -a^3*log(abs(b*x + a))/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

maple [A] time = 0.04, size = 41, normalized size = 0.93

$$\frac{x^{3}}{3b}-\frac{a\,x^{2}}{2b^{2}}-\frac{a^{3}\ln\left(bx+a\right)}{b^{4}}+\frac{a^{2}x}{b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2),x)
```

```
[Out] a^2/b^3*x-1/2*a/b^2*x^2+1/3/b*x^3-a^3*ln(b*x+a)/b^4
```

maxima [A] time = 1.31, size = 42, normalized size = 0.95

$$-\frac{a^{3}\log\left(bx+a\right)}{b^{4}}+\frac{2\,b^{2}x^{3}-3\,abx^{2}+6\,a^{2}x}{6\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] -a^3*log(b*x + a)/b^4 + 1/6*(2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/b^3
```

mupad [B] time = 0.04, size = 40, normalized size = 0.91

$$\frac{x^3}{3\,b}-\frac{a^3\,\ln\left(a+b\,x\right)}{b^4}-\frac{a\,x^2}{2\,b^2}+\frac{a^2\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3),x)
```

```
[Out] x^3/(3*b) - (a^3*log(a + b*x))/b^4 - (a*x^2)/(2*b^2) + (a^2*x)/b^3
```

sympy [A] time = 0.13, size = 37, normalized size = 0.84

$$-\frac{a^{3}\log{\left(a+bx\right)}}{b^{4}}+\frac{a^{2}x}{b^{3}}-\frac{ax^{2}}{2b^{2}}+\frac{x^{3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2),x)
```

```
[Out] -a**3*log(a + b*x)/b**4 + a**2*x/b**3 - a*x**2/(2*b**2) + x**3/(3*b)
```

3.215 $\int \frac{x^4}{ax^2+bx^3}\,dx$

Optimal. Leaf size=31

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{ax^2+bx^3}\,dx &= \int \frac{x^2}{a+bx}\,dx\\ &= \int \left(-\frac{a}{b^2}+\frac{x}{b}+\frac{a^2}{b^2(a+bx)}\right)dx\\ &= -\frac{ax}{b^2}+\frac{x^2}{2b}+\frac{a^2\log(a+bx)}{b^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{a^2\log(a+bx)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

fricas [A] time = 0.47, size = 29, normalized size = 0.94

$$\frac{b^2x^2-2\,abx+2\,a^2\log\left(bx+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

giac [A] time = 0.19, size = 30, normalized size = 0.97

$$\frac{a^2 \log\left(|bx+a|\right)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

maple [A] time = 0.04, size = 30, normalized size = 0.97

$$\frac{x^2}{2b} + \frac{a^2 \ln\left(bx+a\right)}{b^3} - \frac{ax}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2),x)
```

```
[Out] -a/b^2*x+1/2/b*x^2+a^2*ln(b*x+a)/b^3
```

maxima [A] time = 1.37, size = 29, normalized size = 0.94

$$\frac{a^2 \log\left(bx+a\right)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

mupad [B] time = 0.04, size = 29, normalized size = 0.94

$$\frac{2\,a^2\,\ln\left(a+b\,x\right) + b^2\,x^2 - 2\,a\,b\,x}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3),x)
```

```
[Out] (2*a^2*log(a + b*x) + b^2*x^2 - 2*a*b*x)/(2*b^3)
```

sympy [A] time = 0.12, size = 26, normalized size = 0.84

$$\frac{a^2 \log\left(a+bx\right)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

3.216 $\int \frac{x^3}{ax^2+bx^3} \, dx$

Optimal. Leaf size=18

$$\frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

```
[Out] x/b-a*ln(b*x+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{ax^2+bx^3} \, dx &= \int \frac{x}{a+bx} \, dx \\
&= \int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx \\
&= \frac{x}{b} - \frac{a\log(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

fricas [A] time = 0.38, size = 17, normalized size = 0.94

$$\frac{bx - a\log(bx+a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(b*x + a))/b^2
```

giac [A] time = 0.15, size = 19, normalized size = 1.06

$$\frac{x}{b} - \frac{a \log\left(|bx + a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

maple [A] time = 0.05, size = 19, normalized size = 1.06

$$-\frac{a \ln\left(bx + a\right)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2),x)
```

```
[Out] 1/b*x-a*ln(b*x+a)/b^2
```

maxima [A] time = 1.30, size = 18, normalized size = 1.00

$$\frac{x}{b} - \frac{a \log\left(bx + a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] x/b - a*log(b*x + a)/b^2
```

mupad [B] time = 0.04, size = 18, normalized size = 1.00

$$-\frac{a\,\ln\left(a + b\,x\right) - b\,x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3),x)
```

```
[Out] -(a*log(a + b*x) - b*x)/b^2
```

sympy [A] time = 0.11, size = 14, normalized size = 0.78

$$-\frac{a \log\left(a + bx\right)}{b^2} + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

3.217 $\int \frac{x^2}{ax^2+bx^3} \, dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

[Out] ln(b*x+a)/b

Rubi [A] time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*x^2 + b*x^3),x]

[Out] Log[a + b*x]/b

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{ax^2+bx^3} \, dx &= \int \frac{1}{a+bx} \, dx \\ &= \frac{\log(a+bx)}{b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*x^2 + b*x^3),x]

[Out] Log[a + b*x]/b

fricas [A] time = 0.38, size = 10, normalized size = 1.00

$$\frac{\log(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="fricas")

```
[Out] log(b*x + a)/b
```

giac [A] time = 0.15, size = 11, normalized size = 1.10

$$\frac{\log(|bx + a|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="giac")
[Out] log(abs(b*x + a))/b
```

maple [A] time = 0.04, size = 11, normalized size = 1.10

$$\frac{\ln(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2),x)
[Out] 1/b*ln(b*x+a)
```

maxima [A] time = 1.32, size = 10, normalized size = 1.00

$$\frac{\log(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] log(b*x + a)/b
```

mupad [B] time = 0.02, size = 10, normalized size = 1.00

$$\frac{\ln(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3),x)
[Out] log(a + b*x)/b
```

sympy [A] time = 0.07, size = 7, normalized size = 0.70

$$\frac{\log(a + bx)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2),x)
[Out] log(a + b*x)/b
```

3.218 $\int \frac{x}{ax^2+bx^3} \, dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

```
[Out] ln(x)/a-ln(b*x+a)/a
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1584, 36, 29, 31}

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{ax^2+bx^3} \, dx &= \int \frac{1}{x(a+bx)} \, dx \\
&= \frac{\int \frac{1}{x} \, dx}{a} - \frac{b \int \frac{1}{a+bx} \, dx}{a} \\
&= \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 1.00

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3),x]
[Out] Log[x]/a - Log[a + b*x]/a
```

fricas [A] time = 0.39, size = 16, normalized size = 0.89

$$-\frac{\log(bx + a) - \log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="fricas")
[Out] -(log(b*x + a) - log(x))/a
```

giac [A] time = 0.16, size = 20, normalized size = 1.11

$$-\frac{\log(|bx + a|)}{a} + \frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="giac")
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

maple [A] time = 0.05, size = 19, normalized size = 1.06

$$\frac{\ln(x)}{a} - \frac{\ln(bx + a)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2),x)
[Out] 1/a*ln(x)-ln(b*x+a)/a
```

maxima [A] time = 1.30, size = 18, normalized size = 1.00

$$-\frac{\log(bx + a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2),x, algorithm="maxima")
[Out] -log(b*x + a)/a + log(x)/a
```

mupad [B] time = 5.12, size = 15, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3),x)
[Out] -(2*atanh((2*b*x)/a + 1))/a
```

sympy [A] time = 0.16, size = 10, normalized size = 0.56

$$\frac{\log(x) - \log\left(\frac{a}{b} + x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2),x)
[Out] (log(x) - log(a/b + x))/a
```

3.219 $\int \frac{1}{ax^2+bx^3} dx$

Optimal. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-1),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^2+bx^3}\,dx &= \int \frac{1}{x^2(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 28, normalized size = 1.00

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-1),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

fricas [A] time = 0.39, size = 26, normalized size = 0.93

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

giac [A] time = 0.15, size = 30, normalized size = 1.07

$$\frac{b\log\left(|bx+a|\right)}{a^2}-\frac{b\log\left(|x|\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

maple [A] time = 0.05, size = 29, normalized size = 1.04

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(bx+a\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2),x)
```

```
[Out] -1/a/x-1/a^2*b*ln(x)+b*ln(b*x+a)/a^2
```

maxima [A] time = 1.36, size = 28, normalized size = 1.00

$$\frac{b\log\left(bx+a\right)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

mupad [B] time = 0.05, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3),x)
```

```
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

sympy [A] time = 0.20, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

3.220 $\int \frac{1}{x(ax^2+bx^3)}\,dx$

Optimal. Leaf size=42

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(ax^2+bx^3\right)}\,dx&=\int \frac{1}{x^3(a+bx)}\,dx\\&=\int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx\\&=-\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.00

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

fricas [A] time = 0.38, size = 41, normalized size = 0.98

$$-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log(x)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

giac [A] time = 0.15, size = 45, normalized size = 1.07

$$-\frac{b^2 \log (|bx+a|)}{a^3}+\frac{b^2 \log (|x|)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

maple [A] time = 0.05, size = 41, normalized size = 0.98

$$\frac{b^2 \ln(x)}{a^3}-\frac{b^2 \ln (bx+a)}{a^3}+\frac{b}{a^2x}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2),x)
```

```
[Out] -1/2/a/x^2+1/a^2*b/x+1/a^3*b^2*ln(x)-b^2*ln(b*x+a)/a^3
```

maxima [A] time = 1.37, size = 40, normalized size = 0.95

$$-\frac{b^2 \log (bx+a)}{a^3}+\frac{b^2 \log(x)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

mupad [B] time = 0.06, size = 38, normalized size = 0.90

$$-\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)),x)
```

```
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

sympy [A] time = 0.21, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log (x)-\log \left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

3.221 $\int \frac{1}{x^2(ax^2+bx^3)}\,dx$

Optimal. Leaf size=56

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

```
[Out] -1/3/a/x^3+1/2*b/a^2/x^2-b^2/a^3/x-b^3*ln(x)/a^4+b^3*ln(b*x+a)/a^4
```

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/(3*a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[a
+ b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(ax^2+bx^3\right)}\,dx &= \int \frac{1}{x^4(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^4}-\frac{b}{a^2x^3}+\frac{b^2}{a^3x^2}-\frac{b^3}{a^4x}+\frac{b^4}{a^4(a+bx)}\right)dx\\ &= -\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\frac{b^2}{a^3x}-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 56, normalized size = 1.00

$$-\frac{b^3\log(x)}{a^4}+\frac{b^3\log(a+bx)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3)),x]
```

```
[Out] -1/3*1/(a*x^3) + b/(2*a^2*x^2) - b^2/(a^3*x) - (b^3*Log[x])/a^4 + (b^3*Log[
a + b*x])/a^4
```

fricas [A] time = 0.39, size = 54, normalized size = 0.96

$$\frac{6\,b^3x^3\log\left(bx+a\right)-6\,b^3x^3\log(x)-6\,ab^2x^2+3\,a^2bx-2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="fricas")
```

```
[Out] 1/6*(6*b^3*x^3*log(b*x + a) - 6*b^3*x^3*log(x) - 6*a*b^2*x^2 + 3*a^2*b*x -
2*a^3)/(a^4*x^3)
```

giac [A] time = 0.16, size = 56, normalized size = 1.00

$$\frac{b^3\log\left(|bx+a|\right)}{a^4}-\frac{b^3\log\left(|x|\right)}{a^4}-\frac{6\,ab^2x^2-3\,a^2bx+2\,a^3}{6\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="giac")
```

```
[Out] b^3*log(abs(b*x + a))/a^4 - b^3*log(abs(x))/a^4 - 1/6*(6*a*b^2*x^2 - 3*a^2*
b*x + 2*a^3)/(a^4*x^3)
```

maple [A] time = 0.05, size = 53, normalized size = 0.95

$$-\frac{b^3\ln(x)}{a^4}+\frac{b^3\ln\left(bx+a\right)}{a^4}-\frac{b^2}{a^3x}+\frac{b}{2a^2x^2}-\frac{1}{3a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2),x)
```

```
[Out] -1/3/a/x^3+1/2/a^2*b/x^2-1/a^3*b^2/x-1/a^4*b^3*ln(x)+b^3*ln(b*x+a)/a^4
```

maxima [A] time = 1.33, size = 51, normalized size = 0.91

$$\frac{b^3\log\left(bx+a\right)}{a^4}-\frac{b^3\log(x)}{a^4}-\frac{6\,b^2x^2-3\,abx+2\,a^2}{6\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2),x, algorithm="maxima")
```

```
[Out] b^3*log(b*x + a)/a^4 - b^3*log(x)/a^4 - 1/6*(6*b^2*x^2 - 3*a*b*x + 2*a^2)/(
a^3*x^3)
```

mupad [B] time = 0.06, size = 48, normalized size = 0.86

$$\frac{2\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^4}-\frac{\frac{a^3}{3}-\frac{a^2\,b\,x}{2}+a\,b^2\,x^2}{a^4\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3)),x)
```

```
[Out] (2*b^3*atanh((2*b*x)/a + 1))/a^4 - (a^3/3 + a*b^2*x^2 - (a^2*b*x)/2)/(a^4*x
^3)
```

sympy [A] time = 0.24, size = 44, normalized size = 0.79

$$\frac{-2a^2+3abx-6b^2x^2}{6a^3x^3}+\frac{b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2),x)
```

```
[Out] (-2*a**2 + 3*a*b*x - 6*b**2*x**2)/(6*a**3*x**3) + b**3*(-log(x) + log(a/b +
 x))/a**4
```

3.222 $\int \frac{x^8}{\left(ax^2+bx^3\right)^2}\,dx$

Optimal. Leaf size=58

$$-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

```
[Out] 3*a^2*x/b^4-a*x^2/b^3+1/3*x^3/b^2-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

Rubi [A] time = 0.04, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$-\frac{a^4}{b^5(a+bx)}+\frac{3a^2x}{b^4}-\frac{4a^3\log(a+bx)}{b^5}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (3*a^2*x)/b^4 - (a*x^2)/b^3 + x^3/(3*b^2) - a^4/(b^5*(a + b*x)) - (4*a^3*Lo
g[a + b*x])/b^5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{x^4}{(a+bx)^2}\,dx\\
&= \int \left(\frac{3a^2}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{b^2}+\frac{a^4}{b^4(a+bx)^2}-\frac{4a^3}{b^4(a+bx)}\right)dx\\
&= \frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}-\frac{a^4}{b^5(a+bx)}-\frac{4a^3\log(a+bx)}{b^5}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.93

$$\frac{-\frac{3a^4}{a+bx}-12a^3\log(a+bx)+9a^2bx-3ab^2x^2+b^3x^3}{3b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (9*a^2*b*x - 3*a*b^2*x^2 + b^3*x^3 - (3*a^4)/(a + b*x) - 12*a^3*Log[a + b*x
])/(3*b^5)
```

fricas [A] time = 0.39, size = 73, normalized size = 1.26

$$\frac{b^4x^4 - 2\,ab^3x^3 + 6\,a^2b^2x^2 + 9\,a^3bx - 3\,a^4 - 12\left(a^3bx + a^4\right)\log\left(bx + a\right)}{3\left(b^6x + ab^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(b^4*x^4 - 2*a*b^3*x^3 + 6*a^2*b^2*x^2 + 9*a^3*b*x - 3*a^4 - 12*(a^3*b*
x + a^4)*log(b*x + a))/(b^6*x + a*b^5)
```

giac [A] time = 0.17, size = 62, normalized size = 1.07

$$-\frac{4\,a^3\log\left(|bx + a|\right)}{b^5} - \frac{a^4}{(bx + a)b^5} + \frac{b^4x^3 - 3\,ab^3x^2 + 9\,a^2b^2x}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -4*a^3*log(abs(b*x + a))/b^5 - a^4/((b*x + a)*b^5) + 1/3*(b^4*x^3 - 3*a*b^3
*x^2 + 9*a^2*b^2*x)/b^6
```

maple [A] time = 0.05, size = 57, normalized size = 0.98

$$\frac{x^3}{3b^2} - \frac{a\,x^2}{b^3} - \frac{a^4}{(bx + a)\,b^5} - \frac{4a^3\ln\left(bx + a\right)}{b^5} + \frac{3a^2x}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(b*x^3+a*x^2)^2,x)
```

```
[Out] 3*a^2/b^4*x-a/b^3*x^2+1/3/b^2*x^3-a^4/b^5/(b*x+a)-4*a^3*ln(b*x+a)/b^5
```

maxima [A] time = 1.31, size = 59, normalized size = 1.02

$$-\frac{a^4}{b^6x + ab^5} - \frac{4\,a^3\log\left(bx + a\right)}{b^5} + \frac{b^2x^3 - 3\,abx^2 + 9\,a^2x}{3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -a^4/(b^6*x + a*b^5) - 4*a^3*log(b*x + a)/b^5 + 1/3*(b^2*x^3 - 3*a*b*x^2 +
9*a^2*x)/b^4
```

mupad [B] time = 0.04, size = 62, normalized size = 1.07

$$\frac{x^3}{3\,b^2} - \frac{4\,a^3\,\ln\left(a + b\,x\right)}{b^5} - \frac{a\,x^2}{b^3} + \frac{3\,a^2\,x}{b^4} - \frac{a^4}{b\,\left(x\,b^5 + a\,b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*x^2 + b*x^3)^2,x)
```

```
[Out] x^3/(3*b^2) - (4*a^3*log(a + b*x))/b^5 - (a*x^2)/b^3 + (3*a^2*x)/b^4 - a^4/
(b*(a*b^4 + b^5*x))
```

sympy [A] time = 0.21, size = 54, normalized size = 0.93

$$-\frac{a^4}{ab^5+b^6x}-\frac{4a^3\log(a+bx)}{b^5}+\frac{3a^2x}{b^4}-\frac{ax^2}{b^3}+\frac{x^3}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(b*x**3+a*x**2)**2,x)
```

```
[Out] -a**4/(a*b**5 + b**6*x) - 4*a**3*log(a + b*x)/b**5 + 3*a**2*x/b**4 - a*x**2
/b**3 + x**3/(3*b**2)
```

3.223 $\int \frac{x^7}{\left(ax^2+bx^3\right)^2} \, dx$

Optimal. Leaf size=46

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

```
[Out] -2*a*x/b^3+1/2*x^2/b^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}-\frac{2ax}{b^3}+\frac{x^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (-2*a*x)/b^3 + x^2/(2*b^2) + a^3/(b^4*(a + b*x)) + (3*a^2*Log[a + b*x])/b^4
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{\left(ax^2+bx^3\right)^2} \, dx &= \int \frac{x^3}{(a+bx)^2} \, dx \\
&= \int \left(-\frac{2a}{b^3}+\frac{x}{b^2}-\frac{a^3}{b^3(a+bx)^2}+\frac{3a^2}{b^3(a+bx)}\right) dx \\
&= -\frac{2ax}{b^3}+\frac{x^2}{2b^2}+\frac{a^3}{b^4(a+bx)}+\frac{3a^2\log(a+bx)}{b^4}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 0.93

$$\frac{\frac{2a^3}{a+bx}+6a^2\log(a+bx)-4abx+b^2x^2}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (-4*a*b*x + b^2*x^2 + (2*a^3)/(a + b*x) + 6*a^2*Log[a + b*x])/(2*b^4)
```

fricas [A] time = 0.37, size = 62, normalized size = 1.35

$$\frac{b^3x^3 - 3\,ab^2x^2 - 4\,a^2bx + 2\,a^3 + 6\left(a^2bx + a^3\right)\log\left(bx + a\right)}{2\left(b^5x + ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*x^3 - 3*a*b^2*x^2 - 4*a^2*b*x + 2*a^3 + 6*(a^2*b*x + a^3)*log(b*x
+ a))/(b^5*x + a*b^4)
```

giac [A] time = 0.16, size = 48, normalized size = 1.04

$$\frac{3\,a^2\log\left(|bx + a|\right)}{b^4} + \frac{a^3}{(bx + a)b^4} + \frac{b^2x^2 - 4\,abx}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 3*a^2*log(abs(b*x + a))/b^4 + a^3/((b*x + a)*b^4) + 1/2*(b^2*x^2 - 4*a*b*x)
/b^4
```

maple [A] time = 0.05, size = 45, normalized size = 0.98

$$\frac{x^2}{2b^2} + \frac{a^3}{(bx + a)\,b^4} + \frac{3a^2\ln\left(bx + a\right)}{b^4} - \frac{2ax}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(b*x^3+a*x^2)^2,x)
```

```
[Out] -2*a/b^3*x+1/2/b^2*x^2+a^3/b^4/(b*x+a)+3*a^2*ln(b*x+a)/b^4
```

maxima [A] time = 1.32, size = 47, normalized size = 1.02

$$\frac{a^3}{b^5x + ab^4} + \frac{3\,a^2\log\left(bx + a\right)}{b^4} + \frac{bx^2 - 4\,ax}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] a^3/(b^5*x + a*b^4) + 3*a^2*log(b*x + a)/b^4 + 1/2*(b*x^2 - 4*a*x)/b^3
```

mupad [B] time = 0.05, size = 50, normalized size = 1.09

$$\frac{x^2}{2\,b^2} + \frac{3\,a^2\,\ln\left(a + b\,x\right)}{b^4} + \frac{a^3}{b\,\left(x\,b^4 + a\,b^3\right)} - \frac{2\,a\,x}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(a*x^2 + b*x^3)^2,x)
```

```
[Out] x^2/(2*b^2) + (3*a^2*log(a + b*x))/b^4 + a^3/(b*(a*b^3 + b^4*x)) - (2*a*x)/
b^3
```

sympy [A] time = 0.21, size = 44, normalized size = 0.96

$$\frac{a^3}{ab^4 + b^5x} + \frac{3a^2\log\left(a + bx\right)}{b^4} - \frac{2ax}{b^3} + \frac{x^2}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(b*x**3+a*x**2)**2,x)
```

```
[Out] a**3/(a*b**4 + b**5*x) + 3*a**2*log(a + b*x)/b**4 - 2*a*x/b**3 + x**2/(2*b*
*2)
```

3.224 $\int \frac{x^6}{(ax^2+bx^3)^2} dx$

Optimal. Leaf size=33

$$-\frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3} + \frac{x}{b^2}$$

```
[Out] x/b^2-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$-\frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3)^2,x]
```

```
[Out] x/b^2 - a^2/(b^3*(a + b*x)) - (2*a*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{(ax^2+bx^3)^2}\,dx &= \int \frac{x^2}{(a+bx)^2}\,dx \\
&= \int \left(\frac{1}{b^2} + \frac{a^2}{b^2(a+bx)^2} - \frac{2a}{b^2(a+bx)}\right) dx \\
&= \frac{x}{b^2} - \frac{a^2}{b^3(a+bx)} - \frac{2a\log(a+bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.88

$$\frac{-\frac{a^2}{a+bx} - 2a\log(a+bx) + bx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (b*x - a^2/(a + b*x) - 2*a*Log[a + b*x])/b^3
```

fricas [A] time = 0.37, size = 47, normalized size = 1.42

$$\frac{b^2x^2 + abx - a^2 - 2\left(abx + a^2\right)\log\left(bx + a\right)}{b^4x + ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] (b^2*x^2 + a*b*x - a^2 - 2*(a*b*x + a^2)*log(b*x + a))/(b^4*x + a*b^3)
```

giac [A] time = 0.15, size = 34, normalized size = 1.03

$$\frac{x}{b^2} - \frac{2\,a\log\left(|bx + a|\right)}{b^3} - \frac{a^2}{(bx + a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] x/b^2 - 2*a*log(abs(b*x + a))/b^3 - a^2/((b*x + a)*b^3)
```

maple [A] time = 0.05, size = 34, normalized size = 1.03

$$-\frac{a^2}{\left(bx + a\right)b^3} - \frac{2a\ln\left(bx + a\right)}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2)^2,x)
[Out] 1/b^2*x-a^2/b^3/(b*x+a)-2*a*ln(b*x+a)/b^3
```

maxima [A] time = 1.28, size = 36, normalized size = 1.09

$$-\frac{a^2}{b^4x + ab^3} + \frac{x}{b^2} - \frac{2\,a\log\left(bx + a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] -a^2/(b^4*x + a*b^3) + x/b^2 - 2*a*log(b*x + a)/b^3
```

mupad [B] time = 0.04, size = 36, normalized size = 1.09

$$\frac{x}{b^2} - \frac{a^2}{x\,b^4 + a\,b^3} - \frac{2\,a\,\ln\left(a + b\,x\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3)^2,x)
[Out] x/b^2 - a^2/(a*b^3 + b^4*x) - (2*a*log(a + b*x))/b^3
```

sympy [A] time = 0.18, size = 31, normalized size = 0.94

$$-\frac{a^2}{ab^3 + b^4x} - \frac{2a\log\left(a + bx\right)}{b^3} + \frac{x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2)**2,x)
[Out] -a**2/(a*b**3 + b**4*x) - 2*a*log(a + b*x)/b**3 + x/b**2
```

3.225 $\int \frac{x^5}{\left(ax^2+bx^3\right)^2} \, dx$

Optimal. Leaf size=23

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

```
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 43}

$$\frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3)^2,x]
```

```
[Out] a/(b^2*(a + b*x)) + Log[a + b*x]/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(ax^2+bx^3\right)^2} \, dx &= \int \frac{x}{(a+bx)^2} \, dx\\
&= \int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right) dx\\
&= \frac{a}{b^2(a+bx)}+\frac{\log(a+bx)}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{\frac{a}{a+bx}+\log(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a/(a + b*x) + Log[a + b*x])/b^2
```

fricas [A] time = 0.39, size = 28, normalized size = 1.22

$$\frac{(bx+a)\log(bx+a)+a}{b^3x+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] ((b*x + a)*log(b*x + a) + a)/(b^3*x + a*b^2)
```

giac [A] time = 0.15, size = 24, normalized size = 1.04

$$\frac{\log\left(|bx+a|\right)}{b^2}+\frac{a}{(bx+a)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] log(abs(b*x + a))/b^2 + a/((b*x + a)*b^2)
```

maple [A] time = 0.05, size = 24, normalized size = 1.04

$$\frac{a}{(bx+a)\,b^2}+\frac{\ln\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2)^2,x)
[Out] a/b^2/(b*x+a)+ln(b*x+a)/b^2
```

maxima [A] time = 1.33, size = 26, normalized size = 1.13

$$\frac{a}{b^3x+ab^2}+\frac{\log\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] a/(b^3*x + a*b^2) + log(b*x + a)/b^2
```

mupad [B] time = 0.04, size = 23, normalized size = 1.00

$$\frac{\ln\left(a+b\,x\right)}{b^2}+\frac{a}{b^2\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3)^2,x)
[Out] log(a + b*x)/b^2 + a/(b^2*(a + b*x))
```

sympy [A] time = 0.14, size = 20, normalized size = 0.87

$$\frac{a}{ab^2+b^3x}+\frac{\log\left(a+bx\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2)**2,x)
[Out] a/(a*b**2 + b**3*x) + log(a + b*x)/b**2
```

3.226 $\int \frac{x^4}{\left(ax^2+bx^3\right)^2}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

```
[Out] -1/b/(b*x+a)
```

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{1}{(a+bx)^2}\,dx\\ &= -\frac{1}{b(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(b*(a + b*x)))
```

fricas [A] time = 0.38, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] -1/(b^2*x + a*b)
```

giac [A] time = 0.15, size = 12, normalized size = 1.00

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] -1/((b*x + a)*b)
```

maple [A] time = 0.05, size = 13, normalized size = 1.08

$$-\frac{1}{(bx+a)\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^2,x)
[Out] -1/b/(b*x+a)
```

maxima [A] time = 1.30, size = 13, normalized size = 1.08

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] -1/(b^2*x + a*b)
```

mupad [B] time = 5.17, size = 12, normalized size = 1.00

$$-\frac{1}{b\,(a+b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3)^2,x)
[Out] -1/(b*(a + b*x))
```

sympy [A] time = 0.14, size = 10, normalized size = 0.83

$$-\frac{1}{ab+b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**2,x)
[Out] -1/(a*b + b**2*x)
```

3.227 $\int \frac{x^3}{(ax^2+bx^3)^2} dx$

Optimal. Leaf size=29

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

```
[Out] 1/a/(b*x+a)+ln(x)/a^2-ln(b*x+a)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{\log(a+bx)}{a^2}+\frac{\log(x)}{a^2}+\frac{1}{a(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3)^2,x]
[Out] 1/(a*(a + b*x)) + Log[x]/a^2 - Log[a + b*x]/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(ax^2+bx^3)^2} dx &= \int \frac{1}{x(a+bx)^2} dx\\ &= \int \left(\frac{1}{a^2x}-\frac{b}{a(a+bx)^2}-\frac{b}{a^2(a+bx)}\right) dx\\ &= \frac{1}{a(a+bx)}+\frac{\log(x)}{a^2}-\frac{\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 0.83

$$\frac{\frac{a}{a+bx}-\log(a+bx)+\log(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3)^2,x]
[Out] (a/(a + b*x) + Log[x] - Log[a + b*x])/a^2
```

fricas [A] time = 0.39, size = 39, normalized size = 1.34

$$-\frac{(bx+a)\log(bx+a)-(bx+a)\log(x)-a}{a^2bx+a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="fricas")
[Out] -((b*x + a)*log(b*x + a) - (b*x + a)*log(x) - a)/(a^2*b*x + a^3)
```

giac [A] time = 0.15, size = 31, normalized size = 1.07

$$-\frac{\log(|bx+a|)}{a^2}+\frac{\log(|x|)}{a^2}+\frac{1}{(bx+a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="giac")
[Out] -log(abs(b*x + a))/a^2 + log(abs(x))/a^2 + 1/((b*x + a)*a)
```

maple [A] time = 0.06, size = 30, normalized size = 1.03

$$\frac{1}{(bx+a)\,a}+\frac{\ln(x)}{a^2}-\frac{\ln(bx+a)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^2,x)
[Out] 1/a/(b*x+a)+1/a^2*ln(x)-ln(b*x+a)/a^2
```

maxima [A] time = 1.29, size = 28, normalized size = 0.97

$$\frac{1}{abx+a^2}-\frac{\log(bx+a)}{a^2}+\frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^2,x, algorithm="maxima")
[Out] 1/(a*b*x + a^2) - log(b*x + a)/a^2 + log(x)/a^2
```

mupad [B] time = 0.04, size = 26, normalized size = 0.90

$$\frac{1}{a^2+b\,x\,a}-\frac{2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3)^2,x)
[Out] 1/(a^2 + a*b*x) - (2*atanh((2*b*x)/a + 1))/a^2
```

sympy [A] time = 0.22, size = 22, normalized size = 0.76

$$\frac{1}{a^2+abx}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**2,x)
[Out] 1/(a**2 + a*b*x) + (log(x) - log(a/b + x))/a**2
```

3.228 $\int \frac{x^2}{\left(ax^2+bx^3\right)^2} dx$

Optimal. Leaf size=42

$$-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{b}{a^2(a+bx)}-\frac{1}{a^2x}$$

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(ax^2+bx^3\right)^2} dx &= \int \frac{1}{x^2(a+bx)^2} dx\\
&= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right) dx\\
&= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{b}{a+bx}+\frac{1}{x}\right)-2b\log(a+bx)+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

fricas [A] time = 0.39, size = 63, normalized size = 1.50

$$-\frac{2\,abx + a^2 - 2\left(b^2x^2 + abx\right)\log\left(bx + a\right) + 2\left(b^2x^2 + abx\right)\log\left(x\right)}{a^3bx^2 + a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

giac [A] time = 0.15, size = 45, normalized size = 1.07

$$\frac{2\,b\log\left(|bx + a|\right)}{a^3} - \frac{2\,b\log\left(|x|\right)}{a^3} - \frac{2\,bx + a}{\left(bx^2 + ax\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*x + a))/a^3 - 2*b*log(abs(x))/a^3 - (2*b*x + a)/((b*x^2 + a*x
)*a^2)
```

maple [A] time = 0.05, size = 43, normalized size = 1.02

$$-\frac{b}{\left(bx + a\right)a^2} - \frac{2b\ln\left(x\right)}{a^3} + \frac{2b\ln\left(bx + a\right)}{a^3} - \frac{1}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2/a^3*b*ln(x)+2*b*ln(b*x+a)/a^3
```

maxima [A] time = 1.31, size = 45, normalized size = 1.07

$$-\frac{2\,bx + a}{a^2bx^2 + a^3x} + \frac{2\,b\log\left(bx + a\right)}{a^3} - \frac{2\,b\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

mupad [B] time = 5.34, size = 41, normalized size = 0.98

$$\frac{4\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^3} - \frac{\frac{1}{a} + \frac{2\,b\,x}{a^2}}{b\,x^2 + a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3)^2,x)
```

```
[Out] (4*b*atanh((2*b*x)/a + 1))/a^3 - (1/a + (2*b*x)/a^2)/(a*x + b*x^2)
```

sympy [A] time = 0.27, size = 37, normalized size = 0.88

$$\frac{-a - 2bx}{a^3x + a^2bx^2} + \frac{2b\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a - 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

3.229 $\int \frac{x}{(ax^2+bx^3)^2} \, dx$

Optimal. Leaf size=58

$$\frac{3b^2 \log(x)}{a^4} - \frac{3b^2 \log(a+bx)}{a^4} + \frac{b^2}{a^3(a+bx)} + \frac{2b}{a^3x} - \frac{1}{2a^2x^2}$$

```
[Out] -1/2/a^2/x^2+2*b/a^3/x+b^2/a^3/(b*x+a)+3*b^2*ln(x)/a^4-3*b^2*ln(b*x+a)/a^4
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 44}

$$\frac{b^2}{a^3(a+bx)} + \frac{3b^2 \log(x)}{a^4} - \frac{3b^2 \log(a+bx)}{a^4} + \frac{2b}{a^3x} - \frac{1}{2a^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3)^2,x]
```

```
[Out] -1/(2*a^2*x^2) + (2*b)/(a^3*x) + b^2/(a^3*(a + b*x)) + (3*b^2*Log[x])/a^4 - (3*b^2*Log[a + b*x])/a^4
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(ax^2+bx^3)^2} \, dx &= \int \frac{1}{x^3(a+bx)^2} \, dx \\
&= \int \left(\frac{1}{a^2x^3} - \frac{2b}{a^3x^2} + \frac{3b^2}{a^4x} - \frac{b^3}{a^3(a+bx)^2} - \frac{3b^3}{a^4(a+bx)}\right) dx \\
&= -\frac{1}{2a^2x^2} + \frac{2b}{a^3x} + \frac{b^2}{a^3(a+bx)} + \frac{3b^2 \log(x)}{a^4} - \frac{3b^2 \log(a+bx)}{a^4}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 53, normalized size = 0.91

$$\frac{a\left(\frac{2b^2}{a+bx} - \frac{a}{x^2} + \frac{4b}{x}\right) - 6b^2 \log(a+bx) + 6b^2 \log(x)}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3)^2,x]
```

```
[Out] (a*(-(a/x^2) + (4*b)/x + (2*b^2)/(a + b*x)) + 6*b^2*Log[x] - 6*b^2*Log[a + b*x])/(2*a^4)
```

fricas [A] time = 0.39, size = 86, normalized size = 1.48

$$\frac{6\,ab^{2}x^{2}+3\,a^{2}bx-a^{3}-6\left(b^{3}x^{3}+ab^{2}x^{2}\right)\log\left(bx+a\right)+6\left(b^{3}x^{3}+ab^{2}x^{2}\right)\log(x)}{2\left(a^{4}bx^{3}+a^{5}x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*x^2 + 3*a^2*b*x - a^3 - 6*(b^3*x^3 + a*b^2*x^2)*log(b*x + a) +
 6*(b^3*x^3 + a*b^2*x^2)*log(x))/(a^4*b*x^3 + a^5*x^2)
```

giac [A] time = 0.15, size = 64, normalized size = 1.10

$$-\frac{3\,b^{2}\log\left(|bx+a|\right)}{a^{4}}+\frac{3\,b^{2}\log\left(|x|\right)}{a^{4}}+\frac{6\,ab^{2}x^{2}+3\,a^{2}bx-a^{3}}{2\,(bx+a)a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -3*b^2*log(abs(b*x + a))/a^4 + 3*b^2*log(abs(x))/a^4 + 1/2*(6*a*b^2*x^2 + 3
*a^2*b*x - a^3)/((b*x + a)*a^4*x^2)
```

maple [A] time = 0.06, size = 57, normalized size = 0.98

$$\frac{b^{2}}{(bx+a)\,a^{3}}+\frac{3b^{2}\ln(x)}{a^{4}}-\frac{3b^{2}\ln\left(bx+a\right)}{a^{4}}+\frac{2b}{a^{3}x}-\frac{1}{2a^{2}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/2/a^2/x^2+2/a^3*b/x+b^2/a^3/(b*x+a)+3/a^4*b^2*ln(x)-3*b^2*ln(b*x+a)/a^4
```

maxima [A] time = 1.32, size = 64, normalized size = 1.10

$$\frac{6\,b^{2}x^{2}+3\,abx-a^{2}}{2\left(a^{3}bx^{3}+a^{4}x^{2}\right)}-\frac{3\,b^{2}\log\left(bx+a\right)}{a^{4}}+\frac{3\,b^{2}\log(x)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(6*b^2*x^2 + 3*a*b*x - a^2)/(a^3*b*x^3 + a^4*x^2) - 3*b^2*log(b*x + a)/
a^4 + 3*b^2*log(x)/a^4
```

mupad [B] time = 5.31, size = 57, normalized size = 0.98

$$\frac{\frac{3\,b^{2}\,x^{2}}{a^{3}}-\frac{1}{2\,a}+\frac{3\,b\,x}{2\,a^{2}}}{b\,x^{3}+a\,x^{2}}-\frac{6\,b^{2}\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3)^2,x)
```

```
[Out] ((3*b^2*x^2)/a^3 - 1/(2*a) + (3*b*x)/(2*a^2))/(a*x^2 + b*x^3) - (6*b^2*atan
h((2*b*x)/a + 1))/a^4
```

sympy [A] time = 0.31, size = 54, normalized size = 0.93

$$\frac{-a^{2}+3abx+6b^{2}x^{2}}{2a^{4}x^{2}+2a^{3}bx^{3}}+\frac{3b^{2}\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a**2 + 3*a*b*x + 6*b**2*x**2)/(2*a**4*x**2 + 2*a**3*b*x**3) + 3*b**2*(log
(x) - log(a/b + x))/a**4
```

3.230 $\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx$

Optimal. Leaf size=69

$$-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*ln
(b*x+a)/a^5
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] -1/(3*a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (4*b
^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{1}{x^4(a+bx)^2}\,dx\\ &= \int \left(\frac{1}{a^2x^4}-\frac{2b}{a^3x^3}+\frac{3b^2}{a^4x^2}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{4b^4}{a^5(a+bx)}\right)dx\\ &= -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}\end{aligned}$$

Mathematica [A] time = 0.07, size = 66, normalized size = 0.96

$$-\frac{\frac{a\left(a^3-2a^2bx+6ab^2x^2+12b^3x^3\right)}{x^3(a+bx)}-12b^3\log(a+bx)+12b^3\log(x)}{3a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-2),x]
```

```
[Out] -1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 12
*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

fricas [A] time = 0.40, size = 95, normalized size = 1.38

$$-\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4-12\left(b^4x^4+ab^3x^3\right)\log\left(bx+a\right)+12\left(b^4x^4+ab^3x^3\right)\log(x)}{3\left(a^5bx^4+a^6x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3*
x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3)
```

giac [A] time = 0.15, size = 73, normalized size = 1.06

$$\frac{4\,b^3\log\left(|bx+a|\right)}{a^5}-\frac{4\,b^3\log\left(|x|\right)}{a^5}-\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4}{3\left(bx+a\right)a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] 4*b^3*log(abs(b*x + a))/a^5 - 4*b^3*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 + 6
*a^2*b^2*x^2 - 2*a^3*b*x + a^4)/((b*x + a)*a^5*x^3)
```

maple [A] time = 0.05, size = 68, normalized size = 0.99

$$-\frac{b^3}{\left(bx+a\right)a^4}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln\left(bx+a\right)}{a^5}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/3/a^2/x^3+b/a^3/x^2-3/a^4*b^2/x-b^3/a^4/(b*x+a)-4/a^5*b^3*ln(x)+4*b^3*ln
(b*x+a)/a^5
```

maxima [A] time = 1.34, size = 73, normalized size = 1.06

$$-\frac{12\,b^3x^3+6\,ab^2x^2-2\,a^2bx+a^3}{3\left(a^4bx^4+a^5x^3\right)}+\frac{4\,b^3\log\left(bx+a\right)}{a^5}-\frac{4\,b^3\log(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) + 4
*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

mupad [B] time = 0.07, size = 69, normalized size = 1.00

$$\frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5}-\frac{\frac{1}{3\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}-\frac{2\,b\,x}{3\,a^2}}{b\,x^4+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3)^2,x)
```

```
[Out] (8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3)
/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

sympy [A] time = 0.33, size = 66, normalized size = 0.96

$$\frac{-a^3+2a^2bx-6ab^2x^2-12b^3x^3}{3a^5x^3+3a^4bx^4}+\frac{4b^3\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*b
*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

3.231 $\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx$

Optimal. Leaf size=84

$$\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}-\frac{3b^2}{2a^4x^2}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2*b^2/a^4/x^2+4*b^3/a^5/x+b^4/a^5/(b*x+a)+5*b^
4*ln(x)/a^6-5*b^4*ln(b*x+a)/a^6
```

Rubi [A] time = 0.05, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1584, 44}

$$-\frac{3b^2}{2a^4x^2}+\frac{b^4}{a^5(a+bx)}+\frac{4b^3}{a^5x}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}+\frac{2b}{3a^3x^3}-\frac{1}{4a^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] -1/(4*a^2*x^4) + (2*b)/(3*a^3*x^3) - (3*b^2)/(2*a^4*x^2) + (4*b^3)/(a^5*x)
+ b^4/(a^5*(a + b*x)) + (5*b^4*Log[x])/a^6 - (5*b^4*Log[a + b*x])/a^6
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(ax^2+bx^3\right)^2}\,dx &= \int \frac{1}{x^5(a+bx)^2}\,dx\\
&= \int \left(\frac{1}{a^2x^5}-\frac{2b}{a^3x^4}+\frac{3b^2}{a^4x^3}-\frac{4b^3}{a^5x^2}+\frac{5b^4}{a^6x}-\frac{b^5}{a^5(a+bx)^2}-\frac{5b^5}{a^6(a+bx)}\right)dx\\
&= -\frac{1}{4a^2x^4}+\frac{2b}{3a^3x^3}-\frac{3b^2}{2a^4x^2}+\frac{4b^3}{a^5x}+\frac{b^4}{a^5(a+bx)}+\frac{5b^4\log(x)}{a^6}-\frac{5b^4\log(a+bx)}{a^6}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 79, normalized size = 0.94

$$\frac{\frac{a\left(-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4\right)}{x^4(a+bx)}-60b^4\log(a+bx)+60b^4\log(x)}{12a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)^2),x]
```

```
[Out] ((a*(-3*a^4 + 5*a^3*b*x - 10*a^2*b^2*x^2 + 30*a*b^3*x^3 + 60*b^4*x^4))/(x^4
*(a + b*x)) + 60*b^4*Log[x] - 60*b^4*Log[a + b*x])/(12*a^6)
```

fricas [A] time = 0.38, size = 108, normalized size = 1.29

$$\frac{60\,ab^4x^4 + 30\,a^2b^3x^3 - 10\,a^3b^2x^2 + 5\,a^4bx - 3\,a^5 - 60\left(b^5x^5 + ab^4x^4\right)\log\left(bx + a\right) + 60\left(b^5x^5 + ab^4x^4\right)\log(x)}{12\left(a^6bx^5 + a^7x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(60*a*b^4*x^4 + 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5 -
60*(b^5*x^5 + a*b^4*x^4)*log(b*x + a) + 60*(b^5*x^5 + a*b^4*x^4)*log(x))/(a
^6*b*x^5 + a^7*x^4)
```

giac [A] time = 0.15, size = 86, normalized size = 1.02

$$-\frac{5\,b^4\log\left(|bx + a|\right)}{a^6} + \frac{5\,b^4\log\left(|x|\right)}{a^6} + \frac{60\,ab^4x^4 + 30\,a^2b^3x^3 - 10\,a^3b^2x^2 + 5\,a^4bx - 3\,a^5}{12\,(bx + a)a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

```
[Out] -5*b^4*log(abs(b*x + a))/a^6 + 5*b^4*log(abs(x))/a^6 + 1/12*(60*a*b^4*x^4 +
 30*a^2*b^3*x^3 - 10*a^3*b^2*x^2 + 5*a^4*b*x - 3*a^5)/((b*x + a)*a^6*x^4)
```

maple [A] time = 0.05, size = 79, normalized size = 0.94

$$\frac{b^4}{(bx + a)\,a^5} + \frac{5b^4\ln(x)}{a^6} - \frac{5b^4\ln\left(bx + a\right)}{a^6} + \frac{4b^3}{a^5x} - \frac{3b^2}{2a^4x^2} + \frac{2b}{3a^3x^3} - \frac{1}{4a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^2,x)
```

```
[Out] -1/4/a^2/x^4+2/3*b/a^3/x^3-3/2/a^4*b^2/x^2+4/a^5*b^3/x+b^4/a^5/(b*x+a)+5/a^
6*b^4*ln(x)-5*b^4*ln(b*x+a)/a^6
```

maxima [A] time = 1.37, size = 86, normalized size = 1.02

$$\frac{60\,b^4x^4 + 30\,ab^3x^3 - 10\,a^2b^2x^2 + 5\,a^3bx - 3\,a^4}{12\left(a^5bx^5 + a^6x^4\right)} - \frac{5\,b^4\log\left(bx + a\right)}{a^6} + \frac{5\,b^4\log(x)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(60*b^4*x^4 + 30*a*b^3*x^3 - 10*a^2*b^2*x^2 + 5*a^3*b*x - 3*a^4)/(a^5*
b*x^5 + a^6*x^4) - 5*b^4*log(b*x + a)/a^6 + 5*b^4*log(x)/a^6
```

mupad [B] time = 0.08, size = 79, normalized size = 0.94

$$\frac{\frac{5\,b^3\,x^3}{2\,a^4} - \frac{5\,b^2\,x^2}{6\,a^3} - \frac{1}{4\,a} + \frac{5\,b^4\,x^4}{a^5} + \frac{5\,b\,x}{12\,a^2}}{b\,x^5 + a\,x^4} - \frac{10\,b^4\,\mathrm{atanh}\left(\frac{2\,b\,x}{a} + 1\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)^2),x)
```

```
[Out] ((5*b^3*x^3)/(2*a^4) - (5*b^2*x^2)/(6*a^3) - 1/(4*a) + (5*b^4*x^4)/a^5 + (5
*b*x)/(12*a^2))/(a*x^4 + b*x^5) - (10*b^4*atanh((2*b*x)/a + 1))/a^6
```

sympy [A] time = 0.36, size = 80, normalized size = 0.95

$$\frac{-3a^4+5a^3bx-10a^2b^2x^2+30ab^3x^3+60b^4x^4}{12a^6x^4+12a^5bx^5}+\frac{5b^4\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**2,x)
```

```
[Out] (-3*a**4 + 5*a**3*b*x - 10*a**2*b**2*x**2 + 30*a*b**3*x**3 + 60*b**4*x**4)/
(12*a**6*x**4 + 12*a**5*b*x**5) + 5*b**4*(log(x) - log(a/b + x))/a**6
```

3.232 $\int x^2\sqrt{ax^2+bx^3}\,dx$

Optimal. Leaf size=105

$$-\frac{32a^3\left(ax^2+bx^3\right)^{3/2}}{315b^4x^3}+\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}$$

```
[Out] 2/9*(b*x^3+a*x^2)^(3/2)/b-32/315*a^3*(b*x^3+a*x^2)^(3/2)/b^4/x^3+16/105*a^2
*(b*x^3+a*x^2)^(3/2)/b^3/x^2-4/21*a*(b*x^3+a*x^2)^(3/2)/b^2/x
```

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{32a^3\left(ax^2+bx^3\right)^{3/2}}{315b^4x^3}+\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(9*b) - (32*a^3*(a*x^2 + b*x^3)^(3/2))/(315*b^4*x
^3) + (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^2) - (4*a*(a*x^2 + b*x^3)^(
3/2))/(21*b^2*x)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^2\sqrt{ax^2+bx^3}\,dx = \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b} - \frac{(2a)\int x\sqrt{ax^2+bx^3}\,dx}{3b}$$
$$= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b} - \frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x} + \frac{\left(8a^2\right)\int\sqrt{ax^2+bx^3}\,dx}{21b^2}$$
$$= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b} + \frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2} - \frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x} - \frac{\left(16a^3\right)\int\frac{\sqrt{ax^2+bx^3}}{x}\,dx}{105b^3}$$
$$= \frac{2\left(ax^2+bx^3\right)^{3/2}}{9b} - \frac{32a^3\left(ax^2+bx^3\right)^{3/2}}{315b^4x^3} + \frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^2} - \frac{4a\left(ax^2+bx^3\right)^{3/2}}{21b^2x}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.50

$$\frac{2\left(x^2(a+bx)\right)^{3/2}\left(-16a^3+24a^2bx-30ab^2x^2+35b^3x^3\right)}{315b^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(-16*a^3 + 24*a^2*b*x - 30*a*b^2*x^2 + 35*b^3*x^3)
)/(315*b^4*x^3)
```

fricas [A] time = 0.40, size = 62, normalized size = 0.59

$$\frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx^3+ax^2}}{315\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x^3 + a*x^2)/(b^4*x)
```

giac [A] time = 0.16, size = 131, normalized size = 1.25

$$\frac{32\,a^{\frac{9}{2}}\mathrm{sgn}(x)}{315\,b^4}+\frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a\mathrm{sgn}(x)}{b^3}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}\right.}{b^3}}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/315*a^(9/2)*sgn(x)/b^4 + 2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2
)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/b^3 + (35*(b*
x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x +
 a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^3)/b
```

maple [A] time = 0.04, size = 57, normalized size = 0.54

$$-\frac{2\,(bx+a)\left(-35b^3x^3+30a\,b^2x^2-24a^2bx+16a^3\right)\sqrt{b\,x^3+a\,x^2}}{315b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/315*(b*x+a)*(-35*b^3*x^3+30*a*b^2*x^2-24*a^2*b*x+16*a^3)*(b*x^3+a*x^2)^(
1/2)/b^4/x
```

maxima [A] time = 1.46, size = 53, normalized size = 0.50

$$\frac{2\left(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4\right)\sqrt{bx+a}}{315\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt(
b*x + a)/b^4
```

mupad [B] time = 5.49, size = 62, normalized size = 0.59

$$\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(-16\,a^4+8\,a^3\,b\,x-6\,a^2\,b^2\,x^2+5\,a\,b^3\,x^3+35\,b^4\,x^4\right)}{315\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(35*b^4*x^4 - 16*a^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2
 + 8*a^3*b*x))/(315*b^4*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{x^2\,(a+bx)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2*(a + b*x)), x)
```

3.233 $\int x\sqrt{ax^2+bx^3}\,dx$

Optimal. Leaf size=80

$$\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^3}-\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}$$

```
[Out] 16/105*a^2*(b*x^3+a*x^2)^(3/2)/b^3/x^3-8/35*a*(b*x^3+a*x^2)^(3/2)/b^2/x^2+2
/7*(b*x^3+a*x^2)^(3/2)/b/x
```

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^3}-\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(3/2))/(105*b^3*x^3) - (8*a*(a*x^2 + b*x^3)^(3/2))/
(35*b^2*x^2) + (2*(a*x^2 + b*x^3)^(3/2))/(7*b*x)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{ax^2+bx^3}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{(4a)\int\sqrt{ax^2+bx^3}\,dx}{7b}\\
&= -\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}+\frac{\left(8a^2\right)\int\frac{\sqrt{ax^2+bx^3}}{x}\,dx}{35b^2}\\
&= \frac{16a^2\left(ax^2+bx^3\right)^{3/2}}{105b^3x^3}-\frac{8a\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.52

$$\frac{2\left(x^2(a+bx)\right)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3*x^3)
```

fricas [A] time = 0.39, size = 51, normalized size = 0.64

$$\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{105\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2)/(b
^3*x)
```

giac [A] time = 0.16, size = 108, normalized size = 1.35

$$-\frac{16\,a^{\frac{7}{2}}\operatorname{sgn}(x)}{105\,b^3}+\frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a\operatorname{sgn}(x)}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)\operatorname{sgn}(x)}{b^2}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -16/105*a^(7/2)*sgn(x)/b^3 + 2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/
2)*a + 15*sqrt(b*x + a)*a^2)*a*sgn(x)/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x
+ a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b^2)/b
```

maple [A] time = 0.05, size = 46, normalized size = 0.58

$$\frac{2\,(bx+a)\left(15b^2x^2-12abx+8a^2\right)\sqrt{b\,x^3+a\,x^2}}{105b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 2/105*(b*x+a)*(15*b^2*x^2-12*a*b*x+8*a^2)*(b*x^3+a*x^2)^(1/2)/b^3/x
```

maxima [A] time = 1.44, size = 42, normalized size = 0.52

$$\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

mupad [B] time = 5.48, size = 51, normalized size = 0.64

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^3-4\,a^2\,b\,x+3\,a\,b^2\,x^2+15\,b^3\,x^3\right)}{105\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(8*a^3 + 15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x))/(1
05*b^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x)), x)
```

3.234 $\int \sqrt{ax^2+bx^3}\,dx$

Optimal. Leaf size=52

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}$$

```
[Out] -4/15*a*(b*x^3+a*x^2)^(3/2)/b^2/x^3+2/5*(b*x^3+a*x^2)^(3/2)/b/x^2
```

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2002, 2014}

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*
x^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{ax^2+bx^3}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{(2a)\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx}{5b}\\ &= -\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.60

$$\frac{2\left(x^2(a+bx)\right)^{3/2}(3bx-2a)}{15b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2)*(-2*a + 3*b*x))/(15*b^2*x^3)
```

fricas [A] time = 0.38, size = 39, normalized size = 0.75

$$\frac{2\left(3\,b^2x^2 + abx - 2\,a^2\right)\sqrt{bx^3 + ax^2}}{15\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2)/(b^2*x)
```

giac [A] time = 0.18, size = 81, normalized size = 1.56

$$\frac{4\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,b^2} + \frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}} - 3\,\sqrt{bx+a}\,a\right)a\mathrm{sgn}(x)}{b} + \frac{\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}\,a^2\right)\mathrm{sgn}(x)}{b}\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 4/15*a^(5/2)*sgn(x)/b^2 + 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a*s
gn(x)/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*sgn(x)/b)/b
```

maple [A] time = 0.05, size = 35, normalized size = 0.67

$$-\frac{2\,(bx+a)\,(-3bx+2a)\,\sqrt{b\,x^3 + a\,x^2}}{15b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2),x)
[Out] -2/15*(b*x+a)*(-3*b*x+2*a)*(b*x^3+a*x^2)^(1/2)/b^2/x
```

maxima [A] time = 1.42, size = 30, normalized size = 0.58

$$\frac{2\left(3\,b^2x^2 + abx - 2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

mupad [B] time = 5.30, size = 39, normalized size = 0.75

$$\frac{2\,\sqrt{b\,x^3 + a\,x^2}\,\left(-2\,a^2 + a\,b\,x + 3\,b^2\,x^2\right)}{15\,b^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2),x)
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(3*b^2*x^2 - 2*a^2 + a*b*x))/(15*b^2*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^2 + bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**3), x)
```

3.235 $\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx$

Optimal. Leaf size=25

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

```
[Out] 2/3*(b*x^3+a*x^2)^(3/2)/b/x^3
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x,x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3}}{x}\,dx = \frac{2\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{2\left(x^2(a+bx)\right)^{3/2}}{3bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x,x]
```

```
[Out] (2*(x^2*(a + b*x))^(3/2))/(3*b*x^3)
```

fricas [A] time = 0.39, size = 26, normalized size = 1.04

$$\frac{2\sqrt{bx^3+ax^2}\,(bx+a)}{3\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(b*x + a)/(b*x)
```

giac [B] time = 0.15, size = 50, normalized size = 2.00

$$-\frac{2\,a^{\frac{3}{2}}\mathrm{sgn}(x)}{3\,b}+\frac{2\left(3\,\sqrt{bx+a}\,a\mathrm{sgn}(x)+\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)\mathrm{sgn}(x)\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] -2/3*a^(3/2)*sgn(x)/b + 2/3*(3*sqrt(b*x + a)*a*sgn(x) + ((b*x + a)^(3/2) -
3*sqrt(b*x + a)*a)*sgn(x))/b
```

maple [A] time = 0.04, size = 27, normalized size = 1.08

$$\frac{2\,(bx+a)\,\sqrt{b\,x^{3}+a\,x^{2}}}{3bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x,x)
```

```
[Out] 2/3*(b*x+a)*(b*x^3+a*x^2)^(1/2)/b/x
```

maxima [A] time = 1.43, size = 12, normalized size = 0.48

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{b\,x^{3}+a\,x^{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}\,(a+bx)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x, x)
```

3.236 $\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

```
[Out] -2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))*a^(1/2)+2*(b*x^3+a*x^2)^(1/2)/x
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx &= \frac{2\sqrt{ax^2+bx^3}}{x}+a\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\ &= \frac{2\sqrt{ax^2+bx^3}}{x}-(2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.04

$$\frac{2x\left(-\sqrt{a}\sqrt{a+bx}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+a+bx\right)}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^2,x]
```

```
[Out] (2*x*(a + b*x - Sqrt[a]*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/Sqrt
[x^2*(a + b*x)]
```

fricas [A] time = 0.40, size = 111, normalized size = 2.18

$$\left[\frac{\sqrt{a}\,x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}}{x},\ \frac{2\left(\sqrt{-a}\,x\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\right)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*sq
rt(b*x^3 + a*x^2))/x, 2*(sqrt(-a)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*
x)) + sqrt(b*x^3 + a*x^2))/x]
```

giac [A] time = 0.16, size = 67, normalized size = 1.31

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\,\mathrm{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\,\sqrt{a}\right)\mathrm{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x)
 - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

maple [A] time = 0.04, size = 51, normalized size = 1.00

$$\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(-\sqrt{a}\,\ \mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\right)}{\sqrt{bx+a}\,\ x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^2,x)
```

```
[Out] 2*(b*x^3+a*x^2)^(1/2)*(-a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1/2
))/x/(b*x+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3+ax^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^2, x)
```

mupad [B] time = 5.36, size = 73, normalized size = 1.43

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}}{x}+\frac{\sqrt{a}\,\,\mathrm{asin}\left(\frac{\sqrt{a}\,\sqrt{\frac{1}{x}}\,1\mathrm{i}}{\sqrt{b}}\right)\,\sqrt{b\,x^3+a\,x^2}\,\left(\frac{1}{x}\right)^{3/2}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^2,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2))/x + (a^(1/2)*asin((a^(1/2)*(1/x)^(1/2)*1i)/b^(1/2
))*(a*x^2 + b*x^3)^(1/2)*(1/x)^(3/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2 (a + bx)}}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**2, x)
```

3.237 $\int \frac{\sqrt{ax^2+bx^3}}{x^3} dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

```
[Out] -b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(1/2)-(b*x^3+a*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/S
qrt[a]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{x^2}+\frac{1}{2}b\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\
&= -\frac{\sqrt{ax^2+bx^3}}{x^2}-b\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\
&= -\frac{\sqrt{ax^2+bx^3}}{x^2}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 48, normalized size = 0.92

$$-\frac{bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^3,x]
```

```
[Out] -((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[x^2*(a
+ b*x)])
```

fricas [A] time = 0.40, size = 127, normalized size = 2.44

$$\left[\frac{\sqrt{a}bx^2\log\left(\frac{bx^2+2ax-2\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)-2\sqrt{bx^3+ax^2}a}{2ax^2}, \frac{\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{ax}\right)-\sqrt{bx^3+ax^2}a}{ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2
) - 2*sqrt(b*x^3 + a*x^2)*a)/(a*x^2), (sqrt(-a)*b*x^2*arctan(sqrt(b*x^3 + a
*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*a)/(a*x^2)]
```

giac [A] time = 0.26, size = 45, normalized size = 0.87

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}-\frac{\sqrt{bx+a}\,b\,\mathrm{sgn}(x)}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - sqrt(b*x + a)*b*sgn(x
)/x)/b
```

maple [A] time = 0.05, size = 56, normalized size = 1.08

$$-\frac{\sqrt{b\,x^3+a\,x^2}\,\left(bx\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\,\sqrt{a}\right)}{\sqrt{bx+a}\,\sqrt{a}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^3,x)
```

```
[Out] -(b*x^3+a*x^2)^(1/2)*(arctanh((b*x+a)^(1/2)/a^(1/2))*x*b+(b*x+a)^(1/2)*a^(1
/2))/x^2/(b*x+a)^(1/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{bx^3+ax^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,x^3+a\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^3,x)
[Out] int((a*x^2 + b*x^3)^(1/2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(a+bx)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**3,x)
[Out] Integral(sqrt(x**2*(a + b*x))/x**3, x)
```

3.238 $\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx$

Optimal. Leaf size=84

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

```
[Out] 1/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)-1/2*(b*x^3+a*x^2)^(1
/2)/x^3-1/4*b*(b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}-\frac{\sqrt{ax^2+bx^3}}{2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^4,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(2*x^3) - (b*Sqrt[a*x^2 + b*x^3])/(4*a*x^2) + (b^2*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx = -\frac{\sqrt{ax^2+bx^3}}{2x^3}+\frac{1}{4}b\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$$
$$= -\frac{\sqrt{ax^2+bx^3}}{2x^3}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}-\frac{b^2\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{2x^3}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{2x^3}-\frac{b\sqrt{ax^2+bx^3}}{4ax^2}+\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{3/2}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.50

$$-\frac{2b^2\left(x^2(a+bx)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},3;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^4,x]
[Out] (-2*b^2*(x^2*(a + b*x))^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/
(3*a^3*x^3)
```

fricas [A] time = 0.41, size = 149, normalized size = 1.77

$$\left[\frac{\sqrt{a}\,b^2x^3\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\left(abx+2\,a^2\right)}{8\,a^2x^3},-\frac{\sqrt{-a}\,b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+\cdots}}{4\,a^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
[Out] [1/8*(sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) - 2*sqrt(b*x^3 + a*x^2)*(a*b*x + 2*a^2))/(a^2*x^3), -1/4*(sqrt(-a)*b^2*
x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(a*b*x
 + 2*a^2))/(a^2*x^3)]
```

giac [A] time = 0.21, size = 72, normalized size = 0.86

$$-\frac{\frac{b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a}+\frac{(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x)+\sqrt{bx+a}\,ab^3\mathrm{sgn}(x)}{ab^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
[Out] -1/4*(b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + ((b*x + a)^(
3/2)*b^3*sgn(x) + sqrt(b*x + a)*a*b^3*sgn(x))/(a*b^2*x^2))/b
```

maple [A] time = 0.06, size = 73, normalized size = 0.87

$$-\frac{\sqrt{b\,x^3+a\,x^2}\,\left(-a\,b^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\;a^{\frac{5}{2}}+(bx+a)^{\frac{3}{2}}\,a^{\frac{3}{2}}\right)}{4\sqrt{bx+a}\;a^{\frac{5}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^4,x)
```

```
[Out] -1/4*(b*x^3+a*x^2)^(1/2)*((b*x+a)^(3/2)*a^(3/2)-arctanh((b*x+a)^(1/2)/a^(1/
2))*a*x^2*b^2+(b*x+a)^(1/2)*a^(5/2))/x^3/(b*x+a)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^3 + ax^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^3 + a\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2\,(a + bx)}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**4, x)
```

3.239 $\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$

Optimal. Leaf size=112

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

```
[Out] -1/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)-1/3*(b*x^3+a*x^2)^(
1/2)/x^4-1/12*b*(b*x^3+a*x^2)^(1/2)/a/x^3+1/8*b^2*(b*x^3+a*x^2)^(1/2)/a^2/x
^2
```

Rubi [A] time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{\sqrt{ax^2+bx^3}}{3x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(3*x^4) - (b*Sqrt[a*x^2 + b*x^3])/(12*a*x^3) + (b^2*Sq
rt[a*x^2 + b*x^3])/(8*a^2*x^2) - (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]])/(8*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx = -\frac{\sqrt{ax^2+bx^3}}{3x^4}+\frac{1}{6}b\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$$
$$= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}-\frac{b^2\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}+\frac{b^3\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^2}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a^2}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{3x^4}-\frac{b\sqrt{ax^2+bx^3}}{12ax^3}+\frac{b^2\sqrt{ax^2+bx^3}}{8a^2x^2}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{5/2}}$$

Mathematica [C] time = 0.01, size = 42, normalized size = 0.38

$$\frac{2b^3\left(x^2(a+bx)\right)^{3/2}\,{}_2F_1\left(\frac{3}{2},4;\frac{5}{2};\frac{bx}{a}+1\right)}{3a^4x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^3]/x^5,x]
```

```
[Out] (2*b^3*(x^2*(a + b*x))^(3/2)*Hypergeometric2F1[3/2, 4, 5/2, 1 + (b*x)/a])/(
3*a^4*x^3)
```

fricas [A] time = 0.42, size = 175, normalized size = 1.56

$$\left[\frac{3\sqrt{a}\,b^3x^4\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\left(3\,ab^2x^2-2\,a^2bx-8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^3x^4},\ \frac{3\sqrt{-a}\,b^3x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*(3*a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)
, 1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (3*
a*b^2*x^2 - 2*a^2*b*x - 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^3*x^4)]
```

giac [A] time = 0.27, size = 92, normalized size = 0.82

$$\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\text{sgn}(x)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{5}{2}}b^4\text{sgn}(x)-8\,(bx+a)^{\frac{3}{2}}ab^4\text{sgn}(x)-3\,\sqrt{bx+a}\,a^2b^4\text{sgn}(x)}{a^2b^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
+ a)^(5/2)*b^4*sgn(x) - 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a^
2*b^4*sgn(x))/(a^2*b^3*x^3))/b
```

maple [A] time = 0.06, size = 89, normalized size = 0.79

$$\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left(-3a^{2}b^{3}x^{3}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\sqrt{bx+a}\,a^{\frac{9}{2}}-8\left(bx+a\right)^{\frac{3}{2}}a^{\frac{7}{2}}+3\left(bx+a\right)^{\frac{5}{2}}a^{\frac{5}{2}}\right)}{24\sqrt{bx+a}\,a^{\frac{9}{2}}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(1/2)/x^5,x)
```

```
[Out] 1/24*(b*x^3+a*x^2)^(1/2)*(3*(b*x+a)^(5/2)*a^(5/2)-8*(b*x+a)^(3/2)*a^(7/2)-3
*arctanh((b*x+a)^(1/2)/a^(1/2))*a^2*b^3*x^3-3*(b*x+a)^(1/2)*a^(9/2))/x^4/(b
*x+a)^(1/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^{3}+ax^{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^3 + a*x^2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^{3}+a\,x^{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(1/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3)^(1/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^{2}\left(a+bx\right)}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(x**2*(a + b*x))/x**5, x)
```

3.240 $\int x^2 \left(ax^2+bx^3\right)^{3/2} dx$

Optimal. Leaf size=161

$$-\frac{512a^5\left(ax^2+bx^3\right)^{5/2}}{45045b^6x^5}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}$$

```
[Out] 2/15*(b*x^3+a*x^2)^(5/2)/b-512/45045*a^5*(b*x^3+a*x^2)^(5/2)/b^6/x^5+256/90
09*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^4-64/1287*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^3+
32/429*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^2-4/39*a*(b*x^3+a*x^2)^(5/2)/b^2/x
```

Rubi [A] time = 0.23, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2016, 2002, 2014}

$$-\frac{512a^5\left(ax^2+bx^3\right)^{5/2}}{45045b^6x^5}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(15*b) - (512*a^5*(a*x^2 + b*x^3)^(5/2))/(45045*b
^6*x^5) + (256*a^4*(a*x^2 + b*x^3)^(5/2))/(9009*b^5*x^4) - (64*a^3*(a*x^2 +
 b*x^3)^(5/2))/(1287*b^4*x^3) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^2
) - (4*a*(a*x^2 + b*x^3)^(5/2))/(39*b^2*x)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x^2\left(ax^2+bx^3\right)^{3/2}dx=\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{(2a)\int x\left(ax^2+bx^3\right)^{3/2}dx}{3b}$$
$$=\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+\frac{\left(16a^2\right)\int\left(ax^2+bx^3\right)^{3/2}dx}{39b^2}$$
$$=\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}-\frac{\left(32a^3\right)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x}dx}{143b^3}$$
$$=\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{39b^2x}+$$
$$=\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5/2}}{1287b^4x^3}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^2}$$
$$=\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{512a^5\left(ax^2+bx^3\right)^{5/2}}{45045b^6x^5}+\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{9009b^5x^4}-\frac{64a^3\left(ax^2+bx^3\right)^{5}}{1287b^4x^3}$$

Mathematica [A] time = 0.05, size = 80, normalized size = 0.50

$$\frac{2x(a+bx)^3\left(-256a^5+640a^4bx-1120a^3b^2x^2+1680a^2b^3x^3-2310ab^4x^4+3003b^5x^5\right)}{45045b^6\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(-256*a^5 + 640*a^4*b*x - 1120*a^3*b^2*x^2 + 1680*a^2*b^3*
x^3 - 2310*a*b^4*x^4 + 3003*b^5*x^5))/(45045*b^6*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.40, size = 95, normalized size = 0.59

$$\frac{2\left(3003\,b^7x^7+3696\,ab^6x^6+63\,a^2b^5x^5-70\,a^3b^4x^4+80\,a^4b^3x^3-96\,a^5b^2x^2+128\,a^6bx-256\,a^7\right)\sqrt{bx^3+ax^2}}{45045\,b^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x^3 + a*x^2
)/(b^6*x)
```

giac [B] time = 0.18, size = 282, normalized size = 1.75

$$\frac{512\,a^{\frac{15}{2}}\operatorname{sgn}(x)}{45045\,b^6}+\frac{2\left(\frac{65\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}\,a^5\right)a^2\operatorname{sgn}(x)}{b^5}+\frac{30\left(231\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 512/45045*a^(15/2)*sgn(x)/b^6 + 2/45045*(65*(63*(b*x + a)^(11/2) - 385*(b*x
 + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(
b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2*sgn(x)/b^5 + 30*(231*(b*x +
```

```
 a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x
 + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 300
3*sqrt(b*x + a)*a^6)*a*sgn(x)/b^5 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x + a
)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175
*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^
6 - 6435*sqrt(b*x + a)*a^7)*sgn(x)/b^5)/b
```

maple [A] time = 0.05, size = 79, normalized size = 0.49

$$-\frac{2\,(bx+a)\left(-3003x^5b^5+2310a\,b^4x^4-1680a^2b^3x^3+1120a^3b^2x^2-640a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{45045b^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2/45045*(b*x+a)*(-3003*b^5*x^5+2310*a*b^4*x^4-1680*a^2*b^3*x^3+1120*a^3*b^
2*x^2-640*a^4*b*x+256*a^5)*(b*x^3+a*x^2)^(3/2)/b^6/x^3
```

maxima [A] time = 1.52, size = 86, normalized size = 0.53

$$\frac{2\left(3003\,b^7x^7+3696\,ab^6x^6+63\,a^2b^5x^5-70\,a^3b^4x^4+80\,a^4b^3x^3-96\,a^5b^2x^2+128\,a^6bx-256\,a^7\right)\sqrt{bx+a}}{45045\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)/b^6
```

mupad [B] time = 5.24, size = 80, normalized size = 0.50

$$-\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2\,\left(256\,a^5-640\,a^4\,b\,x+1120\,a^3\,b^2\,x^2-1680\,a^2\,b^3\,x^3+2310\,a\,b^4\,x^4-3003\,b^5\,x^5\right)}{45045\,b^6\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(256*a^5 - 3003*b^5*x^5 + 2310*a*b^4*
x^4 + 1120*a^3*b^2*x^2 - 1680*a^2*b^3*x^3 - 640*a^4*b*x))/(45045*b^6*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\left(x^2\,(a+bx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**2*(x**2*(a + b*x))**(3/2), x)
```

3.241 $\int x\left(ax^2+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=136

$$\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}$$

```
[Out] 256/15015*a^4*(b*x^3+a*x^2)^(5/2)/b^5/x^5-128/3003*a^3*(b*x^3+a*x^2)^(5/2)/
b^4/x^4+32/429*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^3-16/143*a*(b*x^3+a*x^2)^(5/2)
/b^2/x^2+2/13*(b*x^3+a*x^2)^(5/2)/b/x
```

Rubi [A] time = 0.17, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2016, 2002, 2014}

$$\frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (256*a^4*(a*x^2 + b*x^3)^(5/2))/(15015*b^5*x^5) - (128*a^3*(a*x^2 + b*x^3)^
(5/2))/(3003*b^4*x^4) + (32*a^2*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^3) - (16*
a*(a*x^2 + b*x^3)^(5/2))/(143*b^2*x^2) + (2*(a*x^2 + b*x^3)^(5/2))/(13*b*x)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int x\left(ax^2+bx^3\right)^{3/2}dx = \frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx} - \frac{(8a)\int\left(ax^2+bx^3\right)^{3/2}dx}{13b}$$
$$= -\frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2} + \frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx} + \frac{(48a^2)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x}dx}{143b^2}$$
$$= \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3} - \frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2} + \frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx} - \frac{(64a^3)\int\frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}dx}{429b^3}$$
$$= -\frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4} + \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3} - \frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2} + \frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx} +$$
$$= \frac{256a^4\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5} - \frac{128a^3\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4} + \frac{32a^2\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3} - \frac{16a\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}$$

Mathematica [A] time = 0.04, size = 69, normalized size = 0.51

$$\frac{2x(a+bx)^3\left(128a^4-320a^3bx+560a^2b^2x^2-840ab^3x^3+1155b^4x^4\right)}{15015b^5\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(a + b*x)^3*(128*a^4 - 320*a^3*b*x + 560*a^2*b^2*x^2 - 840*a*b^3*x^3 +
 1155*b^4*x^4))/(15015*b^5*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.40, size = 84, normalized size = 0.62

$$\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx^3+ax^2}}{15015\,b^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

giac [B] time = 0.19, size = 246, normalized size = 1.81

$$-\frac{256\,a^{\frac{13}{2}}\,\mathrm{sgn}(x)}{15015\,b^5}+\frac{2\left(\frac{143\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{bx+a}\,a^4\right)a^2\mathrm{sgn}(x)}{b^4}+\frac{130\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] -256/15015*a^(13/2)*sgn(x)/b^5 + 2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b*
x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sq
rt(b*x + a)*a^4)*a^2*sgn(x)/b^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^
(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x +
a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a*sgn(x)/b^4 + 15*(231*(b*x + a)^(13/
2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7
/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b
*x + a)*a^6)*sgn(x)/b^4)/b
```

maple [A] time = 0.05, size = 68, normalized size = 0.50

$$\frac{2\,(bx+a)\left(1155x^4b^4-840a\,b^3x^3+560a^2x^2b^2-320a^3xb+128a^4\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{15015b^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 2/15015*(b*x+a)*(1155*b^4*x^4-840*a*b^3*x^3+560*a^2*b^2*x^2-320*a^3*b*x+128
*a^4)*(b*x^3+a*x^2)^(3/2)/b^5/x^3
```

maxima [A] time = 1.55, size = 75, normalized size = 0.55

$$\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx+a}}{15015\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)/b^5
```

mupad [B] time = 5.24, size = 69, normalized size = 0.51

$$\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2\,\left(128\,a^4-320\,a^3\,b\,x+560\,a^2\,b^2\,x^2-840\,a\,b^3\,x^3+1155\,b^4\,x^4\right)}{15015\,b^5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(128*a^4 + 1155*b^4*x^4 - 840*a*b^3*x^
3 + 560*a^2*b^2*x^2 - 320*a^3*b*x))/(15015*b^5*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(x^2\,(a+bx)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x*(x**2*(a + b*x))**(3/2), x)
```

3.242 $\int \left(ax^2+bx^3\right)^{3/2}\,dx$

Optimal. Leaf size=108

$$-\frac{32a^3\left(ax^2+bx^3\right)^{5/2}}{1155b^4x^5}+\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

```
[Out] -32/1155*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^5+16/231*a^2*(b*x^3+a*x^2)^(5/2)/b^3
/x^4-4/33*a*(b*x^3+a*x^2)^(5/2)/b^2/x^3+2/11*(b*x^3+a*x^2)^(5/2)/b/x^2
```

Rubi [A] time = 0.14, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2002, 2016, 2014}

$$-\frac{32a^3\left(ax^2+bx^3\right)^{5/2}}{1155b^4x^5}+\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-32*a^3*(a*x^2 + b*x^3)^(5/2))/(1155*b^4*x^5) + (16*a^2*(a*x^2 + b*x^3)^(5
/2))/(231*b^3*x^4) - (4*a*(a*x^2 + b*x^3)^(5/2))/(33*b^2*x^3) + (2*(a*x^2 +
 b*x^3)^(5/2))/(11*b*x^2)
```

Rule 2002

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*
(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n -
 j)], 0] && NeQ[j*p + 1, 0]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \left(ax^2+bx^3\right)^{3/2} dx = \frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2} - \frac{(6a)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x} dx}{11b}$$

$$= -\frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3} + \frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2} + \frac{\left(8a^2\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2} dx}{33b^2}$$

$$= \frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4} - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3} + \frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2} - \frac{\left(16a^3\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3} dx}{231b^3}$$

$$= -\frac{32a^3\left(ax^2+bx^3\right)^{5/2}}{1155b^4x^5} + \frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4} - \frac{4a\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3} + \frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.54

$$\frac{2x(a+bx)^3\left(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3\right)}{1155b^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(a + b*x)^3*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(1155
*b^4*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.39, size = 73, normalized size = 0.68

$$\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx^3+ax^2}}{1155\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

giac [B] time = 0.23, size = 210, normalized size = 1.94

$$\frac{32\,a^{\frac{11}{2}}\,\mathrm{sgn}(x)}{1155\,b^4} + \frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}\,a^3\right)a^2\mathrm{sgn}(x)}{b^3} + \frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-42\cdots\right.}{b^3}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] 32/1155*a^(11/2)*sgn(x)/b^4 + 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^
(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2*sgn(x)/b^3 + 2
2*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 4
20*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*sgn(x)/b^3 + 5*(63*(b*x +
 a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x +
a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*sgn(x)/b^3
)/b
```

maple [A] time = 0.04, size = 57, normalized size = 0.53

$$-\frac{2\,(bx+a)\left(-105b^3x^3+70a\,b^2x^2-40a^2bx+16a^3\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{1155b^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2/1155*(b*x+a)*(-105*b^3*x^3+70*a*b^2*x^2-40*a^2*b*x+16*a^3)*(b*x^3+a*x^2)
^(3/2)/b^4/x^3
```

maxima [A] time = 1.44, size = 64, normalized size = 0.59

$$\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}}{1155\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^4
*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

mupad [B] time = 5.19, size = 58, normalized size = 0.54

$$-\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2\,\left(16\,a^3-40\,a^2\,b\,x+70\,a\,b^2\,x^2-105\,b^3\,x^3\right)}{1155\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(16*a^3 - 105*b^3*x^3 + 70*a*b^2*x^2
- 40*a^2*b*x))/(1155*b^4*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax^2+bx^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3)**(3/2), x)
```

3.243 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x} \, dx$

Optimal. Leaf size=80

$$\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{315b^3x^5}-\frac{8a\left(ax^2+bx^3\right)^{5/2}}{63b^2x^4}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}$$

```
[Out] 16/315*a^2*(b*x^3+a*x^2)^(5/2)/b^3/x^5-8/63*a*(b*x^3+a*x^2)^(5/2)/b^2/x^4+2
/9*(b*x^3+a*x^2)^(5/2)/b/x^3
```

Rubi [A] time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{315b^3x^5}-\frac{8a\left(ax^2+bx^3\right)^{5/2}}{63b^2x^4}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
[Out] (16*a^2*(a*x^2 + b*x^3)^(5/2))/(315*b^3*x^5) - (8*a*(a*x^2 + b*x^3)^(5/2))/
(63*b^2*x^4) + (2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x} \, dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{(4a)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2} \, dx}{9b}\\
&= -\frac{8a\left(ax^2+bx^3\right)^{5/2}}{63b^2x^4}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}+\frac{\left(8a^2\right)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3} \, dx}{63b^2}\\
&= \frac{16a^2\left(ax^2+bx^3\right)^{5/2}}{315b^3x^5}-\frac{8a\left(ax^2+bx^3\right)^{5/2}}{63b^2x^4}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 0.59

$$\frac{2x(a+bx)^3\left(8a^2-20abx+35b^2x^2\right)}{315b^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x,x]
```

```
[Out] (2*x*(a + b*x)^3*(8*a^2 - 20*a*b*x + 35*b^2*x^2))/(315*b^3*Sqrt[x^2*(a + b*
x)])
```

fricas [A] time = 0.38, size = 62, normalized size = 0.78

$$\frac{2\left(35\,b^4x^4 + 50\,ab^3x^3 + 3\,a^2b^2x^2 - 4\,a^3bx + 8\,a^4\right)\sqrt{bx^3 + ax^2}}{315\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x^3 + a*x^2)/(b^3*x)
```

giac [B] time = 0.24, size = 173, normalized size = 2.16

$$-\frac{16\,a^{\frac{9}{2}}\operatorname{sgn}(x)}{315\,b^3} + \frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}\,a^2\right)a^2\operatorname{sgn}(x)}{b^2} + \frac{18\left(5\,(bx+a)^{\frac{7}{2}} - 21\,(bx+a)^{\frac{5}{2}}a + 35\,(bx+a)^{\frac{3}{2}}a^2 - 35\,\sqrt{bx+a}\,a^3\right)a\operatorname{sgn}(x)}{b^2} + \right.}{315\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

```
[Out] -16/315*a^(9/2)*sgn(x)/b^3 + 2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3
/2)*a + 15*sqrt(b*x + a)*a^2)*a^2*sgn(x)/b^2 + 18*(5*(b*x + a)^(7/2) - 21*(
b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*sgn(x)/
b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*sgn(x)/b^2)/b
```

maple [A] time = 0.04, size = 46, normalized size = 0.58

$$\frac{2\,(bx + a)\left(35b^2x^2 - 20abx + 8a^2\right)\left(b\,x^3 + a\,x^2\right)^{\frac{3}{2}}}{315b^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x,x)
```

```
[Out] 2/315*(b*x+a)*(35*b^2*x^2-20*a*b*x+8*a^2)*(b*x^3+a*x^2)^(3/2)/b^3/x^3
```

maxima [A] time = 1.55, size = 53, normalized size = 0.66

$$\frac{2\left(35\,b^4x^4 + 50\,ab^3x^3 + 3\,a^2b^2x^2 - 4\,a^3bx + 8\,a^4\right)\sqrt{bx + a}}{315\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(
b*x + a)/b^3
```

mupad [B] time = 5.18, size = 47, normalized size = 0.59

$$\frac{2\,\sqrt{b\,x^3 + a\,x^2}\,\left(a + b\,x\right)^2\,\left(8\,a^2 - 20\,a\,b\,x + 35\,b^2\,x^2\right)}{315\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(8*a^2 + 35*b^2*x^2 - 20*a*b*x))/(315*
b^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x, x)
```

3.244 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=52

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}$$

```
[Out] -4/35*a*(b*x^3+a*x^2)^(5/2)/b^2/x^5+2/7*(b*x^3+a*x^2)^(5/2)/b/x^4
```

Rubi [A] time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2014}

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*
x^4)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx &= \frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{(2a)\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx}{7b}\\ &= -\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}+\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.69

$$\frac{2x(a+bx)^3(5bx-2a)}{35b^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^2,x]
```

```
[Out] (2*x*(a + b*x)^3*(-2*a + 5*b*x))/(35*b^2*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.39, size = 50, normalized size = 0.96

$$\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx^3+ax^2}}{35\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*x^2)/(b^2*x
)
```

giac [B] time = 0.18, size = 136, normalized size = 2.62

$$\frac{4\,a^{\frac{7}{2}}\mathrm{sgn}(x)}{35\,b^2}+\frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a^2\mathrm{sgn}(x)}{b}+\frac{14\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}\,a^2\right)a\mathrm{sgn}(x)}{b}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\right)}{}\right)}{105\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 4/35*a^(7/2)*sgn(x)/b^2 + 2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a
^2*sgn(x)/b + 14*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x +
a)*a^2)*a*sgn(x)/b + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x
+ a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*sgn(x)/b)/b
```

maple [A] time = 0.04, size = 35, normalized size = 0.67

$$-\frac{2\,(bx+a)\,(-5bx+2a)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{35b^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^2,x)
```

```
[Out] -2/35*(b*x+a)*(-5*b*x+2*a)*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

maxima [A] time = 1.47, size = 41, normalized size = 0.79

$$\frac{2\left(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3\right)\sqrt{bx+a}}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)/b^2
```

mupad [B] time = 5.17, size = 36, normalized size = 0.69

$$-\frac{2\,\left(2\,a-5\,b\,x\right)\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2}{35\,b^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^2,x)
```

```
[Out] -(2*(2*a - 5*b*x)*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(35*b^2*x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**2,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**2, x)
```

3.245 $\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=25

$$\frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

```
[Out] 2/5*(b*x^3+a*x^2)^(5/2)/b/x^5
```

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2014}

$$\frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (2*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5)
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^3}\,dx = \frac{2(ax^2+bx^3)^{5/2}}{5bx^5}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$\frac{2(x^2(a+bx))^{5/2}}{5bx^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^3,x]
```

```
[Out] (2*(x^2*(a + b*x))^(5/2))/(5*b*x^5)
```

fricas [A] time = 0.40, size = 37, normalized size = 1.48

$$\frac{2\left(b^2x^2+2\,abx+a^2\right)\sqrt{bx^3+ax^2}}{5\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x^3 + a*x^2)/(b*x)
```

giac [B] time = 0.16, size = 89, normalized size = 3.56

$$-\frac{2\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{5\,b}+\frac{2\left(15\,\sqrt{bx+a}\,a^{2}\mathrm{sgn}(x)+10\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}\,a\right)a\mathrm{sgn}(x)+\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\right.}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] -2/5*a^(5/2)*sgn(x)/b + 2/15*(15*sqrt(b*x + a)*a^2*sgn(x) + 10*((b*x + a)^(
3/2) - 3*sqrt(b*x + a)*a)*a*sgn(x) + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2
)*a + 15*sqrt(b*x + a)*a^2)*sgn(x))/b
```

maple [A] time = 0.04, size = 27, normalized size = 1.08

$$\frac{2\,(bx+a)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{5b\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^3,x)
```

```
[Out] 2/5*(b*x+a)*(b*x^3+a*x^2)^(3/2)/b/x^3
```

maxima [A] time = 1.39, size = 28, normalized size = 1.12

$$\frac{2\left(b^{2}x^{2}+2\,abx+a^{2}\right)\sqrt{bx+a}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)/b
```

mupad [B] time = 5.62, size = 28, normalized size = 1.12

$$\frac{2\,\sqrt{b\,x^{3}+a\,x^{2}}\,\left(a+b\,x\right)^{2}}{5\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^3,x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2)/(5*b*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{2}\,(a+bx)\right)^{\frac{3}{2}}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**3,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**3, x)
```

3.246 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=74

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)+\frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}$$

```
[Out] 2/3*(b*x^3+a*x^2)^(3/2)/x^3-2*a^(3/2)*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2)
)+2*a*(b*x^3+a*x^2)^(1/2)/x
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2021, 2008, 206}

$$-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)+\frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*a*Sqrt[a*x^2 + b*x^3])/x + (2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) - 2*a^(3/2)
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx &= \frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}+a\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx\\
&= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}+a^2\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\
&= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}-\left(2a^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\
&= \frac{2a\sqrt{ax^2+bx^3}}{x}+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}-2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 0.92

$$\frac{2x\sqrt{a+bx}\left(\sqrt{a+bx}\,(4a+bx)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^4,x]
```

```
[Out] (2*x*Sqrt[a + b*x]*(Sqrt[a + b*x]*(4*a + b*x) - 3*a^(3/2)*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]]))/(3*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.41, size = 130, normalized size = 1.76

$$\left[\frac{3\,a^{\frac{3}{2}}x\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\,(bx+4\,a)}{3\,x},\ \frac{2\left(3\,\sqrt{-a}\,ax\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\,(b\right.}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)
+ 2*sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x, 2/3*(3*sqrt(-a)*a*x*arctan(sqrt(b*x
^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(b*x + 4*a))/x]
```

giac [A] time = 0.17, size = 85, normalized size = 1.15

$$\frac{2\,a^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+\frac{2}{3}\,(bx+a)^{\frac{3}{2}}\mathrm{sgn}(x)+2\,\sqrt{bx+a}\,a\mathrm{sgn}(x)-\frac{2\left(3\,a^2\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+4\,\sqrt{-a}\,a^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{3\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] 2*a^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*(b*x + a)^(3/2)*
sgn(x) + 2*sqrt(b*x + a)*a*sgn(x) - 2/3*(3*a^2*arctan(sqrt(a)/sqrt(-a)) + 4
*sqrt(-a)*a^(3/2))*sgn(x)/sqrt(-a)
```

maple [A] time = 0.05, size = 61, normalized size = 0.82

$$\frac{2\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(-3a^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+3\sqrt{bx+a}\,a+(bx+a)^{\frac{3}{2}}\right)}{3\,(bx+a)^{\frac{3}{2}}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^4,x)
```

```
[Out] 2/3*(b*x^3+a*x^2)^(3/2)*(-3*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^
(3/2)+3*(b*x+a)^(1/2)*a)/x^3/(b*x+a)^(3/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\,x^2\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^4,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**4,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**4, x)
```

3.247 $\int \frac{(ax^2+bx^3)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=73

$$\frac{3b\sqrt{ax^2+bx^3}}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}$$

```
[Out] -(b*x^3+a*x^2)^(3/2)/x^4-3*b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))*a^(1/2)
+3*b*(b*x^3+a*x^2)^(1/2)/x
```

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2021, 2008, 206}

$$-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{3b\sqrt{ax^2+bx^3}}{x}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (3*b*Sqrt[a*x^2 + b*x^3])/x - (a*x^2 + b*x^3)^(3/2)/x^4 - 3*Sqrt[a]*b*ArcTa
nh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2021

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a
*(n - j)*p)/(c^j*(m + n*p + 1)), Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Inte
gersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx &= -\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{1}{2}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^2}\,dx\\ &= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}+\frac{1}{2}(3ab)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx\\ &= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}-(3ab)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{3b\sqrt{ax^2+bx^3}}{x}-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}-3\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.55

$$\frac{2b\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(2,\frac{5}{2};\frac{7}{2};\frac{bx}{a}+1\right)}{5a^2x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^5,x]
```

```
[Out] (2*b*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[2, 5/2, 7/2, 1 + (b*x)/a])/(5*
a^2*x^5)
```

fricas [A] time = 0.41, size = 136, normalized size = 1.86

$$\left[\frac{3\sqrt{a}\,bx^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\,(2\,bx-a)}{2\,x^2},\frac{3\,\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\right.}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/2*(3*sqrt(a)*b*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x
^2) + 2*sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2, (3*sqrt(-a)*b*x^2*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(2*b*x - a))/x^2]
```

giac [A] time = 0.21, size = 62, normalized size = 0.85

$$\frac{\frac{3\,ab^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\,b^2\mathrm{sgn}(x)-\frac{\sqrt{bx+a}\,ab\mathrm{sgn}(x)}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] (3*a*b^2*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*b
^2*sgn(x) - sqrt(b*x + a)*a*b*sgn(x)/x)/b
```

maple [A] time = 0.06, size = 72, normalized size = 0.99

$$-\frac{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(3abx\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-2\sqrt{bx+a}\,\sqrt{a}\,bx+\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{(bx+a)^{\frac{3}{2}}\,\sqrt{a}\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^5,x)
```

```
[Out] -(b*x^3+a*x^2)^(3/2)*((b*x+a)^(1/2)*a^(3/2)-2*(b*x+a)^(1/2)*x*b*a^(1/2)+3*a
rctanh((b*x+a)^(1/2)/a^(1/2))*x*a*b)/x^4/(b*x+a)^(3/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + ax^2\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3 + a\,x^2\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^5,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\,(a + bx)\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**5,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**5, x)
```

3.248 $\int \frac{(ax^2+bx^3)^{3/2}}{x^6} \, dx$

Optimal. Leaf size=81

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}-\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

```
[Out] -1/2*(b*x^3+a*x^2)^(3/2)/x^5-3/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))
/a^(1/2)-3/4*b*(b*x^3+a*x^2)^(1/2)/x^2
```

Rubi [A] time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2020, 2008, 206}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}-\frac{3b\sqrt{ax^2+bx^3}}{4x^2}-\frac{(ax^2+bx^3)^{3/2}}{2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(4*x^2) - (a*x^2 + b*x^3)^(3/2)/(2*x^5) - (3*b^2
*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^6}\,dx = -\frac{(ax^2+bx^3)^{3/2}}{2x^5} + \frac{1}{4}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^3}\,dx$$
$$= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2} - \frac{(ax^2+bx^3)^{3/2}}{2x^5} + \frac{1}{8}\left(3b^2\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$
$$= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2} - \frac{(ax^2+bx^3)^{3/2}}{2x^5} - \frac{1}{4}\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)$$
$$= -\frac{3b\sqrt{ax^2+bx^3}}{4x^2} - \frac{(ax^2+bx^3)^{3/2}}{2x^5} - \frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4\sqrt{a}}$$

Mathematica [A] time = 0.05, size = 72, normalized size = 0.89

$$-\frac{2a^2+3b^2x^2\sqrt{\frac{bx}{a}+1}\,\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+7abx+5b^2x^2}{4x\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^6,x]
```

```
[Out] -1/4*(2*a^2 + 7*a*b*x + 5*b^2*x^2 + 3*b^2*x^2*Sqrt[1 + (b*x)/a]*ArcTanh[Sqr
t[1 + (b*x)/a]])/(x*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.42, size = 154, normalized size = 1.90

$$\left[\frac{3\sqrt{a}\,b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\left(5\,abx+2\,a^2\right)}{8\,ax^3}, \frac{3\,\sqrt{-a}\,b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)-\sqrt{bx^3+\ldots}}{4\,ax^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) - 2*sqrt(b*x^3 + a*x^2)*(5*a*b*x + 2*a^2))/(a*x^3), 1/4*(3*sqrt(-a)*b
^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) - sqrt(b*x^3 + a*x^2)*(5*
a*b*x + 2*a^2))/(a*x^3)]
```

giac [A] time = 0.26, size = 70, normalized size = 0.86

$$\frac{\frac{3\,b^3\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}(x)-3\,\sqrt{bx+a}\,ab^3\mathrm{sgn}(x)}{b^2x^2}}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] 1/4*(3*b^3*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - (5*(b*x + a)^(3
/2)*b^3*sgn(x) - 3*sqrt(b*x + a)*a*b^3*sgn(x))/(b^2*x^2))/b
```

maple [A] time = 0.06, size = 74, normalized size = 0.91

$$-\frac{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\left(3b^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\sqrt{bx+a}\;a^{\frac{3}{2}}+5\,(bx+a)^{\frac{3}{2}}\sqrt{a}\right)}{4\,(bx+a)^{\frac{3}{2}}\sqrt{a}\;x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^6,x)
```

```
[Out] -1/4*(b*x^3+a*x^2)^(3/2)*(3*arctanh((b*x+a)^(1/2)/a^(1/2))*x^2*b^2+5*(b*x+a
)^(3/2)*a^(1/2)-3*(b*x+a)^(1/2)*a^(3/2))/x^5/(b*x+a)^(3/2)/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^3 + ax^2\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3 + a\,x^2\right)^{3/2}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^6,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^2\,(a + bx)\right)^{\frac{3}{2}}}{x^6}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**6,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**6, x)
```

3.249 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^7} dx$

Optimal. Leaf size=109

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

```
[Out] -1/3*(b*x^3+a*x^2)^(3/2)/x^6+1/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))
/a^(3/2)-1/4*b*(b*x^3+a*x^2)^(1/2)/x^3-1/8*b^2*(b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{8ax^2}-\frac{b\sqrt{ax^2+bx^3}}{4x^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] -(b*Sqrt[a*x^2 + b*x^3])/(4*x^3) - (b^2*Sqrt[a*x^2 + b*x^3])/(8*a*x^2) - (a
*x^2 + b*x^3)^(3/2)/(3*x^6) + (b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/(8*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{(ax^2+bx^3)^{3/2}}{x^7}\,dx = -\frac{(ax^2+bx^3)^{3/2}}{3x^6} + \frac{1}{2}b\int \frac{\sqrt{ax^2+bx^3}}{x^4}\,dx$$
$$= -\frac{b\sqrt{ax^2+bx^3}}{4x^3} - \frac{(ax^2+bx^3)^{3/2}}{3x^6} + \frac{1}{8}b^2\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$$
$$= -\frac{b\sqrt{ax^2+bx^3}}{4x^3} - \frac{b^2\sqrt{ax^2+bx^3}}{8ax^2} - \frac{(ax^2+bx^3)^{3/2}}{3x^6} - \frac{b^3\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a}$$
$$= -\frac{b\sqrt{ax^2+bx^3}}{4x^3} - \frac{b^2\sqrt{ax^2+bx^3}}{8ax^2} - \frac{(ax^2+bx^3)^{3/2}}{3x^6} + \frac{b^3\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{8a}$$
$$= -\frac{b\sqrt{ax^2+bx^3}}{4x^3} - \frac{b^2\sqrt{ax^2+bx^3}}{8ax^2} - \frac{(ax^2+bx^3)^{3/2}}{3x^6} + \frac{b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{3/2}}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.39

$$\frac{2b^3\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},4;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^7,x]
```

```
[Out] (2*b^3*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[5/2, 4, 7/2, 1 + (b*x)/a])/(
5*a^4*x^5)
```

fricas [A] time = 0.42, size = 175, normalized size = 1.61

$$\left[\frac{3\sqrt{a}\,b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right) - 2\left(3\,ab^2x^2+14\,a^2bx+8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^2x^4}, -\frac{3\sqrt{-a}\,b^3x^4\arctan\left(\frac{\sqrt{bx^3+a}}{a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] [1/48*(3*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) - 2*(3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4
), -1/24*(3*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (
3*a*b^2*x^2 + 14*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^2*x^4)]
```

giac [A] time = 0.23, size = 92, normalized size = 0.84

$$-\frac{\frac{3\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a} + \frac{3\,(bx+a)^{\frac{5}{2}}b^4\mathrm{sgn}(x)+8\,(bx+a)^{\frac{3}{2}}ab^4\mathrm{sgn}(x)-3\,\sqrt{bx+a}\,a^2b^4\mathrm{sgn}(x)}{ab^3x^3}}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] -1/24*(3*b^4*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a) + (3*(b*x +
 a)^(5/2)*b^4*sgn(x) + 8*(b*x + a)^(3/2)*a*b^4*sgn(x) - 3*sqrt(b*x + a)*a^2
*b^4*sgn(x))/(a*b^3*x^3))/b
```

maple [A] time = 0.06, size = 87, normalized size = 0.80

$$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(-3a\,b^{3}x^{3}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\sqrt{bx+a}\;a^{\frac{7}{2}}+8\left(bx+a\right)^{\frac{3}{2}}a^{\frac{5}{2}}+3\left(bx+a\right)^{\frac{5}{2}}a^{\frac{3}{2}}\right)}{24\left(bx+a\right)^{\frac{3}{2}}a^{\frac{5}{2}}x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^7,x)
```

```
[Out] -1/24*(b*x^3+a*x^2)^(3/2)*(3*(b*x+a)^(5/2)*a^(3/2)-3*arctanh((b*x+a)^(1/2)/
a^(1/2))*x^3*a*b^3+8*(b*x+a)^(3/2)*a^(5/2)-3*(b*x+a)^(1/2)*a^(7/2))/x^6/(b*
x+a)^(3/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^{3}+ax^{2}\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,x^{3}+a\,x^{2}\right)^{3/2}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^7,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^7, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**7,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**7, x)
```

3.250 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx$

Optimal. Leaf size=137

$$-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}+\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}-\frac{b\sqrt{ax^2+bx^3}}{8x^4}$$

```
[Out] -1/4*(b*x^3+a*x^2)^(3/2)/x^7-3/64*b^4*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2)
)/a^(5/2)-1/8*b*(b*x^3+a*x^2)^(1/2)/x^4-1/32*b^2*(b*x^3+a*x^2)^(1/2)/a/x^3+
3/64*b^3*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.18, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$\frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2}-\frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{5/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{32ax^3}-\frac{b\sqrt{ax^2+bx^3}}{8x^4}-\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] -(b*Sqrt[a*x^2 + b*x^3])/(8*x^4) - (b^2*Sqrt[a*x^2 + b*x^3])/(32*a*x^3) + (
3*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^2) - (a*x^2 + b*x^3)^(3/2)/(4*x^7) - (
3*b^4*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx = -\frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7} + \frac{1}{8}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^5}\,dx$$

$$= -\frac{b\sqrt{ax^2+bx^3}}{8x^4} - \frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7} + \frac{1}{16}b^2\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$$

$$= -\frac{b\sqrt{ax^2+bx^3}}{8x^4} - \frac{b^2\sqrt{ax^2+bx^3}}{32ax^3} - \frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7} - \frac{\left(3b^3\right)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{64a}$$

$$= -\frac{b\sqrt{ax^2+bx^3}}{8x^4} - \frac{b^2\sqrt{ax^2+bx^3}}{32ax^3} + \frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2} - \frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7} + \frac{\left(3b^4\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,d}{128a^2}$$

$$= -\frac{b\sqrt{ax^2+bx^3}}{8x^4} - \frac{b^2\sqrt{ax^2+bx^3}}{32ax^3} + \frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2} - \frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7} - \frac{\left(3b^4\right)\text{Subst}\left(\int \frac{1}{1-a}\right.}{64}$$

$$= -\frac{b\sqrt{ax^2+bx^3}}{8x^4} - \frac{b^2\sqrt{ax^2+bx^3}}{32ax^3} + \frac{3b^3\sqrt{ax^2+bx^3}}{64a^2x^2} - \frac{\left(ax^2+bx^3\right)^{3/2}}{4x^7} - \frac{3b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+b}}\right.}{64a^{5/2}}$$

Mathematica [C] time = 0.02, size = 42, normalized size = 0.31

$$-\frac{2b^4\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},5;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^8,x]
```

```
[Out] (-2*b^4*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[5/2, 5, 7/2, 1 + (b*x)/a])/
(5*a^5*x^5)
```

fricas [A] time = 0.41, size = 197, normalized size = 1.44

$$\left[\frac{3\sqrt{a}\,b^4x^5\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\left(3\,ab^3x^3-2\,a^2b^2x^2-24\,a^3bx-16\,a^4\right)\sqrt{bx^3+ax^2}}{128\,a^3x^5},\ \frac{3\sqrt{-a}\,b^4x^5\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

```
[Out] [1/128*(3*sqrt(a)*b^4*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) + 2*(3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*x^3
+ a*x^2))/(a^3*x^5), 1/64*(3*sqrt(-a)*b^4*x^5*arctan(sqrt(b*x^3 + a*x^2)*sq
rt(-a)/(a*x)) + (3*a*b^3*x^3 - 2*a^2*b^2*x^2 - 24*a^3*b*x - 16*a^4)*sqrt(b*
x^3 + a*x^2))/(a^3*x^5)]
```

giac [A] time = 0.24, size = 109, normalized size = 0.80

$$\frac{\frac{3\,b^5\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a^2}+\frac{3\,(bx+a)^{\frac{7}{2}}b^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{sgn}(x)-11\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{sgn}(x)+3\,\sqrt{bx+a}\,a^3b^5\mathrm{sgn}(x)}{a^2b^4x^4}}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

```
[Out] 1/64*(3*b^5*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^2) + (3*(b*x
+ a)^(7/2)*b^5*sgn(x) - 11*(b*x + a)^(5/2)*a*b^5*sgn(x) - 11*(b*x + a)^(3/2
)*a^2*b^5*sgn(x) + 3*sqrt(b*x + a)*a^3*b^5*sgn(x))/(a^2*b^4*x^4))/b
```

maple [A] time = 0.05, size = 101, normalized size = 0.74

$$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(-3a^{2}b^{4}x^{4}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+3\sqrt{bx+a}\;a^{\frac{11}{2}}-11\left(bx+a\right)^{\frac{3}{2}}a^{\frac{9}{2}}-11\left(bx+a\right)^{\frac{5}{2}}a^{\frac{7}{2}}+3\left(bx+a\right)^{\frac{7}{2}}a^{\frac{5}{2}}\right)}{64\left(bx+a\right)^{\frac{3}{2}}a^{\frac{9}{2}}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^8,x)
```

```
[Out] 1/64*(b*x^3+a*x^2)^(3/2)*(3*(b*x+a)^(7/2)*a^(5/2)-11*(b*x+a)^(5/2)*a^(7/2)-
3*arctanh((b*x+a)^(1/2)/a^(1/2))*a^2*x^4*b^4-11*(b*x+a)^(3/2)*a^(9/2)+3*(b*
x+a)^(1/2)*a^(11/2))/x^7/(b*x+a)^(3/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^{3}+ax^{2}\right)^{\frac{3}{2}}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^8, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^8,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**8,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**8, x)
```

3.251 $\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx$

Optimal. Leaf size=165

$$\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}-\frac{3b\sqrt{ax^2+bx^3}}{40x^5}$$

```
[Out] -1/5*(b*x^3+a*x^2)^(3/2)/x^8+3/128*b^5*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2
))/a^(7/2)-3/40*b*(b*x^3+a*x^2)^(1/2)/x^5-1/80*b^2*(b*x^3+a*x^2)^(1/2)/a/x^
4+1/64*b^3*(b*x^3+a*x^2)^(1/2)/a^2/x^3-3/128*b^4*(b*x^3+a*x^2)^(1/2)/a^3/x^
2
```

Rubi [A] time = 0.24, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2020, 2025, 2008, 206}

$$-\frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2}+\frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3}+\frac{3b^5\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{128a^{7/2}}-\frac{b^2\sqrt{ax^2+bx^3}}{80ax^4}-\frac{3b\sqrt{ax^2+bx^3}}{40x^5}-\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] (-3*b*Sqrt[a*x^2 + b*x^3])/(40*x^5) - (b^2*Sqrt[a*x^2 + b*x^3])/(80*a*x^4)
+ (b^3*Sqrt[a*x^2 + b*x^3])/(64*a^2*x^3) - (3*b^4*Sqrt[a*x^2 + b*x^3])/(128
*a^3*x^2) - (a*x^2 + b*x^3)^(3/2)/(5*x^8) + (3*b^5*ArcTanh[(Sqrt[a]*x)/Sqrt
[a*x^2 + b*x^3]])/(128*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2020

```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*(m + j*p + 1)), x] - Dist[(b*
p*(n - j))/(c^n*(m + j*p + 1)), Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integers
Q[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx = -\frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8} + \frac{1}{10}(3b)\int \frac{\sqrt{ax^2+bx^3}}{x^6}\,dx$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8} + \frac{1}{80}\left(3b^2\right)\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} - \frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8} - \frac{b^3\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{32a}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8} + \frac{\left(3b^4\right)\int \frac{1}{x\sqrt{ax^2+b}}}{128a^2}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

$$= -\frac{3b\sqrt{ax^2+bx^3}}{40x^5} - \frac{b^2\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^3\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^4\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{\left(ax^2+bx^3\right)^{3/2}}{5x^8}$$

Mathematica [C] time = 0.03, size = 42, normalized size = 0.25

$$\frac{2b^5\left(x^2(a+bx)\right)^{5/2}\,{}_2F_1\left(\frac{5}{2},6;\frac{7}{2};\frac{bx}{a}+1\right)}{5a^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(3/2)/x^9,x]
```

```
[Out] (2*b^5*(x^2*(a + b*x))^(5/2)*Hypergeometric2F1[5/2, 6, 7/2, 1 + (b*x)/a])/(
5*a^6*x^5)
```

fricas [A] time = 0.41, size = 219, normalized size = 1.33

$$\left[\frac{15\sqrt{a}\,b^5x^6\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right) - 2\left(15\,ab^4x^4 - 10\,a^2b^3x^3 + 8\,a^3b^2x^2 + 176\,a^4bx + 128\,a^5\right)\sqrt{bx^3+ax^2}}{1280\,a^4x^6},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

```
[Out] [1/1280*(15*sqrt(a)*b^5*x^6*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt
(a))/x^2) - 2*(15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8*a^3*b^2*x^2 + 176*a^4*b*x
+ 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6), -1/640*(15*sqrt(-a)*b^5*x^6*arct
an(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^4*x^4 - 10*a^2*b^3*x^3 + 8
*a^3*b^2*x^2 + 176*a^4*b*x + 128*a^5)*sqrt(b*x^3 + a*x^2))/(a^4*x^6)]
```

giac [A] time = 0.27, size = 126, normalized size = 0.76

$$-\frac{\frac{15\,b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}\,a^3} + \frac{15\,(bx+a)^{\frac{9}{2}}b^6\mathrm{sgn}(x) - 70\,(bx+a)^{\frac{7}{2}}ab^6\mathrm{sgn}(x) + 128\,(bx+a)^{\frac{5}{2}}a^2b^6\mathrm{sgn}(x) + 70\,(bx+a)^{\frac{3}{2}}a^3b^6\mathrm{sgn}(x) - 15\,\sqrt{bx+a}\,a^4b^6\mathrm{sgn}}{a^3b^5x^5}}{640\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

```
[Out] -1/640*(15*b^6*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/(sqrt(-a)*a^3) + (15*(
b*x + a)^(9/2)*b^6*sgn(x) - 70*(b*x + a)^(7/2)*a*b^6*sgn(x) + 128*(b*x + a)
^(5/2)*a^2*b^6*sgn(x) + 70*(b*x + a)^(3/2)*a^3*b^6*sgn(x) - 15*sqrt(b*x + a
)*a^4*b^6*sgn(x))/(a^3*b^5*x^5))/b
```

maple [A] time = 0.05, size = 113, normalized size = 0.68

$$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(-15a^{3}b^{5}x^{5}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-15\sqrt{bx+a}\;a^{\frac{15}{2}}+70\left(bx+a\right)^{\frac{3}{2}}a^{\frac{13}{2}}+128\left(bx+a\right)^{\frac{5}{2}}a^{\frac{11}{2}}-70\left(bx+\right.\right.}{640\left(bx+a\right)^{\frac{3}{2}}a^{\frac{13}{2}}x^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^(3/2)/x^9,x)
```

```
[Out] -1/640*(b*x^3+a*x^2)^(3/2)*(15*(b*x+a)^(9/2)*a^(7/2)-70*(b*x+a)^(7/2)*a^(9/
2)+128*(b*x+a)^(5/2)*a^(11/2)-15*arctanh((b*x+a)^(1/2)/a^(1/2))*a^3*x^5*b^5
+70*(b*x+a)^(3/2)*a^(13/2)-15*(b*x+a)^(1/2)*a^(15/2))/x^8/(b*x+a)^(3/2)/a^(
13/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{3}+ax^{2}\right)^{\frac{3}{2}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(3/2)/x^9, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^3+a\,x^2\right)^{3/2}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^(3/2)/x^9,x)
```

```
[Out] int((a*x^2 + b*x^3)^(3/2)/x^9, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x^{2}\left(a+bx\right)\right)^{\frac{3}{2}}}{x^{9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**(3/2)/x**9,x)
```

```
[Out] Integral((x**2*(a + b*x))**(3/2)/x**9, x)
```

3.252 $\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=103

$$-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}+\frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

```
[Out] 16/35*a^2*(b*x^3+a*x^2)^(1/2)/b^3-32/35*a^3*(b*x^3+a*x^2)^(1/2)/b^4/x-12/35
*a*x*(b*x^3+a*x^2)^(1/2)/b^2+2/7*x^2*(b*x^3+a*x^2)^(1/2)/b
```

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}+\frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^3])/(35*b^3) - (32*a^3*Sqrt[a*x^2 + b*x^3])/(35*b^
4*x) - (12*a*x*Sqrt[a*x^2 + b*x^3])/(35*b^2) + (2*x^2*Sqrt[a*x^2 + b*x^3])/
(7*b)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4}{\sqrt{ax^2+bx^3}}\,dx &= \frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{(6a)\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx}{7b}\\
&= -\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}+\frac{\left(24a^2\right)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{35b^2}\\
&= \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{\left(16a^3\right)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{35b^3}\\
&= \frac{16a^2\sqrt{ax^2+bx^3}}{35b^3}-\frac{32a^3\sqrt{ax^2+bx^3}}{35b^4x}-\frac{12ax\sqrt{ax^2+bx^3}}{35b^2}+\frac{2x^2\sqrt{ax^2+bx^3}}{7b}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.51

$$\frac{2\sqrt{x^2(a+bx)}\,\left(-16a^3+8a^2bx-6ab^2x^2+5b^3x^3\right)}{35b^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)]*(-16*a^3 + 8*a^2*b*x - 6*a*b^2*x^2 + 5*b^3*x^3))/(35
*b^4*x)
```

fricas [A] time = 0.38, size = 51, normalized size = 0.50

$$\frac{2\left(5\,b^3x^3-6\,ab^2x^2+8\,a^2bx-16\,a^3\right)\sqrt{bx^3+ax^2}}{35\,b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(5*b^3*x^3 - 6*a*b^2*x^2 + 8*a^2*b*x - 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^
4*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^3 + a*x^2), x)
```

maple [A] time = 0.05, size = 55, normalized size = 0.53

$$-\frac{2\,(bx+a)\left(-5b^3x^3+6a\,b^2x^2-8a^2bx+16a^3\right)x}{35\sqrt{b\,x^3+a\,x^2}\;b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)*(-5*b^3*x^3+6*a*b^2*x^2-8*a^2*b*x+16*a^3)*x/b^4/(b*x^3+a*x^2)
^(1/2)
```

maxima [A] time = 1.46, size = 53, normalized size = 0.51

$$\frac{2\left(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4\right)}{35\,\sqrt{bx+a}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)/(sqrt(b*x
 + a)*b^4)
```

mupad [B] time = 5.19, size = 51, normalized size = 0.50

$$-\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(16\,a^3-8\,a^2\,b\,x+6\,a\,b^2\,x^2-5\,b^3\,x^3\right)}{35\,b^4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 - 5*b^3*x^3 + 6*a*b^2*x^2 - 8*a^2*b*x))/(
35*b^4*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(a + b*x)), x)
```

3.253 $\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=75

$$\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}-\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

```
[Out] -8/15*a*(b*x^3+a*x^2)^(1/2)/b^2+16/15*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x+2/5*x*(
b*x^3+a*x^2)^(1/2)/b
```

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}-\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-8*a*Sqrt[a*x^2 + b*x^3])/(15*b^2) + (16*a^2*Sqrt[a*x^2 + b*x^3])/(15*b^3*
x) + (2*x*Sqrt[a*x^2 + b*x^3])/(5*b)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx &= \frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{(4a)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{5b}\\ &= -\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{2x\sqrt{ax^2+bx^3}}{5b}+\frac{\left(8a^2\right)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{15b^2}\\ &= -\frac{8a\sqrt{ax^2+bx^3}}{15b^2}+\frac{16a^2\sqrt{ax^2+bx^3}}{15b^3x}+\frac{2x\sqrt{ax^2+bx^3}}{5b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.56

$$\frac{2\sqrt{x^2(a+bx)}\left(8a^2-4abx+3b^2x^2\right)}{15b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^3],x]
[Out] (2*Sqrt[x^2*(a + b*x)]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3*x)
```

fricas [A] time = 0.40, size = 40, normalized size = 0.53

$$\frac{2\left(3\,b^2x^2 - 4\,abx + 8\,a^2\right)\sqrt{bx^3 + ax^2}}{15\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^3 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(b*x^3 + a*x^2), x)
```

maple [A] time = 0.05, size = 44, normalized size = 0.59

$$\frac{2\,(bx + a)\left(3b^2x^2 - 4abx + 8a^2\right)x}{15\sqrt{b\,x^3 + a\,x^2}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^(1/2),x)
[Out] 2/15*(b*x+a)*(3*b^2*x^2-4*a*b*x+8*a^2)*x/b^3/(b*x^3+a*x^2)^(1/2)
```

maxima [A] time = 1.46, size = 42, normalized size = 0.56

$$\frac{2\left(3\,b^3x^3 - ab^2x^2 + 4\,a^2bx + 8\,a^3\right)}{15\,\sqrt{bx + a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)/(sqrt(b*x + a)*b^3)
```

mupad [B] time = 5.20, size = 40, normalized size = 0.53

$$\frac{2\,\sqrt{b\,x^3 + a\,x^2}\,\left(8\,a^2 - 4\,a\,b\,x + 3\,b^2\,x^2\right)}{15\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3)^(1/2),x)
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 + 3*b^2*x^2 - 4*a*b*x))/(15*b^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\,(a + bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2*(a + b*x)), x)
```

3.254 $\int \frac{x^2}{\sqrt{ax^2+bx^3}} dx$

Optimal. Leaf size=49

$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

[Out] 2/3*(b*x^3+a*x^2)^(1/2)/b-4/3*a*(b*x^3+a*x^2)^(1/2)/b^2/x

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}$$

Antiderivative was successfully verified.

[In] Int[x^2/Sqrt[a*x^2 + b*x^3],x]

[Out] (2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{ax^2+bx^3}} dx &= \frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{(2a)\int \frac{x}{\sqrt{ax^2+bx^3}} dx}{3b}\\ &= \frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.61

$$\frac{2(bx-2a)\sqrt{x^2(a+bx)}}{3b^2x}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a*x^2 + b*x^3],x]

[Out] (2*(-2*a + b*x)*Sqrt[x^2*(a + b*x)])/(3*b^2*x)

fricas [A] time = 0.39, size = 28, normalized size = 0.57

$$\frac{2\sqrt{bx^3+ax^2}\,(bx-2\,a)}{3\,b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(b*x - 2*a)/(b^2*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/sqrt(b*x^3 + a*x^2), x)
```

maple [A] time = 0.04, size = 33, normalized size = 0.67

$$-\frac{2\,(bx+a)\,(-bx+2a)\,x}{3\sqrt{b\,x^3+a\,x^2}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^(1/2),x)
[Out] -2/3*(b*x+a)*(-b*x+2*a)*x/b^2/(b*x^3+a*x^2)^(1/2)
```

maxima [A] time = 1.43, size = 30, normalized size = 0.61

$$\frac{2\left(b^2x^2-abx-2\,a^2\right)}{3\,\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*(b^2*x^2 - a*b*x - 2*a^2)/(sqrt(b*x + a)*b^2)
```

mupad [B] time = 5.16, size = 31, normalized size = 0.63

$$-\frac{\left(\frac{4\,a}{3\,b^2}-\frac{2\,x}{3\,b}\right)\sqrt{b\,x^3+a\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3)^(1/2),x)
[Out] -(((4*a)/(3*b^2) - (2*x)/(3*b))*(a*x^2 + b*x^3)^(1/2))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(x**2/sqrt(x**2*(a + b*x)), x)
```

3.255 $\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=23

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

```
[Out] 2*(b*x^3+a*x^2)^(1/2)/b/x
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2\sqrt{ax^2+bx^3}}{bx}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^3])/(b*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{ax^2+bx^3}}{bx}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.91

$$\frac{2\sqrt{x^2(a+bx)}}{bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x)])/(b*x)
```

fricas [A] time = 0.39, size = 21, normalized size = 0.91

$$\frac{2\sqrt{bx^3+ax^2}}{bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x^3 + a*x^2)/(b*x)
```

giac [A] time = 0.20, size = 26, normalized size = 1.13

$$\frac{2}{\sqrt{\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/(sqrt(b/x + a/x^2) - sqrt(a)/x)
```

maple [A] time = 0.04, size = 25, normalized size = 1.09

$$\frac{2\,(bx+a)\,x}{\sqrt{b\,x^3+a\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 2*x*(b*x+a)/b/(b*x^3+a*x^2)^(1/2)
```

maxima [A] time = 1.41, size = 12, normalized size = 0.52

$$\frac{2\,\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

mupad [B] time = 5.14, size = 17, normalized size = 0.74

$$\frac{2\,|x|\,\sqrt{a+b\,x}}{b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] (2*abs(x)*(a + b*x)^(1/2))/(b*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x)), x)
```

3.256 $\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=30

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

```
[Out] -2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 1.53

$$-\frac{2x\sqrt{a+bx}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a +
b*x)])
```

fricas [A] time = 0.41, size = 74, normalized size = 2.47

$$\left[\frac{\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)}{\sqrt{a}}, \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)/sqrt(a), 2*sqrt(-
a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))/a]
```

giac [A] time = 0.16, size = 45, normalized size = 1.50

$$-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} + \frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(sqrt(b*x + a)/sqrt(-
a))/(sqrt(-a)*sgn(x))
```

maple [A] time = 0.04, size = 39, normalized size = 1.30

$$-\frac{2\sqrt{bx+a}\;x\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{b\,x^3+a\,x^2}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/(b*x^3+a*x^2)^(1/2)*x*(b*x+a)^(1/2)/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**3), x)
```

3.257 $\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=54

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

```
[Out] b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)-(b*x^3+a*x^2)^(1/2)/a/x^2
```

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]
])/a^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{ax^2}-\frac{b\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{2a}\\ &= -\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{a}\\ &= -\frac{\sqrt{ax^2+bx^3}}{ax^2}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 66, normalized size = 1.22

$$\frac{2bx(a+bx)\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{2\sqrt{\frac{bx}{a}+1}}-\frac{a}{2bx}\right)}{a^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*b*x*(a + b*x)*(-1/2*a/(b*x) + ArcTanh[Sqrt[1 + (b*x)/a]]/(2*Sqrt[1 + (b*
x)/a])))/(a^2*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.41, size = 127, normalized size = 2.35

$$\left[\frac{\sqrt{a}\,bx^2\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\,a}{2\,a^2x^2},-\frac{\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+ax^2}\,a}{a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x^2*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2
) - 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2), -(sqrt(-a)*b*x^2*arctan(sqrt(b*x^3
+ a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a^2*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,
2,2]%%%}] at parameters values [62.4600259969,-13,46]-1/a*sqrt(a*(1/x)^2+b/
x)-2*b/4/a/sqrt(a)*ln(abs(2*sqrt(a)*(sqrt(a*(1/x)^2+b/x)-sqrt(a)/x)-b))
```

maple [A] time = 0.05, size = 55, normalized size = 1.02

$$-\frac{\sqrt{bx+a}\,\left(-abx\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\,a^{\frac{3}{2}}\right)}{\sqrt{b\,x^3+a\,x^2}\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -(b*x+a)^(1/2)*((b*x+a)^(1/2)*a^(3/2)-arctanh((b*x+a)^(1/2)/a^(1/2))*x*a*b)
/(b*x^3+a*x^2)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax^2}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x))), x)
```

3.258 $\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=87

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

```
[Out] -3/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)-1/2*(b*x^3+a*x^2)^(
1/2)/a/x^3+3/4*b*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(2*a*x^3) + (3*b*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^2) - (3
*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(4*a^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx &= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}-\frac{(3b)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{4a}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}+\frac{\left(3b^2\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{\left(3b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{4a^2}\\
&= -\frac{\sqrt{ax^2+bx^3}}{2ax^3}+\frac{3b\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.46

$$-\frac{2b^2\sqrt{x^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{bx}{a}+1\right)}{a^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*b^2*Sqrt[x^2*(a + b*x)]*Hypergeometric2F1[1/2, 3, 3/2, 1 + (b*x)/a])/(a
^3*x)
```

fricas [A] time = 0.41, size = 153, normalized size = 1.76

$$\left[\frac{3\sqrt{a}\,b^2x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\left(3\,abx-2\,a^2\right)}{8\,a^3x^3}, \frac{3\,\sqrt{-a}\,b^2x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3+}}{4\,a^3x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*sqrt(a)*b^2*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x - 2*a^2))/(a^3*x^3), 1/4*(3*sqrt(-a)
*b^2*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*(
3*a*b*x - 2*a^2))/(a^3*x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,
2,2]%%%}] at parameters values [62.4600259969,-13,46]2*(-4*a/16/a^2/x+6*b/1
6/a^2)*sqrt(a*(1/x)^2+b/x)+6*b^2/16/a^2/sqrt(a)*ln(abs(2*sqrt(a)*(sqrt(a*(1
/x)^2+b/x)-sqrt(a)/x)-b))
```

maple [A] time = 0.05, size = 77, normalized size = 0.89

$$-\frac{\sqrt{bx+a}\,\left(3a\,b^2x^2\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\sqrt{bx+a}\,a^{\frac{3}{2}}bx+2\sqrt{bx+a}\,a^{\frac{5}{2}}\right)}{4\sqrt{b\,x^3+a\,x^2}\,a^{\frac{7}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/4*(b*x+a)^(1/2)*(2*(b*x+a)^(1/2)*a^(5/2)-3*(b*x+a)^(1/2)*a^(3/2)*x*b+3*a
rctanh((b*x+a)^(1/2)/a^(1/2))*a*x^2*b^2)/x/(b*x^3+a*x^2)^(1/2)/a^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx^3+ax^2}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

mupad [B] time = 5.41, size = 44, normalized size = 0.51

$$-\frac{2\sqrt{\frac{a}{b\,x}+1}\;{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{2};\ \frac{7}{2};\ -\frac{a}{b\,x}\right)}{5\,x\,\sqrt{b\,x^3+a\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] -(2*(a/(b*x) + 1)^(1/2)*hypergeom([1/2, 5/2], 7/2, -a/(b*x)))/(5*x*(a*x^2 +
 b*x^3)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x))), x)
```

3.259 $\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=115

$$\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

```
[Out] 5/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(7/2)-1/3*(b*x^3+a*x^2)^(1
/2)/a/x^4+5/12*b*(b*x^3+a*x^2)^(1/2)/a^2/x^3-5/8*b^2*(b*x^3+a*x^2)^(1/2)/a^
3/x^2
```

Rubi [A] time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$-\frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}+\frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^3]/(3*a*x^4) + (5*b*Sqrt[a*x^2 + b*x^3])/(12*a^2*x^3) - (
5*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^2) + (5*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*
x^2 + b*x^3]])/(8*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{ax^2+bx^3}}{3ax^4} - \frac{(5b)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{6a}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} + \frac{\left(5b^2\right)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a^2}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} - \frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2} - \frac{\left(5b^3\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{16a^3}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} - \frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2} + \frac{\left(5b^3\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+b}}\right.}{8a^3}$$
$$= -\frac{\sqrt{ax^2+bx^3}}{3ax^4} + \frac{5b\sqrt{ax^2+bx^3}}{12a^2x^3} - \frac{5b^2\sqrt{ax^2+bx^3}}{8a^3x^2} + \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{7/2}}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.35

$$\frac{2b^3\sqrt{x^2(a+bx)}\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};\frac{bx}{a}+1\right)}{a^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (2*b^3*Sqrt[x^2*(a + b*x)]*Hypergeometric2F1[1/2, 4, 3/2, 1 + (b*x)/a])/(a^
4*x)
```

fricas [A] time = 0.42, size = 175, normalized size = 1.52

$$\left[\frac{15\sqrt{a}\,b^3x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right) - 2\left(15\,ab^2x^2 - 10\,a^2bx + 8\,a^3\right)\sqrt{bx^3+ax^2}}{48\,a^4x^4}, -\frac{15\sqrt{-a}\,b^3x^4\arctan\left(\frac{\sqrt{bx}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*sqrt(a)*b^3*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a
))/x^2) - 2*(15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x
^4), -1/24*(15*sqrt(-a)*b^3*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x))
+ (15*a*b^2*x^2 - 10*a^2*b*x + 8*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*x^4)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,
2,2]%%%}] at parameters values [62.4600259969,-13,46]2*((-16*a^2/96/a^3/x+2
0*a*b/96/a^3)/x-30*b^2/96/a^3)*sqrt(a*(1/x)^2+b/x)-10*b^3/32/a^3/sqrt(a)*ln
(abs(2*sqrt(a)*(sqrt(a*(1/x)^2+b/x)-sqrt(a)/x)-b))
```

maple [A] time = 0.06, size = 95, normalized size = 0.83

$$-\frac{\sqrt{bx+a}\,\left(-15a\,b^{3}x^{3}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+15\sqrt{bx+a}\;a^{\frac{3}{2}}b^{2}x^{2}-10\sqrt{bx+a}\;a^{\frac{5}{2}}bx+8\sqrt{bx+a}\;a^{\frac{7}{2}}\right)}{24\sqrt{b\,x^{3}+a\,x^{2}}\;a^{\frac{9}{2}}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/24/x^2*(b*x+a)^(1/2)*(15*(b*x+a)^(1/2)*a^(3/2)*x^2*b^2-15*arctanh((b*x+a
)^(1/2)/a^(1/2))*x^3*a*b^3-10*(b*x+a)^(1/2)*a^(5/2)*x*b+8*(b*x+a)^(1/2)*a^(
7/2))/(b*x^3+a*x^2)^(1/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax^2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x))), x)
```

3.260 $\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

```
[Out] -2*x^4/b/(b*x^3+a*x^2)^(1/2)-16/5*a*(b*x^3+a*x^2)^(1/2)/b^3+32/5*a^2*(b*x^3
+a*x^2)^(1/2)/b^4/x+12/5*x*(b*x^3+a*x^2)^(1/2)/b^2
```

Rubi [A] time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 1588}

$$\frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x}+\frac{12x\sqrt{ax^2+bx^3}}{5b^2}-\frac{16a\sqrt{ax^2+bx^3}}{5b^3}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^4)/(b*Sqrt[a*x^2 + b*x^3]) - (16*a*Sqrt[a*x^2 + b*x^3])/(5*b^3) + (32
*a^2*Sqrt[a*x^2 + b*x^3])/(5*b^4*x) + (12*x*Sqrt[a*x^2 + b*x^3])/(5*b^2)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^6}{\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2x^4}{b\sqrt{ax^2+bx^3}} + \frac{6\int \frac{x^3}{\sqrt{ax^2+bx^3}}\,dx}{b}$$

$$= -\frac{2x^4}{b\sqrt{ax^2+bx^3}} + \frac{12x\sqrt{ax^2+bx^3}}{5b^2} - \frac{(24a)\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{5b^2}$$

$$= -\frac{2x^4}{b\sqrt{ax^2+bx^3}} - \frac{16a\sqrt{ax^2+bx^3}}{5b^3} + \frac{12x\sqrt{ax^2+bx^3}}{5b^2} + \frac{\left(16a^2\right)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{5b^3}$$

$$= -\frac{2x^4}{b\sqrt{ax^2+bx^3}} - \frac{16a\sqrt{ax^2+bx^3}}{5b^3} + \frac{32a^2\sqrt{ax^2+bx^3}}{5b^4x} + \frac{12x\sqrt{ax^2+bx^3}}{5b^2}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.51

$$\frac{2x\left(16a^3+8a^2bx-2ab^2x^2+b^3x^3\right)}{5b^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*(16*a^3 + 8*a^2*b*x - 2*a*b^2*x^2 + b^3*x^3))/(5*b^4*Sqrt[x^2*(a + b*x
)])
```

fricas [A] time = 0.39, size = 60, normalized size = 0.61

$$\frac{2\left(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3\right)\sqrt{bx^3+ax^2}}{5\left(b^5x^2+ab^4x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*sqrt(b*x^3 + a*x^2)/(b^5*x
^2 + a*b^4*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/(b*x^3 + a*x^2)^(3/2), x)
```

maple [A] time = 0.05, size = 56, normalized size = 0.57

$$\frac{2\,(bx+a)\left(b^3x^3-2a\,b^2x^2+8a^2bx+16a^3\right)x^3}{5\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] 2/5*(b*x+a)*(b^3*x^3-2*a*b^2*x^2+8*a^2*b*x+16*a^3)*x^3/b^4/(b*x^3+a*x^2)^(3
/2)
```

maxima [A] time = 1.48, size = 41, normalized size = 0.42

$$\frac{2\left(b^{3}x^{3} - 2\,ab^{2}x^{2} + 8\,a^{2}bx + 16\,a^{3}\right)}{5\,\sqrt{bx + a}\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)/(sqrt(b*x + a)*b^4)
```

mupad [B] time = 5.28, size = 57, normalized size = 0.58

$$\frac{2\,\sqrt{b\,x^3 + a\,x^2}\,\left(16\,a^3 + 8\,a^2\,b\,x - 2\,a\,b^2\,x^2 + b^3\,x^3\right)}{5\,b^4\,x\,\left(a + b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] (2*(a*x^2 + b*x^3)^(1/2)*(16*a^3 + b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x))/(5*b
^4*x*(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{6}}{\left(x^{2}\left(a + bx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**6/(x**2*(a + b*x))**(3/2), x)
```

3.261 $\int \frac{x^5}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=72

$$-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}+\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

```
[Out] -2*x^3/b/(b*x^3+a*x^2)^(1/2)+8/3*(b*x^3+a*x^2)^(1/2)/b^2-16/3*a*(b*x^3+a*x^
2)^(1/2)/b^3/x
```

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2015, 2016, 1588}

$$\frac{8\sqrt{ax^2+bx^3}}{3b^2}-\frac{16a\sqrt{ax^2+bx^3}}{3b^3x}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*x^3)/(b*Sqrt[a*x^2 + b*x^3]) + (8*Sqrt[a*x^2 + b*x^3])/(3*b^2) - (16*a*
Sqrt[a*x^2 + b*x^3])/(3*b^3*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2x^3}{b\sqrt{ax^2+bx^3}} + \frac{4\int \frac{x^2}{\sqrt{ax^2+bx^3}}\,dx}{b}$$
$$= -\frac{2x^3}{b\sqrt{ax^2+bx^3}} + \frac{8\sqrt{ax^2+bx^3}}{3b^2} - \frac{(8a)\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{3b^2}$$
$$= -\frac{2x^3}{b\sqrt{ax^2+bx^3}} + \frac{8\sqrt{ax^2+bx^3}}{3b^2} - \frac{16a\sqrt{ax^2+bx^3}}{3b^3x}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.54

$$\frac{2x\left(-8a^2-4abx+b^2x^2\right)}{3b^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(a*x^2 + b*x^3)^(3/2),x]
[Out] (2*x*(-8*a^2 - 4*a*b*x + b^2*x^2))/(3*b^3*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.39, size = 49, normalized size = 0.68

$$\frac{2\left(b^2x^2-4\,abx-8\,a^2\right)\sqrt{bx^3+ax^2}}{3\left(b^4x^2+ab^3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*sqrt(b*x^3 + a*x^2)/(b^4*x^2 + a*b^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] integrate(x^5/(b*x^3 + a*x^2)^(3/2), x)
```

maple [A] time = 0.05, size = 46, normalized size = 0.64

$$-\frac{2\,(bx+a)\left(-b^2x^2+4abx+8a^2\right)x^3}{3\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^3+a*x^2)^(3/2),x)
[Out] -2/3*(b*x+a)*(-b^2*x^2+4*a*b*x+8*a^2)*x^3/b^3/(b*x^3+a*x^2)^(3/2)
```

maxima [A] time = 1.44, size = 30, normalized size = 0.42

$$\frac{2\left(b^2x^2-4\,abx-8\,a^2\right)}{3\,\sqrt{bx+a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)/(sqrt(b*x + a)*b^3)
```

mupad [B] time = 5.22, size = 47, normalized size = 0.65

$$-\frac{2\sqrt{b\,x^3+a\,x^2}\,\left(8\,a^2+4\,a\,b\,x-b^2\,x^2\right)}{3\,b^3\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2 - b^2*x^2 + 4*a*b*x))/(3*b^3*x*(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**5/(x**2*(a + b*x))**(3/2), x)
```

3.262 $\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=47

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

[Out] -2*x^2/b/(b*x^3+a*x^2)^(1/2)+4*(b*x^3+a*x^2)^(1/2)/b^2/x

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2015, 1588}

$$\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[x^4/(a*x^2 + b*x^3)^(3/2),x]

[Out] (-2*x^2)/(b*Sqrt[a*x^2 + b*x^3]) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2015

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n -
 j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{2\int \frac{x}{\sqrt{ax^2+bx^3}}\,dx}{b}\\ &= -\frac{2x^2}{b\sqrt{ax^2+bx^3}}+\frac{4\sqrt{ax^2+bx^3}}{b^2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.55

$$\frac{2x(2a+bx)}{b^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(a*x^2 + b*x^3)^(3/2),x]

```
[Out] (2*x*(2*a + b*x))/(b^2*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.39, size = 38, normalized size = 0.81

$$\frac{2\sqrt{bx^3+ax^2}\,(bx+2\,a)}{b^3x^2+ab^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] 2*sqrt(b*x^3 + a*x^2)*(b*x + 2*a)/(b^3*x^2 + a*b^2*x)
```

giac [A] time = 0.28, size = 28, normalized size = 0.60

$$\frac{2\left(\frac{1}{b}+\frac{2\,a}{b^2x}\right)}{\sqrt{\frac{b}{x}+\frac{a}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] 2*(1/b + 2*a/(b^2*x))/sqrt(b/x + a/x^2)
```

maple [A] time = 0.05, size = 34, normalized size = 0.72

$$\frac{2\,(bx+a)\,(bx+2a)\,x^3}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^3+a*x^2)^(3/2),x)
[Out] 2*(b*x+a)*(b*x+2*a)*x^3/b^2/(b*x^3+a*x^2)^(3/2)
```

maxima [A] time = 1.54, size = 19, normalized size = 0.40

$$\frac{2\,(bx+2\,a)}{\sqrt{bx+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] 2*(b*x + 2*a)/(sqrt(b*x + a)*b^2)
```

mupad [B] time = 5.17, size = 35, normalized size = 0.74

$$\frac{2\,\left(2\,a+b\,x\right)\,\sqrt{b\,x^3+a\,x^2}}{b^2\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^3)^(3/2),x)
[Out] (2*(2*a + b*x)*(a*x^2 + b*x^3)^(1/2))/(b^2*x*(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x**4/(x**2*(a + b*x))**(3/2), x)
```

3.263 $\int \frac{x^3}{(ax^2+bx^3)^{3/2}} dx$

Optimal. Leaf size=21

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

```
[Out] -2*x/b/(b*x^3+a*x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$-\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a*x^2 + b*x^3)^(3/2),x]
[Out] (-2*x)/(b*Sqrt[a*x^2 + b*x^3])
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{(ax^2+bx^3)^{3/2}} dx = -\frac{2x}{b\sqrt{ax^2+bx^3}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.90

$$-\frac{2x}{b\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a*x^2 + b*x^3)^(3/2),x]
[Out] (-2*x)/(b*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.38, size = 29, normalized size = 1.38

$$-\frac{2\sqrt{bx^3+ax^2}}{b^2x^2+abx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] -2*sqrt(b*x^3 + a*x^2)/(b^2*x^2 + a*b*x)
```

giac [A] time = 0.23, size = 37, normalized size = 1.76

$$\frac{2}{\left(\sqrt{a}\left(\sqrt{\frac{b}{x}+\frac{a}{x^2}}-\frac{\sqrt{a}}{x}\right)-b\right)\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] 2/((sqrt(a)*(sqrt(b/x + a/x^2) - sqrt(a)/x) - b)*sqrt(a))
```

maple [A] time = 0.04, size = 27, normalized size = 1.29

$$-\frac{2\left(bx+a\right)x^3}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^3+a*x^2)^(3/2),x)
[Out] -2*(b*x+a)*x^3/b/(b*x^3+a*x^2)^(3/2)
```

maxima [A] time = 1.48, size = 12, normalized size = 0.57

$$-\frac{2}{\sqrt{bx+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] -2/(sqrt(b*x + a)*b)
```

mupad [B] time = 5.07, size = 28, normalized size = 1.33

$$-\frac{2\sqrt{b\,x^3+a\,x^2}}{b\,x\,\left(a+b\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^3)^(3/2),x)
[Out] -(2*(a*x^2 + b*x^3)^(1/2))/(b*x*(a + b*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3}{\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**3/(x**2*(a + b*x))**(3/2), x)
```

3.264 $\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=52

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

```
[Out] -2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(3/2)+2*x/a/(b*x^3+a*x^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2023, 2008, 206}

$$\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x)/(a*Sqrt[a*x^2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]
)/a^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(ax^2+bx^3\right)^{3/2}}\,dx &= \frac{2x}{a\sqrt{ax^2+bx^3}}+\frac{\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{a}\\
&= \frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^3}}\right)}{a}\\
&= \frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 35, normalized size = 0.67

$$\frac{2x\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx}{a}+1\right)}{a\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (2*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x)/a])/(a*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.43, size = 156, normalized size = 3.00

$$\left[\frac{\left(bx^2+ax\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}\,a}{a^2bx^2+a^3x},\frac{2\left(\left(bx^2+ax\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^3}\right.}{a^2bx^2+a^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [((b*x^2 + a*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))
/x^2) + 2*sqrt(b*x^3 + a*x^2)*a)/(a^2*b*x^2 + a^3*x), 2*((b*x^2 + a*x)*sqrt
(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^3 + a*x^2)*a)/(a
^2*b*x^2 + a^3*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Unable to divide, perhaps due to rounding error%%%{%%%{1,[1]%%%},[2,
2]%%%}+%%%{%%{[-2,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,3]%%%}+%%%{1,[0,4]%%%} / %%
%{%%%{1,[2]%%%},[2,0]%%%}+%%%{%%{[%%%{-2,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%}
,[1,1]%%%}+%%%{%%%{1,[1]%%%},[0,2]%%%} Error: Bad Argument Value
```

maple [A] time = 0.05, size = 54, normalized size = 1.04

$$-\frac{2\left(bx+a\right)\left(\sqrt{bx+a}\;a\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-a^{\frac{3}{2}}\right)x^3}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -2*x^3*(b*x+a)*(arctanh((b*x+a)^(1/2)/a^(1/2))*a*(b*x+a)^(1/2)-a^(3/2))/(b*
x^3+a*x^2)^(3/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(b*x^3 + a*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\left(b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^3)^(3/2),x)
[Out] int(x^2/(a*x^2 + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**3+a*x**2)**(3/2),x)
[Out] Integral(x**2/(x**2*(a + b*x))**(3/2), x)
```

3.265 $\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$\frac{3b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

```
[Out] 3*b*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(5/2)+2/a/(b*x^3+a*x^2)^(1/2)-
3*(b*x^3+a*x^2)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2023, 2025, 2008, 206}

$$-\frac{3\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}+\frac{2}{a\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] 2/(a*Sqrt[a*x^2 + b*x^3]) - (3*Sqrt[a*x^2 + b*x^3])/(a^2*x^2) + (3*b*ArcTan
h[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(5/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{x}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{a\sqrt{ax^2+bx^3}} + \frac{3\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{a}$$

$$= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} - \frac{(3b)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{2a^2}$$

$$= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^3}}\right)}{a^2}$$

$$= \frac{2}{a\sqrt{ax^2+bx^3}} - \frac{3\sqrt{ax^2+bx^3}}{a^2x^2} + \frac{3b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.01, size = 36, normalized size = 0.48

$$-\frac{2bx\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{bx}{a}+1\right)}{a^2\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^2 + b*x^3)^(3/2),x]
```

```
[Out] (-2*b*x*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x)/a])/(a^2*Sqrt[x^2*(a + b*
x)])
```

fricas [A] time = 0.41, size = 189, normalized size = 2.52

$$\left[\frac{3\left(b^2x^3+abx^2\right)\sqrt{a}\,\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\,\sqrt{bx^3+ax^2}\left(3\,abx+a^2\right)}{2\left(a^3bx^3+a^4x^2\right)}, -\frac{3\left(b^2x^3+abx^2\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^3\cdots}}{\cdots}\right.}{a^3bx^3\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(b^2*x^3 + a*b*x^2)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x
^2)*sqrt(a))/x^2) - 2*sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4
*x^2), -(3*(b^2*x^3 + a*b*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)
/(a*x)) + sqrt(b*x^3 + a*x^2)*(3*a*b*x + a^2))/(a^3*b*x^3 + a^4*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,%%%{-4,[1,0,0]%%%}+%%%{-2,[0,1,1]%%%},0,%%%{1,[0,
2,2]%%%}] at parameters values [62.4600259969,-13,46]-4*a^2/4/a^4*sqrt(a*(1
/x)^2+b/x)+2*(-b^2/a^2/(-a*(sqrt(a*(1/x)^2+b/x)-sqrt(a)/x)+sqrt(a)*b)-3*b/4
/a^2/sqrt(a)*ln(abs(2*sqrt(a)*(sqrt(a*(1/x)^2+b/x)-sqrt(a)/x)-b)))
```

maple [A] time = 0.06, size = 62, normalized size = 0.83

$$\frac{(bx+a)\left(3\sqrt{bx+a}\, bx \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\sqrt{a}\, bx-a^{\frac{3}{2}}\right)x^2}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}} a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] x^2*(b*x+a)*(3*(b*x+a)^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*x*b-3*x*b*a^(1/
2)-a^(3/2))/(b*x^3+a*x^2)^(3/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(b*x^3 + a*x^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(b\,x^3+a\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] int(x/(a*x^2 + b*x^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(x/(x**2*(a + b*x))**(3/2), x)
```

3.266 $\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

```
[Out] -15/4*b^2*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(7/2)+2/a/x/(b*x^3+a*x^2
)^(1/2)-5/2*(b*x^3+a*x^2)^(1/2)/a^2/x^3+15/4*b*(b*x^3+a*x^2)^(1/2)/a^3/x^2
```

Rubi [A] time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2006, 2025, 2008, 206}

$$-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^(-3/2),x]
```

```
[Out] 2/(a*x*Sqrt[a*x^2 + b*x^3]) - (5*Sqrt[a*x^2 + b*x^3])/(2*a^2*x^3) + (15*b*S
qrt[a*x^2 + b*x^3])/(4*a^3*x^2) - (15*b^2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 +
b*x^3]])/(4*a^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2006

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Dist[(n*p + n - j + 1)/
(a*(n - j)*(p + 1)), Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a, b
}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{ax\sqrt{ax^2+bx^3}}+\frac{5\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{a}$$
$$= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}-\frac{(15b)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{4a^2}$$
$$= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}+\frac{\left(15b^2\right)\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx}{8a^3}$$
$$= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{\left(15b^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+}}\right.}{4a^3}$$
$$= \frac{2}{ax\sqrt{ax^2+bx^3}}-\frac{5\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{15b\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{15b^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}$$

Mathematica [C] time = 0.02, size = 38, normalized size = 0.35

$$\frac{2b^2x\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{bx}{a}+1\right)}{a^3\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^(-3/2),x]
[Out] (2*b^2*x*Hypergeometric2F1[-1/2, 3, 1/2, 1 + (b*x)/a])/(a^3*Sqrt[x^2*(a + b
*x)])
```

fricas [A] time = 0.43, size = 219, normalized size = 1.99

$$\left[\frac{15\left(b^3x^4+ab^2x^3\right)\sqrt{a}\,\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\left(15\,ab^2x^2+5\,a^2bx-2\,a^3\right)\sqrt{bx^3+ax^2}}{8\left(a^4bx^4+a^5x^3\right)},\frac{15\left(b^3x^4+ab^2x^3\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] [1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 +
a*x^2)*sqrt(a))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 + a*
x^2))/(a^4*b*x^4 + a^5*x^3), 1/4*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(-a)*arctan(
sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sq
rt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 0.06, size = 76, normalized size = 0.69

$$-\frac{(bx+a)\left(15\sqrt{bx+a}\,b^2x^2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-15\sqrt{a}\,b^2x^2-5a^{\frac{3}{2}}bx+2a^{\frac{5}{2}}\right)x}{4\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/4*x*(b*x+a)*(15*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*x^2*b^2-5*a
^(3/2)*x*b-15*x^2*b^2*a^(1/2)+2*a^(5/2))/(b*x^3+a*x^2)^(3/2)/a^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^(-3/2), x)
```

mupad [B] time = 5.43, size = 42, normalized size = 0.38

$$-\frac{2\,x\,{\left(\frac{a}{b\,x}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a}{b\,x}\right)}{7\,{\left(b\,x^3+a\,x^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^3)^(3/2),x)
```

```
[Out] -(2*x*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -a/(b*x)))/(7*(a*x^2 +
 b*x^3)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^2+bx^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral((a*x**2 + b*x**3)**(-3/2), x)
```

3.267 $\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=138

$$\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

```
[Out] 35/8*b^3*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(9/2)+2/a/x^2/(b*x^3+a*x^
2)^(1/2)-7/3*(b*x^3+a*x^2)^(1/2)/a^2/x^4+35/12*b*(b*x^3+a*x^2)^(1/2)/a^3/x^
3-35/8*b^2*(b*x^3+a*x^2)^(1/2)/a^4/x^2
```

Rubi [A] time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2008, 206}

$$-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b^3\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{8a^{9/2}}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] 2/(a*x^2*Sqrt[a*x^2 + b*x^3]) - (7*Sqrt[a*x^2 + b*x^3])/(3*a^2*x^4) + (35*b
*Sqrt[a*x^2 + b*x^3])/(12*a^3*x^3) - (35*b^2*Sqrt[a*x^2 + b*x^3])/(8*a^4*x^
2) + (35*b^3*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(9/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{ax^2\sqrt{ax^2+bx^3}}+\frac{7\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx}{a}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}-\frac{(35b)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{6a^2}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}+\frac{\left(35b^2\right)\int \frac{1}{x\sqrt{ax^2+bx^3}}\,dx}{8a^3}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}-\frac{\left(35b^3\right)\int \frac{1}{\sqrt{ax^2\cdots}}}{16a^4}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{\left(35b^3\right)\text{Subst}\cdots}{\cdots}$$

$$= \frac{2}{ax^2\sqrt{ax^2+bx^3}}-\frac{7\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{35b\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{35b^2\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{35b^3\tanh^{-1}\left(\cdots\right.}{8a^{9/2}}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.28

$$-\frac{2b^3x\,{}_2F_1\left(-\frac{1}{2},4;\frac{1}{2};\frac{bx}{a}+1\right)}{a^4\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] (-2*b^3*x*Hypergeometric2F1[-1/2, 4, 1/2, 1 + (b*x)/a])/(a^4*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.40, size = 241, normalized size = 1.75

$$\left[\frac{105\left(b^4x^5+ab^3x^4\right)\sqrt{a}\,\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)-2\left(105\,ab^3x^3+35\,a^2b^2x^2-14\,a^3bx+8\,a^4\right)\sqrt{bx^3+ax^2}}{48\left(a^5bx^5+a^6x^4\right)},\ -\frac{10\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(105*(b^4*x^5 + a*b^3*x^4)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3
+ a*x^2)*sqrt(a))/x^2) - 2*(105*a*b^3*x^3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8
*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4), -1/24*(105*(b^4*x^5 + a*b
^3*x^4)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*x)) + (105*a*b^3*x^
3 + 35*a^2*b^2*x^2 - 14*a^3*b*x + 8*a^4)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 +
a^6*x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

maple [A] time = 0.06, size = 86, normalized size = 0.62

$$-\frac{(bx+a)\left(-105\sqrt{bx+a}\,b^3x^3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+105\sqrt{a}\,b^3x^3+35a^{\frac{3}{2}}b^2x^2-14a^{\frac{5}{2}}bx+8a^{\frac{7}{2}}\right)}{24\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/24*(b*x+a)*(-105*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*x^3*b^3-14
*a^(5/2)*x*b+35*a^(3/2)*x^2*b^2+105*b^3*x^3*a^(1/2)+8*a^(7/2))/(b*x^3+a*x^2
)^(3/2)/a^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^3)^(3/2)),x)
```

```
[Out] int(1/(x*(a*x^2 + b*x^3)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x*(x**2*(a + b*x))**(3/2)), x)
```

3.268 $\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}+\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{2}{ax^3\sqrt{ax^2+b}}$$

```
[Out] -315/64*b^4*arctanh(x*a^(1/2)/(b*x^3+a*x^2)^(1/2))/a^(11/2)+2/a/x^3/(b*x^3+
a*x^2)^(1/2)-9/4*(b*x^3+a*x^2)^(1/2)/a^2/x^5+21/8*b*(b*x^3+a*x^2)^(1/2)/a^3
/x^4-105/32*b^2*(b*x^3+a*x^2)^(1/2)/a^4/x^3+315/64*b^3*(b*x^3+a*x^2)^(1/2)/
a^5/x^2
```

Rubi [A] time = 0.23, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2023, 2025, 2008, 206}

$$\frac{315b^3\sqrt{ax^2+bx^3}}{64a^5x^2}-\frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3}-\frac{315b^4\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^3}}\right)}{64a^{11/2}}+\frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4}-\frac{9\sqrt{ax^2+bx^3}}{4a^2x^5}+\frac{2}{ax^3\sqrt{ax^2+bx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

```
[Out] 2/(a*x^3*Sqrt[a*x^2 + b*x^3]) - (9*Sqrt[a*x^2 + b*x^3])/(4*a^2*x^5) + (21*b
*Sqrt[a*x^2 + b*x^3])/(8*a^3*x^4) - (105*b^2*Sqrt[a*x^2 + b*x^3])/(32*a^4*x
^3) + (315*b^3*Sqrt[a*x^2 + b*x^3])/(64*a^5*x^2) - (315*b^4*ArcTanh[(Sqrt[a
]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a^(11/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2023

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &&
!IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1
]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2}{ax^3\sqrt{ax^2+bx^3}} + \frac{9\int \frac{1}{x^4\sqrt{ax^2+bx^3}}\,dx}{a}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} - \frac{(63b)\int \frac{1}{x^3\sqrt{ax^2+bx^3}}\,dx}{8a^2}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} + \frac{\left(105b^2\right)\int \frac{1}{x^2\sqrt{ax^2+bx^3}}\,dx}{16a^3}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} - \frac{\left(315b^3\right)\int}{}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{a}}{64a}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{a}}{64a}$$
$$= \frac{2}{ax^3\sqrt{ax^2+bx^3}} - \frac{9\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{21b\sqrt{ax^2+bx^3}}{8a^3x^4} - \frac{105b^2\sqrt{ax^2+bx^3}}{32a^4x^3} + \frac{315b^3\sqrt{a}}{64a}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.23

$$\frac{2b^4x\,{}_2F_1\left(-\frac{1}{2},5;\frac{1}{2};\frac{bx}{a}+1\right)}{a^5\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
[Out] (2*b^4*x*Hypergeometric2F1[-1/2, 5, 1/2, 1 + (b*x)/a])/(a^5*Sqrt[x^2*(a + b
*x)])
```

fricas [A] time = 0.42, size = 263, normalized size = 1.58

$$\left[\frac{315\left(b^5x^6+ab^4x^5\right)\sqrt{a}\,\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{a}}{x^2}\right)+2\left(315\,ab^4x^4+105\,a^2b^3x^3-42\,a^3b^2x^2+24\,a^4bx-16\,a^5\right)}{128\left(a^6bx^6+a^7x^5\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
[Out] [1/128*(315*(b^5*x^6 + a*b^4*x^5)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3
 + a*x^2)*sqrt(a))/x^2) + 2*(315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x
^2 + 24*a^4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5), 1/64*
(315*(b^5*x^6 + a*b^4*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(a*
x)) + (315*a*b^4*x^4 + 105*a^2*b^3*x^3 - 42*a^3*b^2*x^2 + 24*a^4*b*x - 16*a
^5)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

maple [A] time = 0.06, size = 100, normalized size = 0.60

$$-\frac{(bx+a)\left(315\sqrt{bx+a}\; b^4x^4 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-315\sqrt{a}\; b^4x^4-105a^{\frac{3}{2}}b^3x^3+42a^{\frac{5}{2}}b^2x^2-24a^{\frac{7}{2}}bx+16a^{\frac{9}{2}}\right)}{64\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}a^{\frac{11}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^3+a*x^2)^(3/2),x)
```

```
[Out] -1/64*(b*x+a)*(315*arctanh((b*x+a)^(1/2)/a^(1/2))*(b*x+a)^(1/2)*x^4*b^4-24*
a^(7/2)*x*b+42*a^(5/2)*x^2*b^2-105*a^(3/2)*x^3*b^3-315*x^4*b^4*a^(1/2)+16*a
^(9/2))/x/(b*x^3+a*x^2)^(3/2)/a^(11/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

mupad [B] time = 5.68, size = 44, normalized size = 0.27

$$-\frac{2\left(\frac{a}{b\,x}+1\right)^{3/2}{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{11}{2};\;\frac{13}{2};\;-\frac{a}{b\,x}\right)}{11\,x\left(b\,x^3+a\,x^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^3)^(3/2)),x)
```

```
[Out] -(2*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 11/2], 13/2, -a/(b*x)))/(11*x*(a*x^
2 + b*x^3)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**3+a*x**2)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2*(a + b*x))**(3/2)), x)
```

3.269 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=125

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a\sqrt{x}\,\sqrt{ax^2+bx^3}}{12b^2}+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

```
[Out] -5/8*a^3*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(7/2)+1/3*x^(3/2)*(
b*x^3+a*x^2)^(1/2)/b+5/8*a^2*(b*x^3+a*x^2)^(1/2)/b^3/x^(1/2)-5/12*a*x^(1/2)
*(b*x^3+a*x^2)^(1/2)/b^2
```

Rubi [A] time = 0.17, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}-\frac{5a\sqrt{x}\,\sqrt{ax^2+bx^3}}{12b^2}+\frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (5*a^2*Sqrt[a*x^2 + b*x^3])/(8*b^3*Sqrt[x]) - (5*a*Sqrt[x]*Sqrt[a*x^2 + b*x
^3])/(12*b^2) + (x^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a^3*ArcTanh[(Sqrt[
b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(8*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{7/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{(5a)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx}{6b}$$
$$= -\frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} + \frac{\left(5a^2\right)\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx}{8b^2}$$
$$= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{\left(5a^3\right)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{16b^3}$$
$$= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+b}}\right.}{8b^3}$$
$$= \frac{5a^2\sqrt{ax^2+bx^3}}{8b^3\sqrt{x}} - \frac{5a\sqrt{x}\sqrt{ax^2+bx^3}}{12b^2} + \frac{x^{3/2}\sqrt{ax^2+bx^3}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.13, size = 104, normalized size = 0.83

$$\frac{\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{\frac{bx}{a}+1}\left(15a^2-10abx+8b^2x^2\right)-15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\right)}{24b^{7/2}x\sqrt{\frac{bx}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[1 + (b*x)/a]*(15*a^2 - 10*a*b*x
+ 8*b^2*x^2) - 15*a^(5/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]]))/(24*b^(7/2)*
x*Sqrt[1 + (b*x)/a])
```

fricas [A] time = 0.41, size = 180, normalized size = 1.44

$$\left[\frac{15\,a^3\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)+2\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx^3+ax^2}\,\sqrt{x}}{48\,b^4x}, \frac{15\,a^3\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^3}}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*
sqrt(x))/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*sqr
t(x))/(b^4*x), 1/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/
(b*x^(3/2))) + (8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^3 + a*x^2)*sqrt
(x))/(b^4*x)]
```

giac [A] time = 0.20, size = 64, normalized size = 0.51

$$\frac{1}{24}\sqrt{bx+a}\left(2\,x\left(\frac{4\,x}{b}-\frac{5\,a}{b^2}\right)+\frac{15\,a^2}{b^3}\right)\sqrt{x}+\frac{5\,a^3\log\left(\left|-\sqrt{b}\sqrt{x}+\sqrt{bx+a}\right|\right)}{8\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x + a)*(2*x*(4*x/b - 5*a/b^2) + 15*a^2/b^3)*sqrt(x) + 5/8*a^3*l
og(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(7/2)
```

maple [A] time = 0.07, size = 103, normalized size = 0.82

$$-\frac{\left(-16b^{\frac{9}{2}}x^4+4a\,b^{\frac{7}{2}}x^3-10a^2b^{\frac{5}{2}}x^2-30a^3b^{\frac{3}{2}}x+15\sqrt{(bx+a)\,x}\;a^3b\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\;\sqrt{b}}{2\sqrt{b}}\right)\right)\sqrt{x}}{48\sqrt{b\,x^3+a\,x^2}\;b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/48*x^(1/2)*(-16*b^(9/2)*x^4+4*b^(7/2)*x^3*a-10*b^(5/2)*x^2*a^2-30*b^(3/2
)*x*a^3+15*(x*(b*x+a))^(1/2)*ln(1/2*(2*(b*x^2+a*x)^(1/2)*b^(1/2)+2*b*x+a)/b
^(1/2))*a^3*b)/(b*x^3+a*x^2)^(1/2)/b^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{7}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(7/2)/sqrt(b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^{7/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(7/2)/(a*x^2 + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{7}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x)), x)
```

3.270 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=95

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b}$$

```
[Out] 3/4*a^2*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(5/2)-3/4*a*(b*x^3+a
*x^2)^(1/2)/b^2/x^(1/2)+1/2*x^(1/2)*(b*x^3+a*x^2)^(1/2)/b
```

Rubi [A] time = 0.13, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}}+\frac{\sqrt{x}\,\sqrt{ax^2+bx^3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (-3*a*Sqrt[a*x^2 + b*x^3])/(4*b^2*Sqrt[x]) + (Sqrt[x]*Sqrt[a*x^2 + b*x^3])/
(2*b) + (3*a^2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(4*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b} - \frac{(3a)\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx}{4b}$$
$$= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b} + \frac{(3a^2)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{8b^2}$$
$$= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b} + \frac{(3a^2)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^2}$$
$$= -\frac{3a\sqrt{ax^2+bx^3}}{4b^2\sqrt{x}} + \frac{\sqrt{x}\sqrt{ax^2+bx^3}}{2b} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}$$

Mathematica [A] time = 0.05, size = 90, normalized size = 0.95

$$\frac{3a^{5/2}x\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}\,x^{3/2}\left(-3a^2-abx+2b^2x^2\right)}{4b^{5/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[b]*x^(3/2)*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*x*Sqrt[1 + (b*x)/
a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.42, size = 159, normalized size = 1.67

$$\left[\frac{3\,a^2\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)+2\,\sqrt{bx^3+ax^2}\,\left(2\,b^2x-3\,ab\right)\sqrt{x}}{8\,b^3x}, -\frac{3\,a^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)-}{4\,b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sq
rt(x))/x) + 2*sqrt(b*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x), -1/4*
(3*a^2*sqrt(-b)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) - sqrt(b
*x^3 + a*x^2)*(2*b^2*x - 3*a*b)*sqrt(x))/(b^3*x)]
```

giac [A] time = 0.18, size = 52, normalized size = 0.55

$$\frac{1}{4}\sqrt{bx+a}\,\sqrt{x}\left(\frac{2\,x}{b}-\frac{3\,a}{b^2}\right)-\frac{3\,a^2\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{4\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(b*x + a)*sqrt(x)*(2*x/b - 3*a/b^2) - 3/4*a^2*log(abs(-sqrt(b)*sqrt
(x) + sqrt(b*x + a)))/b^(5/2)
```

maple [A] time = 0.05, size = 92, normalized size = 0.97

$$\frac{\left(4b^{\frac{7}{2}}x^3-2a\,b^{\frac{5}{2}}x^2-6a^2b^{\frac{3}{2}}x+3\sqrt{(bx+a)\,x}\;a^2b\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\;\sqrt{b}}{2\sqrt{b}}\right)\right)\sqrt{x}}{8\sqrt{b\,x^3+a\,x^2}\;b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/8*x^(1/2)*(4*b^(7/2)*x^3-2*b^(5/2)*x^2*a-6*b^(3/2)*x*a^2+3*((b*x+a)*x)^(1
/2)*ln(1/2*(2*b*x+a+2*(b*x^2+a*x)^(1/2)*b^(1/2))/b^(1/2))*a^2*b)/(b*x^3+a*x
^2)^(1/2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^3 + ax^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(5/2)/sqrt(b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{5/2}}{\sqrt{b\,x^3 + a\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(5/2)/(a*x^2 + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\,(a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x)), x)
```

3.271 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=60

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

```
[Out] -a*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(3/2)+(b*x^3+a*x^2)^(1/2)
/b/x^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] Sqrt[a*x^2 + b*x^3]/(b*Sqrt[x]) - (a*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 +
 b*x^3]])/b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\,dx &= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx}{2b}\\
&= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b}\\
&= \frac{\sqrt{ax^2+bx^3}}{b\sqrt{x}}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.22

$$\frac{\sqrt{b}\,x^{3/2}(a+bx)-a^{3/2}x\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^3],x]
```

```
[Out] (Sqrt[b]*x^(3/2)*(a + b*x) - a^(3/2)*x*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*S
qrt[x])/Sqrt[a]])/(b^(3/2)*Sqrt[x^2*(a + b*x)])
```

fricas [A] time = 0.42, size = 131, normalized size = 2.18

$$\left[\frac{a\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)+2\,\sqrt{bx^3+ax^2}\,b\sqrt{x}}{2\,b^2x},\ \frac{a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)+\sqrt{bx^3+ax^2}\,b\sqrt{x}}{b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x
))/x) + 2*sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt
(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2))) + sqrt(b*x^3 + a*x^2)*b*sqrt(x))/(b^2
*x)]
```

giac [A] time = 0.18, size = 38, normalized size = 0.63

$$\frac{a\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{b^{\frac{3}{2}}}+\frac{\sqrt{bx+a}\,\sqrt{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/b^(3/2) + sqrt(b*x + a)*sqrt(x
)/b
```

maple [A] time = 0.05, size = 78, normalized size = 1.30

$$-\frac{\left(-2b^{\frac{5}{2}}x^2-2a\,b^{\frac{3}{2}}x+\sqrt{(bx+a)\,x}\;ab\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\,\sqrt{b}}{2\sqrt{b}}\right)\right)\sqrt{x}}{2\sqrt{b\,x^3+a\,x^2}\;b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -1/2*x^(1/2)*(-2*b^(5/2)*x^2-2*a*b^(3/2)*x+a*((b*x+a)*x)^(1/2)*ln(1/2*(2*b*
x+a+2*(b*x^2+a*x)^(1/2)*b^(1/2))/b^(1/2))*b)/(b*x^3+a*x^2)^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{3}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{3/2}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x^2 + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x)), x)
```

3.272 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=34

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

[Out] 2*arctanh(x^(3/2)*b^(1/2)/(b*x^3+a*x^2)^(1/2))/b^(1/2)

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]

[Out] (2*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/Sqrt[b]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^3}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^2+bx^3}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 1.62

$$\frac{2\sqrt{a}\,x\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x^2(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^3],x]

[Out] (2*Sqrt[a]*x*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]
Sqrt[x^2(a + b*x)])

fricas [A] time = 0.40, size = 77, normalized size = 2.26

$$\left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\,\sqrt{b}\,\sqrt{x}}{x}\right)}{\sqrt{b}},\ -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^3+ax^2}\,\sqrt{-b}}{bx^{\frac{3}{2}}}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b)*sqrt(x))/x)/sqrt(b), -2
*sqrt(-b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b)/(b*x^(3/2)))/b]
```

giac [A] time = 0.17, size = 23, normalized size = 0.68

$$-\frac{2\,\log\left(\left|-\sqrt{b}\,\sqrt{x}+\sqrt{bx+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(abs(-sqrt(b)*sqrt(x) + sqrt(b*x + a)))/sqrt(b)
```

maple [B] time = 0.05, size = 58, normalized size = 1.71

$$\frac{\sqrt{(bx+a)\,x}\,\sqrt{x}\,\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\,\sqrt{b}}{2\sqrt{b}}\right)}{\sqrt{b\,x^3+a\,x^2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] 1/(b*x^3+a*x^2)^(1/2)*x^(1/2)*((b*x+a)*x)^(1/2)*ln(1/2*(2*b*x+a+2*(b*x^2+a*
x)^(1/2)*b^(1/2))/b^(1/2))/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(b*x^3 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sqrt{x}}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x^2 + b*x^3)^(1/2),x)
```

```
[Out] int(x^(1/2)/(a*x^2 + b*x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x^2 (a + bx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(sqrt(x)/sqrt(x**2*(a + b*x)), x)
```

3.273 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=25

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

[Out] -2*(b*x^3+a*x^2)^(1/2)/a/x^(3/2)

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2014}

$$-\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]

[Out] (-2*Sqrt[a*x^2 + b*x^3])/(a*x^(3/2))

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{ax^2+bx^3}}{ax^{3/2}}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.92

$$-\frac{2\sqrt{x^2(a+bx)}}{ax^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^3]),x]

[Out] (-2*Sqrt[x^2*(a + b*x)])/(a*x^(3/2))

fricas [A] time = 0.40, size = 21, normalized size = 0.84

$$-\frac{2\sqrt{bx^3+ax^2}}{ax^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(b*x^3 + a*x^2)/(a*x^(3/2))

giac [A] time = 0.24, size = 30, normalized size = 1.20

$$\frac{4\sqrt{b}}{\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 4*sqrt(b)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)
```

maple [A] time = 0.04, size = 27, normalized size = 1.08

$$-\frac{2 (bx+a) \sqrt{x}}{\sqrt{b\,x^3+a\,x^2}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x)
[Out] -2*(b*x+a)*x^(1/2)/a/(b*x^3+a*x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax^2}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{x}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)),x)
[Out] int(1/(x^(1/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**3+a*x**2)**(1/2),x)
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x))), x)
```

3.274 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=56

$$\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

```
[Out] -2/3*(b*x^3+a*x^2)^(1/2)/a/x^(5/2)+4/3*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(3/2)
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}-\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(3*a*x^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*x
^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}-\frac{(2b)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^2+bx^3}}{3ax^{5/2}}+\frac{4b\sqrt{ax^2+bx^3}}{3a^2x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 0.55

$$-\frac{2(a-2bx)\sqrt{x^2(a+bx)}}{3a^2x^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*(a - 2*b*x)*Sqrt[x^2*(a + b*x)])/(3*a^2*x^(5/2))
```

fricas [A] time = 0.38, size = 29, normalized size = 0.52

$$\frac{2\sqrt{bx^3+ax^2}\,(2\,bx-a)}{3\,a^2x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^3 + a*x^2)*(2*b*x - a)/(a^2*x^(5/2))
```

giac [A] time = 0.21, size = 55, normalized size = 0.98

$$\frac{8\left(3\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)b^{\frac{3}{2}}}{3\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)*b^(3/2)/((sqrt(b)*sqrt(x) -
 sqrt(b*x + a))^2 - a)^3
```

maple [A] time = 0.06, size = 33, normalized size = 0.59

$$-\frac{2\,(bx+a)\,(-2bx+a)}{3\sqrt{b\,x^3+a\,x^2}\;a^2\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)*(-2*b*x+a)/x^(1/2)/a^2/(b*x^3+a*x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx^3+ax^2}\,x^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{x^{3/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^{\frac{3}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x))), x)
```

3.275 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=86

$$-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

```
[Out] -2/5*(b*x^3+a*x^2)^(1/2)/a/x^(7/2)+8/15*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(5/2)-1
6/15*b^2*(b*x^3+a*x^2)^(1/2)/a^3/x^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(5*a*x^(7/2)) + (8*b*Sqrt[a*x^2 + b*x^3])/(15*a^2*
x^(5/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(15*a^3*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}-\frac{(4b)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx}{5a}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}+\frac{\left(8b^2\right)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{15a^2}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{5ax^{7/2}}+\frac{8b\sqrt{ax^2+bx^3}}{15a^2x^{5/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{15a^3x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.51

$$-\frac{2\sqrt{x^2(a+bx)}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
[Out] (-2*Sqrt[x^2*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^(7/2))
```

fricas [A] time = 0.38, size = 40, normalized size = 0.47

$$-\frac{2\left(8\,b^2x^2-4\,abx+3\,a^2\right)\sqrt{bx^3+ax^2}}{15\,a^3x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] -2/15*(8*b^2*x^2 - 4*a*b*x + 3*a^2)*sqrt(b*x^3 + a*x^2)/(a^3*x^(7/2))
```

giac [A] time = 0.20, size = 77, normalized size = 0.90

$$\frac{32\left(10\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4-5\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2+a^2\right)b^{\frac{5}{2}}}{15\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 32/15*(10*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^4 - 5*a*(sqrt(b)*sqrt(x) - sqrt
(b*x + a))^2 + a^2)*b^(5/2)/((sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^5
```

maple [A] time = 0.04, size = 46, normalized size = 0.53

$$-\frac{2\,(bx+a)\left(8b^2x^2-4abx+3a^2\right)}{15\sqrt{b\,x^3+a\,x^2}\;a^3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x)
[Out] -2/15*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/x^(3/2)/a^3/(b*x^3+a*x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^3+ax^2}\,x^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{5/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)),x)
[Out] int(1/(x^(5/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x))), x)
```

3.276 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal. Leaf size=116

$$\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

```
[Out] -2/7*(b*x^3+a*x^2)^(1/2)/a/x^(9/2)+12/35*b*(b*x^3+a*x^2)^(1/2)/a^2/x^(7/2)-
16/35*b^2*(b*x^3+a*x^2)^(1/2)/a^3/x^(5/2)+32/35*b^3*(b*x^3+a*x^2)^(1/2)/a^4
/x^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^3])/(7*a*x^(9/2)) + (12*b*Sqrt[a*x^2 + b*x^3])/(35*a^2
*x^(7/2)) - (16*b^2*Sqrt[a*x^2 + b*x^3])/(35*a^3*x^(5/2)) + (32*b^3*Sqrt[a*
x^2 + b*x^3])/(35*a^4*x^(3/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^3}}\,dx &= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}-\frac{(6b)\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^3}}\,dx}{7a}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}+\frac{\left(24b^2\right)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^3}}\,dx}{35a^2}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}-\frac{\left(16b^3\right)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^3}}\,dx}{35a^3}\\
&= -\frac{2\sqrt{ax^2+bx^3}}{7ax^{9/2}}+\frac{12b\sqrt{ax^2+bx^3}}{35a^2x^{7/2}}-\frac{16b^2\sqrt{ax^2+bx^3}}{35a^3x^{5/2}}+\frac{32b^3\sqrt{ax^2+bx^3}}{35a^4x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 55, normalized size = 0.47

$$\frac{2\sqrt{x^2(a+bx)}\left(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3\right)}{35a^4x^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
[Out] (2*Sqrt[x^2*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^(9/2))
```

fricas [A] time = 0.39, size = 51, normalized size = 0.44

$$\frac{2\left(16\,b^3x^3-8\,ab^2x^2+6\,a^2bx-5\,a^3\right)\sqrt{bx^3+ax^2}}{35\,a^4x^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^3 + a*x^2)/(a^
4*x^(9/2))
```

giac [A] time = 0.21, size = 103, normalized size = 0.89

$$\frac{64\left(35\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^6-21\,a\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^4+7\,a^2\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a^3\right)b^{\frac{7}{2}}}{35\left(\left(\sqrt{b}\sqrt{x}-\sqrt{bx+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
[Out] 64/35*(35*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^6 - 21*a*(sqrt(b)*sqrt(x) - sqr
t(b*x + a))^4 + 7*a^2*(sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a^3)*b^(7/2)/((
sqrt(b)*sqrt(x) - sqrt(b*x + a))^2 - a)^7
```

maple [A] time = 0.04, size = 57, normalized size = 0.49

$$-\frac{2\,(bx+a)\left(-16b^3x^3+8a\,b^2x^2-6a^2bx+5a^3\right)}{35\sqrt{b\,x^3+a\,x^2}\;a^4x^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x)
[Out] -2/35*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^(5/2)/a^4/(b*x^3+
a*x^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx^3+ax^2}\,x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^3 + a*x^2)*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^{7/2}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x))), x)
```

3.277 $\int x^{1-3n} \left(ax^2+bx^3\right)^n \, dx$

Optimal. Leaf size=61

$$\frac{x^{2-3n} \left(\frac{bx}{a}+1\right)^{-n} \left(ax^2+bx^3\right)^n \, _2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}$$

```
[Out] x^(2-3*n)*(b*x^3+a*x^2)^n*hypergeom([-n, 2-n],[3-n],-b*x/a)/(2-n)/((1+b*x/a
)^n)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 66, 64}

$$\frac{x^{2-3n} \left(\frac{bx}{a}+1\right)^{-n} \left(ax^2+bx^3\right)^n \, _2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] (x^(2 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)/
a)])/((2 - n)*(1 + (b*x)/a)^n)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int x^{1-3n} \left(ax^2+bx^3\right)^n \, dx &= \left(x^{-2n}(a+bx)^{-n} \left(ax^2+bx^3\right)^n\right) \int x^{1-n}(a+bx)^n \, dx \\
&= \left(x^{-2n} \left(1+\frac{bx}{a}\right)^{-n} \left(ax^2+bx^3\right)^n\right) \int x^{1-n} \left(1+\frac{bx}{a}\right)^n \, dx \\
&= \frac{x^{2-3n} \left(1+\frac{bx}{a}\right)^{-n} \left(ax^2+bx^3\right)^n \, _2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 0.97

$$\frac{x^{2-3n}\left(x^2(a+bx)\right)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(2-n,-n;3-n;-\frac{bx}{a}\right)}{2-n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] (x^(2 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[2 - n, -n, 3 - n, -((b*x)/
a)])/((2 - n)*(1 + (b*x)/a)^n)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^3+ax^2\right)^n x^{-3n+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+ax^2\right)^n x^{-3n+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int x^{-3n+1}\left(b\,x^3+a\,x^2\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] int(x^(1-3*n)*(b*x^3+a*x^2)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+ax^2\right)^n x^{-3n+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^{1-3n}\left(b\,x^3+a\,x^2\right)^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1 - 3*n)*(a*x^2 + b*x^3)^n,x)
```

```
[Out] int(x^(1 - 3*n)*(a*x^2 + b*x^3)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{1-3n}\left(x^2\left(a+bx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Integral(x**(1 - 3*n)*(x**2*(a + b*x))**n, x)
```

3.278 $\int x^{-3n} \left(ax^2+bx^3\right)^n \, dx$

Optimal. Leaf size=48

$$\frac{x^{-3n-1} \left(ax^2+bx^3\right)^{n+1} {}_2F_1\left(1,2;2-n;-\frac{bx}{a}\right)}{a(1-n)}$$

```
[Out] x^(-1-3*n)*(b*x^3+a*x^2)^(1+n)*hypergeom([1, 2],[2-n],-b*x/a)/a/(1-n)
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2032, 66, 64}

$$\frac{x^{1-3n} \left(\frac{bx}{a}+1\right)^{-n} \left(ax^2+bx^3\right)^n {}_2F_1\left(1-n,-n;2-n;-\frac{bx}{a}\right)}{1-n}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^3)^n/x^(3*n),x]
```

```
[Out] (x^(1 - 3*n)*(a*x^2 + b*x^3)^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)/
a)])/((1 - n)*(1 + (b*x)/a)^n)
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int x^{-3n} \left(ax^2+bx^3\right)^n \, dx &= \left(x^{-2n}(a+bx)^{-n} \left(ax^2+bx^3\right)^n\right) \int x^{-n}(a+bx)^n \, dx \\
&= \left(x^{-2n} \left(1+\frac{bx}{a}\right)^{-n} \left(ax^2+bx^3\right)^n\right) \int x^{-n} \left(1+\frac{bx}{a}\right)^n \, dx \\
&= \frac{x^{1-3n} \left(1+\frac{bx}{a}\right)^{-n} \left(ax^2+bx^3\right)^n {}_2F_1\left(1-n,-n;2-n;-\frac{bx}{a}\right)}{1-n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 59, normalized size = 1.23

$$\frac{x^{1-3n}\left(x^2(a+bx)\right)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(1-n,-n;2-n;-\frac{bx}{a}\right)}{1-n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^3)^n/x^(3*n),x]
```

```
[Out] (x^(1 - 3*n)*(x^2*(a + b*x))^n*Hypergeometric2F1[1 - n, -n, 2 - n, -((b*x)/
a)])/((1 - n)*(1 + (b*x)/a)^n)
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\left(bx^3+ax^2\right)^n}{x^{3\,n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x^2)^n/x^(3*n), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+ax^2\right)^n}{x^{3\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int x^{-3n}\left(b\,x^3+a\,x^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a*x^2)^n/(x^(3*n)),x)
```

```
[Out] int((b*x^3+a*x^2)^n/(x^(3*n)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(bx^3+ax^2\right)^n}{x^{3\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a*x^2)^n/(x^(3*n)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n/x^(3*n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(b\,x^3+a\,x^2\right)^n}{x^{3\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n),x)
```

```
[Out] int((a*x^2 + b*x^3)^n/x^(3*n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-3n} \left(x^2 \left(a + bx\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a*x**2)**n/(x**(3*n)),x)
```

```
[Out] Integral(x**(-3*n)*(x**2*(a + b*x))**n, x)
```

3.279 $\int x^{-1-3n} \left(ax^2+bx^3\right)^n \, dx$

Optimal. Leaf size=54

$$-\frac{x^{-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n {}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}$$

```
[Out] -(b*x^3+a*x^2)^n*hypergeom([-n, -n],[1-n],-b*x/a)/n/(x^(3*n))/((1+b*x/a)^n)
```

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2032, 66, 64}

$$-\frac{x^{-3n}\left(\frac{bx}{a}+1\right)^{-n}\left(ax^2+bx^3\right)^n {}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((a*x^2 + b*x^3)^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3*
n)*(1 + (b*x)/a)^n))
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Dist[(c^IntPar
t[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*
x)/c)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ
[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0] && ((RationalQ[m] && !(EqQ[n, -
2^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-3n}\left(ax^2+bx^3\right)^n \, dx &= \left(x^{-2n}(a+bx)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-1-n}(a+bx)^n \, dx \\
&= \left(x^{-2n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n\right)\int x^{-1-n}\left(1+\frac{bx}{a}\right)^n \, dx \\
&= -\frac{x^{-3n}\left(1+\frac{bx}{a}\right)^{-n}\left(ax^2+bx^3\right)^n {}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 52, normalized size = 0.96

$$-\frac{x^{-3n}\left(x^2(a+bx)\right)^n\left(\frac{bx}{a}+1\right)^{-n}{}_2F_1\left(-n,-n;1-n;-\frac{bx}{a}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((x^2*(a + b*x))^n*Hypergeometric2F1[-n, -n, 1 - n, -((b*x)/a)])/(n*x^(3*
n)*(1 + (b*x)/a)^n))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+ax^2\right)^n x^{-3\,n-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+ax^2\right)^n x^{-3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

maple [F] time = 0.50, size = 0, normalized size = 0.00

$$\int x^{-3n-1}\left(b\,x^3+a\,x^2\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3*n-1)*(b*x^3+a*x^2)^n,x)
```

```
[Out] int(x^(-3*n-1)*(b*x^3+a*x^2)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+ax^2\right)^n x^{-3\,n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^3+a\,x^2\right)^n}{x^{3\,n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 1),x)
```

```
[Out] int((a*x^2 + b*x^3)^n/x^(3*n + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-3n-1}\left(x^2\left(a+bx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Integral(x**(-3*n - 1)*(x**2*(a + b*x))**n, x)
```

3.280 $\int x^{-2-3n} \left(ax^2+bx^3\right)^n \, dx$

Optimal. Leaf size=32

$$-\frac{x^{-3(n+1)} \left(ax^2+bx^3\right)^{n+1}}{a(n+1)}$$

```
[Out] -(b*x^3+a*x^2)^(1+n)/a/(1+n)/(x^(3+3*n))
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2014}

$$-\frac{x^{-3(n+1)} \left(ax^2+bx^3\right)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((a*x^2 + b*x^3)^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-2-3n} \left(ax^2+bx^3\right)^n \, dx = -\frac{x^{-3(1+n)} \left(ax^2+bx^3\right)^{1+n}}{a(1+n)}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.94

$$-\frac{x^{-3(n+1)} \left(x^2(a+bx)\right)^{n+1}}{a(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^2*(a + b*x))^(1 + n)/(a*(1 + n)*x^(3*(1 + n))))
```

fricas [A] time = 0.42, size = 38, normalized size = 1.19

$$-\frac{\left(bx^2+ax\right)\left(bx^3+ax^2\right)^n x^{-3n-2}}{an+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] -(b*x^2 + a*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 2)/(a*n + a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3+ax^2\right)^n x^{-3n-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

maple [A] time = 0.05, size = 36, normalized size = 1.12

$$-\frac{(bx+a)\,x^{-3n-1}\left(b\,x^3+a\,x^2\right)^n}{(n+1)\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] -(b*x+a)*x^(-3*n-1)/a/(n+1)*(b*x^3+a*x^2)^n
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^3+ax^2\right)^n x^{-3\,n-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 2), x)
```

mupad [B] time = 5.28, size = 54, normalized size = 1.69

$$-\left(b\,x^3+a\,x^2\right)^n\left(\frac{x}{x^{3\,n+2}\,\left(n+1\right)}+\frac{b\,x^2}{a\,x^{3\,n+2}\,\left(n+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 2),x)
```

```
[Out] -(a*x^2 + b*x^3)^n*(x/(x^(3*n + 2)*(n + 1)) + (b*x^2)/(a*x^(3*n + 2)*(n + 1
)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-3n-2}\left(x^2\left(a+bx\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Integral(x**(-3*n - 2)*(x**2*(a + b*x))**n, x)
```

3.281 $\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx$

Optimal. Leaf size=70

$$\frac{bx^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a(n+2)}$$

```
[Out] -x^(-4-3*n)*(b*x^3+a*x^2)^(1+n)/a/(2+n)+b*(b*x^3+a*x^2)^(1+n)/a^2/(1+n)/(2+
n)/(x^(3+3*n))
```

Rubi [A] time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{bx^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+1)(n+2)}-\frac{x^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-4 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(2 + n))) + (b*(a*x^2 + b*x^3)^
(1 + n))/(a^2*(1 + n)*(2 + n)*x^(3*(1 + n)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx&=-\frac{x^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a(2+n)}-\frac{b\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx}{a(2+n)}\\&=-\frac{x^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a(2+n)}+\frac{bx^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a^2(1+n)(2+n)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.63

$$-\frac{x^{-3n-4}(an+a-bx)\left(x^2(a+bx)\right)^{n+1}}{a^2(n+1)(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-4 - 3*n)*(a + a*n - b*x)*(x^2*(a + b*x))^(1 + n))/(a^2*(1 + n)*(2 +
n)))
```

fricas [A] time = 0.42, size = 70, normalized size = 1.00

$$-\frac{\left(abnx^2 - b^2x^3 + \left(a^2n + a^2\right)x\right)\left(bx^3 + ax^2\right)^n x^{-3\,n-3}}{a^2n^2 + 3\,a^2n + 2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] -(a*b*n*x^2 - b^2*x^3 + (a^2*n + a^2)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 3)/(a^
2*n^2 + 3*a^2*n + 2*a^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + ax^2\right)^n x^{-3\,n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

maple [A] time = 0.05, size = 50, normalized size = 0.71

$$-\frac{(an - bx + a)\,(bx + a)\,x^{-3n-2}\left(b\,x^3 + a\,x^2\right)^n}{(n+2)\,(n+1)\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] -(b*x^3+a*x^2)^n*x^(-2-3*n)*(a*n-b*x+a)*(b*x+a)/(n+2)/(n+1)/a^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^3 + ax^2\right)^n x^{-3\,n-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 3), x)
```

mupad [B] time = 5.28, size = 98, normalized size = 1.40

$$-\left(b\,x^3 + a\,x^2\right)^n \left(\frac{x\,(n+1)}{x^{3\,n+3}\,\left(n^2 + 3\,n + 2\right)} - \frac{b^2\,x^3}{a^2\,x^{3\,n+3}\,\left(n^2 + 3\,n + 2\right)} + \frac{b\,n\,x^2}{a\,x^{3\,n+3}\,\left(n^2 + 3\,n + 2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 3),x)
```

```
[Out] -(a*x^2 + b*x^3)^n*((x*(n + 1))/(x^(3*n + 3)*(3*n + n^2 + 2)) - (b^2*x^3)/(
a^2*x^(3*n + 3)*(3*n + n^2 + 2)) + (b*n*x^2)/(a*x^(3*n + 3)*(3*n + n^2 + 2)
))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Timed out
```

3.282 $\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx$

Optimal. Leaf size=116

$$-\frac{2b^2x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^3(n+1)(n+2)(n+3)}+\frac{2bx^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+2)(n+3)}-\frac{x^{-3n-5}\left(ax^2+bx^3\right)^{n+1}}{a(n+3)}$$

```
[Out] -x^(-5-3*n)*(b*x^3+a*x^2)^(1+n)/a/(3+n)+2*b*x^(-4-3*n)*(b*x^3+a*x^2)^(1+n)/
a^2/(2+n)/(3+n)-2*b^2*(b*x^3+a*x^2)^(1+n)/a^3/(2+n)/(n^2+4*n+3)/(x^(3+3*n))
```

Rubi [A] time = 0.09, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$-\frac{2b^2x^{-3(n+1)}\left(ax^2+bx^3\right)^{n+1}}{a^3(n+1)(n+2)(n+3)}+\frac{2bx^{-3n-4}\left(ax^2+bx^3\right)^{n+1}}{a^2(n+2)(n+3)}-\frac{x^{-3n-5}\left(ax^2+bx^3\right)^{n+1}}{a(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -((x^(-5 - 3*n)*(a*x^2 + b*x^3)^(1 + n))/(a*(3 + n))) + (2*b*x^(-4 - 3*n)*(
a*x^2 + b*x^3)^(1 + n))/(a^2*(2 + n)*(3 + n)) - (2*b^2*(a*x^2 + b*x^3)^(1 +
 n))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-4-3n}\left(ax^2+bx^3\right)^n\,dx&=-\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)}-\frac{(2b)\int x^{-3-3n}\left(ax^2+bx^3\right)^n\,dx}{a(3+n)}\\&=-\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)}+\frac{2bx^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a^2(2+n)(3+n)}+\frac{\left(2b^2\right)\int x^{-2-3n}\left(ax^2+bx^3\right)^n\,dx}{a^2(2+n)(3+n)}\\&=-\frac{x^{-5-3n}\left(ax^2+bx^3\right)^{1+n}}{a(3+n)}+\frac{2bx^{-4-3n}\left(ax^2+bx^3\right)^{1+n}}{a^2(2+n)(3+n)}-\frac{2b^2x^{-3(1+n)}\left(ax^2+bx^3\right)^{1+n}}{a^3(1+n)(2+n)(3+n)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.62

$$-\frac{x^{-3(n+1)}(a+bx)\left(x^2(a+bx)\right)^n\left(a^2\left(n^2+3n+2\right)-2ab(n+1)x+2b^2x^2\right)}{a^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-4 - 3*n)*(a*x^2 + b*x^3)^n,x]
```

```
[Out] -(((a + b*x)*(x^2*(a + b*x))^n*(a^2*(2 + 3*n + n^2) - 2*a*b*(1 + n)*x + 2*b
^2*x^2))/(a^3*(1 + n)*(2 + n)*(3 + n)*x^(3*(1 + n))))
```

fricas [A] time = 0.41, size = 111, normalized size = 0.96

$$\frac{\left(2\,ab^{2}nx^{3}-2\,b^{3}x^{4}-\left(a^{2}bn^{2}+a^{2}bn\right)x^{2}-\left(a^{3}n^{2}+3\,a^{3}n+2\,a^{3}\right)x\right)\left(bx^{3}+ax^{2}\right)^{n}x^{-3\,n-4}}{a^{3}n^{3}+6\,a^{3}n^{2}+11\,a^{3}n+6\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="fricas")
```

```
[Out] (2*a*b^2*n*x^3 - 2*b^3*x^4 - (a^2*b*n^2 + a^2*b*n)*x^2 - (a^3*n^2 + 3*a^3*n
 + 2*a^3)*x)*(b*x^3 + a*x^2)^n*x^(-3*n - 4)/(a^3*n^3 + 6*a^3*n^2 + 11*a^3*n
 + 6*a^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^{3}+ax^{2}\right)^{n}x^{-3\,n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

maple [A] time = 0.05, size = 84, normalized size = 0.72

$$-\frac{(bx+a)\left(a^{2}n^{2}-2abnx+2b^{2}x^{2}+3a^{2}n-2abx+2a^{2}\right)x^{-3n-3}\left(b\,x^{3}+a\,x^{2}\right)^{n}}{(n+3)\,(n+2)\,(n+1)\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-4-3*n)*(b*x^3+a*x^2)^n,x)
```

```
[Out] -x^(-3-3*n)*(b*x+a)*(a^2*n^2-2*a*b*n*x+2*b^2*x^2+3*a^2*n-2*a*b*x+2*a^2)*(b*
x^3+a*x^2)^n/(n+3)/(n+2)/(n+1)/a^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^{3}+ax^{2}\right)^{n}x^{-3\,n-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-4-3*n)*(b*x^3+a*x^2)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a*x^2)^n*x^(-3*n - 4), x)
```

mupad [B] time = 5.36, size = 157, normalized size = 1.35

$$-\left(b\,x^{3}+a\,x^{2}\right)^{n}\left(\frac{x\,\left(n^{2}+3\,n+2\right)}{x^{3\,n+4}\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}+\frac{2\,b^{3}\,x^{4}}{a^{3}\,x^{3\,n+4}\,\left(n^{3}+6\,n^{2}+11\,n+6\right)}-\frac{2\,b^{2}\,n\,x^{3}}{a^{2}\,x^{3\,n+4}\,\left(n^{3}+6\,n^{2}+11\,n\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^3)^n/x^(3*n + 4),x)
```

```
[Out] -(a*x^2 + b*x^3)^n*((x*(3*n + n^2 + 2))/(x^(3*n + 4)*(11*n + 6*n^2 + n^3 +
6)) + (2*b^3*x^4)/(a^3*x^(3*n + 4)*(11*n + 6*n^2 + n^3 + 6)) - (2*b^2*n*x^3
)/(a^2*x^(3*n + 4)*(11*n + 6*n^2 + n^3 + 6)) + (b*n*x^2*(n + 1))/(a*x^(3*n
+ 4)*(11*n + 6*n^2 + n^3 + 6)))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-4-3*n)*(b*x**3+a*x**2)**n,x)
```

```
[Out] Timed out
```

3.283 $\int \frac{x^{11}}{(ax^2+bx^5)^3} dx$

Optimal. Leaf size=19

$$\frac{x^6}{6a(a+bx^3)^2}$$

```
[Out] 1/6*x^6/a/(b*x^3+a)^2
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 264}

$$\frac{x^6}{6a(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^11/(a*x^2 + b*x^5)^3,x]
```

```
[Out] x^6/(6*a*(a + b*x^3)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int \frac{x^{11}}{(ax^2+bx^5)^3} dx &= \int \frac{x^5}{(a+bx^3)^3} dx \\ &= \frac{x^6}{6a(a+bx^3)^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{a+2bx^3}{6b^2(a+bx^3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^11/(a*x^2 + b*x^5)^3,x]
```

```
[Out] -1/6*(a + 2*b*x^3)/(b^2*(a + b*x^3)^2)
```

fricas [B] time = 0.36, size = 36, normalized size = 1.89

$$-\frac{2bx^3+a}{6(b^4x^6+2ab^3x^3+a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

giac [A] time = 0.18, size = 22, normalized size = 1.16

$$-\frac{2\,bx^3 + a}{6\left(bx^3 + a\right)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="giac")
```

```
[Out] -1/6*(2*b*x^3 + a)/((b*x^3 + a)^2*b^2)
```

maple [A] time = 0.06, size = 31, normalized size = 1.63

$$\frac{a}{6\left(b\,x^3 + a\right)^2 b^2} - \frac{1}{3\left(b\,x^3 + a\right) b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(b*x^5+a*x^2)^3,x)
```

```
[Out] 1/6*a/b^2/(b*x^3+a)^2-1/3/b^2/(b*x^3+a)
```

maxima [B] time = 1.29, size = 36, normalized size = 1.89

$$-\frac{2\,bx^3 + a}{6\left(b^4x^6 + 2\,ab^3x^3 + a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11/(b*x^5+a*x^2)^3,x, algorithm="maxima")
```

```
[Out] -1/6*(2*b*x^3 + a)/(b^4*x^6 + 2*a*b^3*x^3 + a^2*b^2)
```

mupad [B] time = 5.11, size = 37, normalized size = 1.95

$$-\frac{\frac{a}{6\,b^2} + \frac{x^3}{3\,b}}{a^2 + 2\,a\,b\,x^3 + b^2\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11/(a*x^2 + b*x^5)^3,x)
```

```
[Out] -(a/(6*b^2) + x^3/(3*b))/(a^2 + b^2*x^6 + 2*a*b*x^3)
```

sympy [B] time = 0.40, size = 36, normalized size = 1.89

$$\frac{-a - 2bx^3}{6a^2b^2 + 12ab^3x^3 + 6b^4x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11/(b*x**5+a*x**2)**3,x)
```

```
[Out] (-a - 2*b*x**3)/(6*a**2*b**2 + 12*a*b**3*x**3 + 6*b**4*x**6)
```

3.284 $\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=80

$$\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

```
[Out] 16/45*a^2*(b*x^5+a*x^2)^(1/2)/b^3/x-8/45*a*x^2*(b*x^5+a*x^2)^(1/2)/b^2+2/15
*x^5*(b*x^5+a*x^2)^(1/2)/b
```

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (16*a^2*Sqrt[a*x^2 + b*x^5])/(45*b^3*x) - (8*a*x^2*Sqrt[a*x^2 + b*x^5])/(45
*b^2) + (2*x^5*Sqrt[a*x^2 + b*x^5])/(15*b)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^9}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2x^5\sqrt{ax^2+bx^5}}{15b}-\frac{(4a)\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx}{5b}\\ &= -\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}+\frac{\left(8a^2\right)\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx}{15b^2}\\ &= \frac{16a^2\sqrt{ax^2+bx^5}}{45b^3x}-\frac{8ax^2\sqrt{ax^2+bx^5}}{45b^2}+\frac{2x^5\sqrt{ax^2+bx^5}}{15b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.58

$$\frac{2\sqrt{x^2\left(a+bx^3\right)}\left(8a^2-4abx^3+3b^2x^6\right)}{45b^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*Sqrt[x^2*(a + b*x^3)]*(8*a^2 - 4*a*b*x^3 + 3*b^2*x^6))/(45*b^3*x)
```

fricas [A] time = 0.39, size = 42, normalized size = 0.52

$$\frac{2\left(3\,b^2x^6 - 4\,abx^3 + 8\,a^2\right)\sqrt{bx^5 + ax^2}}{45\,b^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/45*(3*b^2*x^6 - 4*a*b*x^3 + 8*a^2)*sqrt(b*x^5 + a*x^2)/(b^3*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^9/sqrt(b*x^5 + a*x^2), x)
```

maple [A] time = 0.05, size = 48, normalized size = 0.60

$$\frac{2\left(b\,x^3 + a\right)\left(3b^2x^6 - 4ab\,x^3 + 8a^2\right)x}{45\sqrt{b\,x^5 + a\,x^2}\;b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(b*x^5+a*x^2)^(1/2),x)
[Out] 2/45*(b*x^3+a)*(3*b^2*x^6-4*a*b*x^3+8*a^2)*x/b^3/(b*x^5+a*x^2)^(1/2)
```

maxima [A] time = 1.43, size = 46, normalized size = 0.58

$$\frac{2\left(3\,b^3x^9 - ab^2x^6 + 4\,a^2bx^3 + 8\,a^3\right)}{45\,\sqrt{bx^3 + a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/45*(3*b^3*x^9 - a*b^2*x^6 + 4*a^2*b*x^3 + 8*a^3)/(sqrt(b*x^3 + a)*b^3)
```

mupad [B] time = 5.18, size = 42, normalized size = 0.52

$$\frac{2\,\sqrt{b\,x^5 + a\,x^2}\,\left(8\,a^2 - 4\,a\,b\,x^3 + 3\,b^2\,x^6\right)}{45\,b^3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(a*x^2 + b*x^5)^(1/2),x)
[Out] (2*(a*x^2 + b*x^5)^(1/2)*(8*a^2 + 3*b^2*x^6 - 4*a*b*x^3))/(45*b^3*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^9}{\sqrt{x^2 \left(a+b x^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**9/sqrt(x**2*(a + b*x**3)), x)
```

3.285 $\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=52

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

```
[Out] -4/9*a*(b*x^5+a*x^2)^(1/2)/b^2/x+2/9*x^2*(b*x^5+a*x^2)^(1/2)/b
```

Rubi [A] time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 1588}

$$\frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-4*a*Sqrt[a*x^2 + b*x^5])/(9*b^2*x) + (2*x^2*Sqrt[a*x^2 + b*x^5])/(9*b)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2x^2\sqrt{ax^2+bx^5}}{9b}-\frac{(2a)\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx}{3b}\\ &= -\frac{4a\sqrt{ax^2+bx^5}}{9b^2x}+\frac{2x^2\sqrt{ax^2+bx^5}}{9b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.65

$$\frac{2\left(bx^3-2a\right)\sqrt{x^2\left(a+bx^3\right)}}{9b^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(-2*a + b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*b^2*x)
```

fricas [A]   time = 0.39, size = 30, normalized size = 0.58

$$\frac{2\sqrt{bx^5+ax^2}\left(bx^3-2a\right)}{9b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 - 2*a)/(b^2*x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^6/sqrt(b*x^5 + a*x^2), x)
```

maple [A]   time = 0.06, size = 37, normalized size = 0.71

$$-\frac{2\left(b\,x^3+a\right)\left(-b\,x^3+2a\right)x}{9\sqrt{b\,x^5+a\,x^2}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(b*x^5+a*x^2)^(1/2),x)
[Out] -2/9*(b*x^3+a)*(-b*x^3+2*a)*x/b^2/(b*x^5+a*x^2)^(1/2)
```

maxima [A]   time = 1.44, size = 34, normalized size = 0.65

$$\frac{2\left(b^2x^6-abx^3-2a^2\right)}{9\sqrt{bx^3+a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(b^2*x^6 - a*b*x^3 - 2*a^2)/(sqrt(b*x^3 + a)*b^2)
```

mupad [B]   time = 5.33, size = 33, normalized size = 0.63

$$-\frac{\sqrt{b\,x^5+a\,x^2}\,\left(\frac{4\,a}{9\,b^2}-\frac{2\,x^3}{9\,b}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(a*x^2 + b*x^5)^(1/2),x)
[Out] -((a*x^2 + b*x^5)^(1/2)*((4*a)/(9*b^2) - (2*x^3)/(9*b)))/x
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**6/sqrt(x**2*(a + b*x**3)), x)
```

3.286 $\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=25

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

```
[Out] 2/3*(b*x^5+a*x^2)^(1/2)/b/x
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1588}

$$\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^5])/(3*b*x)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax^2+bx^5}}\,dx=\frac{2\sqrt{ax^2+bx^5}}{3bx}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\sqrt{x^2\left(a+bx^3\right)}}{3bx}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[x^2*(a + b*x^3)])/(3*b*x)
```

fricas [A] time = 0.37, size = 21, normalized size = 0.84

$$\frac{2\,\sqrt{bx^5+ax^2}}{3\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x^5 + a*x^2)/(b*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x^3/sqrt(b*x^5 + a*x^2), x)
```

maple [A] time = 0.05, size = 27, normalized size = 1.08

$$\frac{2\left(b\,x^3+a\right)x}{3\sqrt{b\,x^5+a\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^5+a*x^2)^(1/2),x)
[Out] 2/3*(b*x^3+a)*x/b/(b*x^5+a*x^2)^(1/2)
```

maxima [A] time = 1.40, size = 14, normalized size = 0.56

$$\frac{2\,\sqrt{bx^3+a}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/3*sqrt(b*x^3 + a)/b
```

mupad [B] time = 5.19, size = 21, normalized size = 0.84

$$\frac{2\,\sqrt{b\,x^5+a\,x^2}}{3\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2 + b*x^5)^(1/2),x)
[Out] (2*(a*x^2 + b*x^5)^(1/2))/(3*b*x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**3/sqrt(x**2*(a + b*x**3)), x)
```

3.287 $\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=32

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

```
[Out] -2/3*arctanh(x*a^(1/2)/(b*x^5+a*x^2)^(1/2))/a^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[a])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx &= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^5}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 1.69

$$-\frac{2x\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-2*x*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*Sqrt[x^2
*(a + b*x^3)])
```

fricas [A] time = 0.41, size = 75, normalized size = 2.34

$$\left[\frac{\log\left(\frac{bx^4+2\,ax-2\,\sqrt{bx^5+ax^2}\,\sqrt{a}}{x^4}\right)}{3\,\sqrt{a}}, \frac{2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{bx^5+ax^2}\,\sqrt{-a}}{ax}\right)}{3\,a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/3*log((b*x^4 + 2*a*x - 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4)/sqrt(a), 2/3*
sqrt(-a)*arctan(sqrt(b*x^5 + a*x^2)*sqrt(-a)/(a*x))/a]
```

giac [A] time = 0.19, size = 47, normalized size = 1.47

$$-\frac{2\,\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)\mathrm{sgn}\left(x\right)}{3\,\sqrt{-a}} + \frac{2\,\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*arctan(sqrt(b*x^3 + a)/
sqrt(-a))/(sqrt(-a)*sgn(x))
```

maple [A] time = 0.05, size = 43, normalized size = 1.34

$$-\frac{2\sqrt{b\,x^3+a}\;x\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)}{3\sqrt{b\,x^5+a\,x^2}\;\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/3/(b*x^5+a*x^2)^(1/2)*x*(b*x^3+a)^(1/2)*arctanh((b*x^3+a)^(1/2)/a^(1/2))
/a^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(1/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**5), x)
```

3.288 $\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=59

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

```
[Out] 1/3*b*arctanh(x*a^(1/2)/(b*x^5+a*x^2)^(1/2))/a^(3/2)-1/3*(b*x^5+a*x^2)^(1/2
)/a/x^4
```

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2008, 206}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}-\frac{\sqrt{ax^2+bx^5}}{3ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^5]/(3*a*x^4) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^5]
])/(3*a^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{ax^2+bx^5}}\,dx &= -\frac{\sqrt{ax^2+bx^5}}{3ax^4}-\frac{b\int \frac{1}{\sqrt{ax^2+bx^5}}\,dx}{2a}\\
&= -\frac{\sqrt{ax^2+bx^5}}{3ax^4}+\frac{b\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^5}}\right)}{3a}\\
&= -\frac{\sqrt{ax^2+bx^5}}{3ax^4}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^5}}\right)}{3a^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 1.20

$$\frac{2b\sqrt{x^2\left(a+bx^3\right)}\left(\frac{\tanh^{-1}\left(\sqrt{\frac{bx^3}{a}+1}\right)}{2\sqrt{\frac{bx^3}{a}+1}}-\frac{a}{2bx^3}\right)}{3a^2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (2*b*Sqrt[x^2*(a + b*x^3)]*(-1/2*a/(b*x^3) + ArcTanh[Sqrt[1 + (b*x^3)/a]]/(
2*Sqrt[1 + (b*x^3)/a])))/(3*a^2*x)
```

fricas [A] time = 0.42, size = 127, normalized size = 2.15

$$\left[\frac{\sqrt{a}\,bx^4\log\left(\frac{bx^4+2\,ax+2\,\sqrt{bx^5+ax^2}\,\sqrt{a}}{x^4}\right)-2\,\sqrt{bx^5+ax^2}\,a}{6\,a^2x^4},-\frac{\sqrt{-a}\,bx^4\arctan\left(\frac{\sqrt{bx^5+ax^2}\,\sqrt{-a}}{ax}\right)+\sqrt{bx^5+ax^2}\,a}{3\,a^2x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*b*x^4*log((b*x^4 + 2*a*x + 2*sqrt(b*x^5 + a*x^2)*sqrt(a))/x^4
) - 2*sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4), -1/3*(sqrt(-a)*b*x^4*arctan(sqrt(b*
x^5 + a*x^2)*sqrt(-a)/(a*x)) + sqrt(b*x^5 + a*x^2)*a)/(a^2*x^4)]
```

giac [A] time = 0.21, size = 57, normalized size = 0.97

$$-\frac{b\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}\,a\mathrm{sgn}(x)}-\frac{\sqrt{\frac{b}{x}+\frac{a}{x^4}}}{3\,ax\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*b*arctan(sqrt(b*x^3 + a)/sqrt(-a))/(sqrt(-a)*a*sgn(x)) - 1/3*sqrt(b/x
+ a/x^4)/(a*x*sgn(x))
```

maple [A] time = 0.05, size = 66, normalized size = 1.12

$$-\frac{\sqrt{b\,x^3+a}\,\left(-ab\,x^3\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)+\sqrt{b\,x^3+a}\,a^{\frac{3}{2}}\right)}{3\sqrt{b\,x^5+a\,x^2}\,a^{\frac{5}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -1/3/x^2*(b*x^3+a)^(1/2)*(-b*arctanh((b*x^3+a)^(1/2)/a^(1/2))*a*x^3+(b*x^3+
a)^(1/2)*a^(3/2))/(b*x^5+a*x^2)^(1/2)/a^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{bx^5+ax^2}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^3*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2*(a + b*x**3))), x)
```

3.289 $\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=238

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{4\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] 2/5*(b*x^5+a*x^2)^(1/2)/b-4/15*a*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x
+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6
^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(4/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^
(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2032, 218}

$$\frac{2\sqrt{ax^2+bx^5}}{5b}-\frac{4\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[a*x^2 + b*x^5])/(5*b) - (4*Sqrt[2 + Sqrt[3]]*a*x*(a^(1/3) + b^(1/3)
*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3)
 + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(5*3^(1/4)*b^(4/3)*Sqrt[(
a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[
a*x^2 + b*x^5])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{(2a)\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx}{5b} \\
&= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{\left(2ax\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{5b\sqrt{ax^2+bx^5}} \\
&= \frac{2\sqrt{ax^2+bx^5}}{5b} - \frac{4\sqrt{2+\sqrt{3}}\,ax\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right.\right.}{5\sqrt[4]{3}\,b^{4/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.29

$$
\frac{2x^2\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)+a+bx^3\right)}{5b\sqrt{x^2\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^2*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3,
-((b*x^3)/a)]))/(5*b*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{bx^5+ax^2}\,x^2}{bx^3+a},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^2/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{x^4}{\sqrt{bx^5+ax^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

maple [A] time = 1.29, size = 248, normalized size = 1.04

$$\frac{2\left(3b^2x^4+3abx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(-2bx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}-\left(-a\,b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{2\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\,\sqrt{-\frac{i\left(2bx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}+\left(-a\,b^2\right)^{\frac{1}{3}}\right.}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\right.}{15\sqrt{b\,x^5+a\,x^2}\;b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 2/15*x*(I*a*3^(1/2)*(-a*b^2)^(1/3)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*
b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2
)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(I*3^(1/2)*(-a*b^2)^(1/3)+2*b*x+(-a*b^2)^(
1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(I*3^
(1/2)*(-a*b^2)^(1/3)-2*b*x-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^
(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))+3*b^2*x^4+3*a*b*x)/(b*x^5+a*x^2)^(1/
2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^4/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**2*(a + b*x**3)), x)
```

3.290 $\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=212

$$\frac{2\sqrt{2+\sqrt{3}}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}$$

```
[Out] 2/3*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3
)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)
-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^
(3/4)/b^(1/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a
^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2032, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,\sqrt[3]{b}\,\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[
((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -
7 - 4*Sqrt[3]])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 +
 Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$
$$= \frac{2\sqrt{2+\sqrt{3}}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Mathematica [C] time = 0.01, size = 52, normalized size = 0.25

$$\frac{x^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^2 + b*x^5],x]
[Out] (x^2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/3, 1/2, 4/3, -((b*x^3)/a)])/Sq
rt[x^2*(a + b*x^3)]
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}}{bx^4+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^4 + a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(x/sqrt(b*x^5 + a*x^2), x)
```

maple [A] time = 0.05, size = 231, normalized size = 1.09

$$-\frac{i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(-2bx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}-\left(-a\,b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(2bx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}+\left(-a\,b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\,x\,\text{EllipticF}\left(\cdots\right.}{3\sqrt{b\,x^5+a\,x^2}\;b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -1/3*I/(b*x^5+a*x^2)^(1/2)*x*3^(1/2)*(-a*b^2)^(1/3)/b*(-I*(-2*b*x+I*3^(1/2)
*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a
*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(2*b*x+I*3^(1/2)*(-a*b
^2)^(1/3)+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/
2)*2^(1/2)*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a
*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^5 + ax^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x}{\sqrt{b\,x^5 + a\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^2 \left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2*(a + b*x**3)), x)
```

3.291 $\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=243

$$-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

```
[Out] -1/2*(b*x^5+a*x^2)^(1/2)/a/x^3-1/6*b^(2/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF(
(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2
*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(
1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a/(b*x^5+a*x^2)^(1/2)/(a^(1/3)
*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2025, 2032, 218}

$$-\frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{2ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -Sqrt[a*x^2 + b*x^5]/(2*a*x^3) - (Sqrt[2 + Sqrt[3]]*b^(2/3)*x*(a^(1/3) + b^
(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^
(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/
((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*3^(1/4)*a*Sqrt[(a
^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a
*x^2 + b*x^5])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{ax^2+bx^5}}\,dx = -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{b\int \frac{x}{\sqrt{ax^2+bx^5}}\,dx}{4a}$$
$$= -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{\left(bx\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{4a\sqrt{ax^2+bx^5}}$$
$$= -\frac{\sqrt{ax^2+bx^5}}{2ax^3} - \frac{\sqrt{2+\sqrt{3}}\,b^{2/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+}{\left(1+\sqrt{3}\right)\sqrt[3]{a}+}\right.\right.}{2\sqrt[4]{3}\,a\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.23

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};-\frac{bx^3}{a}\right)}{2x\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^2 + b*x^5]),x]
[Out] -1/2*(Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-2/3, 1/2, 1/3, -((b*x^3)/a)])/
(x*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}}{bx^7+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^7 + a*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5+ax^2}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

maple [A] time = 0.52, size = 248, normalized size = 1.02

$$\frac{-6b\,x^{3}+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{-\frac{i\left(-2bx+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}-\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\,\sqrt{-\frac{2\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}{\left(-a\,b^{2}\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\,\sqrt{-\frac{i\left(2bx+i\sqrt{3}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^{2}\right)^{\frac{1}{3}}}}\;x^{2}\,\mathrm{El}}{12\sqrt{b\,x^{5}+a\,x^{2}}\;ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/12/x*(I*3^(1/2)*(-a*b^2)^(1/3)*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b
^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)
^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1
/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I*(-2*b*
x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^
(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*x^2-6*b*x^3-6*a)/(b*x^5+a*x^2)^(1/2)
/a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^{5}+ax^{2}}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^2), x)
```

mupad [B] time = 5.71, size = 44, normalized size = 0.18

$$-\frac{2\sqrt{\frac{a}{b\,x^{3}}+1}\;{}_2\mathrm{F}_1\!\left(\frac{1}{2},\frac{7}{6};\ \frac{13}{6};\ -\frac{a}{b\,x^{3}}\right)}{7\,x\,\sqrt{b\,x^{5}+a\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] -(2*(a/(b*x^3) + 1)^(1/2)*hypergeom([1/2, 7/6], 13/6, -a/(b*x^3)))/(7*x*(a*
x^2 + b*x^5)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\sqrt{x^{2}\left(a+bx^{3}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2*(a + b*x**3))), x)
```

3.292 $\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=514

$$-\frac{8\sqrt{2}\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}x\left(\sqrt[3]{a}\right.}{}$$

```
[Out] -8/7*a*x*(b*x^3+a)/b^(5/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1
/2)+2/7*x*(b*x^5+a*x^2)^(1/2)/b-8/21*a^(4/3)*x*(a^(1/3)+b^(1/3)*x)*Elliptic
F((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)
+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(
1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+
b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)+4/7*3^(1/4)*a^(4/3)*x*(
a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(
1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3
)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(
b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2024, 2032, 303, 218, 1877}

$$-\frac{8\sqrt{2}\,a^{4/3}x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)\middle|-7-4\sqrt{3}\right)}{7\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}x\left(\sqrt[3]{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-8*a*x*(a + b*x^3))/(7*b^(5/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*
x^2 + b*x^5]) + (2*x*Sqrt[a*x^2 + b*x^5])/(7*b) + (4*3^(1/4)*Sqrt[2 - Sqrt[
3]]*a^(4/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(
2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqr
t[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqr
t[3]])/(7*b^(5/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/
3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - (8*Sqrt[2]*a^(4/3)*x*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a
^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)
/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(7*3^(1/4)*b^(5/3)*
Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]
*Sqrt[a*x^2 + b*x^5])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{ax^2+bx^5}}\,dx = \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{(4a)\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx}{7b}$$

$$= \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{\left(4ax\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{7b\sqrt{ax^2+bx^5}}$$

$$= \frac{2x\sqrt{ax^2+bx^5}}{7b} - \frac{\left(4ax\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{7b^{4/3}\sqrt{ax^2+bx^5}} - \frac{\left(4\sqrt{2\left(2-\sqrt{3}\right)}\,a^{4/3}x\sqrt{a+bx^3}\right)\int \cdots}{7b^{4/3}\sqrt{ax^2+bx^5}}$$

$$= -\frac{8ax\left(a+bx^3\right)}{7b^{5/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} + \frac{2x\sqrt{ax^2+bx^5}}{7b} + \frac{4\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,a^{4/3}x\left(\sqrt[3]{a}+\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.03, size = 68, normalized size = 0.13

$$\frac{2x^3\left(-a\sqrt{\frac{bx^3}{a}+1}\,_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)+a+bx^3\right)}{7b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^3*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3,
-((b*x^3)/a)]))/(7*b*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\,x^3}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^3/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

maple [A] time = 0.54, size = 676, normalized size = 1.32

$$-\frac{2\left(-3b^3x^5-3a\,b^2x^2+3i\sqrt{-\frac{2\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}{\left(-a\,b^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(2bx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}+\left(-a\,b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{i\left(-2bx+i\sqrt{3}\,\left(-a\,b^2\right)^{\frac{1}{3}}-\left(-a\,b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a\,b^2\right)^{\frac{1}{3}}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/21*x*(3*I*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*
(-I*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))
^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(
-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-
3))^(1/2))*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a
*b^2)^(1/3))^(1/2)*(-a*b^2)^(2/3)*3^(1/2)*a-2*I*(-2*(-b*x+(-a*b^2)^(1/3))/(
-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*
b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2)*(-I
*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(
1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2
)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-a*b^2)^(2/3)*3^(1/2
```

```
)*a-3*b^3*x^5+3*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/
2)*(-I*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/
3))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3
)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/
2)-3))^(1/2))*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/
(-a*b^2)^(1/3))^(1/2)*(-a*b^2)^(2/3)*a-3*a*b^2*x^2)/(b*x^5+a*x^2)^(1/2)/b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{bx^5 + ax^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^5/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{b\,x^5 + a\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^5/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{x^2 \left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**5/sqrt(x**2*(a + b*x**3)), x)
```

3.293 $\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=484

$$\frac{2\sqrt{2}\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\right.}{}$$

```
[Out] 2*x*(b*x^3+a)/b^(2/3)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1/2)+2
/3*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/
(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^(1/2)*((a^(2/3)-a^(1/3)*b^
(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/
3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3
^(1/2)))^2)^(1/2)-3^(1/4)*a^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticE((b^(1/3)*
x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*
6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^
(1/3)*(1+3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b
^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {2032, 303, 218, 1877}

$$\frac{2\sqrt{2}\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x*(a + b*x^3))/(b^(2/3)*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 +
 b*x^5]) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[
(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])
*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) +
b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (
2*Sqrt[2]*a^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x
 + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticF[ArcSin[((1
 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 -
 4*Sqrt[3]])/(3^(1/4)*b^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sq
rt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx &= \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}\\
&= \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}\,\sqrt{ax^2+bx^5}} + \frac{\left(\sqrt{2\left(2-\sqrt{3}\right)}\,\sqrt[3]{a}\,x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{a+bx^3}}\,dx}{\sqrt[3]{b}\,\sqrt{ax^2+bx^5}}\\
&= \frac{2x\left(a+bx^3\right)}{b^{2/3}\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} - \frac{\sqrt[4]{3}\,\sqrt{2-\sqrt{3}}\,\sqrt[3]{a}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}}}}{b^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.03, size = 55, normalized size = 0.11

$$
\frac{x^3\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};-\frac{bx^3}{a}\right)}{2\sqrt{x^2\left(a+bx^3\right)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^3*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 2/3, 5/3, -((b*x^3)/a)])/(2
*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{bx^5+ax^2}}{bx^3+a}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

maple [A] time = 0.06, size = 394, normalized size = 0.81

$$-\frac{i\sqrt{3}\left(-a b^2\right)^{\frac{2}{3}}\sqrt{-\frac{i\left(-2bx+i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}-\left(-a b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a b^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-a b^2\right)^{\frac{1}{3}}\right)}{\left(-a b^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(2bx+i\sqrt{3}\left(-a b^2\right)^{\frac{1}{3}}+\left(-a b^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-a b^2\right)^{\frac{1}{3}}}}\left(i\sqrt{3}\ \mathrm{Elli}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -1/6*I*x*3^(1/2)*(-a*b^2)^(2/3)*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^
2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^
(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/
3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(I*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(-2*
b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),
2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*3^(1/2)-3*EllipticE(1/6*3^(1/2)*2^
(1/2)*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)
^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))+2*EllipticF(1/6*3^(1
/2)*2^(1/2)*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-
a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2)))/(b*x^5+a*x^2)
^(1/2)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^2/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**2/sqrt(x**2*(a + b*x**3)), x)
```

3.294 $\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=510

$$\frac{\sqrt{2}\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\cdots\right.}{\cdots}$$

```
[Out] b^(1/3)*x*(b*x^3+a)/a/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^5+a*x^2)^(1/2)-(
b*x^5+a*x^2)^(1/2)/a/x^2+1/3*b^(1/3)*x*(a^(1/3)+b^(1/3)*x)*EllipticF((b^(1/
3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*2^
(1/2)*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2
)))^2)^(1/2)*3^(3/4)/a^(2/3)/(b*x^5+a*x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*
x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(1/4)*b^(1/3)*x*(a^(1/3)+
b^(1/3)*x)*EllipticE((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+
3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)-1/2*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3
)*x+b^(2/3)*x^2)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^5+a*
x^2)^(1/2)/(a^(1/3)*(a^(1/3)+b^(1/3)*x)/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2025, 2032, 303, 218, 1877}

$$\frac{\sqrt{2}\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\,F\left(\sin^{-1}\left(\frac{\sqrt[3]{b}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{a}}{\sqrt[3]{b}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{a}}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{a}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (b^(1/3)*x*(a + b*x^3))/(a*((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a*x^2 +
 b*x^5]) - Sqrt[a*x^2 + b*x^5]/(a*x^2) - (3^(1/4)*Sqrt[2 - Sqrt[3]]*b^(1/3)
*x*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(
(1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*a^(1/
3) + b^(1/3)*x)/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(2*a
^(2/3)*Sqrt[(a^(1/3)*(a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3
)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (Sqrt[2]*b^(1/3)*x*(a^(1/3) + b^(1/3)*x)*Sqr
t[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/((1 + Sqrt[3])*a^(1/3) + b^(1
/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/((1 + Sqrt[3
])*a^(1/3) + b^(1/3)*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*a^(2/3)*Sqrt[(a^(1/3)*(
a^(1/3) + b^(1/3)*x))/((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)^2]*Sqrt[a*x^2 + b
*x^5])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 303

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]
```

Rule 1877

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sq
rt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{ax^2+bx^5}}\,dx &= -\frac{\sqrt{ax^2+bx^5}}{ax^2}+\frac{b\int \frac{x^2}{\sqrt{ax^2+bx^5}}\,dx}{2a}\\
&= -\frac{\sqrt{ax^2+bx^5}}{ax^2}+\frac{\left(bx\sqrt{a+bx^3}\right)\int \frac{x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt{ax^2+bx^5}}\\
&= -\frac{\sqrt{ax^2+bx^5}}{ax^2}+\frac{\left(b^{2/3}x\sqrt{a+bx^3}\right)\int \frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x}{\sqrt{a+bx^3}}\,dx}{2a\sqrt{ax^2+bx^5}}+\frac{\left(\sqrt{\frac{1}{2}\left(2-\sqrt{3}\right)}\,b^{2/3}x\sqrt{a+bx^3}\right)\int}{a^{2/3}\sqrt{ax^2+bx^5}}\\
&= \frac{\sqrt[3]{b}\,x\left(a+bx^3\right)}{a\left(\left(1+\sqrt{3}\right)\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}}-\frac{\sqrt{ax^2+bx^5}}{ax^2}-\frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\,\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{}}{2a^{2/3}\sqrt{}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.10

$$-\frac{\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] -((Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/3, 1/2, 2/3, -((b*x^3)/a)])/Sqr
t[x^2*(a + b*x^3)])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}}{bx^6+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)/(b*x^6 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5+ax^2}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

maple [A] time = 0.54, size = 673, normalized size = 1.32

$$-12b^2x^3-12ab+3i\sqrt{-\frac{i\left(-2bx+i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}-\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-ab^2\right)^{\frac{1}{3}}}}\sqrt{-\frac{2\left(-bx+\left(-ab^2\right)^{\frac{1}{3}}\right)}{\left(-ab^2\right)^{\frac{1}{3}}\left(i\sqrt{3}-3\right)}}\sqrt{-\frac{i\left(2bx+i\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}+\left(-ab^2\right)^{\frac{1}{3}}\right)\sqrt{3}}{\left(-ab^2\right)^{\frac{1}{3}}}}\left(-a\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/12*(3*I*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*
b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(
1/2)*(-I*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(
1/3))^(1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1
/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(
1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*x-2*I*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)
^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/
3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-I*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)
+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2)*EllipticF(1/6*3^(1/2)*2^(1/2
)*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/
3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*3^(1/2)*x
```

```
+3*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1
/3))^(1/2)*(-2*(-b*x+(-a*b^2)^(1/3))/(-a*b^2)^(1/3)/(I*3^(1/2)-3))^(1/2)*(-
I*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(
1/2)*EllipticE(1/6*3^(1/2)*2^(1/2)*(-I*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a
*b^2)^(1/3))*3^(1/2)/(-a*b^2)^(1/3))^(1/2),2^(1/2)*(I*3^(1/2)/(I*3^(1/2)-3)
)^(1/2))*(-a*b^2)^(2/3)*x-12*b^2*x^3-12*a*b)/(b*x^5+a*x^2)^(1/2)/a/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\, x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\sqrt{b\,x^5 + a\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^2\left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(x**2*(a + b*x**3))), x)
```

3.295 $\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=265

$$\frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}\,b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}}+\frac{x^{5/2}\sqrt{ax^2}}{5b}$$

```
[Out] 1/5*x^(5/2)*(b*x^5+a*x^2)^(1/2)/b-7/20*a*(b*x^5+a*x^2)^(1/2)/b^2/x^(1/2)+7/
120*a^(5/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/
(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a
^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^
2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/
3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*
3^(3/4)/b^2/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2024, 2032, 329, 225}

$$\frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{40\sqrt[4]{3}\,b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}}+\frac{x^{5/2}\sqrt{ax^2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-7*a*Sqrt[a*x^2 + b*x^5])/(20*b^2*Sqrt[x]) + (x^(5/2)*Sqrt[a*x^2 + b*x^5])
/(5*b) + (7*a^(5/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b
^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[Ar
cCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x
)], (2 + Sqrt[3])/4])/(40*3^(1/4)*b^2*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x)
)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{13/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} - \frac{(7a)\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx}{10b}$$

$$= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{\left(7a^2\right)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{40b^2}$$

$$= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{\left(7a^2x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{40b^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{\left(7a^2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{20b^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{7a\sqrt{ax^2+bx^5}}{20b^2\sqrt{x}} + \frac{x^{5/2}\sqrt{ax^2+bx^5}}{5b} + \frac{7a^{5/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\cdots\right.\right.}{40\sqrt[4]{3}\,b^2\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax\cdots}}$$

Mathematica [C] time = 0.05, size = 86, normalized size = 0.32

$$\frac{x^{3/2}\left(7a^2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)-7a^2-3abx^3+4b^2x^6\right)}{20b^2\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(13/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(-7*a^2 - 3*a*b*x^3 + 4*b^2*x^6 + 7*a^2*Sqrt[1 + (b*x^3)/a]*Hyperg
eometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a)]))/(20*b^2*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\,x^{\frac{9}{2}}}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(9/2)/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

maple [C] time = 0.85, size = 2017, normalized size = 7.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -1/20/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^3/(-a*b^2)^(1/3)*(14*I*(-(I*3
^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((2*b*x+I*3^(1/2)*
(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*3^(1/2)*x^2*a^2*b^2-28*I*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^
(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^
2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipti
cF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/
2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*x*a^2*b+14*
I*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a^2-4*I*(-a*b^2)^(1/3)*((b*x^3+a)*x)^(1
/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)
^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3)))^(1/2)*3^(1/2)*x^3
*b^2-14*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((2*
b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1
)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/
(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*
3^(1/2)-3))^(1/2))*x^2*a^2*b^2+28*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b
^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-
a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ell
ipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a^2*b-14*(-a*
b^2)^(2/3)*(-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)*x*b/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/
(I*3^(1/2)-3))^(1/2))*a^2+12*x^3*((b*x^3+a)*x)^(1/2)*b^2*(-a*b^2)^(1/3)*(1/
b^2*x*(-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))
```

```
*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3)))^(1/2)+7*I*(-a*b^2)^(1/3)
*((b*x^3+a)*x)^(1/2)*(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^
2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3)))^
(1/2)*3^(1/2)*a*b-21*a*((b*x^3+a)*x)^(1/2)*b*(-a*b^2)^(1/3)*(1/b^2*x*(-b*x+
(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3
^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3)))^(1/2))/((b*x^3+a)*x)^(1/2)/(I*3^(1/2
)-3)/(1/b^2*x*(-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2
)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{13}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(13/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(13/2)/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{13/2}}{\sqrt{b\,x^5 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(13/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(13/2)/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(13/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Timed out
```

3.296 $\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=525

$$\frac{5\left(1-\sqrt{3}\right)a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{5\sqrt[4]{3}\,a^{4/3}x^{3/2}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] -5/8*a*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/b^(5/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2))
)/(b*x^5+a*x^2)^(1/2)+1/4*x^(3/2)*(b*x^5+a*x^2)^(1/2)/b+5/8*3^(1/4)*a^(4/3)
*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1
/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+
b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*
b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/b^(5/3)/(b*
x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)+5/48*a^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*
(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*
(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*
x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2
^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/
3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(
a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2024, 2032, 329, 308, 225, 1881}

$$\frac{5\left(1-\sqrt{3}\right)a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{16\sqrt[4]{3}\,b^{5/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{5\sqrt[4]{3}\,a^{4/3}x^{3/2}\left(\sqrt[3]{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (-5*(1 + Sqrt[3])*a*x^(3/2)*(a + b*x^3))/(8*b^(5/3)*(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) + (x^(3/2)*Sqrt[a*x^2 + b*x^5])/(4*b) + (
5*3^(1/4)*a^(4/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(
1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcC
os[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)]
, (2 + Sqrt[3])/4])/(8*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) + (5*(1 - Sqrt[3])*a
^(4/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^
(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3)
 + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt
[3])/4])/(16*3^(1/4)*b^(5/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3
) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{11/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} - \frac{(5a)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{8b}$$

$$= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} - \frac{\left(5ax\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{8b\sqrt{ax^2+bx^5}}$$

$$= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} - \frac{\left(5ax\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{4b\sqrt{ax^2+bx^5}}$$

$$= \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} + \frac{\left(5ax\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{8b^{5/3}\sqrt{ax^2+bx^5}} + \frac{\left(5\left(1-\sqrt{3}\right)\right.}{}$$

$$= -\frac{5\left(1+\sqrt{3}\right)ax^{3/2}\left(a+bx^3\right)}{8b^{5/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} + \frac{x^{3/2}\sqrt{ax^2+bx^5}}{4b} + \frac{5\sqrt[4]{3}\,a^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\right.}{}$$

Mathematica [C] time = 0.03, size = 70, normalized size = 0.13

$$\frac{x^{7/2}\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)+a+bx^3\right)}{4b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(11/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(7/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/
6, -((b*x^3)/a)]))/(4*b*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\,x^{\frac{7}{2}}}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(7/2)/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

maple [C] time = 0.67, size = 2586, normalized size = 4.93

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/4*x^(3/2)*(b*x^3+a)*(-5*I*(-a*b^2)^(1/3)*3^(1/2)*x^2*a*b-5*I*(-a*b^2)^(1/
3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I
*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/
2)-3))^(1/2))*3^(1/2)*x^2*a*b-10*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a
*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
pticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^
(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b+15*(-a*b
^2)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((
2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-
b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(
I*3^(1/2)-3))^(1/2))*x^2*a*b-5*I*(-a*b^2)^(2/3)*3^(1/2)*x*a+20*(-a*b^2)^(2/
3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I
*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3))
)^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b
*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/
2)-3))^(1/2))*x*a-30*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1
+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-
(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3
^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*
3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a+10*I*(-a*b^2)^(2/3)*(-(I
*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2
)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*
b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^
(1/2))*3^(1/2)*x*a-5*I*3^(1/2)*x^3*a*b^2+5*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/
(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^
(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^
2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipti
cE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/
2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a^2*b+10*(-
(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1
/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*a^2*b-15*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^
(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+
(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*a^2*b+I*((b*x^3+a)*x)^(1/2)*((-b*x+(-a*b^2)^(1
/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*
b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*3^(1/2)*x^2*b^2-3*((b*x^3+a)*x)^(1/
2)*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(
-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*x^2*b^2+15*a*b
^2*x^3+15*(-a*b^2)^(1/3)*x^2*a*b+15*(-a*b^2)^(2/3)*x*a)/(b*x^5+a*x^2)^(1/2)
```

```
/b^3/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1
/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2
)^(1/3))/b^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(11/2)/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11/2}}{\sqrt{b\,x^5 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(11/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(11/2)/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{11}{2}}}{\sqrt{x^2\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**(11/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.297 $\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}} dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

```
[Out] -1/3*a*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a*x^2)^(1/2))/b^(3/2)+1/3*x^(1/2)*(b*
x^5+a*x^2)^(1/2)/b
```

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (Sqrt[x]*Sqrt[a*x^2 + b*x^5])/(3*b) - (a*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x
^2 + b*x^5]])/(3*b^(3/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{9/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx}{2b}\\
&= \frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b}\\
&= \frac{\sqrt{x}\sqrt{ax^2+bx^5}}{3b}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 81, normalized size = 1.25

$$\frac{\sqrt{b}\,x^{5/2}\left(a+bx^3\right)-ax\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)}{3b^{3/2}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(9/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (Sqrt[b]*x^(5/2)*(a + b*x^3) - a*x*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2)
)/Sqrt[a + b*x^3]])/(3*b^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

fricas [A] time = 0.55, size = 148, normalized size = 2.28

$$\left[\frac{a\sqrt{b}\,\log\left(-8\,b^2x^6-8\,abx^3+4\,\sqrt{bx^5+ax^2}\left(2\,bx^3+a\right)\sqrt{b}\,\sqrt{x}-a^2\right)+4\,\sqrt{bx^5+ax^2}\,b\sqrt{x}}{12\,b^2},\ \frac{a\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{b\cdots}}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(a*sqrt(b)*log(-8*b^2*x^6 - 8*a*b*x^3 + 4*sqrt(b*x^5 + a*x^2)*(2*b*x^
3 + a)*sqrt(b)*sqrt(x) - a^2) + 4*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2, 1/6*(
a*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(-b)*sqrt(x)/(2*b*x^3 + a)) + 2
*sqrt(b*x^5 + a*x^2)*b*sqrt(x))/b^2]
```

giac [A] time = 0.25, size = 44, normalized size = 0.68

$$\frac{\sqrt{bx^3+a}\,x^{\frac{3}{2}}}{3\,b}+\frac{a\log\left(\left|-\sqrt{b}\,x^{\frac{3}{2}}+\sqrt{bx^3+a}\right|\right)}{3\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(b*x^3 + a)*x^(3/2)/b + 1/3*a*log(abs(-sqrt(b)*x^(3/2) + sqrt(b*x^3
 + a)))/b^(3/2)
```

maple [C] time = 1.14, size = 3347, normalized size = 51.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/3/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^3*(6*I*3^(1/2)*(-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(
1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*
3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(
1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a
*b^2-6*I*3^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(
1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1
```

```
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2-12*I*3^(
1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x
)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b+12*I*3^(1/2)*(-a*b^2)^(1/3)*(-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*
b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*
b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
*b*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*x*a*b+6*I*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)
-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^
2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*
x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*
x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a
-6*I*3^(1/2)*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-
3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a-6*(-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/
2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*x^2*a*b^2+6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x
)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3
^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2+12*(-
a*b^2)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)
*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*x*a*b-12*(-a*b^2)^(1/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^
2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a
*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Elli
pticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^
(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2))*x*a*b+I*3^(1/2)*((b*x^3+a)*x)^(1/2)*((-b*x+(-a*b^2)^(1/3))*(2*
b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/
3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*x*b^2-6*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)
+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1
/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/
2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a+6*(-a*b
^2)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((
2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)
^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*
```

```
(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a-3*x*((b*x^3+a)*x)^(1/2)
*b^2*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))
*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2))/((b*x^3+a)*
x)^(1/2)/(I*3^(1/2)-3)/((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/
3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{bx^5 + ax^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(9/2)/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^{9/2}}{\sqrt{b\,x^5 + a\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(9/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(9/2)/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{9}{2}}}{\sqrt{x^2 \left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**(9/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.298 $\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=237

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\,b\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}$$

```
[Out] 1/2*(b*x^5+a*x^2)^(1/2)/b/x^(1/2)-1/12*a^(2/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*
((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF(
(1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/
2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x
*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2024, 2032, 329, 225}

$$\frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}}-\frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\,b\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] Sqrt[a*x^2 + b*x^5]/(2*b*Sqrt[x]) - (a^(2/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*
Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
+ (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(4*3^(1/4)*b*Sqrt[(b^(1/3)*x
*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 +
 b*x^5])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
```

```
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] && !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{7/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{4b} \\
&= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{\left(ax\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{4b\sqrt{ax^2+bx^5}} \\
&= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{\left(ax\sqrt{a+bx^3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{2b\sqrt{ax^2+bx^5}} \\
&= \frac{\sqrt{ax^2+bx^5}}{2b\sqrt{x}} - \frac{a^{2/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\right.\right.}{4\sqrt[4]{3}\,b\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}
\end{aligned}$$

Mathematica [C] time = 0.04, size = 70, normalized size = 0.30

$$\frac{x^{3/2}\left(-a\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)+a+bx^3\right)}{2b\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(7/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(a + b*x^3 - a*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6
, -((b*x^3)/a)]))/(2*b*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^5+ax^2}\,x^{\frac{3}{2}}}{bx^3+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*x^(3/2)/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

maple [C] time = 1.14, size = 1793, normalized size = 7.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 1/2/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/b^2/(-a*b^2)^(1/3)*(2*I*(-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a
*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2
*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*3^(1/2)*x^2*a*b^2-4*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/
(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/
3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(
1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x*a*b+2*I*(-(I*3^
(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(
-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((
-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3
))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/
2))*(-a*b^2)^(2/3)*3^(1/2)*a-2*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^
(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b
^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/
2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^2*a*b^2+4*(-a*b^2)^(1/3)*(-(I*3
^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*
(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*x*a*b-2*(-a*b^2)^(2/3)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a+I*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*
3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a
*b^2)^(1/3))/b^2*x)^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*x)^(1/2)*b-3*((
b*x^3+a)*x)^(1/2)*b*(-a*b^2)^(1/3)*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*
(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1
/3))/b^2*x)^(1/2))/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/((-b*x+(-a*b^2)^(1/3))
*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)
^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(7/2)/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{7/2}}{\sqrt{b\,x^5 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(7/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(7/2)/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{7}{2}}}{\sqrt{x^2\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(x**(7/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.299 $\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=492

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt[3]{a}\,x^{3/2}\left(\sqrt[3]{a}+\cdots\right.}{\cdots}$$

```
[Out] x^(3/2)*(b*x^3+a)*(1+3^(1/2))/b^(2/3)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(b*x^
5+a*x^2)^(1/2)-3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3
)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1
/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1
/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)/b^(2/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*
x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)-1/6*a^(1/3)*x^(3/2)*(a^(1/3)+b^
(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))
^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*E
llipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)
))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*
x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/b^(2/3)/(b*
x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/
2)))^2)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2032, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{a}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}\,b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{\sqrt[4]{3}\sqrt[3]{a}\,x^{3/2}\left(\sqrt[3]{a}+\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] ((1 + Sqrt[3])*x^(3/2)*(a + b*x^3))/(b^(2/3)*(a^(1/3) + (1 + Sqrt[3])*b^(1/
3)*x)*Sqrt[a*x^2 + b*x^5]) - (3^(1/4)*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(b^(2/3)*Sqrt[(b^(1/3)*x*(a
^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*
x^5]) - ((1 - Sqrt[3])*a^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3)
- a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*E
llipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3]
)*b^(1/3)*x)], (2 + Sqrt[3])/4])/(2*3^(1/4)*b^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3
) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
```

```
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$

$$= \frac{\left(2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax^2+bx^5}}$$

$$= -\frac{\left(x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{b^{2/3}\sqrt{ax^2+bx^5}} - \frac{\left(\left(1-\sqrt{3}\right)a^{2/3}x\sqrt{a+bx^3}\right)\text{Subs}}{b^{2/3}\sqrt{ax^2+b}}$$

$$= \frac{\left(1+\sqrt{3}\right)x^{3/2}\left(a+bx^3\right)}{b^{2/3}\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} - \frac{\sqrt[4]{3}\,\sqrt[3]{a}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,E}{b^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)}}}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.12

$$\frac{2x^{7/2}\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};-\frac{bx^3}{a}\right)}{5\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(5/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x^(7/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/2, 5/6, 11/6, -((b*x^3)/
a)])/(5*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{bx^5+ax^2}\,\sqrt{x}}{bx^3+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^3 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^{\frac{5}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

maple [C] time = 1.17, size = 2374, normalized size = 4.83

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2*x^(3/2)*(b*x^3+a)*(-I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
)*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x^2*b+2*I*(-(I*3
^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*
(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*(
(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1
/2))*(-a*b^2)^(2/3)*3^(1/2)*x-2*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2
)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3
^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*
b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1
/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*x^2*b+3*(-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a
```

```
*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2
*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))
*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2)
)*(-a*b^2)^(1/3)*x^2*b+I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
)*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*3^(1/2)*a*b+4*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(
-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)
*x-6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x
+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3
)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(
-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(
-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(
1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x-I*3^(1/2)*x^3*b^2+2*(-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(
-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2
)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)
*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(
(I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*a*b-3*(-(I*
3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)
*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(
1/2))*a*b-I*(-a*b^2)^(1/3)*3^(1/2)*x^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*x+3*b^2*x
^3+3*(-a*b^2)^(1/3)*x^2*b+3*(-a*b^2)^(2/3)*x)/(b*x^5+a*x^2)^(1/2)/b^2/((b*x
^3+a)*x)^(1/2)/(I*3^(1/2)-3)/((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^
2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b
^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{bx^5 + ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(x^(5/2)/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{5/2}}{\sqrt{b\,x^5 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(5/2)/(a*x^2 + b*x^5)^(1/2),x)
[Out] int(x^(5/2)/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{5}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(5/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.300 $\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=36

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

```
[Out] 2/3*arctanh(x^(5/2)*b^(1/2)/(b*x^5+a*x^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a*x^2 + b*x^5]])/(3*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{3/2}}{\sqrt{ax^2+bx^5}}\,dx &= \frac{2}{3}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\right)\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{ax^2+bx^5}}\right)}{3\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.64

$$\frac{2x\sqrt{a+bx^3}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)}{3\sqrt{b}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2)/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*x*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*Sqrt[b]
*Sqrt[x^2*(a + b*x^3)])
```

fricas [A] time = 0.54, size = 101, normalized size = 2.81

$$\left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-4\,\sqrt{bx^5+ax^2}\left(2\,bx^3+a\right)\sqrt{b}\,\sqrt{x}-a^2\right)}{6\,\sqrt{b}},\,-\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{bx^5+ax^2}\,\sqrt{-b}\,\sqrt{x}}{2\,bx^3+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - 4*sqrt(b*x^5 + a*x^2)*(2*b*x^3 + a)*sqrt(
b)*sqrt(x) - a^2)/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(b*x^5 + a*x^2)*sqrt(
-b)*sqrt(x)/(2*b*x^3 + a))/b]
```

giac [A] time = 0.20, size = 41, normalized size = 1.14

$$-\frac{2\,\arctan\left(\frac{\sqrt{b+\frac{a}{x^3}}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}+\frac{2\,\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right)}{3\,\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*arctan(sqrt(b + a/x^3)/sqrt(-b))/sqrt(-b) + 2/3*arctan(sqrt(b)/sqrt(-b
))/sqrt(-b)
```

maple [C] time = 1.24, size = 480, normalized size = 13.33

$$-\frac{4\left(b\,x^3+a\right)\left(i\sqrt{3}-1\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)bx}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)^2\sqrt{\frac{2bx+i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+\left(-a\,b^2\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{-2bx+i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}-\cdots}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\cdots\right)\right)}}\cdots}{\sqrt{b\,x^5+a\,x^2}\,\sqrt{\left(b\,x^3+a\right)x}\,\left(i\sqrt{3}-3\right)\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -4*x^(3/2)*(b*x^3+a)*(I*3^(1/2)-1)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*
b^2)^(1/3))*b*x)^(1/2)*(-b*x+(-a*b^2)^(1/3))^2*((2*b*x+I*3^(1/2)*(-a*b^2)^(
1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*
3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))
^(1/2)*(EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^
(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))-Elli
pticPi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^
(1/2)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2
)-3))^(1/2)))/(b*x^5+a*x^2)^(1/2)/b^2/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/((-
b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x
+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^(3/2)/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{3/2}}{\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^(3/2)/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{\frac{3}{2}}}{\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x**(3/2)/sqrt(x**2*(a + b*x**3)), x)
```

3.301 $\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=203

$$\frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}$$

```
[Out] 1/3*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)
+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b
^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/
3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
a^(1/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 329, 225}

$$\frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,\sqrt[3]{a}\,\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*
x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1
- Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4
])/(3^(1/4)*a^(1/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
```

```
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx = \frac{\left(x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{\sqrt{ax^2+bx^5}}$$
$$= \frac{\left(2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{\sqrt{ax^2+bx^5}}$$
$$= \frac{x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Mathematica [C] time = 0.01, size = 55, normalized size = 0.27

$$\frac{2x^{3/2}\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/Sqrt[a*x^2 + b*x^5],x]
[Out] (2*x^(3/2)*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[1/6, 1/2, 7/6, -((b*x^3)/a
)])/Sqrt[x^2*(a + b*x^3)]
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{bx^5+ax^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

maple [C] time = 1.22, size = 437, normalized size = 2.15

$$-\frac{4\left(b\,x^{3}+a\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)bx}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{2bx+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}+\left(-a\,b^{2}\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{-2bx+i\sqrt{3}\left(-a\,b^{2}\right)^{\frac{1}{3}}-\left(-a\,b^{2}\right)^{\frac{1}{3}}}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^{2}\right)^{\frac{1}{3}}\right)}}\left(i\sqrt{3}\,b^{2}x^{2}-b^{2}x^{2}-2i\cdots\right.}{\sqrt{b\,x^{5}+a\,x^{2}}\,\left(-a\,b^{2}\right)^{\frac{1}{3}}\sqrt{\left(b\,x^{3}+a\right)x}\,\left(i\sqrt{3}-3\right)\sqrt{\frac{\left(-bx\cdots\right.}{\cdots}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -4/(b*x^5+a*x^2)^(1/2)*x^(3/2)*(b*x^3+a)/(-a*b^2)^(1/3)/b*(-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1
/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3
^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1
/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(I*3^(
1/2)*x^2*b^2-2*I*(-a*b^2)^(1/3)*3^(1/2)*x*b+I*(-a*b^2)^(2/3)*3^(1/2)-b^2*x^
2+2*(-a*b^2)^(1/3)*x*b-(-a*b^2)^(2/3))/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/((
-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*
x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{bx^{5}+ax^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x)/sqrt(b*x^5 + a*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{x}}{\sqrt{b\,x^{5}+a\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] int(x^(1/2)/(a*x^2 + b*x^5)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt{x^{2}\left(a+bx^{3}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x)/sqrt(x**2*(a + b*x**3)), x)
```

3.302 $\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=519

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}-\frac{2\sqrt[4]{3}\,\sqrt[3]{b}\,x^{3/2}\left(\sqrt[3]{a}\right.}{}$$

```
[Out] 2*b^(1/3)*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/a/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))/(
b*x^5+a*x^2)^(1/2)-2*(b*x^5+a*x^2)^(1/2)/a/x^(3/2)-2*3^(1/4)*b^(1/3)*x^(3/2
)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))*EllipticE((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)
*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(2/3)/(b*x^5+a*x
^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)
^(1/2)-1/3*b^(1/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/
2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(
1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*
(1-3^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))^2)^(1/2)*3^(3/4)/a^(2/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+
b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2025, 2032, 329, 308, 225, 1881}

$$-\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[4]{3}\,a^{2/3}\sqrt{\frac{\sqrt[3]{b}\,x\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)^2}}\,\sqrt{ax^2+bx^5}}-\frac{2\sqrt[4]{3}\,\sqrt[3]{b}\,x^{3/2}\left(\sqrt[3]{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (2*(1 + Sqrt[3])*b^(1/3)*x^(3/2)*(a + b*x^3))/(a*(a^(1/3) + (1 + Sqrt[3])*b
^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(a*x^(3/2)) - (2*3
^(1/4)*b^(1/3)*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[
(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (
2 + Sqrt[3])/4])/(a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) +
 (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5]) - ((1 - Sqrt[3])*b^(1/3)*
x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x
^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 -
 Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4]
)/(3^(1/4)*a^(2/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + S
qrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
```

```
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}} + \frac{(2b)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{a}$$
$$= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}} + \frac{\left(2bx\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{a\sqrt{ax^2+bx^5}}$$
$$= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}} + \frac{\left(4bx\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{a\sqrt{ax^2+bx^5}}$$
$$= -\frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}} - \frac{\left(2\sqrt[3]{b}\,x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{a\sqrt{ax^2+bx^5}} - \frac{\left(2\left(1-\right.\right.}{}$$
$$= \frac{2\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x^{3/2}\left(a+bx^3\right)}{a\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} - \frac{2\sqrt{ax^2+bx^5}}{ax^{3/2}} - \frac{2\sqrt[4]{3}\,\sqrt[3]{b}\,x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}\,x\right)\sqrt{}}{a^{2/3}}$$

Mathematica [C] time = 0.03, size = 55, normalized size = 0.11

$$-\frac{2\sqrt{x}\sqrt{\frac{bx^3}{a}+1}\;{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};-\frac{bx^3}{a}\right)}{\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*Sqrt[a*x^2 + b*x^5]),x]
[Out] (-2*Sqrt[x]*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-1/6, 1/2, 5/6, -((b*x^3)
/a)])/Sqrt[x^2*(a + b*x^3)]
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\sqrt{x}}{bx^6+ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^6 + a*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5+ax^2}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

maple [C] time = 0.91, size = 2860, normalized size = 5.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2*x^(1/2)*(4*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1
/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-
a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3
^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)
-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1
/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-a*b^2)^(2/3)*3^(1/2)*x+2*I
*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3
^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b
^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)
-3))^(1/2))*((b*x^3+a)*x)^(1/2)*3^(1/2)*a*b-4*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)
^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b
^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*Ellipt
icF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1
/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2
)*(-a*b^2)^(1/3)*x^2*b+6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
)*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))
/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1
/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/
(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-a*b^2)^(1/3)*x^2*
b-2*I*((b*x^3+a)*x)^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*x^2*b+8*(-(I*3^(1/2)-3)/(I
*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/
3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^
(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(
1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/
2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3
+a)*x)^(1/2)*(-a*b^2)^(2/3)*x-12*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^
2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*
3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a
*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(
1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-a*b^2)^(2
/3)*x+I*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/
3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*3^(1/2)*x
^3*b^2-2*I*((b*x^3+a)*x)^(1/2)*(-a*b^2)^(2/3)*3^(1/2)*x+4*(-(I*3^(1/2)-3)/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1
/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3
^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^
(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1
/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^
3+a)*x)^(1/2)*a*b-6*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x
)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*a*b-2*I*((b*x^3+a)*x)^(1
/2)*3^(1/2)*x^3*b^2-3*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3
)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(
1/2)*x^3*b^2-2*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(
1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(
-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*
3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
```

```
1/2))/(I*3^(1/2)-3))^(1/2))*((b*x^3+a)*x)^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*x^2*
b+6*((b*x^3+a)*x)^(1/2)*x^3*b^2+I*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(
-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/
3))/b^2*x)^(1/2)*3^(1/2)*a*b+6*((b*x^3+a)*x)^(1/2)*(-a*b^2)^(1/3)*x^2*b+6*(
(b*x^3+a)*x)^(1/2)*(-a*b^2)^(2/3)*x-3*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/
2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)
^(1/3))/b^2*x)^(1/2)*a*b)/(b*x^5+a*x^2)^(1/2)/b/a/(I*3^(1/2)-3)/((-b*x+(-a*
b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/
2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\,\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*sqrt(x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{b\,x^5 + a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(1/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x}\,\sqrt{x^2\left(a + bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(sqrt(x)*sqrt(x**2*(a + b*x**3))), x)
```

3.303 $\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=27

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

```
[Out] -2/3*(b*x^5+a*x^2)^(1/2)/a/x^(5/2)
```

Rubi [A] time = 0.04, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {2014}

$$-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(3*a*x^(5/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx=-\frac{2\sqrt{ax^2+bx^5}}{3ax^{5/2}}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$-\frac{2\sqrt{x^2\left(a+bx^3\right)}}{3ax^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(3/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[x^2*(a + b*x^3)])/(3*a*x^(5/2))
```

fricas [A] time = 0.38, size = 21, normalized size = 0.78

$$-\frac{2\sqrt{bx^5+ax^2}}{3\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(b*x^5 + a*x^2)/(a*x^(5/2))
```

giac [A] time = 0.20, size = 23, normalized size = 0.85

$$-\frac{2\sqrt{b+\frac{a}{x^3}}}{3\,a}+\frac{2\sqrt{b}}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(b + a/x^3)/a + 2/3*sqrt(b)/a
```

maple [A] time = 0.04, size = 29, normalized size = 1.07

$$-\frac{2\left(b\,x^{3}+a\right)}{3\sqrt{b\,x^{5}+a\,x^{2}}\;a\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/3*(b*x^3+a)/x^(1/2)/a/(b*x^5+a*x^2)^(1/2)
```

maxima [A] time = 1.42, size = 26, normalized size = 0.96

$$-\frac{2\left(bx^{4}+ax\right)}{3\sqrt{bx^{3}+a}\,ax^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(3/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*(b*x^4 + a*x)/(sqrt(b*x^3 + a)*a*x^(5/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^{3/2}\,\sqrt{b\,x^{5}+a\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^(3/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{3}{2}}\sqrt{x^{2}\left(a+bx^{3}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(3/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(3/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.304 $\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=235

$$-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

```
[Out] -2/5*(b*x^5+a*x^2)^(1/2)/a/x^(7/2)-2/15*b*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(
1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(
1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a
^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/
4*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^
(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/a^(4/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*
x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2025, 2032, 329, 225}

$$-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[4]{3}a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}-\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(5*a*x^(7/2)) - (2*b*x^(3/2)*(a^(1/3) + b^(1/3)*x)
*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*
b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3)
 + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(5*3^(1/4)*a^(4/3)*Sqrt[(b^
(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[
a*x^2 + b*x^5])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
```

```
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx &= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{(2b)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{5a}\\ &= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{\left(2bx\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{5a\sqrt{ax^2+bx^5}}\\ &= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{\left(4bx\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{5a\sqrt{ax^2+bx^5}}\\ &= -\frac{2\sqrt{ax^2+bx^5}}{5ax^{7/2}}-\frac{2bx^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\left(\frac{\sqrt[3]{a}+\left(1-\sqrt{3}\right)\sqrt[3]{b}x}{\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x}\right)\middle|\right.}{5\sqrt[4]{3}\,a^{4/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.24

$$-\frac{2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};-\frac{bx^3}{a}\right)}{5x^{3/2}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(5/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-5/6, 1/2, 1/6, -((b*x^3)/a)])/(5
*x^(3/2)*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\sqrt{x}}{bx^8+ax^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^8 + a*x^5), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\, x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

maple [C] time = 1.08, size = 1795, normalized size = 7.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 2/5/(b*x^5+a*x^2)^(1/2)/x^(3/2)*(b*x^3+a)/(-a*b^2)^(1/3)/a*(4*I*(-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b
^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b
*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b
*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
3^(1/2)*x^5*b^2-8*I*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x
)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*
x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(
1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*
3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)*3^(1/2)*x^4*b+4*I*(-(I*3^(1/2
)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b
^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b
*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b
*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*
(-a*b^2)^(2/3)*3^(1/2)*x^3-4*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2
)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-
3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)
-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*b^2+8*(-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*
b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(
-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*El
lipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*
3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-a*b^2)^(1/3)
*x^4*b-4*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2
*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b
*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I
*3^(1/2)-3))^(1/2))*(-a*b^2)^(2/3)*x^3-I*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^
(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b
^2)^(1/3))/b^2*x)^(1/2)*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*x)^(1/2)+3*((b*x^
3+a)*x)^(1/2)*(-a*b^2)^(1/3)*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^
2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b
^2*x)^(1/2))/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/((-b*x+(-a*b^2)^(1/3))*(2*b*
x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)
-(-a*b^2)^(1/3))/b^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\, x^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(5/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{5/2}\, \sqrt{b\, x^5 + a\, x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(5/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(5/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{5}{2}} \sqrt{x^2 \left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(5/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(5/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.305 $\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=555

$$\frac{4\left(1-\sqrt{3}\right)b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{7\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{8\sqrt[4]{3}b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\right.}{}$$

```
[Out] -8/7*b^(4/3)*x^(3/2)*(b*x^3+a)*(1+3^(1/2))/a^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2
)))/(b*x^5+a*x^2)^(1/2)-2/7*(b*x^5+a*x^2)^(1/2)/a/x^(9/2)+8/7*b*(b*x^5+a*x^
2)^(1/2)/a^2/x^(3/2)+8/7*3^(1/4)*b^(4/3)*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1
/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1
/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticE((1-(a^
(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4
*6^(1/2)+1/4*2^(1/2))*((a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(
1/3)*x*(1+3^(1/2)))^2)^(1/2)/a^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3
)+b^(1/3)*x)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)+4/21*b^(4/3)*x^(3/2)*
(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x*(
1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(a^(1/3)+b^(1/3)*x*(1+
3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2/(a^(1/3)+b^(1/3)*x
*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(1-3^(1/2))*((a^(2/3)-a^(1/
3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/
a^(5/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3)+b^(1/3)
*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 555, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2025, 2032, 329, 308, 225, 1881}

$$-\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}\left(a+bx^3\right)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)\sqrt{ax^2+bx^5}}+\frac{4\left(1-\sqrt{3}\right)b^{4/3}x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\cdots}{\left(1+\sqrt{3}\right)\cdots}\right.\right.}{7\sqrt[4]{3}a^{5/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-8*(1 + Sqrt[3])*b^(4/3)*x^(3/2)*(a + b*x^3))/(7*a^2*(a^(1/3) + (1 + Sqrt[
3])*b^(1/3)*x)*Sqrt[a*x^2 + b*x^5]) - (2*Sqrt[a*x^2 + b*x^5])/(7*a*x^(9/2))
 + (8*b*Sqrt[a*x^2 + b*x^5])/(7*a^2*x^(3/2)) + (8*3^(1/4)*b^(4/3)*x^(3/2)*(
a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1
/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticE[ArcCos[(a^(1/3) + (1 - Sqrt[3])
*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(7*a^(5
/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)^2]*Sqrt[a*x^2 + b*x^5]) + (4*(1 - Sqrt[3])*b^(4/3)*x^(3/2)*(a^(1/3) + b
^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 +
Sqrt[3])*b^(1/3)*x)^2]*EllipticF[ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)
/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)], (2 + Sqrt[3])/4])/(7*3^(1/4)*a^(5/3)
*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)
^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
```

```
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 308

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[((Sqrt[3] - 1)*s^2)/(2*r^2), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 1881

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[((1 + Sqrt[3])*d*s^3*x*Sqr
t[a + b*x^6])/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2)), x] - Simp[(3^(1/4)*d*s*x
*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]*El
lipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + S
qrt[3])/4])/(2*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sq
rt[a + b*x^6]), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{x^{7/2}\sqrt{ax^2+bx^5}}\,dx = -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} - \frac{(4b)\int \frac{1}{\sqrt{x}\sqrt{ax^2+bx^5}}\,dx}{7a}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{(8b^2)\int \frac{x^{5/2}}{\sqrt{ax^2+bx^5}}\,dx}{7a^2}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{\left(8b^2x\sqrt{a+bx^3}\right)\int \frac{x^{3/2}}{\sqrt{a+bx^3}}\,dx}{7a^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} - \frac{\left(16b^2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{a+bx^6}}\,dx, x, \sqrt{x}\right)}{7a^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} + \frac{\left(8b^{4/3}x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{\left(-1+\sqrt{3}\right)a^{2/3}-2b^{2/3}x^4}{\sqrt{a+bx^6}}\,dx, x, \cdots}{7a^2\sqrt{ax^2+bx^5}}$$

$$= -\frac{8\left(1+\sqrt{3}\right)b^{4/3}x^{3/2}\left(a+bx^3\right)}{7a^2\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}\,x\right)\sqrt{ax^2+bx^5}} - \frac{2\sqrt{ax^2+bx^5}}{7ax^{9/2}} + \frac{8b\sqrt{ax^2+bx^5}}{7a^2x^{3/2}} + \frac{8\sqrt[4]{3}\,b^{4/3}x\cdots}{\cdots}$$

Mathematica [C] time = 0.03, size = 57, normalized size = 0.10

$$-\frac{2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};-\frac{bx^3}{a}\right)}{7x^{5/2}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(7/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-7/6, 1/2, -1/6, -((b*x^3)/a)])/(
7*x^(5/2)*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\,\sqrt{x}}{bx^9+ax^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^9 + a*x^6), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5+ax^2}\,x^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

maple [C] time = 1.09, size = 3048, normalized size = 5.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 2/7*(-8*I*3^(1/2)*((b*x^3+a)*x)^(1/2)*x^6*b^2+16*I*(-a*b^2)^(2/3)*3^(1/2)*(
(b*x^3+a)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)
^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4-16*(-a*b^2)^(1/3)*((b*x^3+a)*x)^(1/2)*(-(
I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/
2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a
*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))
^(1/2))*x^5*b+24*(-a*b^2)^(1/3)*((b*x^3+a)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-
a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)
*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*
EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((
I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*b+4*I*3^
(1/2)*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3)
)*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*x^6*b^2+32*
(-a*b^2)^(2/3)*((b*x^3+a)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*
b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+
I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(
-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^
(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3
^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^4-48*(-a*b^2)^(2/3)*((b*x^3
+a)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)
*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b
^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)
/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2)
)/(I*3^(1/2)-3))^(1/2))*x^4-8*I*(-a*b^2)^(2/3)*3^(1/2)*((b*x^3+a)*x)^(1/2)*
x^4+3*I*3^(1/2)*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*
b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*x
^3*a*b-8*I*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^
(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+
(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/
2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2
)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^5*b+16*(
(b*x^3+a)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)
^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x
+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(
I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1
/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3
^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a*b-24*((b*x^3+a)*x)^(1/2)*(-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)
^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+
I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)
))^(1/2)*EllipticE((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)
^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3
*a*b-I*3^(1/2)*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b
```

```
^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*a^
2+24*((b*x^3+a)*x)^(1/2)*x^6*b^2-12*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)
*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(
1/3))/b^2*x)^(1/2)*x^6*b^2+24*(-a*b^2)^(1/3)*((b*x^3+a)*x)^(1/2)*x^5*b+8*I*
3^(1/2)*((b*x^3+a)*x)^(1/2)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*EllipticE((-(I*3^(1/2)-3
)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-
1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*x^3*a*b+24*(-a*b^2)^(2/3)*((b*x^3+a)
*x)^(1/2)*x^4-9*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*
b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*x
^3*a*b-8*I*(-a*b^2)^(1/3)*3^(1/2)*((b*x^3+a)*x)^(1/2)*x^5*b+3*((-b*x+(-a*b^
2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)
*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*a^2)/(b*x^5+a*x^2)^(1/2)/x^(5/
2)/a^2/(I*3^(1/2)-3)/((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)
+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\, x^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(7/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(7/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{7/2}\, \sqrt{b\, x^5 + a\, x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(7/2)*(a*x^2 + b*x^5)^(1/2)),x)
```

```
[Out] int(1/(x^(7/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{7}{2}} \sqrt{x^2 \left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(7/2)/(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(1/(x**(7/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.306 $\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=56

$$\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

```
[Out] -2/9*(b*x^5+a*x^2)^(1/2)/a/x^(11/2)+4/9*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(5/2)
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2016, 2014}

$$\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}-\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(9*a*x^(11/2)) + (4*b*Sqrt[a*x^2 + b*x^5])/(9*a^2*
x^(5/2))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{9/2}\sqrt{ax^2+bx^5}}\,dx &= -\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}-\frac{(2b)\int \frac{1}{x^{3/2}\sqrt{ax^2+bx^5}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^2+bx^5}}{9ax^{11/2}}+\frac{4b\sqrt{ax^2+bx^5}}{9a^2x^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.62

$$-\frac{2\left(a-2bx^3\right)\sqrt{x^2\left(a+bx^3\right)}}{9a^2x^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(9/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*(a - 2*b*x^3)*Sqrt[x^2*(a + b*x^3)])/(9*a^2*x^(11/2))
```

fricas [A] time = 0.39, size = 31, normalized size = 0.55

$$\frac{2\,\sqrt{bx^5+ax^2}\left(2\,bx^3-a\right)}{9\,a^2x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(2*b*x^3 - a)/(a^2*x^(11/2))
```

giac [A] time = 0.23, size = 38, normalized size = 0.68

$$-\frac{2\left(b+\frac{a}{x^3}\right)^{\frac{3}{2}}}{9\,a^2}+\frac{2\,\sqrt{b+\frac{a}{x^3}}\,b}{3\,a^2}-\frac{4\,b^{\frac{3}{2}}}{9\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
[Out] -2/9*(b + a/x^3)^(3/2)/a^2 + 2/3*sqrt(b + a/x^3)*b/a^2 - 4/9*b^(3/2)/a^2
```

maple [A] time = 0.04, size = 37, normalized size = 0.66

$$-\frac{2\left(b\,x^3+a\right)\left(-2b\,x^3+a\right)}{9\sqrt{b\,x^5+a\,x^2}\;a^2x^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x)
[Out] -2/9*(b*x^3+a)*(-2*b*x^3+a)/x^(7/2)/a^2/(b*x^5+a*x^2)^(1/2)
```

maxima [A] time = 1.42, size = 38, normalized size = 0.68

$$\frac{2\left(2\,b^2x^7+abx^4-a^2x\right)}{9\,\sqrt{bx^3+a}\,a^2x^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(9/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] 2/9*(2*b^2*x^7 + a*b*x^4 - a^2*x)/(sqrt(b*x^3 + a)*a^2*x^(11/2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^{9/2}\,\sqrt{b\,x^5+a\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(9/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{9}{2}}\sqrt{x^2\left(a+bx^3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(9/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(9/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.307 $\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx$

Optimal. Leaf size=265

$$\frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}-\frac{2\sqrt{ax^2+}}{11ax^{13}}$$

```
[Out] -2/11*(b*x^5+a*x^2)^(1/2)/a/x^(13/2)+16/55*b*(b*x^5+a*x^2)^(1/2)/a^2/x^(7/2
)+16/165*b^2*x^(3/2)*(a^(1/3)+b^(1/3)*x)*((a^(1/3)+b^(1/3)*x*(1-3^(1/2)))^2
/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)/(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))*(
a^(1/3)+b^(1/3)*x*(1+3^(1/2)))*EllipticF((1-(a^(1/3)+b^(1/3)*x*(1-3^(1/2)))
^2/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*((a^(2
/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/(a^(1/3)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
*3^(3/4)/a^(7/3)/(b*x^5+a*x^2)^(1/2)/(b^(1/3)*x*(a^(1/3)+b^(1/3)*x)/(a^(1/3
)+b^(1/3)*x*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2025, 2032, 329, 225}

$$\frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}F\left(\cos^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}{\left(1+\sqrt{3}\right)\sqrt[3]{b}x+\sqrt[3]{a}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{55\sqrt[4]{3}a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\sqrt{ax^2+bx^5}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}-\frac{2\sqrt{ax^2+}}{11ax^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^5])/(11*a*x^(13/2)) + (16*b*Sqrt[a*x^2 + b*x^5])/(55*a
^2*x^(7/2)) + (16*b^2*x^(3/2)*(a^(1/3) + b^(1/3)*x)*Sqrt[(a^(2/3) - a^(1/3)
*b^(1/3)*x + b^(2/3)*x^2)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*EllipticF[
ArcCos[(a^(1/3) + (1 - Sqrt[3])*b^(1/3)*x)/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)
*x)], (2 + Sqrt[3])/4])/(55*3^(1/4)*a^(7/3)*Sqrt[(b^(1/3)*x*(a^(1/3) + b^(1
/3)*x))/(a^(1/3) + (1 + Sqrt[3])*b^(1/3)*x)^2]*Sqrt[a*x^2 + b*x^5])
```

Rule 225

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(x*(s + r*x^2)*Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]*EllipticF[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s
+ (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4])/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqr
t[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]), x]] /; FreeQ[{a, b}, x
]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2025

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
&&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[m
 + j*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^{11/2}\sqrt{ax^2+bx^5}}\,dx &= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}-\frac{(8b)\int \frac{1}{x^{5/2}\sqrt{ax^2+bx^5}}\,dx}{11a}\\ &= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}+\frac{\left(16b^2\right)\int \frac{\sqrt{x}}{\sqrt{ax^2+bx^5}}\,dx}{55a^2}\\ &= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}+\frac{\left(16b^2x\sqrt{a+bx^3}\right)\int \frac{1}{\sqrt{x}\sqrt{a+bx^3}}\,dx}{55a^2\sqrt{ax^2+bx^5}}\\ &= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}+\frac{\left(32b^2x\sqrt{a+bx^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^6}}\,dx,x,\sqrt{x}\right)}{55a^2\sqrt{ax^2+bx^5}}\\ &= -\frac{2\sqrt{ax^2+bx^5}}{11ax^{13/2}}+\frac{16b\sqrt{ax^2+bx^5}}{55a^2x^{7/2}}+\frac{16b^2x^{3/2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)\sqrt{\frac{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\,F\left(\cos^{-1}\ldots\right.}{55\sqrt[4]{3}\,a^{7/3}\sqrt{\frac{\sqrt[3]{b}x\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\left(\sqrt[3]{a}+\left(1+\sqrt{3}\right)\sqrt[3]{b}x\right)^2}}\ldots}\end{aligned}$$

Mathematica [C] time = 0.02, size = 57, normalized size = 0.22

$$-\frac{2\sqrt{\frac{bx^3}{a}+1}\,{}_2F_1\left(-\frac{11}{6},\frac{1}{2};-\frac{5}{6};-\frac{bx^3}{a}\right)}{11x^{9/2}\sqrt{x^2\left(a+bx^3\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^(11/2)*Sqrt[a*x^2 + b*x^5]),x]
```

```
[Out] (-2*Sqrt[1 + (b*x^3)/a]*Hypergeometric2F1[-11/6, 1/2, -5/6, -((b*x^3)/a)])/
(11*x^(9/2)*Sqrt[x^2*(a + b*x^3)])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{bx^5+ax^2}\sqrt{x}}{bx^{11}+ax^8},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*x^5 + a*x^2)*sqrt(x)/(b*x^11 + a*x^8), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\, x^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

maple [C] time = 1.24, size = 2009, normalized size = 7.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] -2/55/(b*x^5+a*x^2)^(1/2)/x^(9/2)*(b*x^3+a)/(-a*b^2)^(1/3)/a^2*(32*I*3^(1/2
)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),
((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/
2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*
b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*
b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3)))^(1/2)*x^8*b^3-64*I*(-a*b^2)^(1/3)*3^(1/2)*EllipticF((-(I*3^(1/2)-3)/
(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)
/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-
a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(
1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)
-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*x^7*b^2+32*I*(-
a*b^2)^(2/3)*3^(1/2)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)
^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2))*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2
*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-
1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*x^6*b-32*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/
2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^
(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(
1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1
/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2
)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*x^8*b^3+64*(-a*b^2)^(1/
3)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)
,((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1
/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a
*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2
*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^
(1/3)))^(1/2)*x^7*b^2-32*(-a*b^2)^(2/3)*EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2
)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(
1/2))/(I*3^(1/2)-3))^(1/2))*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1
/3))*b*x)^(1/2)*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))/(1+I*3^(1/
2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)*x^6*b-8*I*(-a*b^2)^(1/3)
*3^(1/2)*((b*x^3+a)*x)^(1/2)*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^
2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b
^2*x)^(1/2)*x^3*b+24*b*((b*x^3+a)*x)^(1/2)*x^3*(-a*b^2)^(1/3)*((-b*x+(-a*b^
2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)
*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)+5*I*(-a*b^2)^(1/3)*3^(1/2)*((b
```

```
*x^3+a)*x)^(1/2)*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a
*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)*
a-15*((b*x^3+a)*x)^(1/2)*a*(-a*b^2)^(1/3)*((-b*x+(-a*b^2)^(1/3))*(2*b*x+I*3
^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)-(-a*
b^2)^(1/3))/b^2*x)^(1/2))/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/((-b*x+(-a*b^2)
^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/2)*(
-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^5 + ax^2}\, x^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(11/2)/(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^5 + a*x^2)*x^(11/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^{11/2}\, \sqrt{b\, x^5 + a\, x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(11/2)*(a*x^2 + b*x^5)^(1/2)),x)
[Out] int(1/(x^(11/2)*(a*x^2 + b*x^5)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{\frac{11}{2}} \sqrt{x^2 \left(a + bx^3\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(11/2)/(b*x**5+a*x**2)**(1/2),x)
[Out] Integral(1/(x**(11/2)*sqrt(x**2*(a + b*x**3))), x)
```

3.308 $\int \frac{x}{ax^3+bx^4}\,dx$

Optimal. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 44}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a*x^3 + b*x^4),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{ax^3+bx^4}\,dx &= \int \frac{1}{x^2(a+bx)}\,dx\\
&= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\
&= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a*x^3 + b*x^4),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

fricas [A] time = 0.38, size = 26, normalized size = 0.93

$$\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="fricas")
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

giac [A] time = 0.15, size = 30, normalized size = 1.07

$$\frac{b\log\left(|bx+a|\right)}{a^2}-\frac{b\log\left(|x|\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="giac")
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

maple [A] time = 0.05, size = 29, normalized size = 1.04

$$-\frac{b\ln(x)}{a^2}+\frac{b\ln\left(bx+a\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x^3),x)
[Out] -1/a^2*b*ln(x)+1/a^2*b*ln(b*x+a)-1/a/x
```

maxima [A] time = 1.36, size = 28, normalized size = 1.00

$$\frac{b\log\left(bx+a\right)}{a^2}-\frac{b\log(x)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3),x, algorithm="maxima")
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

mupad [B] time = 5.21, size = 25, normalized size = 0.89

$$\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^3 + b*x^4),x)
[Out] (2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

sympy [A] time = 0.21, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x**3),x)
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

3.309 $\int \frac{1}{ax^3+bx^4}\,dx$

Optimal. Leaf size=42

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

```
[Out] -1/2/a/x^2+b/a^2/x+b^2*ln(x)/a^3-b^2*ln(b*x+a)/a^3
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 44}

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^4)^(-1),x]
```

```
[Out] -1/(2*a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^3+bx^4}\,dx &= \int \frac{1}{x^3(a+bx)}\,dx\\ &= \int \left(\frac{1}{ax^3}-\frac{b}{a^2x^2}+\frac{b^2}{a^3x}-\frac{b^3}{a^3(a+bx)}\right)dx\\ &= -\frac{1}{2ax^2}+\frac{b}{a^2x}+\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 42, normalized size = 1.00

$$\frac{b^2\log(x)}{a^3}-\frac{b^2\log(a+bx)}{a^3}+\frac{b}{a^2x}-\frac{1}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^4)^(-1),x]
```

```
[Out] -1/2*1/(a*x^2) + b/(a^2*x) + (b^2*Log[x])/a^3 - (b^2*Log[a + b*x])/a^3
```

fricas [A] time = 0.39, size = 41, normalized size = 0.98

$$-\frac{2\,b^2x^2\log\left(bx+a\right)-2\,b^2x^2\log(x)-2\,abx+a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*x^2*log(b*x + a) - 2*b^2*x^2*log(x) - 2*a*b*x + a^2)/(a^3*x^2)
```

giac [A] time = 0.14, size = 45, normalized size = 1.07

$$-\frac{b^2\log\left(|bx+a|\right)}{a^3}+\frac{b^2\log\left(|x|\right)}{a^3}+\frac{2\,abx-a^2}{2\,a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(b*x + a))/a^3 + b^2*log(abs(x))/a^3 + 1/2*(2*a*b*x - a^2)/(a^3
*x^2)
```

maple [A] time = 0.05, size = 41, normalized size = 0.98

$$\frac{b^2\ln(x)}{a^3}-\frac{b^2\ln\left(bx+a\right)}{a^3}+\frac{b}{a^2x}-\frac{1}{2a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x^3),x)
```

```
[Out] 1/a^3*b^2*ln(x)-1/a^3*b^2*ln(b*x+a)+1/a^2*b/x-1/2/a/x^2
```

maxima [A] time = 1.36, size = 40, normalized size = 0.95

$$-\frac{b^2\log\left(bx+a\right)}{a^3}+\frac{b^2\log(x)}{a^3}+\frac{2\,bx-a}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3),x, algorithm="maxima")
```

```
[Out] -b^2*log(b*x + a)/a^3 + b^2*log(x)/a^3 + 1/2*(2*b*x - a)/(a^2*x^2)
```

mupad [B] time = 5.72, size = 38, normalized size = 0.90

$$-\frac{\frac{a^2}{2}-a\,b\,x}{a^3\,x^2}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^4),x)
```

```
[Out] - (a^2/2 - a*b*x)/(a^3*x^2) - (2*b^2*atanh((2*b*x)/a + 1))/a^3
```

sympy [A] time = 0.22, size = 31, normalized size = 0.74

$$\frac{-a+2bx}{2a^2x^2}+\frac{b^2\left(\log\left(x\right)-\log\left(\frac{a}{b}+x\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x**3),x)
```

```
[Out] (-a + 2*b*x)/(2*a**2*x**2) + b**2*(log(x) - log(a/b + x))/a**3
```

3.310 $\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=112

$$-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}+\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}-\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{x\sqrt{ax^3+bx^4}}{3b}$$

```
[Out] -5/8*a^3*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(7/2)-5/12*a*(b*x^4+a*x
^3)^(1/2)/b^2+5/8*a^2*(b*x^4+a*x^3)^(1/2)/b^3/x+1/3*x*(b*x^4+a*x^3)^(1/2)/b
```

Rubi [A] time = 0.18, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2029, 206}

$$\frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x}-\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}-\frac{5a\sqrt{ax^3+bx^4}}{12b^2}+\frac{x\sqrt{ax^3+bx^4}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (-5*a*Sqrt[a*x^3 + b*x^4])/(12*b^2) + (5*a^2*Sqrt[a*x^3 + b*x^4])/(8*b^3*x)
 + (x*Sqrt[a*x^3 + b*x^4])/(3*b) - (5*a^3*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3
+ b*x^4]])/(8*b^(7/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{ax^3+bx^4}}\,dx = \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{(5a)\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx}{6b}$$
$$= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{x\sqrt{ax^3+bx^4}}{3b} + \frac{\left(5a^2\right)\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx}{8b^2}$$
$$= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{\left(5a^3\right)\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{16b^3}$$
$$= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{\left(5a^3\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^3}$$
$$= -\frac{5a\sqrt{ax^3+bx^4}}{12b^2} + \frac{5a^2\sqrt{ax^3+bx^4}}{8b^3x} + \frac{x\sqrt{ax^3+bx^4}}{3b} - \frac{5a^3\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{8b^{7/2}}$$

Mathematica [A] time = 0.20, size = 94, normalized size = 0.84

$$\frac{\sqrt{x^3(a+bx)}\left(\sqrt{b}\sqrt{x}\left(15a^2-10abx+8b^2x^2\right)-\frac{15a^{5/2}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{\frac{bx}{a}+1}}\right)}{24b^{7/2}x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[x^3*(a + b*x)]*(Sqrt[b]*Sqrt[x]*(15*a^2 - 10*a*b*x + 8*b^2*x^2) - (15
*a^(5/2)*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/Sqrt[1 + (b*x)/a]))/(24*b^(7/2
)*x^(3/2))
```

fricas [A] time = 0.40, size = 171, normalized size = 1.53

$$\left[\frac{15\,a^3\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\,\sqrt{b}}{x}\right)+2\left(8\,b^3x^2-10\,ab^2x+15\,a^2b\right)\sqrt{bx^4+ax^3}}{48\,b^4x}, \frac{15\,a^3\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^4+ax^3}\,\sqrt{}}{bx^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(15*a^3*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))
/x) + 2*(8*b^3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x), 1
/24*(15*a^3*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) + (8*b^
3*x^2 - 10*a*b^2*x + 15*a^2*b)*sqrt(b*x^4 + a*x^3))/(b^4*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^4+ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/sqrt(b*x^4 + a*x^3), x)
```

maple [A] time = 0.05, size = 120, normalized size = 1.07

$$\frac{\sqrt{(bx+a)\,x}\,\left(16\sqrt{b\,x^2+ax}\;b^{\frac{7}{2}}x^2-15a^3b\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\;\sqrt{b}}{2\sqrt{b}}\right)-20\sqrt{b\,x^2+ax}\;a\,b^{\frac{5}{2}}x+30\sqrt{b\,x^2+ax}\;a^2b^{\frac{3}{2}}\right)}{48\sqrt{b\,x^4+a\,x^3}\;b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/48*x*((b*x+a)*x)^(1/2)*(16*x^2*(b*x^2+a*x)^(1/2)*b^(7/2)-20*(b*x^2+a*x)^(
1/2)*b^(5/2)*x*a+30*(b*x^2+a*x)^(1/2)*b^(3/2)*a^2-15*ln(1/2*(2*b*x+a+2*(b*x
^2+a*x)^(1/2)*b^(1/2))/b^(1/2))*a^3*b)/(b*x^4+a*x^3)^(1/2)/b^(9/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{bx^4+ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/sqrt(b*x^4 + a*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x^4/(a*x^3 + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(x**3*(a + b*x)), x)
```

3.311 $\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=86

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\sqrt{ax^3+bx^4}}{2b}$$

```
[Out] 3/4*a^2*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(5/2)+1/2*(b*x^4+a*x^3)^
(1/2)/b-3/4*a*(b*x^4+a*x^3)^(1/2)/b^2/x
```

Rubi [A] time = 0.13, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2029, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}-\frac{3a\sqrt{ax^3+bx^4}}{4b^2x}+\frac{\sqrt{ax^3+bx^4}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(2*b) - (3*a*Sqrt[a*x^3 + b*x^4])/(4*b^2*x) + (3*a^2*Ar
cTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/(4*b^(5/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{ax^3+bx^4}}\,dx = \frac{\sqrt{ax^3+bx^4}}{2b} - \frac{(3a)\int \frac{x^2}{\sqrt{ax^3+bx^4}}\,dx}{4b}$$
$$= \frac{\sqrt{ax^3+bx^4}}{2b} - \frac{3a\sqrt{ax^3+bx^4}}{4b^2x} + \frac{(3a^2)\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx}{8b^2}$$
$$= \frac{\sqrt{ax^3+bx^4}}{2b} - \frac{3a\sqrt{ax^3+bx^4}}{4b^2x} + \frac{(3a^2)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^2}$$
$$= \frac{\sqrt{ax^3+bx^4}}{2b} - \frac{3a\sqrt{ax^3+bx^4}}{4b^2x} + \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{4b^{5/2}}$$

Mathematica [A] time = 0.07, size = 92, normalized size = 1.07

$$\frac{3a^{5/2}x^{3/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)+\sqrt{b}\,x^2\left(-3a^2-abx+2b^2x^2\right)}{4b^{5/2}\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(-3*a^2 - a*b*x + 2*b^2*x^2) + 3*a^(5/2)*x^(3/2)*Sqrt[1 + (b*x
)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(4*b^(5/2)*Sqrt[x^3*(a + b*x)])
```

fricas [A] time = 0.42, size = 150, normalized size = 1.74

$$\left[\frac{3\,a^2\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\,\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}\left(2\,b^2x-3\,ab\right)}{8\,b^3x},\,-\frac{3\,a^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^4+ax^3}\,\sqrt{-b}}{bx^2}\right)-\sqrt{bx^4\cdots}}{4\,b^3x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(3*a^2*sqrt(b)*x*log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x
) + 2*sqrt(b*x^4 + a*x^3)*(2*b^2*x - 3*a*b))/(b^3*x), -1/4*(3*a^2*sqrt(-b)*
x*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2)) - sqrt(b*x^4 + a*x^3)*(2*b^2
*x - 3*a*b))/(b^3*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^4+ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

maple [A] time = 0.05, size = 98, normalized size = 1.14

$$\frac{\sqrt{(bx+a)\,x}\,\left(3a^2b\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\,\sqrt{b}}{2\sqrt{b}}\right)+4\sqrt{b\,x^2+ax}\,b^{\frac{5}{2}}x-6\sqrt{b\,x^2+ax}\,a\,b^{\frac{3}{2}}\right)x}{8\sqrt{b\,x^4+a\,x^3}\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/8*x*((b*x+a)*x)^(1/2)*(4*x*(b*x^2+a*x)^(1/2)*b^(5/2)-6*(b*x^2+a*x)^(1/2)*
b^(3/2)*a+3*ln(1/2*(2*b*x+a+2*(b*x^2+a*x)^(1/2)*b^(1/2))/b^(1/2))*a^2*b)/(b
*x^4+a*x^3)^(1/2)/b^(7/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx^4 + ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(b*x^4 + a*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{b\,x^4 + a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x^3/(a*x^3 + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{x^3\,(a + bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**3*(a + b*x)), x)
```

3.312 $\int \frac{x^2}{\sqrt{ax^3+bx^4}} \, dx$

Optimal. Leaf size=56

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

```
[Out] -a*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(3/2)+(b*x^4+a*x^3)^(1/2)/b/x
```

Rubi [A] time = 0.08, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2024, 2029, 206}

$$\frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] Sqrt[a*x^3 + b*x^4]/(b*x) - (a*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/
b^(3/2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{ax^3+bx^4}} \, dx &= \frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\int \frac{x}{\sqrt{ax^3+bx^4}} \, dx}{2b}\\
&= \frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2} \, dx,x,\frac{x^2}{\sqrt{ax^3+bx^4}}\right)}{b}\\
&= \frac{\sqrt{ax^3+bx^4}}{bx}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 1.34

$$\frac{\sqrt{b}\,x^2(a+bx)-a^{3/2}x^{3/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{b^{3/2}\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (Sqrt[b]*x^2*(a + b*x) - a^(3/2)*x^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]
*Sqrt[x])/Sqrt[a]])/(b^(3/2)*Sqrt[x^3*(a + b*x)])
```

fricas [A] time = 0.41, size = 122, normalized size = 2.18

$$\left[\frac{a\sqrt{b}\,x\log\left(\frac{2\,bx^2+ax-2\,\sqrt{bx^4+ax^3}\,\sqrt{b}}{x}\right)+2\,\sqrt{bx^4+ax^3}\,b}{2\,b^2x},\ \frac{a\sqrt{-b}\,x\arctan\left(\frac{\sqrt{bx^4+ax^3}\,\sqrt{-b}}{bx^2}\right)+\sqrt{bx^4+ax^3}\,b}{b^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(b)*x*log((2*b*x^2 + a*x - 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x) +
2*sqrt(b*x^4 + a*x^3)*b)/(b^2*x), (a*sqrt(-b)*x*arctan(sqrt(b*x^4 + a*x^3)*
sqrt(-b)/(b*x^2)) + sqrt(b*x^4 + a*x^3)*b)/(b^2*x)]
```

giac [A] time = 0.23, size = 48, normalized size = 0.86

$$\frac{\frac{a\sqrt{b+\frac{a}{x}}\,x}{b}+\frac{a^2\arctan\left(\frac{\sqrt{b+\frac{a}{x}}}{\sqrt{-b}}\right)}{\sqrt{-b}\,b}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] (a*sqrt(b + a/x)*x/b + a^2*arctan(sqrt(b + a/x)/sqrt(-b))/(sqrt(-b)*b))/a
```

maple [A] time = 0.05, size = 78, normalized size = 1.39

$$\frac{\sqrt{(bx+a)\,x}\,\left(-ab\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\,\sqrt{b}}{2\sqrt{b}}\right)+2\sqrt{b\,x^2+ax}\,b^{\frac{3}{2}}\right)x}{2\sqrt{b\,x^4+a\,x^3}\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/2*x*((b*x+a)*x)^(1/2)*(2*(b*x^2+a*x)^(1/2)*b^(3/2)-a*ln(1/2*(2*b*x+a+2*(b
*x^2+a*x)^(1/2)*b^(1/2))/b^(1/2))*b)/(b*x^4+a*x^3)^(1/2)/b^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx^4+ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/sqrt(b*x^4 + a*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x^2/(a*x^3 + b*x^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(x**3*(a + b*x)), x)
```

3.313 $\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

```
[Out] 2*arctanh(x^2*b^(1/2)/(b*x^4+a*x^3)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2029, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a*x^3 + b*x^4]])/Sqrt[b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{ax^3+bx^4}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x^2}{\sqrt{ax^3+bx^4}}\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x^2}{\sqrt{ax^3+bx^4}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.84

$$\frac{2\sqrt{a}\,x^{3/2}\sqrt{\frac{bx}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\,\sqrt{x^3(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a*x^3 + b*x^4],x]
```

```
[Out] (2*Sqrt[a]*x^(3/2)*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(S
qrt[b]*Sqrt[x^3*(a + b*x)])
```

fricas [A] time = 0.39, size = 74, normalized size = 2.31

$$\left[\frac{\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^4+ax^3}\,\sqrt{b}}{x}\right)}{\sqrt{b}}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{bx^4+ax^3}\,\sqrt{-b}}{bx^2}\right)}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x^2 + a*x + 2*sqrt(b*x^4 + a*x^3)*sqrt(b))/x)/sqrt(b), -2*sqrt(-b
)*arctan(sqrt(b*x^4 + a*x^3)*sqrt(-b)/(b*x^2))/b]
```

giac [A] time = 0.28, size = 23, normalized size = 0.72

$$-\frac{2\,\arctan\left(\frac{\sqrt{b+\frac{a}{x}}}{\sqrt{-b}}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2*arctan(sqrt(b + a/x)/sqrt(-b))/sqrt(-b)
```

maple [B] time = 0.05, size = 56, normalized size = 1.75

$$\frac{\sqrt{(bx+a)\,x}\;x\ln\left(\frac{2bx+a+2\sqrt{b\,x^2+ax}\,\sqrt{b}}{2\sqrt{b}}\right)}{\sqrt{b\,x^4+a\,x^3}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] 1/(b*x^4+a*x^3)^(1/2)*x*((b*x+a)*x)^(1/2)*ln(1/2*(2*b*x+a+2*(b*x^2+a*x)^(1/
2)*b^(1/2))/b^(1/2))/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx^4+ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(b*x^4 + a*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\sqrt{b\,x^4+a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] int(x/(a*x^3 + b*x^4)^(1/2), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**3*(a + b*x)), x)
```

3.314 $\int \frac{1}{\sqrt{ax^3+bx^4}} dx$

Optimal. Leaf size=23

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

[Out] -2*(b*x^4+a*x^3)^(1/2)/a/x^2

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2000}

$$-\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*x^3 + b*x^4],x]

[Out] (-2*Sqrt[a*x^3 + b*x^4])/(a*x^2)

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{ax^3+bx^4}} \, dx = -\frac{2\sqrt{ax^3+bx^4}}{ax^2}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{x^3(a+bx)}}{ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x^3 + b*x^4],x]

[Out] (-2*Sqrt[x^3*(a + b*x)])/(a*x^2)

fricas [A] time = 0.38, size = 21, normalized size = 0.91

$$-\frac{2\sqrt{bx^4+ax^3}}{ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(b*x^4 + a*x^3)/(a*x^2)

giac [A] time = 0.21, size = 27, normalized size = 1.17

$$\frac{2}{\left(\sqrt{b}\,x-\sqrt{bx^2+ax}\right)\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/((sqrt(b)*x - sqrt(b*x^2 + a*x))*sgn(x))
```

maple [A] time = 0.05, size = 25, normalized size = 1.09

$$-\frac{2\,(bx+a)\,x}{\sqrt{b\,x^4+a\,x^3}\;a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] -2*(b*x+a)*x/a/(b*x^4+a*x^3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+ax^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*x^4 + a*x^3), x)
```

mupad [B] time = 5.14, size = 21, normalized size = 0.91

$$-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b*x^4)^(1/2),x)
```

```
[Out] -(2*(a*x^3 + b*x^4)^(1/2))/(a*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**3 + b*x**4), x)
```

3.315 $\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=52

$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

```
[Out] -2/3*(b*x^4+a*x^3)^(1/2)/a/x^3+4/3*b*(b*x^4+a*x^3)^(1/2)/a^2/x^2
```

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}-\frac{2\sqrt{ax^3+bx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(3*a*x^3) + (4*b*Sqrt[a*x^3 + b*x^4])/(3*a^2*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}-\frac{(2b)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{3a}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{3ax^3}+\frac{4b\sqrt{ax^3+bx^4}}{3a^2x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.56

$$-\frac{2(a-2bx)\sqrt{x^3(a+bx)}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*(a - 2*b*x)*Sqrt[x^3*(a + b*x)])/(3*a^2*x^3)
```

fricas [A] time = 0.38, size = 29, normalized size = 0.56

$$\frac{2\sqrt{bx^4+ax^3}\,(2\,bx-a)}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] 2/3*sqrt(b*x^4 + a*x^3)*(2*b*x - a)/(a^2*x^3)
```

giac [A] time = 0.23, size = 27, normalized size = 0.52

$$-\frac{2\left(\left(b+\frac{a}{x}\right)^{\frac{3}{2}}-3\sqrt{b+\frac{a}{x}}\,b\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] -2/3*((b + a/x)^(3/2) - 3*sqrt(b + a/x)*b)/a^2
```

maple [A] time = 0.05, size = 30, normalized size = 0.58

$$-\frac{2\,(bx+a)\,(-2bx+a)}{3\sqrt{b\,x^4+a\,x^3}\;a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x^4+a*x^3)^(1/2),x)
[Out] -2/3*(b*x+a)*(-2*b*x+a)/a^2/(b*x^4+a*x^3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+ax^3}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x), x)
```

mupad [B] time = 5.06, size = 42, normalized size = 0.81

$$-\frac{2\,a\,\sqrt{b\,x^4+a\,x^3}-4\,b\,x\,\sqrt{b\,x^4+a\,x^3}}{3\,a^2\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*x^3 + b*x^4)^(1/2)),x)
[Out] -(2*a*(a*x^3 + b*x^4)^(1/2) - 4*b*x*(a*x^3 + b*x^4)^(1/2))/(3*a^2*x^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x**4+a*x**3)**(1/2),x)
[Out] Integral(1/(x*sqrt(x**3*(a + b*x))), x)
```

3.316 $\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

```
[Out] -2/5*(b*x^4+a*x^3)^(1/2)/a/x^4+8/15*b*(b*x^4+a*x^3)^(1/2)/a^2/x^3-16/15*b^2
*(b*x^4+a*x^3)^(1/2)/a^3/x^2
```

Rubi [A] time = 0.09, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{2\sqrt{ax^3+bx^4}}{5ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(5*a*x^4) + (8*b*Sqrt[a*x^3 + b*x^4])/(15*a^2*x^3)
 - (16*b^2*Sqrt[a*x^3 + b*x^4])/(15*a^3*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4}-\frac{(4b)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{5a}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}+\frac{\left(8b^2\right)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{15a^2}\\ &= -\frac{2\sqrt{ax^3+bx^4}}{5ax^4}+\frac{8b\sqrt{ax^3+bx^4}}{15a^2x^3}-\frac{16b^2\sqrt{ax^3+bx^4}}{15a^3x^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.52

$$-\frac{2\sqrt{x^3(a+bx)}\left(3a^2-4abx+8b^2x^2\right)}{15a^3x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a*x^3 + b*x^4]),x]
[Out] (-2*Sqrt[x^3*(a + b*x)]*(3*a^2 - 4*a*b*x + 8*b^2*x^2))/(15*a^3*x^4)
```

fricas [A] time = 0.40, size = 40, normalized size = 0.50

$$-\frac{2\sqrt{bx^4+ax^3}\left(8\,b^2x^2-4\,abx+3\,a^2\right)}{15\,a^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
[Out] -2/15*sqrt(b*x^4 + a*x^3)*(8*b^2*x^2 - 4*a*b*x + 3*a^2)/(a^3*x^4)
```

giac [A] time = 0.24, size = 43, normalized size = 0.54

$$-\frac{2\left(3\left(b+\frac{a}{x}\right)^{\frac{5}{2}}-10\left(b+\frac{a}{x}\right)^{\frac{3}{2}}b+15\sqrt{b+\frac{a}{x}}\,b^2\right)}{15\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
[Out] -2/15*(3*(b + a/x)^(5/2) - 10*(b + a/x)^(3/2)*b + 15*sqrt(b + a/x)*b^2)/a^3
```

maple [A] time = 0.04, size = 46, normalized size = 0.58

$$-\frac{2\,(bx+a)\left(8b^2x^2-4abx+3a^2\right)}{15\sqrt{b\,x^4+a\,x^3}\;a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x^4+a*x^3)^(1/2),x)
[Out] -2/15*(b*x+a)*(8*b^2*x^2-4*a*b*x+3*a^2)/x/a^3/(b*x^4+a*x^3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+ax^3}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^2), x)
```

mupad [B] time = 5.14, size = 40, normalized size = 0.50

$$-\frac{2\sqrt{b\,x^4+a\,x^3}\;\left(3\,a^2-4\,a\,b\,x+8\,b^2\,x^2\right)}{15\,a^3\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a*x^3 + b*x^4)^(1/2)),x)
[Out] -(2*(a*x^3 + b*x^4)^(1/2)*(3*a^2 + 8*b^2*x^2 - 4*a*b*x))/(15*a^3*x^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**3*(a + b*x))), x)
```

3.317 $\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=108

$$\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

```
[Out] -2/7*(b*x^4+a*x^3)^(1/2)/a/x^5+12/35*b*(b*x^4+a*x^3)^(1/2)/a^2/x^4-16/35*b^
2*(b*x^4+a*x^3)^(1/2)/a^3/x^3+32/35*b^3*(b*x^4+a*x^3)^(1/2)/a^4/x^2
```

Rubi [A] time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2016, 2000}

$$\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{2\sqrt{ax^3+bx^4}}{7ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(7*a*x^5) + (12*b*Sqrt[a*x^3 + b*x^4])/(35*a^2*x^4
) - (16*b^2*Sqrt[a*x^3 + b*x^4])/(35*a^3*x^3) + (32*b^3*Sqrt[a*x^3 + b*x^4]
)/(35*a^4*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}-\frac{(6b)\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx}{7a}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}+\frac{\left(24b^2\right)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{35a^2}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}-\frac{\left(16b^3\right)\int \frac{1}{\sqrt{ax^3+bx^4}}\,dx}{35a^3}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{7ax^5}+\frac{12b\sqrt{ax^3+bx^4}}{35a^2x^4}-\frac{16b^2\sqrt{ax^3+bx^4}}{35a^3x^3}+\frac{32b^3\sqrt{ax^3+bx^4}}{35a^4x^2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.49

$$\frac{2\sqrt{x^3(a+bx)}\left(-5a^3+6a^2bx-8ab^2x^2+16b^3x^3\right)}{35a^4x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (2*Sqrt[x^3*(a + b*x)]*(-5*a^3 + 6*a^2*b*x - 8*a*b^2*x^2 + 16*b^3*x^3))/(35
*a^4*x^5)
```

fricas [A] time = 0.39, size = 51, normalized size = 0.47

$$\frac{2\left(16\,b^3x^3 - 8\,ab^2x^2 + 6\,a^2bx - 5\,a^3\right)\sqrt{bx^4 + ax^3}}{35\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/35*(16*b^3*x^3 - 8*a*b^2*x^2 + 6*a^2*b*x - 5*a^3)*sqrt(b*x^4 + a*x^3)/(a^
4*x^5)
```

giac [A] time = 0.27, size = 57, normalized size = 0.53

$$-\frac{2\left(5\left(b + \frac{a}{x}\right)^{\frac{7}{2}} - 21\left(b + \frac{a}{x}\right)^{\frac{5}{2}}b + 35\left(b + \frac{a}{x}\right)^{\frac{3}{2}}b^2 - 35\sqrt{b + \frac{a}{x}}\,b^3\right)}{35\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/35*(5*(b + a/x)^(7/2) - 21*(b + a/x)^(5/2)*b + 35*(b + a/x)^(3/2)*b^2 -
35*sqrt(b + a/x)*b^3)/a^4
```

maple [A] time = 0.05, size = 57, normalized size = 0.53

$$-\frac{2\,(bx+a)\left(-16b^3x^3 + 8a\,b^2x^2 - 6a^2bx + 5a^3\right)}{35\sqrt{b\,x^4 + a\,x^3}\;a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] -2/35*(b*x+a)*(-16*b^3*x^3+8*a*b^2*x^2-6*a^2*b*x+5*a^3)/x^2/a^4/(b*x^4+a*x^
3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4 + ax^3}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^3), x)
```

mupad [B] time = 5.14, size = 92, normalized size = 0.85

$$\frac{12\,b\,\sqrt{b\,x^4 + a\,x^3}}{35\,a^2\,x^4} - \frac{2\,\sqrt{b\,x^4 + a\,x^3}}{7\,a\,x^5} - \frac{16\,b^2\,\sqrt{b\,x^4 + a\,x^3}}{35\,a^3\,x^3} + \frac{32\,b^3\,\sqrt{b\,x^4 + a\,x^3}}{35\,a^4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] (12*b*(a*x^3 + b*x^4)^(1/2))/(35*a^2*x^4) - (2*(a*x^3 + b*x^4)^(1/2))/(7*a*
x^5) - (16*b^2*(a*x^3 + b*x^4)^(1/2))/(35*a^3*x^3) + (32*b^3*(a*x^3 + b*x^4
)^(1/2))/(35*a^4*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3\sqrt{x^3(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**3*(a + b*x))), x)
```

3.318 $\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx$

Optimal. Leaf size=136

$$-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

```
[Out] -2/9*(b*x^4+a*x^3)^(1/2)/a/x^6+16/63*b*(b*x^4+a*x^3)^(1/2)/a^2/x^5-32/105*b
^2*(b*x^4+a*x^3)^(1/2)/a^3/x^4+128/315*b^3*(b*x^4+a*x^3)^(1/2)/a^4/x^3-256/
315*b^4*(b*x^4+a*x^3)^(1/2)/a^5/x^2
```

Rubi [A] time = 0.17, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{2016, 2000\}$

$$-\frac{256b^4\sqrt{ax^3+bx^4}}{315a^5x^2}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{2\sqrt{ax^3+bx^4}}{9ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^4])/(9*a*x^6) + (16*b*Sqrt[a*x^3 + b*x^4])/(63*a^2*x^5
) - (32*b^2*Sqrt[a*x^3 + b*x^4])/(105*a^3*x^4) + (128*b^3*Sqrt[a*x^3 + b*x^
4])/(315*a^4*x^3) - (256*b^4*Sqrt[a*x^3 + b*x^4])/(315*a^5*x^2)
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rule 2016

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p
 + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), In
t[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p
}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/
(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\sqrt{ax^3+bx^4}}\,dx &= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}-\frac{(8b)\int \frac{1}{x^3\sqrt{ax^3+bx^4}}\,dx}{9a}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}+\frac{\left(16b^2\right)\int \frac{1}{x^2\sqrt{ax^3+bx^4}}\,dx}{21a^2}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}-\frac{\left(64b^3\right)\int \frac{1}{x\sqrt{ax^3+bx^4}}\,dx}{105a^3}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}+\frac{\left(128b^4\right)\ldots}{3\ldots}\\
&= -\frac{2\sqrt{ax^3+bx^4}}{9ax^6}+\frac{16b\sqrt{ax^3+bx^4}}{63a^2x^5}-\frac{32b^2\sqrt{ax^3+bx^4}}{105a^3x^4}+\frac{128b^3\sqrt{ax^3+bx^4}}{315a^4x^3}-\frac{256b^4\sqrt{a\ldots}}{315\ldots}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 64, normalized size = 0.47

$$-\frac{2\sqrt{x^3(a+bx)}\left(35a^4-40a^3bx+48a^2b^2x^2-64ab^3x^3+128b^4x^4\right)}{315a^5x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[a*x^3 + b*x^4]),x]
```

```
[Out] (-2*Sqrt[x^3*(a + b*x)]*(35*a^4 - 40*a^3*b*x + 48*a^2*b^2*x^2 - 64*a*b^3*x^
3 + 128*b^4*x^4))/(315*a^5*x^6)
```

fricas [A] time = 0.39, size = 62, normalized size = 0.46

$$-\frac{2\left(128\,b^4x^4-64\,ab^3x^3+48\,a^2b^2x^2-40\,a^3bx+35\,a^4\right)\sqrt{bx^4+ax^3}}{315\,a^5x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(128*b^4*x^4 - 64*a*b^3*x^3 + 48*a^2*b^2*x^2 - 40*a^3*b*x + 35*a^4)*
sqrt(b*x^4 + a*x^3)/(a^5*x^6)
```

giac [A] time = 0.23, size = 71, normalized size = 0.52

$$-\frac{2\left(35\left(b+\frac{a}{x}\right)^{\frac{9}{2}}-180\left(b+\frac{a}{x}\right)^{\frac{7}{2}}b+378\left(b+\frac{a}{x}\right)^{\frac{5}{2}}b^2-420\left(b+\frac{a}{x}\right)^{\frac{3}{2}}b^3+315\sqrt{b+\frac{a}{x}}\,b^4\right)}{315\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="giac")
```

```
[Out] -2/315*(35*(b + a/x)^(9/2) - 180*(b + a/x)^(7/2)*b + 378*(b + a/x)^(5/2)*b^
2 - 420*(b + a/x)^(3/2)*b^3 + 315*sqrt(b + a/x)*b^4)/a^5
```

maple [A] time = 0.05, size = 68, normalized size = 0.50

$$-\frac{2\,(bx+a)\left(128b^4x^4-64a\,b^3x^3+48b^2x^2a^2-40bx\,a^3+35a^4\right)}{315\sqrt{b\,x^4+a\,x^3}\;a^5x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(b*x^4+a*x^3)^(1/2),x)
```

```
[Out] -2/315*(b*x+a)*(128*b^4*x^4-64*a*b^3*x^3+48*a^2*b^2*x^2-40*a^3*b*x+35*a^4)/
x^3/a^5/(b*x^4+a*x^3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4+ax^3}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(b*x^4+a*x^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a*x^3)*x^4), x)
```

mupad [B] time = 5.14, size = 116, normalized size = 0.85

$$\frac{16\,b\,\sqrt{b\,x^4+a\,x^3}}{63\,a^2\,x^5}-\frac{2\,\sqrt{b\,x^4+a\,x^3}}{9\,a\,x^6}-\frac{32\,b^2\,\sqrt{b\,x^4+a\,x^3}}{105\,a^3\,x^4}+\frac{128\,b^3\,\sqrt{b\,x^4+a\,x^3}}{315\,a^4\,x^3}-\frac{256\,b^4\,\sqrt{b\,x^4+a\,x^3}}{315\,a^5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a*x^3 + b*x^4)^(1/2)),x)
```

```
[Out] (16*b*(a*x^3 + b*x^4)^(1/2))/(63*a^2*x^5) - (2*(a*x^3 + b*x^4)^(1/2))/(9*a*
x^6) - (32*b^2*(a*x^3 + b*x^4)^(1/2))/(105*a^3*x^4) + (128*b^3*(a*x^3 + b*x
^4)^(1/2))/(315*a^4*x^3) - (256*b^4*(a*x^3 + b*x^4)^(1/2))/(315*a^5*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{x^3\,(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(b*x**4+a*x**3)**(1/2),x)
```

```
[Out] Integral(1/(x**4*sqrt(x**3*(a + b*x))), x)
```

3.319 $\int \frac{1}{x^3+bx^5}\,dx$

Optimal. Leaf size=26

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1593, 266, 44}

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3 + b*x^5)^(-1),x]
```

```
[Out] -1/(2*x^2) - b*Log[x] + (b*Log[1 + b*x^2])/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3+bx^5}\,dx &= \int \frac{1}{x^3\left(1+bx^2\right)}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(1+bx)}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{1+bx}\right)dx,x,x^2\right)\\ &= -\frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1+bx^2\right)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{1}{2}b\log\left(bx^2+1\right)-b\log(x)-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 + b*x^5)^(-1),x]
```

```
[Out] -1/2*1/x^2 - b*Log[x] + (b*Log[1 + b*x^2])/2
```

fricas [A] time = 0.39, size = 28, normalized size = 1.08

$$\frac{bx^2 \log\left(bx^2+1\right) - 2\,bx^2 \log(x) - 1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(b*x^2*log(b*x^2 + 1) - 2*b*x^2*log(x) - 1)/x^2
```

giac [A] time = 0.15, size = 32, normalized size = 1.23

$$-\frac{1}{2}\,b\log\left(x^2\right) + \frac{1}{2}\,b\log\left(\left|bx^2+1\right|\right) + \frac{bx^2-1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="giac")
```

```
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 + 1)) + 1/2*(b*x^2 - 1)/x^2
```

maple [A] time = 0.04, size = 23, normalized size = 0.88

$$-b\ln(x) + \frac{b\ln\left(b\,x^2+1\right)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+x^3),x)
```

```
[Out] -1/2/x^2-b*ln(x)+1/2*b*ln(b*x^2+1)
```

maxima [A] time = 1.34, size = 22, normalized size = 0.85

$$\frac{1}{2}\,b\log\left(bx^2+1\right) - b\log(x) - \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+x^3),x, algorithm="maxima")
```

```
[Out] 1/2*b*log(b*x^2 + 1) - b*log(x) - 1/2/x^2
```

mupad [B] time = 0.05, size = 22, normalized size = 0.85

$$\frac{b\ \ln\left(b\,x^2+1\right)}{2} - b\ \ln(x) - \frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5 + x^3),x)
```

```
[Out] (b*log(b*x^2 + 1))/2 - b*log(x) - 1/(2*x^2)
```

sympy [A] time = 0.22, size = 22, normalized size = 0.85

$$-b\log\left(x\right) + \frac{b\log\left(x^2+\frac{1}{b}\right)}{2} - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+x**3),x)
```

```
[Out] -b*log(x) + b*log(x**2 + 1/b)/2 - 1/(2*x**2)
```

3.320 $\int \frac{1}{-x^3+bx^5}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

```
[Out] 1/2/x^2-b*ln(x)+1/2*b*ln(-b*x^2+1)
```

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 44}

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + b*x^5)^(-1),x]
```

```
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-x^3+bx^5}\,dx &= \int \frac{1}{x^3\left(-1+bx^2\right)}\,dx\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^2(-1+bx)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{x^2}-\frac{b}{x}+\frac{b^2}{-1+bx}\right)dx,x,x^2\right)\\
&= \frac{1}{2x^2}-b\log(x)+\frac{1}{2}b\log\left(1-bx^2\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{1}{2}b\log\left(1-bx^2\right)-b\log(x)+\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + b*x^5)^(-1),x]
[Out] 1/(2*x^2) - b*Log[x] + (b*Log[1 - b*x^2])/2
```

fricas [A] time = 0.38, size = 28, normalized size = 1.04

$$\frac{bx^2\log\left(bx^2-1\right)-2\,bx^2\log(x)+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="fricas")
[Out] 1/2*(b*x^2*log(b*x^2 - 1) - 2*b*x^2*log(x) + 1)/x^2
```

giac [A] time = 0.18, size = 32, normalized size = 1.19

$$-\frac{1}{2}\,b\log\left(x^2\right)+\frac{1}{2}\,b\log\left(\left|bx^2-1\right|\right)+\frac{bx^2+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="giac")
[Out] -1/2*b*log(x^2) + 1/2*b*log(abs(b*x^2 - 1)) + 1/2*(b*x^2 + 1)/x^2
```

maple [A] time = 0.05, size = 23, normalized size = 0.85

$$-b\ln(x)+\frac{b\ln\left(b\,x^2-1\right)}{2}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5-x^3),x)
[Out] 1/2*b*ln(b*x^2-1)+1/2/x^2-b*ln(x)
```

maxima [A] time = 1.32, size = 22, normalized size = 0.81

$$\frac{1}{2}\,b\log\left(bx^2-1\right)-b\log(x)+\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5-x^3),x, algorithm="maxima")
[Out] 1/2*b*log(b*x^2 - 1) - b*log(x) + 1/2/x^2
```

mupad [B] time = 5.21, size = 22, normalized size = 0.81

$$\frac{b\,\ln\left(b\,x^2-1\right)}{2}-b\,\ln(x)+\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5 - x^3),x)
[Out] (b*log(b*x^2 - 1))/2 - b*log(x) + 1/(2*x^2)
```

sympy [A] time = 0.23, size = 22, normalized size = 0.81

$$-b\log(x)+\frac{b\log\left(x^2-\frac{1}{b}\right)}{2}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5-x**3),x)
[Out] -b*log(x) + b*log(x**2 - 1/b)/2 + 1/(2*x**2)
```

3.321 $\int \frac{1}{ax+bx} dx$

Optimal. Leaf size=8

$$\frac{\log(x)}{a+b}$$

[Out] ln(x)/(a+b)

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 29}

$$\frac{\log(x)}{a+b}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x)^(-1),x]

[Out] Log[x]/(a + b)

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx} dx &= \int \frac{1}{(a+b)x} dx \\ &= \frac{\int \frac{1}{x} dx}{a+b} \\ &= \frac{\log(x)}{a+b}\end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.75

$$\frac{\log(ax+bx)}{a+b}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x)^(-1),x]

[Out] Log[a*x + b*x]/(a + b)

fricas [A] time = 0.38, size = 8, normalized size = 1.00

$$\frac{\log(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="fricas")
```

```
[Out] log(x)/(a + b)
```

giac [A] time = 0.15, size = 15, normalized size = 1.88

$$\frac{\log\left(|ax + bx|\right)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="giac")
```

```
[Out] log(abs(a*x + b*x))/(a + b)
```

maple [A] time = 0.04, size = 9, normalized size = 1.12

$$\frac{\ln(x)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x),x)
```

```
[Out] ln(x)/(a+b)
```

maxima [A] time = 1.33, size = 14, normalized size = 1.75

$$\frac{\log\left(ax + bx\right)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x, algorithm="maxima")
```

```
[Out] log(a*x + b*x)/(a + b)
```

mupad [B] time = 5.27, size = 8, normalized size = 1.00

$$\frac{\ln(x)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x),x)
```

```
[Out] log(x)/(a + b)
```

sympy [A] time = 0.07, size = 5, normalized size = 0.62

$$\frac{\log\left(x\right)}{a + b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x),x)
```

```
[Out] log(x)/(a + b)
```

3.322 $\int \frac{1}{(ax+bx)^2} dx$

Optimal. Leaf size=10

$$-\frac{1}{x(a+b)^2}$$

[Out] -1/(a+b)^2/x

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 30}

$$-\frac{1}{x(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x)^(-2),x]

[Out] -(1/((a + b)^2*x))

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax+bx)^2} dx &= \int \frac{1}{(a+b)^2 x^2} dx\\ &= \frac{\int \frac{1}{x^2} dx}{(a+b)^2}\\ &= -\frac{1}{(a+b)^2 x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{x(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x)^(-2),x]

[Out] -(1/((a + b)^2*x))

fricas [A] time = 0.36, size = 18, normalized size = 1.80

$$-\frac{1}{(a^2+2ab+b^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="fricas")
```

```
[Out] -1/((a^2 + 2*a*b + b^2)*x)
```

giac [A] time = 0.15, size = 16, normalized size = 1.60

$$-\frac{1}{(ax+bx)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="giac")
```

```
[Out] -1/((a*x + b*x)*(a + b))
```

maple [A] time = 0.03, size = 11, normalized size = 1.10

$$-\frac{1}{(a+b)^2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x)^2,x)
```

```
[Out] -1/(a+b)^2/x
```

maxima [A] time = 1.34, size = 16, normalized size = 1.60

$$-\frac{1}{(ax+bx)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^2,x, algorithm="maxima")
```

```
[Out] -1/((a*x + b*x)*(a + b))
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$-\frac{1}{x\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x)^2,x)
```

```
[Out] -1/(x*(a + b)^2)
```

sympy [A] time = 0.08, size = 15, normalized size = 1.50

$$-\frac{1}{x\left(a^2+2ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)**2,x)
```

```
[Out] -1/(x*(a**2 + 2*a*b + b**2))
```

3.323 $\int \frac{1}{(ax+bx)^3}\,dx$

Optimal. Leaf size=12

$$-\frac{1}{2x^2(a+b)^3}$$

```
[Out] -1/2/(a+b)^3/x^2
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6, 12, 30}

$$-\frac{1}{2x^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x)^(-3),x]
```

```
[Out] -1/(2*(a + b)^3*x^2)
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax+bx)^3}\,dx &= \int \frac{1}{(a+b)^3x^3}\,dx\\ &= \frac{\int \frac{1}{x^3}\,dx}{(a+b)^3}\\ &= -\frac{1}{2(a+b)^3x^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$-\frac{1}{2x^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x)^(-3),x]
```

```
[Out] -1/2*1/((a + b)^3*x^2)
```

fricas [B] time = 0.37, size = 26, normalized size = 2.17

$$-\frac{1}{2\left(a^3+3\,a^2b+3\,ab^2+b^3\right)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="fricas")
[Out] -1/2/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*x^2)
```

giac [A] time = 0.15, size = 16, normalized size = 1.33

$$-\frac{1}{2\left(ax+bx\right)^{2}(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="giac")
[Out] -1/2/((a*x + b*x)^2*(a + b))
```

maple [A] time = 0.05, size = 11, normalized size = 0.92

$$-\frac{1}{2\left(a+b\right)^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x)^3,x)
[Out] -1/2/(a+b)^3/x^2
```

maxima [A] time = 1.31, size = 16, normalized size = 1.33

$$-\frac{1}{2\left(ax+bx\right)^{2}(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)^3,x, algorithm="maxima")
[Out] -1/2/((a*x + b*x)^2*(a + b))
```

mupad [B] time = 0.04, size = 26, normalized size = 2.17

$$-\frac{1}{2\,x^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x)^3,x)
[Out] -1/(2*x^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))
```

sympy [B] time = 0.09, size = 27, normalized size = 2.25

$$-\frac{1}{2x^{2}\left(a^{3}+3a^{2}b+3ab^{2}+b^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x)**3,x)
[Out] -1/(2*x**2*(a**3 + 3*a**2*b + 3*a*b**2 + b**3))
```

3.324 $\int \frac{1}{ax^2+bx^2}\,dx$

Optimal. Leaf size=10

$$-\frac{1}{x(a+b)}$$

```
[Out] -1/(a+b)/x
```

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$-\frac{1}{x(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^2)^(-1),x]
```

```
[Out] -(1/((a + b)*x))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^2+bx^2}\,dx &= \int \frac{1}{(a+b)x^2}\,dx\\ &= \frac{\int \frac{1}{x^2}\,dx}{a+b}\\ &= -\frac{1}{(a+b)x}\end{aligned}$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$-\frac{1}{x(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^2)^(-1),x]
```

```
[Out] -(1/((a + b)*x))
```

fricas [A] time = 0.37, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="fricas")
[Out] -1/((a + b)*x)
```

giac [A] time = 0.15, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="giac")
[Out] -1/((a + b)*x)
```

maple [A] time = 0.04, size = 11, normalized size = 1.10

$$-\frac{1}{(a+b)\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x^2),x)
[Out] -1/(a+b)/x
```

maxima [A] time = 1.36, size = 10, normalized size = 1.00

$$-\frac{1}{(a+b)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^2),x, algorithm="maxima")
[Out] -1/((a + b)*x)
```

mupad [B] time = 0.03, size = 10, normalized size = 1.00

$$-\frac{1}{x\ (a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b*x^2),x)
[Out] -1/(x*(a + b))
```

sympy [A] time = 0.08, size = 7, normalized size = 0.70

$$-\frac{1}{x\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x**2),x)
[Out] -1/(x*(a + b))
```

3.325 $\int \frac{1}{ax^n+bx^n}\,dx$

Optimal. Leaf size=20

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

```
[Out] x^(1-n)/(a+b)/(1-n)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-1),x]
```

```
[Out] x^(1 - n)/((a + b)*(1 - n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax^n+bx^n}\,dx &= \int \frac{x^{-n}}{a+b}\,dx\\ &= \frac{\int x^{-n}\,dx}{a+b}\\ &= \frac{x^{1-n}}{(a+b)(1-n)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^{1-n}}{(1-n)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-1),x]
```

```
[Out] x^(1 - n)/((a + b)*(1 - n))
```

fricas [A] time = 0.41, size = 22, normalized size = 1.10

$$-\frac{x}{((a+b)n-a-b)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="fricas")
```

```
[Out] -x/(((a + b)*n - a - b)*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax^n+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x^n + b*x^n), x)
```

maple [A] time = 0.04, size = 19, normalized size = 0.95

$$-\frac{x\,x^{-n}}{(n-1)\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n),x)
```

```
[Out] -x/(n-1)/(x^n)/(a+b)
```

maxima [A] time = 1.35, size = 21, normalized size = 1.05

$$-\frac{x}{(a(n-1)+b(n-1))x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n),x, algorithm="maxima")
```

```
[Out] -x/((a*(n - 1) + b*(n - 1))*x^n)
```

mupad [B] time = 5.18, size = 19, normalized size = 0.95

$$-\frac{x^{1-n}}{(a+b)\,(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n + b*x^n),x)
```

```
[Out] -x^(1 - n)/((a + b)*(n - 1))
```

sympy [A] time = 0.64, size = 32, normalized size = 1.60

$$\begin{cases} -\frac{x}{anx^n-ax^n+bnx^n-bx^n} & \text{for } n\neq 1 \\ \frac{\log(x)}{a+b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n),x)
```

```
[Out] Piecewise((-x/(a*n*x**n - a*x**n + b*n*x**n - b*x**n), Ne(n, 1)), (log(x)/(
a + b), True))
```

3.326 $\int \frac{1}{(ax^n+bx^n)^2} dx$

Optimal. Leaf size=20

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

```
[Out] x^(1-2*n)/(a+b)^2/(1-2*n)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {6, 12, 30}

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-2),x]
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax^n+bx^n)^2}\,dx &= \int \frac{x^{-2n}}{(a+b)^2}\,dx\\ &= \frac{\int x^{-2n}\,dx}{(a+b)^2}\\ &= \frac{x^{1-2n}}{(a+b)^2(1-2n)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^{1-2n}}{(1-2n)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-2),x]
[Out] x^(1 - 2*n)/((a + b)^2*(1 - 2*n))
```

fricas [A] time = 0.41, size = 36, normalized size = 1.80

$$\frac{x}{\left(a^2+2\,ab+b^2-2\left(a^2+2\,ab+b^2\right)n\right)x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="fricas")
[Out] x/((a^2 + 2*a*b + b^2 - 2*(a^2 + 2*a*b + b^2)*n)*x^(2*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ax^n+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="giac")
[Out] integrate((a*x^n + b*x^n)^(-2), x)
```

maple [A] time = 0.04, size = 21, normalized size = 1.05

$$-\frac{x\,x^{-2n}}{(2n-1)\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n)^2,x)
[Out] -x/(2*n-1)/(x^n)^2/(a+b)^2
```

maxima [A] time = 1.36, size = 40, normalized size = 2.00

$$-\frac{x}{\left(a^2(2\,n-1)+2\,ab(2\,n-1)+b^2(2\,n-1)\right)x^{2\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^2,x, algorithm="maxima")
[Out] -x/((a^2*(2*n - 1) + 2*a*b*(2*n - 1) + b^2*(2*n - 1))*x^(2*n))
```

mupad [B] time = 5.14, size = 21, normalized size = 1.05

$$-\frac{x^{1-2\,n}}{(a+b)^2\,(2\,n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n + b*x^n)^2,x)
[Out] -x^(1 - 2*n)/((a + b)^2*(2*n - 1))
```

sympy [A] time = 1.03, size = 82, normalized size = 4.10

$$\begin{cases} -\frac{x}{2a^2nx^{2n}-a^2x^{2n}+4abnx^{2n}-2abx^{2n}+2b^2nx^{2n}-b^2x^{2n}} & \text{for } n \neq \frac{1}{2} \\ \frac{\log{(x)}}{a^2+2ab+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n)**2,x)
```

```
[Out] Piecewise((-x/(2*a**2*n*x**(2*n) - a**2*x**(2*n) + 4*a*b*n*x**(2*n) - 2*a*b
*x**(2*n) + 2*b**2*n*x**(2*n) - b**2*x**(2*n)), Ne(n, 1/2)), (log(x)/(a**2
+ 2*a*b + b**2), True))
```

3.327 $\int \frac{1}{(ax^n+bx^n)^3}\,dx$

Optimal. Leaf size=20

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

```
[Out] x^(1-3*n)/(a+b)^3/(1-3*n)
```

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = $\{6, 12, 30\}$

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^n + b*x^n)^(-3),x]
```

```
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

Rule 6

```
Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a +
 b)*v + w)^p, x] /; FreeQ[{a, b}, x] &&  !FreeQ[v, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(ax^n+bx^n)^3}\,dx &= \int \frac{x^{-3n}}{(a+b)^3}\,dx\\ &= \frac{\int x^{-3n}\,dx}{(a+b)^3}\\ &= \frac{x^{1-3n}}{(a+b)^3(1-3n)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 20, normalized size = 1.00

$$\frac{x^{1-3n}}{(1-3n)(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^n + b*x^n)^(-3),x]
```

```
[Out] x^(1 - 3*n)/((a + b)^3*(1 - 3*n))
```

fricas [B] time = 0.39, size = 52, normalized size = 2.60

$$\frac{x}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}-3\,\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)n\right)x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] x/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*n)*x^
(3*n))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^{n}+bx^{n}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^n + b*x^n)^(-3), x)
```

maple [A] time = 0.04, size = 21, normalized size = 1.05

$$-\frac{x\,x^{-3n}}{\left(3n-1\right)\left(a+b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n+b*x^n)^3,x)
```

```
[Out] -x/(3*n-1)/(x^n)^3/(a+b)^3
```

maxima [B] time = 1.40, size = 53, normalized size = 2.65

$$-\frac{x}{\left(a^{3}(3\,n-1)+3\,a^{2}b(3\,n-1)+3\,ab^{2}(3\,n-1)+b^{3}(3\,n-1)\right)x^{3\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^n+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] -x/((a^3*(3*n - 1) + 3*a^2*b*(3*n - 1) + 3*a*b^2*(3*n - 1) + b^3*(3*n - 1))
*x^(3*n))
```

mupad [B] time = 5.12, size = 21, normalized size = 1.05

$$-\frac{x^{1-3\,n}}{\left(a+b\right)^{3}\,\left(3\,n-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^n + b*x^n)^3,x)
```

```
[Out] -x^(1 - 3*n)/((a + b)^3*(3*n - 1))
```

sympy [A] time = 1.35, size = 119, normalized size = 5.95

$$\begin{cases} -\frac{x}{3a^{3}nx^{3n}-a^{3}x^{3n}+9a^{2}bnx^{3n}-3a^{2}bx^{3n}+9ab^{2}nx^{3n}-3ab^{2}x^{3n}+3b^{3}nx^{3n}-b^{3}x^{3n}} & \text{for } n\neq\frac{1}{3} \\ \frac{\log(x)}{a^{3}+3a^{2}b+3ab^{2}+b^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**n+b*x**n)**3,x)
```

```
[Out] Piecewise((-x/(3*a**3*n*x**(3*n) - a**3*x**(3*n) + 9*a**2*b*n*x**(3*n) - 3*
a**2*b*x**(3*n) + 9*a*b**2*n*x**(3*n) - 3*a*b**2*x**(3*n) + 3*b**3*n*x**(3*
n) - b**3*x**(3*n)), Ne(n, 1/3)), (log(x)/(a**3 + 3*a**2*b + 3*a*b**2 + b**
3), True))
```

3.328 $\int \left(ax+bx^{14}\right)^{12} dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

```
[Out] 1/169*(b*x^13+a)^13/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 261}

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^14)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12} dx &= \int x^{12}\left(a+bx^{13}\right)^{12} dx\\ &= \frac{\left(a+bx^{13}\right)^{13}}{169b}\end{aligned}$$

Mathematica [B] time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+\frac{22}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^14)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{169}x^{169}b^{12}+\frac{1}{13}x^{156}b^{11}a+\frac{6}{13}x^{143}b^{10}a^2+\frac{22}{13}x^{130}b^9a^3+\frac{55}{13}x^{117}b^8a^4+\frac{99}{13}x^{104}b^7a^5+\frac{132}{13}x^{91}b^6a^6+\frac{132}{13}x^{78}b^5a^7+\frac{99}{13}x^{65}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^
9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 1
32/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b
^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12
```

giac [B] time = 0.14, size = 134, normalized size = 8.38

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}+\frac{9}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

maple [B] time = 0.05, size = 135, normalized size = 8.44

$$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{2}b^{10}x^{143}+\frac{22}{13}a^{3}b^{9}x^{130}+\frac{55}{13}a^{4}b^{8}x^{117}+\frac{99}{13}a^{5}b^{7}x^{104}+\frac{132}{13}a^{6}b^{6}x^{91}+\frac{132}{13}a^{7}b^{5}x^{78}+\frac{99}{13}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^14+a*x)^12,x)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*
x^26+1/13*a^12*x^13
```

maxima [B] time = 1.31, size = 134, normalized size = 8.38

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^{2}b^{10}x^{143}+\frac{22}{13}\,a^{3}b^{9}x^{130}+\frac{55}{13}\,a^{4}b^{8}x^{117}+\frac{99}{13}\,a^{5}b^{7}x^{104}+\frac{132}{13}\,a^{6}b^{6}x^{91}+\frac{132}{13}\,a^{7}b^{5}x^{78}+\frac{9}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

mupad [B] time = 5.16, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{13}}{13}+\frac{6\,a^{11}\,b\,x^{26}}{13}+\frac{22\,a^{10}\,b^2\,x^{39}}{13}+\frac{55\,a^9\,b^3\,x^{52}}{13}+\frac{99\,a^8\,b^4\,x^{65}}{13}+\frac{132\,a^7\,b^5\,x^{78}}{13}+\frac{132\,a^6\,b^6\,x^{91}}{13}+\frac{99\,a^5\,b^7\,x^{104}}{13}+\frac{55\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^14)^12,x)
```

```
[Out] (a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
+ (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (13
2*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^
4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

sympy [B] time = 0.12, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^2x^{39}}{13}+\frac{55a^9b^3x^{52}}{13}+\frac{99a^8b^4x^{65}}{13}+\frac{132a^7b^5x^{78}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{55a^4b^8x^{117}}{13}+22a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**14+a*x)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

3.329 $\int x^{12} \left(ax+bx^{26}\right)^{12} \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 261}

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int x^{12} \left(ax+bx^{26}\right)^{12} \, dx &= \int x^{24} \left(a+bx^{25}\right)^{12} \, dx \\ &= \frac{\left(a+bx^{25}\right)^{13}}{325b} \end{aligned}$$

Mathematica [B] time = 0.02, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{325}x^{325}b^{12}+\frac{1}{25}x^{300}b^{11}a+\frac{6}{25}x^{275}b^{10}a^2+\frac{22}{25}x^{250}b^9a^3+\frac{11}{5}x^{225}b^8a^4+\frac{99}{25}x^{200}b^7a^5+\frac{132}{25}x^{175}b^6a^6+\frac{132}{25}x^{150}b^5a^7+\frac{99}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

giac [B] time = 0.15, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}ab^{11}x^{300}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}+\frac{9}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}a\,b^{11}x^{300}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}+\frac{99}{25}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^26+a*x)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*a*b^11*x^300+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

maxima [B] time = 1.37, size = 134, normalized size = 8.38

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}ab^{11}x^{300}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}+\frac{9}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

mupad [B] time = 5.16, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+\frac{11\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a*x + b*x^26)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

<u>**sympy**</u> **[B]** time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{22}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**26+a*x)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

3.330 $\int x^{24} \left(ax+bx^{38}\right)^{12}\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx &= \int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{33}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

fricas [B] time = 0.33, size = 134, normalized size = 8.38

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^2+\frac{22}{37}x^{370}b^9a^3+\frac{55}{37}x^{333}b^8a^4+\frac{99}{37}x^{296}b^7a^5+\frac{132}{37}x^{259}b^6a^6+\frac{132}{37}x^{222}b^5a^7+\frac{99}{37}x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [B] time = 0.15, size = 134, normalized size = 8.38

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^38+a*x)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [B] time = 1.31, size = 134, normalized size = 8.38

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

mupad [B] time = 5.15, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(a*x + b*x^38)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

sympy [B] time = 0.14, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^2x^{111}}{37}+\frac{55a^9b^3x^{148}}{37}+\frac{99a^8b^4x^{185}}{37}+\frac{132a^7b^5x^{222}}{37}+\frac{132a^6b^6x^{259}}{37}+\frac{99a^5b^7x^{296}}{37}+\frac{55a^4b^8x^{333}}{37}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**38+a*x)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.331 $\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

```
[Out] 1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1584, 261}

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{12(-1+m)}\left(ax+bx^{2+12m}\right)^{12}\,dx &= \int x^{12+12(-1+m)}\left(a+bx^{1+12m}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 24, normalized size = 0.89

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{156bm+13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(12*(-1 + m))*(a*x + b*x^(2 + 12*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

fricas [B] time = 0.41, size = 231, normalized size = 8.56

$$13\,a^{12}x^{12}x^{12\,m+2}+78\,a^{11}bx^{11}x^{24\,m+4}+286\,a^{10}b^{2}x^{10}x^{36\,m+6}+715\,a^{9}b^{3}x^{9}x^{48\,m+8}+1287\,a^{8}b^{4}x^{8}x^{60\,m+10}+1716$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(13*a^12*x^12*x^(12*m + 2) + 78*a^11*b*x^11*x^(24*m + 4) + 286*a^10*b^
2*x^10*x^(36*m + 6) + 715*a^9*b^3*x^9*x^(48*m + 8) + 1287*a^8*b^4*x^8*x^(60
*m + 10) + 1716*a^7*b^5*x^7*x^(72*m + 12) + 1716*a^6*b^6*x^6*x^(84*m + 14)
+ 1287*a^5*b^7*x^5*x^(96*m + 16) + 715*a^4*b^8*x^4*x^(108*m + 18) + 286*a^3
*b^9*x^3*x^(120*m + 20) + 78*a^2*b^10*x^2*x^(132*m + 22) + 13*a*b^11*x*x^(1
44*m + 24) + b^12*x^(156*m + 26))/((12*m + 1)*x^13)
```

giac [B] time = 0.51, size = 285, normalized size = 10.56

$$\frac{13\,a^{12}x^{12}e^{\left(12\,m\log(x)+2\,\log(x)\right)} + 78\,a^{11}bx^{11}e^{\left(24\,m\log(x)+4\,\log(x)\right)} + 286\,a^{10}b^{2}x^{10}e^{\left(36\,m\log(x)+6\,\log(x)\right)} + 715\,a^{9}b^{3}x^{9}e^{\left(48\,m\,\mathrm{l}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="giac")
```

```
[Out] 1/13*(13*a^12*x^12*e^(12*m*log(x) + 2*log(x)) + 78*a^11*b*x^11*e^(24*m*log(
x) + 4*log(x)) + 286*a^10*b^2*x^10*e^(36*m*log(x) + 6*log(x)) + 715*a^9*b^3
*x^9*e^(48*m*log(x) + 8*log(x)) + 1287*a^8*b^4*x^8*e^(60*m*log(x) + 10*log(
x)) + 1716*a^7*b^5*x^7*e^(72*m*log(x) + 12*log(x)) + 1716*a^6*b^6*x^6*e^(84
*m*log(x) + 14*log(x)) + 1287*a^5*b^7*x^5*e^(96*m*log(x) + 16*log(x)) + 715
*a^4*b^8*x^4*e^(108*m*log(x) + 18*log(x)) + 286*a^3*b^9*x^3*e^(120*m*log(x)
 + 20*log(x)) + 78*a^2*b^10*x^2*e^(132*m*log(x) + 22*log(x)) + 13*a*b^11*x*
e^(144*m*log(x) + 24*log(x)) + b^12*e^(156*m*log(x) + 26*log(x)))/(12*m*x^1
3 + x^13)
```

maple [B] time = 0.11, size = 339, normalized size = 12.56

$$\frac{a^{12}x^{12m+2}}{(12m+1)\,x}+\frac{6a^{11}b\,x^{24m+4}}{(12m+1)\,x^{2}}+\frac{22a^{10}b^{2}x^{36m+6}}{(12m+1)\,x^{3}}+\frac{55a^{9}b^{3}x^{48m+8}}{(12m+1)\,x^{4}}+\frac{99a^{8}b^{4}x^{60m+10}}{(12m+1)\,x^{5}}+\frac{132a^{7}b^{5}x^{72m+12}}{(12m+1)\,x^{6}}+\frac{132a^{6}b^{6}x^{84m+14}}{(12m+1)\,x^{7}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(12*m-12)*(a*x+b*x^(2+12*m))^12,x)
```

```
[Out] 1/13/(12*m+1)*b^12/x^13*(x^(2+12*m))^13+1/(12*m+1)*a*b^11/x^12*(x^(2+12*m))
^12+6/(12*m+1)*a^2*b^10/x^11*(x^(2+12*m))^11+22/(12*m+1)*a^3*b^9/x^10*(x^(2
+12*m))^10+55/(12*m+1)*a^4*b^8/x^9*(x^(2+12*m))^9+99/(12*m+1)*a^5*b^7/x^8*(
x^(2+12*m))^8+132/(12*m+1)*a^6*b^6/x^7*(x^(2+12*m))^7+132/(12*m+1)*a^7*b^5/
x^6*(x^(2+12*m))^6+99/(12*m+1)*a^8*b^4/x^5*(x^(2+12*m))^5+55/(12*m+1)*a^9*b
^3/x^4*(x^(2+12*m))^4+22/(12*m+1)*a^10*b^2/x^3*(x^(2+12*m))^3+6/(12*m+1)*a^
11*b/x^2*(x^(2+12*m))^2+1/(12*m+1)*a^12/x*x^(2+12*m)
```

maxima [B] time = 1.43, size = 275, normalized size = 10.19

$$\frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)}+\frac{ab^{11}x^{144\,m+12}}{12\,m+1}+\frac{6\,a^{2}b^{10}x^{132\,m+11}}{12\,m+1}+\frac{22\,a^{3}b^{9}x^{120\,m+10}}{12\,m+1}+\frac{55\,a^{4}b^{8}x^{108\,m+9}}{12\,m+1}+\frac{99\,a^{5}b^{7}x^{96\,m+8}}{12\,m+1}+\frac{132\,a^{6}b^{6}x^{84\,m}}{12\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-12+12*m)*(a*x+b*x^(2+12*m))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6*
a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1) +
 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1) +
 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m + 1)
+ 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m + 1) +
 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m + 1) + a
^12*x^(12*m + 1)/(12*m + 1)
```

mupad [B] time = 5.87, size = 287, normalized size = 10.63

$$\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{13\,a^{12}\,x\,x^{12\,m}}{156\,m+13}+\frac{78\,a^{11}\,b\,x^{24\,m}\,x^2}{156\,m+13}+\frac{13\,a\,b^{11}\,x^{144\,m}\,x^{12}}{156\,m+13}+\frac{286\,a^{10}\,b^2\,x^{36\,m}\,x^3}{156\,m+13}+\frac{715\,a^9\,b^3\,x^{48\,m}\,x^4}{156\,m+13}+\frac{128}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(12*m - 12)*(a*x + b*x^(12*m + 2))^12,x)
```

```
[Out] (b^12*x^(156*m)*x^13)/(156*m + 13) + (13*a^12*x*x^(12*m))/(156*m + 13) + (7
8*a^11*b*x^(24*m)*x^2)/(156*m + 13) + (13*a*b^11*x^(144*m)*x^12)/(156*m + 1
3) + (286*a^10*b^2*x^(36*m)*x^3)/(156*m + 13) + (715*a^9*b^3*x^(48*m)*x^4)/
(156*m + 13) + (1287*a^8*b^4*x^(60*m)*x^5)/(156*m + 13) + (1716*a^7*b^5*x^(
72*m)*x^6)/(156*m + 13) + (1716*a^6*b^6*x^(84*m)*x^7)/(156*m + 13) + (1287*
a^5*b^7*x^(96*m)*x^8)/(156*m + 13) + (715*a^4*b^8*x^(108*m)*x^9)/(156*m + 1
3) + (286*a^3*b^9*x^(120*m)*x^10)/(156*m + 13) + (78*a^2*b^10*x^(132*m)*x^1
1)/(156*m + 13)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-12+12*m)*(a*x+b*x**(2+12*m))**12,x)
```

```
[Out] Timed out
```

3.332 $\int \left(ax+bx^{14}\right)^{12}\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

```
[Out] 1/169*(b*x^13+a)^13/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 261}

$$\frac{\left(a+bx^{13}\right)^{13}}{169b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^14)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax+bx^{14}\right)^{12}\,dx &= \int x^{12}\left(a+bx^{13}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{13}\right)^{13}}{169b}\end{aligned}$$

Mathematica [B] time = 0.00, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6}{13}a^{11}bx^{26}+\frac{22}{13}a^{10}b^2x^{39}+\frac{55}{13}a^9b^3x^{52}+\frac{99}{13}a^8b^4x^{65}+\frac{132}{13}a^7b^5x^{78}+\frac{132}{13}a^6b^6x^{91}+\frac{99}{13}a^5b^7x^{104}+\frac{55}{13}a^4b^8x^{117}+\frac{22}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^14)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

fricas [B] time = 0.35, size = 134, normalized size = 8.38

$$\frac{1}{169}x^{169}b^{12}+\frac{1}{13}x^{156}b^{11}a+\frac{6}{13}x^{143}b^{10}a^2+\frac{22}{13}x^{130}b^9a^3+\frac{55}{13}x^{117}b^8a^4+\frac{99}{13}x^{104}b^7a^5+\frac{132}{13}x^{91}b^6a^6+\frac{132}{13}x^{78}b^5a^7+\frac{99}{13}x^{65}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^
9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 1
32/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b
^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12
```

giac [B] time = 0.17, size = 134, normalized size = 8.38

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^2b^{10}x^{143}+\frac{22}{13}\,a^3b^9x^{130}+\frac{55}{13}\,a^4b^8x^{117}+\frac{99}{13}\,a^5b^7x^{104}+\frac{132}{13}\,a^6b^6x^{91}+\frac{132}{13}\,a^7b^5x^{78}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

maple [B] time = 0.05, size = 135, normalized size = 8.44

$$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^2b^{10}x^{143}+\frac{22}{13}a^3b^9x^{130}+\frac{55}{13}a^4b^8x^{117}+\frac{99}{13}a^5b^7x^{104}+\frac{132}{13}a^6b^6x^{91}+\frac{132}{13}a^7b^5x^{78}+\frac{99}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^14+a*x)^12,x)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*
x^26+1/13*a^12*x^13
```

maxima [B] time = 1.33, size = 134, normalized size = 8.38

$$\frac{1}{169}\,b^{12}x^{169}+\frac{1}{13}\,ab^{11}x^{156}+\frac{6}{13}\,a^2b^{10}x^{143}+\frac{22}{13}\,a^3b^9x^{130}+\frac{55}{13}\,a^4b^8x^{117}+\frac{99}{13}\,a^5b^7x^{104}+\frac{132}{13}\,a^6b^6x^{91}+\frac{132}{13}\,a^7b^5x^{78}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^14+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/169*b^12*x^169 + 1/13*a*b^11*x^156 + 6/13*a^2*b^10*x^143 + 22/13*a^3*b^9*
x^130 + 55/13*a^4*b^8*x^117 + 99/13*a^5*b^7*x^104 + 132/13*a^6*b^6*x^91 + 1
32/13*a^7*b^5*x^78 + 99/13*a^8*b^4*x^65 + 55/13*a^9*b^3*x^52 + 22/13*a^10*b
^2*x^39 + 6/13*a^11*b*x^26 + 1/13*a^12*x^13
```

mupad [B] time = 0.00, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{13}}{13}+\frac{6\,a^{11}\,b\,x^{26}}{13}+\frac{22\,a^{10}\,b^2\,x^{39}}{13}+\frac{55\,a^9\,b^3\,x^{52}}{13}+\frac{99\,a^8\,b^4\,x^{65}}{13}+\frac{132\,a^7\,b^5\,x^{78}}{13}+\frac{132\,a^6\,b^6\,x^{91}}{13}+\frac{99\,a^5\,b^7\,x^{104}}{13}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + b*x^14)^12,x)
```

```
[Out] (a^12*x^13)/13 + (b^12*x^169)/169 + (6*a^11*b*x^26)/13 + (a*b^11*x^156)/13
+ (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x^52)/13 + (99*a^8*b^4*x^65)/13 + (13
2*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)/13 + (99*a^5*b^7*x^104)/13 + (55*a^
4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13 + (6*a^2*b^10*x^143)/13
```

sympy [B] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^2x^{39}}{13}+\frac{55a^9b^3x^{52}}{13}+\frac{99a^8b^4x^{65}}{13}+\frac{132a^7b^5x^{78}}{13}+\frac{132a^6b^6x^{91}}{13}+\frac{99a^5b^7x^{104}}{13}+\frac{55a^4b^8x^{117}}{13}+\frac{22a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**14+a*x)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

3.333 $\int \left(ax^2 + bx^{27}\right)^{12} dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^{25}\right)^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1593, 261}

$$\frac{\left(a + bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^27)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^2 + bx^{27}\right)^{12} dx &= \int x^{24}\left(a + bx^{25}\right)^{12} dx\\ &= \frac{\left(a + bx^{25}\right)^{13}}{325b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^27)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{325}x^{325}b^{12}+\frac{1}{25}x^{300}b^{11}a+\frac{6}{25}x^{275}b^{10}a^2+\frac{22}{25}x^{250}b^9a^3+\frac{11}{5}x^{225}b^8a^4+\frac{99}{25}x^{200}b^7a^5+\frac{132}{25}x^{175}b^6a^6+\frac{132}{25}x^{150}b^5a^7+\frac{99}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

giac [B] time = 0.15, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}a\,b^{11}x^{300}+\frac{6}{25}a^2b^{10}x^{275}+\frac{22}{25}a^3b^9x^{250}+\frac{11}{5}a^4b^8x^{225}+\frac{99}{25}a^5b^7x^{200}+\frac{132}{25}a^6b^6x^{175}+\frac{132}{25}a^7b^5x^{150}+\frac{99}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^27+a*x^2)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*a*b^11*x^300+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

maxima [B] time = 1.34, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^2b^{10}x^{275}+\frac{22}{25}\,a^3b^9x^{250}+\frac{11}{5}\,a^4b^8x^{225}+\frac{99}{25}\,a^5b^7x^{200}+\frac{132}{25}\,a^6b^6x^{175}+\frac{132}{25}\,a^7b^5x^{150}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^27+a*x^2)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

mupad [B] time = 5.19, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^2\,x^{75}}{25}+\frac{11\,a^9\,b^3\,x^{100}}{5}+\frac{99\,a^8\,b^4\,x^{125}}{25}+\frac{132\,a^7\,b^5\,x^{150}}{25}+\frac{132\,a^6\,b^6\,x^{175}}{25}+\frac{99\,a^5\,b^7\,x^{200}}{25}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2 + b*x^27)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

sympy [B] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^2x^{75}}{25}+\frac{11a^9b^3x^{100}}{5}+\frac{99a^8b^4x^{125}}{25}+\frac{132a^7b^5x^{150}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{99a^5b^7x^{200}}{25}+\frac{11a^4b^8x^{22}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**27+a*x**2)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

3.334 $\int \left(ax^3+bx^{40}\right)^{12}\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

[Out] 1/481*(b*x^37+a)^13/b

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = $\{1593, 261\}$

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

[In] Int[(a*x^3 + b*x^40)^12,x]

[Out] (a + b*x^37)^13/(481*b)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^3+bx^{40}\right)^{12}\,dx &= \int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{33}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^3 + b*x^40)^12,x]

[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^2+\frac{22}{37}x^{370}b^9a^3+\frac{55}{37}x^{333}b^8a^4+\frac{99}{37}x^{296}b^7a^5+\frac{132}{37}x^{259}b^6a^6+\frac{132}{37}x^{222}b^5a^7+\frac{99}{37}x^1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [B] time = 0.16, size = 134, normalized size = 8.38

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^40+a*x^3)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [B] time = 1.35, size = 134, normalized size = 8.38

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^40+a*x^3)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

mupad [B] time = 5.16, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^{2}\,x^{111}}{37}+\frac{55\,a^{9}\,b^{3}\,x^{148}}{37}+\frac{99\,a^{8}\,b^{4}\,x^{185}}{37}+\frac{132\,a^{7}\,b^{5}\,x^{222}}{37}+\frac{132\,a^{6}\,b^{6}\,x^{259}}{37}+\frac{99\,a^{5}\,b^{7}\,x^{296}}{37}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3 + b*x^40)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

sympy [B] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**40+a*x**3)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.335 $\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

```
[Out] 1/13*(a+b*x^(1+12*m))^13/b/(1+12*m)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 261}

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{13b(12m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b*(1 + 12*m))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^m+bx^{1+13m}\right)^{12}\,dx &= \int x^{12m}\left(a+bx^{1+12m}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{1+12m}\right)^{13}}{13b(1+12m)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 0.89

$$\frac{\left(a+bx^{12m+1}\right)^{13}}{156bm+13b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + 13*m))^12,x]
```

```
[Out] (a + b*x^(1 + 12*m))^13/(13*b + 156*b*m)
```

fricas [B] time = 0.42, size = 205, normalized size = 7.59

$$\frac{b^{12}x^{13}x^{156\,m}+13\,ab^{11}x^{12}x^{144\,m}+78\,a^2b^{10}x^{11}x^{132\,m}+286\,a^3b^9x^{10}x^{120\,m}+715\,a^4b^8x^9x^{108\,m}+1287\,a^5b^7x^8x^{96\,m}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

giac [B] time = 0.27, size = 205, normalized size = 7.59

$$\frac{b^{12}x^{13}x^{156\,m} + 13\,ab^{11}x^{12}x^{144\,m} + 78\,a^{2}b^{10}x^{11}x^{132\,m} + 286\,a^{3}b^{9}x^{10}x^{120\,m} + 715\,a^{4}b^{8}x^{9}x^{108\,m} + 1287\,a^{5}b^{7}x^{8}x^{96\,m} + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="giac")
```

```
[Out] 1/13*(b^12*x^13*x^(156*m) + 13*a*b^11*x^12*x^(144*m) + 78*a^2*b^10*x^11*x^(
132*m) + 286*a^3*b^9*x^10*x^(120*m) + 715*a^4*b^8*x^9*x^(108*m) + 1287*a^5*
b^7*x^8*x^(96*m) + 1716*a^6*b^6*x^7*x^(84*m) + 1716*a^7*b^5*x^6*x^(72*m) +
1287*a^8*b^4*x^5*x^(60*m) + 715*a^9*b^3*x^4*x^(48*m) + 286*a^10*b^2*x^3*x^(
36*m) + 78*a^11*b*x^2*x^(24*m) + 13*a^12*x*x^(12*m))/(12*m + 1)
```

maple [B] time = 0.08, size = 287, normalized size = 10.63

$$\frac{b^{12}x^{13}x^{156m}}{13+156m}+\frac{a\,b^{11}x^{12}x^{144m}}{12m+1}+\frac{6a^{2}b^{10}x^{11}x^{132m}}{12m+1}+\frac{22a^{3}b^{9}x^{10}x^{120m}}{12m+1}+\frac{55a^{4}b^{8}x^{9}x^{108m}}{12m+1}+\frac{99a^{5}b^{7}x^{8}x^{96m}}{12m+1}+\frac{132a^{6}b^{6}x^{7}x^{84m}}{12m+1}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(1+13*m))^12,x)
```

```
[Out] 1/13*b^12*x^13/(12*m+1)*(x^m)^156+a*b^11*x^12/(12*m+1)*(x^m)^144+6*a^2*b^10
*x^11/(12*m+1)*(x^m)^132+22*a^3*b^9*x^10/(12*m+1)*(x^m)^120+55*a^4*b^8*x^9/
(12*m+1)*(x^m)^108+99*a^5*b^7*x^8/(12*m+1)*(x^m)^96+132*a^6*b^6*x^7/(12*m+1
)*(x^m)^84+132*a^7*b^5*x^6/(12*m+1)*(x^m)^72+99*a^8*b^4*x^5/(12*m+1)*(x^m)^
60+55*a^9*b^3*x^4/(12*m+1)*(x^m)^48+22*a^10*b^2*x^3/(12*m+1)*(x^m)^36+6*a^1
1*b*x^2/(12*m+1)*(x^m)^24+a^12/(12*m+1)*x*(x^m)^12
```

maxima [B] time = 1.48, size = 275, normalized size = 10.19

$$\frac{b^{12}x^{156\,m+13}}{13\,(12\,m+1)}+\frac{ab^{11}x^{144\,m+12}}{12\,m+1}+\frac{6\,a^{2}b^{10}x^{132\,m+11}}{12\,m+1}+\frac{22\,a^{3}b^{9}x^{120\,m+10}}{12\,m+1}+\frac{55\,a^{4}b^{8}x^{108\,m+9}}{12\,m+1}+\frac{99\,a^{5}b^{7}x^{96\,m+8}}{12\,m+1}+\frac{132\,a^{6}b^{6}x^{84\,m\cdots}}{12\,m+1}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+13*m))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*m + 13)/(12*m + 1) + a*b^11*x^(144*m + 12)/(12*m + 1) + 6*
a^2*b^10*x^(132*m + 11)/(12*m + 1) + 22*a^3*b^9*x^(120*m + 10)/(12*m + 1) +
 55*a^4*b^8*x^(108*m + 9)/(12*m + 1) + 99*a^5*b^7*x^(96*m + 8)/(12*m + 1) +
 132*a^6*b^6*x^(84*m + 7)/(12*m + 1) + 132*a^7*b^5*x^(72*m + 6)/(12*m + 1)
+ 99*a^8*b^4*x^(60*m + 5)/(12*m + 1) + 55*a^9*b^3*x^(48*m + 4)/(12*m + 1) +
 22*a^10*b^2*x^(36*m + 3)/(12*m + 1) + 6*a^11*b*x^(24*m + 2)/(12*m + 1) + a
^12*x^(12*m + 1)/(12*m + 1)
```

mupad [B] time = 5.95, size = 285, normalized size = 10.56

$$\frac{b^{12}\,x^{156\,m}\,x^{13}}{156\,m+13}+\frac{a^{12}\,x\,x^{12\,m}}{12\,m+1}+\frac{6\,a^{11}\,b\,x^{24\,m}\,x^{2}}{12\,m+1}+\frac{a\,b^{11}\,x^{144\,m}\,x^{12}}{12\,m+1}+\frac{22\,a^{10}\,b^{2}\,x^{36\,m}\,x^{3}}{12\,m+1}+\frac{55\,a^{9}\,b^{3}\,x^{48\,m}\,x^{4}}{12\,m+1}+\frac{99\,a^{8}\,b^{4}\,x^{60\,m}\,x^{5}}{12\,m+1}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^(13*m + 1))^12,x)
```

```
[Out] (b^12*x^(156*m)*x^13)/(156*m + 13) + (a^12*x*x^(12*m))/(12*m + 1) + (6*a^11
*b*x^(24*m)*x^2)/(12*m + 1) + (a*b^11*x^(144*m)*x^12)/(12*m + 1) + (22*a^10
*b^2*x^(36*m)*x^3)/(12*m + 1) + (55*a^9*b^3*x^(48*m)*x^4)/(12*m + 1) + (99*
a^8*b^4*x^(60*m)*x^5)/(12*m + 1) + (132*a^7*b^5*x^(72*m)*x^6)/(12*m + 1) +
(132*a^6*b^6*x^(84*m)*x^7)/(12*m + 1) + (99*a^5*b^7*x^(96*m)*x^8)/(12*m + 1
) + (55*a^4*b^8*x^(108*m)*x^9)/(12*m + 1) + (22*a^3*b^9*x^(120*m)*x^10)/(12
*m + 1) + (6*a^2*b^10*x^(132*m)*x^11)/(12*m + 1)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(1+13*m))**12,x)
```

```
[Out] Timed out
```

3.336 $\int \left(ax^m+bx^{1+6m}\right)^5\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^{5m+1}\right)^6}{6b(5m+1)}$$

```
[Out] 1/6*(a+b*x^(1+5*m))^6/b/(1+5*m)
```

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 261}

$$\frac{\left(a+bx^{5m+1}\right)^6}{6b(5m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + 6*m))^5,x]
```

```
[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(ax^m+bx^{1+6m}\right)^5\,dx &= \int x^{5m}\left(a+bx^{1+5m}\right)^5\,dx\\ &= \frac{\left(a+bx^{1+5m}\right)^6}{6b(1+5m)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\frac{\left(a+bx^{5m+1}\right)^6}{6b(5m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + 6*m))^5,x]
```

```
[Out] (a + b*x^(1 + 5*m))^6/(6*b*(1 + 5*m))
```

fricas [B] time = 0.41, size = 93, normalized size = 3.44

$$\frac{b^5x^6x^{30\,m}+6\,ab^4x^5x^{25\,m}+15\,a^2b^3x^4x^{20\,m}+20\,a^3b^2x^3x^{15\,m}+15\,a^4bx^2x^{10\,m}+6\,a^5xx^{5\,m}}{6\,(5\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 20
*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1)
```

giac [B] time = 0.20, size = 93, normalized size = 3.44

$$\frac{b^5x^6x^{30\,m} + 6\,ab^4x^5x^{25\,m} + 15\,a^2b^3x^4x^{20\,m} + 20\,a^3b^2x^3x^{15\,m} + 15\,a^4bx^2x^{10\,m} + 6\,a^5xx^{5\,m}}{6\,(5\,m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="giac")
```

```
[Out] 1/6*(b^5*x^6*x^(30*m) + 6*a*b^4*x^5*x^(25*m) + 15*a^2*b^3*x^4*x^(20*m) + 20
*a^3*b^2*x^3*x^(15*m) + 15*a^4*b*x^2*x^(10*m) + 6*a^5*x*x^(5*m))/(5*m + 1)
```

maple [B] time = 0.06, size = 126, normalized size = 4.67

$$\frac{b^5x^6x^{30m}}{6+30m} + \frac{a\,b^4x^5x^{25m}}{5m+1} + \frac{5a^2b^3x^4x^{20m}}{2\,(5m+1)} + \frac{10a^3b^2x^3x^{15m}}{3\,(5m+1)} + \frac{5a^4b\,x^2x^{10m}}{2\,(5m+1)} + \frac{a^5x\,x^{5m}}{5m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(1+6*m))^5,x)
```

```
[Out] 1/6*b^5*x^6/(1+5*m)*(x^m)^30+a*b^4*x^5/(1+5*m)*(x^m)^25+5/2*a^2*b^3*x^4/(1+
5*m)*(x^m)^20+10/3*a^3*b^2*x^3/(1+5*m)*(x^m)^15+5/2*a^4*b*x^2/(1+5*m)*(x^m)
^10+a^5/(1+5*m)*x*(x^m)^5
```

maxima [B] time = 1.41, size = 121, normalized size = 4.48

$$\frac{b^5x^{30\,m+6}}{6\,(5\,m+1)} + \frac{ab^4x^{25\,m+5}}{5\,m+1} + \frac{5\,a^2b^3x^{20\,m+4}}{2\,(5\,m+1)} + \frac{10\,a^3b^2x^{15\,m+3}}{3\,(5\,m+1)} + \frac{5\,a^4bx^{10\,m+2}}{2\,(5\,m+1)} + \frac{a^5x^{5\,m+1}}{5\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(1+6*m))^5,x, algorithm="maxima")
```

```
[Out] 1/6*b^5*x^(30*m + 6)/(5*m + 1) + a*b^4*x^(25*m + 5)/(5*m + 1) + 5/2*a^2*b^3
*x^(20*m + 4)/(5*m + 1) + 10/3*a^3*b^2*x^(15*m + 3)/(5*m + 1) + 5/2*a^4*b*x
^(10*m + 2)/(5*m + 1) + a^5*x^(5*m + 1)/(5*m + 1)
```

mupad [B] time = 5.44, size = 124, normalized size = 4.59

$$\frac{b^5\,x^{30\,m}\,x^6}{30\,m+6} + \frac{a^5\,x\,x^{5\,m}}{5\,m+1} + \frac{5\,a^4\,b\,x^{10\,m}\,x^2}{10\,m+2} + \frac{a\,b^4\,x^{25\,m}\,x^5}{5\,m+1} + \frac{5\,a^2\,b^3\,x^{20\,m}\,x^4}{10\,m+2} + \frac{10\,a^3\,b^2\,x^{15\,m}\,x^3}{15\,m+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^(6*m + 1))^5,x)
```

```
[Out] (b^5*x^(30*m)*x^6)/(30*m + 6) + (a^5*x*x^(5*m))/(5*m + 1) + (5*a^4*b*x^(10*
m)*x^2)/(10*m + 2) + (a*b^4*x^(25*m)*x^5)/(5*m + 1) + (5*a^2*b^3*x^(20*m)*x
^4)/(10*m + 2) + (10*a^3*b^2*x^(15*m)*x^3)/(15*m + 3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(1+6*m))**5,x)
```

```
[Out] Timed out
```

3.337 $\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

[Out] -1/2/b/(1-3*m)/(a+b*x^(1-3*m))^2

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 261}

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(b*x^(1 - 2*m) + a*x^m)^(-3),x]

[Out] -1/(2*b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(bx^{1-2m}+ax^m\right)^3}\,dx &= \int \frac{x^{-3m}}{\left(a+bx^{1-3m}\right)^3}\,dx\\ &= -\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 27, normalized size = 1.00

$$-\frac{1}{2b(1-3m)\left(a+bx^{1-3m}\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(b*x^(1 - 2*m) + a*x^m)^(-3),x]

[Out] -1/2*1/(b*(1 - 3*m)*(a + b*x^(1 - 3*m))^2)

fricas [B] time = 0.41, size = 82, normalized size = 3.04

$$-\frac{2\,axx^{3\,m}+bx^2}{2\left(2\left(3\,a^3bm-a^3b\right)xx^{3\,m}+\left(3\,a^2b^2m-a^2b^2\right)x^2+\left(3\,a^4m-a^4\right)x^{6\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*x*x^(3*m) + b*x^2)/(2*(3*a^3*b*m - a^3*b)*x*x^(3*m) + (3*a^2*b^2*
m - a^2*b^2)*x^2 + (3*a^4*m - a^4)*x^(6*m))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^m + bx^{-2m+1}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="giac")
```

```
[Out] integrate((a*x^m + b*x^(-2*m + 1))^(-3), x)
```

maple [A] time = 0.07, size = 39, normalized size = 1.44

$$-\frac{\left(2a\,x^{3m} + bx\right)x}{2\,(3m-1)\left(a\,x^{3m} + bx\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(1-2*m)+a*x^m)^3,x)
```

```
[Out] -1/2*x*(2*a*(x^m)^3+b*x)/(-1+3*m)/a^2/(a*(x^m)^3+b*x)^2
```

maxima [B] time = 1.40, size = 66, normalized size = 2.44

$$-\frac{2\,axx^{3\,m} + bx^2}{2\left(2\,a^3b(3\,m-1)xx^{3\,m} + a^2b^2(3\,m-1)x^2 + a^4(3\,m-1)x^{6\,m}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^(1-2*m)+a*x^m)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*x*x^(3*m) + b*x^2)/(2*a^3*b*(3*m - 1)*x*x^(3*m) + a^2*b^2*(3*m -
1)*x^2 + a^4*(3*m - 1)*x^(6*m))
```

mupad [B] time = 5.21, size = 38, normalized size = 1.41

$$-\frac{x\,\left(b\,x+2\,a\,x^{3\,m}\right)}{2\,a^2\,\left(3\,m-1\right)\,\left(b\,x+a\,x^{3\,m}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^m + b*x^(1 - 2*m))^3,x)
```

```
[Out] -(x*(b*x + 2*a*x^(3*m)))/(2*a^2*(3*m - 1)*(b*x + a*x^(3*m))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**(1-2*m)+a*x**m)**3,x)
```

```
[Out] Timed out
```

3.338 $\int \frac{1}{\frac{b}{x}+ax}\,dx$

Optimal. Leaf size=15

$$\frac{\log(ax^2+b)}{2a}$$

```
[Out] 1/2*ln(a*x^2+b)/a
```

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log(ax^2+b)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x}+ax}\,dx &= \int \frac{x}{b+ax^2}\,dx\\ &= \frac{\log(b+ax^2)}{2a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(ax^2+b)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^2]/(2*a)
```

fricas [A] time = 0.37, size = 13, normalized size = 0.87

$$\frac{\log(ax^2+b)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="fricas")
[Out] 1/2*log(a*x^2 + b)/a
```

giac [A] time = 0.15, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|ax^2+b\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="giac")
[Out] 1/2*log(abs(a*x^2 + b))/a
```

maple [A] time = 0.04, size = 14, normalized size = 0.93

$$\frac{\ln\left(a\,x^2+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x+a*x),x)
[Out] 1/2/a*ln(a*x^2+b)
```

maxima [A] time = 1.29, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x, algorithm="maxima")
[Out] 1/2*log(a*x^2 + b)/a
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^2+b\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x),x)
[Out] log(b + a*x^2)/(2*a)
```

sympy [A] time = 0.12, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^2+b\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x),x)
[Out] log(a*x**2 + b)/(2*a)
```

3.339 $\int \frac{1}{\frac{b}{x^2}+ax} \, dx$

Optimal. Leaf size=15

$$\frac{\log \left(ax^3+b\right)}{3a}$$

[Out] 1/3*ln(a*x^3+b)/a

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log \left(ax^3+b\right)}{3a}$$

Antiderivative was successfully verified.

[In] Int[(b/x^2 + a*x)^(-1),x]

[Out] Log[b + a*x^3]/(3*a)

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x^2}+ax} \, dx &= \int \frac{x^2}{b+ax^3} \, dx\\ &= \frac{\log \left(b+ax^3\right)}{3a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log \left(ax^3+b\right)}{3a}$$

Antiderivative was successfully verified.

[In] Integrate[(b/x^2 + a*x)^(-1),x]

[Out] Log[b + a*x^3]/(3*a)

fricas [A] time = 0.38, size = 13, normalized size = 0.87

$$\frac{\log \left(ax^3+b\right)}{3 \, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="fricas")
[Out] 1/3*log(a*x^3 + b)/a
```

giac [A] time = 0.15, size = 14, normalized size = 0.93

$$\frac{\log\left(|ax^3 + b|\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="giac")
[Out] 1/3*log(abs(a*x^3 + b))/a
```

maple [A] time = 0.04, size = 14, normalized size = 0.93

$$\frac{\ln\left(a\,x^3 + b\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^2+a*x),x)
[Out] 1/3/a*ln(a*x^3+b)
```

maxima [A] time = 1.27, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^3 + b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^2+a*x),x, algorithm="maxima")
[Out] 1/3*log(a*x^3 + b)/a
```

mupad [B] time = 5.12, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^3 + b\right)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x^2),x)
[Out] log(b + a*x^3)/(3*a)
```

sympy [A] time = 0.15, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^3 + b\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**2+a*x),x)
[Out] log(a*x**3 + b)/(3*a)
```

3.340 $\int \frac{1}{\frac{b}{x^3}+ax}\,dx$

Optimal. Leaf size=15

$$\frac{\log\left(ax^4+b\right)}{4a}$$

```
[Out] 1/4*ln(a*x^4+b)/a
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log\left(ax^4+b\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^4]/(4*a)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{b}{x^3}+ax}\,dx &= \int \frac{x^3}{b+ax^4}\,dx\\ &= \frac{\log\left(b+ax^4\right)}{4a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log\left(ax^4+b\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3 + a*x)^(-1),x]
```

```
[Out] Log[b + a*x^4]/(4*a)
```

fricas [A] time = 0.38, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^4+b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="fricas")
[Out] 1/4*log(a*x^4 + b)/a
```

giac [A] time = 0.15, size = 14, normalized size = 0.93

$$\frac{\log\left(\left|ax^4 + b\right|\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="giac")
[Out] 1/4*log(abs(a*x^4 + b))/a
```

maple [A] time = 0.04, size = 14, normalized size = 0.93

$$\frac{\ln\left(a\,x^4 + b\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3+a*x),x)
[Out] 1/4*ln(a*x^4+b)/a
```

maxima [A] time = 1.37, size = 13, normalized size = 0.87

$$\frac{\log\left(ax^4 + b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x),x, algorithm="maxima")
[Out] 1/4*log(a*x^4 + b)/a
```

mupad [B] time = 0.05, size = 13, normalized size = 0.87

$$\frac{\ln\left(a\,x^4 + b\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x^3),x)
[Out] log(b + a*x^4)/(4*a)
```

sympy [A] time = 0.16, size = 10, normalized size = 0.67

$$\frac{\log\left(ax^4 + b\right)}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3+a*x),x)
[Out] log(a*x**4 + b)/(4*a)
```

3.341 $\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^4}{4b\left(ax^2+b\right)^2}$$

```
[Out] 1/4*x^4/b/(a*x^2+b)^2
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 264}

$$\frac{x^4}{4b\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x + a*x)^(-3),x]
[Out] x^4/(4*b*(b + a*x^2)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x}+ax\right)^3}\,dx &= \int \frac{x^3}{\left(b+ax^2\right)^3}\,dx\\ &= \frac{x^4}{4b\left(b+ax^2\right)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{2ax^2+b}{4a^2\left(ax^2+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x + a*x)^(-3),x]
[Out] -1/4*(b + 2*a*x^2)/(a^2*(b + a*x^2)^2)
```

fricas [B] time = 0.37, size = 36, normalized size = 1.89

$$-\frac{2\,ax^2+b}{4\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="fricas")
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

giac [A] time = 0.19, size = 22, normalized size = 1.16

$$-\frac{2\,ax^2+b}{4\left(ax^2+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="giac")
[Out] -1/4*(2*a*x^2 + b)/((a*x^2 + b)^2*a^2)
```

maple [A] time = 0.05, size = 31, normalized size = 1.63

$$\frac{b}{4\left(a\,x^2+b\right)^2a^2}-\frac{1}{2\left(a\,x^2+b\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x+a*x)^3,x)
[Out] -1/2/a^2/(a*x^2+b)+1/4/a^2*b/(a*x^2+b)^2
```

maxima [B] time = 1.33, size = 36, normalized size = 1.89

$$-\frac{2\,ax^2+b}{4\left(a^4x^4+2\,a^3bx^2+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)^3,x, algorithm="maxima")
[Out] -1/4*(2*a*x^2 + b)/(a^4*x^4 + 2*a^3*b*x^2 + a^2*b^2)
```

mupad [B] time = 0.04, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{4\,a^2}+\frac{x^2}{2\,a}}{a^2\,x^4+2\,a\,b\,x^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b/x)^3,x)
[Out] -(b/(4*a^2) + x^2/(2*a))/(b^2 + a^2*x^4 + 2*a*b*x^2)
```

sympy [B] time = 0.28, size = 36, normalized size = 1.89

$$\frac{-2ax^2-b}{4a^4x^4+8a^3bx^2+4a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x+a*x)**3,x)
[Out] (-2*a*x**2 - b)/(4*a**4*x**4 + 8*a**3*b*x**2 + 4*a**2*b**2)
```

3.342 $\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^{10}}{10b\left(ax^5+b\right)^2}$$

```
[Out] 1/10*x^10/b/(a*x^5+b)^2
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 264}

$$\frac{x^{10}}{10b\left(ax^5+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b/x^3 + a*x^2)^(-3),x]
```

```
[Out] x^10/(10*b*(b + a*x^5)^2)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^3}+ax^2\right)^3}\,dx &= \int \frac{x^9}{\left(b+ax^5\right)^3}\,dx\\ &= \frac{x^{10}}{10b\left(b+ax^5\right)^2}\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.26

$$-\frac{2ax^5+b}{10a^2\left(ax^5+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/x^3 + a*x^2)^(-3),x]
```

```
[Out] -1/10*(b + 2*a*x^5)/(a^2*(b + a*x^5)^2)
```

fricas [**B**] time = 0.36, size = 36, normalized size = 1.89

$$-\frac{2\,ax^5+b}{10\left(a^4x^{10}+2\,a^3bx^5+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="fricas")
[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

giac [**A**] time = 0.17, size = 22, normalized size = 1.16

$$-\frac{2\,ax^5+b}{10\left(ax^5+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="giac")
[Out] -1/10*(2*a*x^5 + b)/((a*x^5 + b)^2*a^2)
```

maple [**A**] time = 0.05, size = 31, normalized size = 1.63

$$\frac{b}{10\left(a\,x^5+b\right)^2a^2}-\frac{1}{5\left(a\,x^5+b\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^3+a*x^2)^3,x)
[Out] -1/5/a^2/(a*x^5+b)+1/10/a^2*b/(a*x^5+b)^2
```

maxima [**B**] time = 1.29, size = 36, normalized size = 1.89

$$-\frac{2\,ax^5+b}{10\left(a^4x^{10}+2\,a^3bx^5+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^3+a*x^2)^3,x, algorithm="maxima")
[Out] -1/10*(2*a*x^5 + b)/(a^4*x^10 + 2*a^3*b*x^5 + a^2*b^2)
```

mupad [**B**] time = 0.06, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{10\,a^2}+\frac{x^5}{5\,a}}{a^2\,x^{10}+2\,a\,b\,x^5+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2 + b/x^3)^3,x)
[Out] -(b/(10*a^2) + x^5/(5*a))/(b^2 + a^2*x^10 + 2*a*b*x^5)
```

sympy [**B**] time = 0.50, size = 36, normalized size = 1.89

$$\frac{-2ax^5-b}{10a^4x^{10}+20a^3bx^5+10a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**3+a*x**2)**3,x)
[Out] (-2*a*x**5 - b)/(10*a**4*x**10 + 20*a**3*b*x**5 + 10*a**2*b**2)
```

3.343 $\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx$

Optimal. Leaf size=19

$$\frac{x^{16}}{16b\left(ax^8+b\right)^2}$$

[Out] 1/16*x^16/b/(a*x^8+b)^2

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1593, 264}

$$\frac{x^{16}}{16b\left(ax^8+b\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(b/x^5 + a*x^3)^(-3),x]

[Out] x^16/(16*b*(b + a*x^8)^2)

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(\frac{b}{x^5}+ax^3\right)^3}\,dx &= \int \frac{x^{15}}{\left(b+ax^8\right)^3}\,dx\\ &= \frac{x^{16}}{16b\left(b+ax^8\right)^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.26

$$-\frac{2ax^8+b}{16a^2\left(ax^8+b\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(b/x^5 + a*x^3)^(-3),x]

[Out] -1/16*(b + 2*a*x^8)/(a^2*(b + a*x^8)^2)

fricas [B] time = 0.37, size = 36, normalized size = 1.89

$$-\frac{2\,ax^8+b}{16\,\left(a^4x^{16}+2\,a^3bx^8+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="fricas")
[Out] -1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

giac [A] time = 0.15, size = 22, normalized size = 1.16

$$-\frac{2\,ax^8+b}{16\,\left(ax^8+b\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="giac")
[Out] -1/16*(2*a*x^8 + b)/((a*x^8 + b)^2*a^2)
```

maple [A] time = 0.06, size = 31, normalized size = 1.63

$$\frac{b}{16\left(a\,x^8+b\right)^2a^2}-\frac{1}{8\left(a\,x^8+b\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/x^5+a*x^3)^3,x)
[Out] -1/8/a^2/(a*x^8+b)+1/16/a^2*b/(a*x^8+b)^2
```

maxima [B] time = 1.31, size = 36, normalized size = 1.89

$$-\frac{2\,ax^8+b}{16\,\left(a^4x^{16}+2\,a^3bx^8+a^2b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x^5+a*x^3)^3,x, algorithm="maxima")
[Out] -1/16*(2*a*x^8 + b)/(a^4*x^16 + 2*a^3*b*x^8 + a^2*b^2)
```

mupad [B] time = 5.18, size = 37, normalized size = 1.95

$$-\frac{\frac{b}{16\,a^2}+\frac{x^8}{8\,a}}{a^2\,x^{16}+2\,a\,b\,x^8+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3 + b/x^5)^3,x)
[Out] -(b/(16*a^2) + x^8/(8*a))/(b^2 + a^2*x^16 + 2*a*b*x^8)
```

sympy [B] time = 0.74, size = 36, normalized size = 1.89

$$\frac{-2ax^8-b}{16a^4x^{16}+32a^3bx^8+16a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/x**5+a*x**3)**3,x)
[Out] (-2*a*x**8 - b)/(16*a**4*x**16 + 32*a**3*b*x**8 + 16*a**2*b**2)
```

3.344 $\int \left(\frac{a}{x}+bx\right)^2 dx$

Optimal. Leaf size=24

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

```
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x}+bx\right)^2 dx &= \int \frac{\left(a+bx^2\right)^2}{x^2}\,dx\\ &= \int \left(2ab+\frac{a^2}{x^2}+b^2x^2\right)dx\\ &= -\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{a^2}{x}+2abx+\frac{b^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^2,x]
```

```
[Out] -(a^2/x) + 2*a*b*x + (b^2*x^3)/3
```

fricas [A] time = 0.37, size = 25, normalized size = 1.04

$$\frac{b^2x^4+6\,abx^2-3\,a^2}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="fricas")
[Out] 1/3*(b^2*x^4 + 6*a*b*x^2 - 3*a^2)/x
```

giac [A] time = 0.15, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="giac")
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

maple [A] time = 0.04, size = 23, normalized size = 0.96

$$\frac{b^2x^3}{3} + 2abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^2,x)
[Out] -a^2/x+2*a*b*x+1/3*b^2*x^3
```

maxima [A] time = 1.36, size = 22, normalized size = 0.92

$$\frac{1}{3}\,b^2x^3 + 2\,abx - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^2,x, algorithm="maxima")
[Out] 1/3*b^2*x^3 + 2*a*b*x - a^2/x
```

mupad [B] time = 0.04, size = 22, normalized size = 0.92

$$\frac{b^2\,x^3}{3} - \frac{a^2}{x} + 2\,a\,b\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + a/x)^2,x)
[Out] (b^2*x^3)/3 - a^2/x + 2*a*b*x
```

sympy [A] time = 0.10, size = 19, normalized size = 0.79

$$-\frac{a^2}{x} + 2abx + \frac{b^2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**2,x)
[Out] -a**2/x + 2*a*b*x + b**2*x**3/3
```

3.345 $\int \left(\frac{a}{x}+bx\right)^3 dx$

Optimal. Leaf size=40

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

```
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1593, 266, 43}

$$3a^2b\log(x)-\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^3,x]
```

```
[Out] -a^3/(2*x^2) + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{a}{x}+bx\right)^3 dx &= \int \frac{\left(a+bx^2\right)^3}{x^3}\,dx\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{(a+bx)^3}{x^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(3ab^2+\frac{a^3}{x^2}+\frac{3a^2b}{x}+b^3x\right)dx,x,x^2\right)\\
&= -\frac{a^3}{2x^2}+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}+3a^2b\log(x)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 1.00

$$-\frac{a^3}{2x^2}+3a^2b\log(x)+\frac{3}{2}ab^2x^2+\frac{b^3x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^3,x]
[Out] -1/2*a^3/x^2 + (3*a*b^2*x^2)/2 + (b^3*x^4)/4 + 3*a^2*b*Log[x]
```

fricas [A] time = 0.38, size = 38, normalized size = 0.95

$$\frac{b^3x^6 + 6\,ab^2x^4 + 12\,a^2bx^2\log(x) - 2\,a^3}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="fricas")
[Out] 1/4*(b^3*x^6 + 6*a*b^2*x^4 + 12*a^2*b*x^2*log(x) - 2*a^3)/x^2
```

giac [A] time = 0.17, size = 46, normalized size = 1.15

$$\frac{1}{4}\,b^3x^4 + \frac{3}{2}\,ab^2x^2 + \frac{3}{2}\,a^2b\log\left(x^2\right) - \frac{3\,a^2bx^2 + a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="giac")
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3/2*a^2*b*log(x^2) - 1/2*(3*a^2*b*x^2 + a^3)/
x^2
```

maple [A] time = 0.04, size = 35, normalized size = 0.88

$$\frac{b^3x^4}{4} + \frac{3a\,b^2x^2}{2} + 3a^2b\ln(x) - \frac{a^3}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^3,x)
[Out] -1/2*a^3/x^2+3/2*a*b^2*x^2+1/4*b^3*x^4+3*a^2*b*ln(x)
```

maxima [A] time = 1.32, size = 34, normalized size = 0.85

$$\frac{1}{4}\,b^3x^4 + \frac{3}{2}\,ab^2x^2 + 3\,a^2b\log(x) - \frac{a^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^3,x, algorithm="maxima")
[Out] 1/4*b^3*x^4 + 3/2*a*b^2*x^2 + 3*a^2*b*log(x) - 1/2*a^3/x^2
```

mupad [B] time = 0.04, size = 34, normalized size = 0.85

$$\frac{b^3\,x^4}{4} - \frac{a^3}{2\,x^2} + \frac{3\,a\,b^2\,x^2}{2} + 3\,a^2\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + a/x)^3,x)
[Out] (b^3*x^4)/4 - a^3/(2*x^2) + (3*a*b^2*x^2)/2 + 3*a^2*b*log(x)
```

sympy [A] time = 0.15, size = 37, normalized size = 0.92

$$-\frac{a^3}{2x^2} + 3a^2b\log{(x)} + \frac{3ab^2x^2}{2} + \frac{b^3x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**3,x)
```

```
[Out] -a**3/(2*x**2) + 3*a**2*b*log(x) + 3*a*b**2*x**2/2 + b**3*x**4/4
```

3.346 $\int \left(\frac{a}{x}+bx\right)^4\,dx$

Optimal. Leaf size=50

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 270}

$$6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[(a/x + b*x)^4,x]
```

```
[Out] -a^4/(3*x^3) - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{a}{x}+bx\right)^4\,dx &= \int \frac{\left(a+bx^2\right)^4}{x^4}\,dx\\ &= \int \left(6a^2b^2+\frac{a^4}{x^4}+\frac{4a^3b}{x^2}+4ab^3x^2+b^4x^4\right)dx\\ &= -\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 50, normalized size = 1.00

$$-\frac{a^4}{3x^3}-\frac{4a^3b}{x}+6a^2b^2x+\frac{4}{3}ab^3x^3+\frac{b^4x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a/x + b*x)^4,x]
```

```
[Out] -1/3*a^4/x^3 - (4*a^3*b)/x + 6*a^2*b^2*x + (4*a*b^3*x^3)/3 + (b^4*x^5)/5
```

fricas [A] time = 0.38, size = 48, normalized size = 0.96

$$\frac{3\,b^4x^8+20\,ab^3x^6+90\,a^2b^2x^4-60\,a^3bx^2-5\,a^4}{15\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^4*x^8 + 20*a*b^3*x^6 + 90*a^2*b^2*x^4 - 60*a^3*b*x^2 - 5*a^4)/x^3
```

giac [A] time = 0.15, size = 45, normalized size = 0.90

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{12\,a^3bx^2+a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="giac")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 1/3*(12*a^3*b*x^2 + a^4)/x^3
```

maple [A] time = 0.05, size = 45, normalized size = 0.90

$$\frac{b^4x^5}{5}+\frac{4a\,b^3x^3}{3}+6a^2b^2x-\frac{4a^3b}{x}-\frac{a^4}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x)^4,x)
```

```
[Out] -1/3*a^4/x^3-4*a^3*b/x+6*a^2*b^2*x+4/3*a*b^3*x^3+1/5*b^4*x^5
```

maxima [A] time = 1.32, size = 44, normalized size = 0.88

$$\frac{1}{5}\,b^4x^5+\frac{4}{3}\,ab^3x^3+6\,a^2b^2x-\frac{4\,a^3b}{x}-\frac{a^4}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*b^4*x^5 + 4/3*a*b^3*x^3 + 6*a^2*b^2*x - 4*a^3*b/x - 1/3*a^4/x^3
```

mupad [B] time = 0.05, size = 47, normalized size = 0.94

$$\frac{b^4\,x^5}{5}-\frac{\frac{a^4}{3}+4\,b\,a^3\,x^2}{x^3}+6\,a^2\,b^2\,x+\frac{4\,a\,b^3\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + a/x)^4,x)
```

```
[Out] (b^4*x^5)/5 - (a^4/3 + 4*a^3*b*x^2)/x^3 + 6*a^2*b^2*x + (4*a*b^3*x^3)/3
```

sympy [A] time = 0.19, size = 49, normalized size = 0.98

$$6a^2b^2x+\frac{4ab^3x^3}{3}+\frac{b^4x^5}{5}+\frac{-a^4-12a^3bx^2}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x)**4,x)
```

```
[Out] 6*a**2*b**2*x + 4*a*b**3*x**3/3 + b**4*x**5/5 + (-a**4 - 12*a**3*b*x**2)/(3
*x**3)
```

3.347 $\int \frac{1}{\frac{1}{x^2}+x^3} dx$

Optimal. Leaf size=185

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\right.}$$

```
[Out] 1/5*ln(1+x)-1/20*ln(1+x^2-1/2*x*(5^(1/2)+1))*(-5^(1/2)+1)-1/20*ln(1+x^2-1/2
*x*(-5^(1/2)+1))*(5^(1/2)+1)-1/10*arctan(1/5*(25-10*5^(1/2))^(1/2)+2*x*2^(1
/2)/(5+5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/10*arctan(1/5*x*(50+10*5^(1/2
))^(1/2)-1/5*(25+10*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {1593, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1-\sqrt{5}\right)x+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+\frac{1}{5}\log(x+1)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-2) + x^3)^(-1),x]
```

```
[Out] -(Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + 2*Sqrt[2/(5 + Sqrt
[5])]*x])/5 - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] - Sqrt[
(2*(5 + Sqrt[5]))/5]*x])/5 + Log[1 + x]/5 - ((1 + Sqrt[5])*Log[1 - ((1 - Sq
rt[5])*x)/2 + x^2])/20 - ((1 - Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])
/20
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{\frac{1}{x^2}+x^3}\,dx = \int \frac{x^2}{1+x^5}\,dx$$

$$= \frac{2}{5}\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)-\frac{1}{4}\left(1+\sqrt{5}\right)x}{1-\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx+\frac{2}{5}\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)-\frac{1}{4}\left(1-\sqrt{5}\right)x}{1-\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx+\frac{1}{5}\int \frac{1}{1+x}\,dx$$

$$= \frac{1}{5}\log(1+x)+\frac{\int \frac{1}{1+\frac{1}{2}\left(-1-\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}}-\frac{\int \frac{1}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)x+x^2}\,dx}{2\sqrt{5}}+\frac{1}{20}\left(-1-\sqrt{5}\right)\int \frac{\frac{1}{2}\left(-1+\sqrt{5}\right)}{1+\frac{1}{2}\left(-1+\sqrt{5}\right)}$$

$$= \frac{1}{5}\log(1+x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(2-x-\sqrt{5}x+2x^2\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(2-x+\sqrt{5}x+2x^2\right)+$$

$$= \sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}-4x}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}-4x\right)\right.$$

Mathematica [A] time = 0.14, size = 144, normalized size = 0.78

$$\frac{1}{20}\left(-\left(1+\sqrt{5}\right)\log\left(x^2+\frac{1}{2}\left(\sqrt{5}-1\right)x+1\right)+\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}\left(1+\sqrt{5}\right)x+1\right)+4\log(x+1)-2\sqrt{2\left(5+\right.}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(-2) + x^3)^(-1),x]
```

```
[Out] (-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 + Sqrt[5] - 4*x)/Sqrt[10 - 2*Sqrt[5]]]
- 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-1 + Sqrt[5] + 4*x)/Sqrt[2*(5 + Sqrt[5])]]
 + 4*Log[1 + x] - (1 + Sqrt[5])*Log[1 + ((-1 + Sqrt[5])*x)/2 + x^2] + (-1 +
 Sqrt[5])*Log[1 - ((1 + Sqrt[5])*x)/2 + x^2])/20
```

fricas [B] time = 1.25, size = 637, normalized size = 3.44

$$-\frac{1}{20}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)\log\left(\frac{1}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-5}+\sqrt{5}+1\right)^2+x\right)+\frac{1}{20}\left(\sqrt{5}+2\sqrt{-\frac{3}{16}\left(2\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}-\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="fricas")
```

```
[Out] -1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(1/16*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + x) + 1/20*(sqrt(5) + 2*sqrt(-3/16*(2*sq
rt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*sqrt(sqrt(5)
- 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 3/16*(2
*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(sqrt(5) - 5)
 + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(
5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 1/2*sqrt
(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqrt(1/2)*s
qrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) -
1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/2)*sqrt(
sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x - 1) + 1/20*(sqrt(5)
- 2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 1/8*(2*sqr
t(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sq
rt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + sqrt(1/
2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2) - 1)*log(-1/16*(2*sqrt(1/2)*sqrt(
sqrt(5) - 5) + sqrt(5) + 1)^2 - 1/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(
5) - 1)^2 - 1/2*sqrt(-3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2
+ 1/8*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(2*sqrt(1/2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1) - 3/16*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)
^2 + sqrt(1/2)*sqrt(sqrt(5) - 5) + 1/2*sqrt(5) - 5/2)*(sqrt(5) - 1) + 2*x -
 1) + 1/20*(2*sqrt(1/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(1/16*(2*sqrt(1
/2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + x) + 1/5*log(x + 1)
```

giac [A] time = 0.18, size = 112, normalized size = 0.61

$$\frac{1}{20}\left(\sqrt{5}-1\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{5}+1\right)+1\right)-\frac{1}{20}\left(\sqrt{5}+1\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{5}-1\right)+1\right)-\frac{1}{10}\sqrt{-2\sqrt{5}+10}\,\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="giac")
```

```
[Out] 1/20*(sqrt(5) - 1)*log(x^2 - 1/2*x*(sqrt(5) + 1) + 1) - 1/20*(sqrt(5) + 1)*
log(x^2 + 1/2*x*(sqrt(5) - 1) + 1) - 1/10*sqrt(-2*sqrt(5) + 10)*arctan((4*x
 + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10)) + 1/10*sqrt(2*sqrt(5) + 10)*arctan((4
*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10)) + 1/5*log(abs(x + 1))
```

maple [A] time = 0.11, size = 156, normalized size = 0.84

$$\frac{2\sqrt{5}\,\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{10-2\sqrt{5}}}\right)}{5\sqrt{10-2\sqrt{5}}}-\frac{2\sqrt{5}\,\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{10+2\sqrt{5}}}\right)}{5\sqrt{10+2\sqrt{5}}}+\frac{\ln\left(x+1\right)}{5}+\frac{\sqrt{5}\,\ln\left(2x^2-\sqrt{5}\,x-x+2\right)}{20}-\frac{\ln\left(2x^2-\sqrt{5}\,x\right.}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^2+x^3),x)
```

```
[Out] -1/20*ln(2*x^2+5^(1/2)*x-x+2)*5^(1/2)-1/20*ln(2*x^2+5^(1/2)*x-x+2)-2/5/(10+
2*5^(1/2))^(1/2)*arctan((4*x+5^(1/2)-1)/(10+2*5^(1/2))^(1/2))*5^(1/2)+1/20*
ln(2*x^2-5^(1/2)*x-x+2)*5^(1/2)-1/20*ln(2*x^2-5^(1/2)*x-x+2)+2/5/(10-2*5^(1
/2))^(1/2)*arctan((4*x-5^(1/2)-1)/(10-2*5^(1/2))^(1/2))*5^(1/2)+1/5*ln(x+1)
```

maxima [A] time = 2.90, size = 124, normalized size = 0.67

$$-\frac{2\sqrt{5}\arctan\left(\frac{4x+\sqrt{5}-1}{\sqrt{2\sqrt{5}+10}}\right)}{5\sqrt{2\sqrt{5}+10}}+\frac{2\sqrt{5}\arctan\left(\frac{4x-\sqrt{5}-1}{\sqrt{-2\sqrt{5}+10}}\right)}{5\sqrt{-2\sqrt{5}+10}}+\frac{\log\left(2x^2-x\left(\sqrt{5}+1\right)+2\right)}{5\left(\sqrt{5}+1\right)}-\frac{\log\left(2x^2+x\left(\sqrt{5}-1\right)+2\right)}{5\left(\sqrt{5}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^2+x^3),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*arctan((4*x + sqrt(5) - 1)/sqrt(2*sqrt(5) + 10))/sqrt(2*sqrt(5
) + 10) + 2/5*sqrt(5)*arctan((4*x - sqrt(5) - 1)/sqrt(-2*sqrt(5) + 10))/sqr
t(-2*sqrt(5) + 10) + 1/5*log(2*x^2 - x*(sqrt(5) + 1) + 2)/(sqrt(5) + 1) - 1
/5*log(2*x^2 + x*(sqrt(5) - 1) + 2)/(sqrt(5) - 1) + 1/5*log(x + 1)
```

mupad [B] time = 5.91, size = 197, normalized size = 1.06

$$\frac{\ln(x+1)}{5}-\ln\left(1-\frac{x\left(\sqrt{2}\sqrt{-\sqrt{5}-5}-\sqrt{5}+1\right)^3}{64}\right)\left(\frac{\sqrt{2}\sqrt{-\sqrt{5}-5}}{20}-\frac{\sqrt{5}}{20}+\frac{1}{20}\right)+\ln\left(\frac{x\left(\sqrt{2}\sqrt{-\sqrt{5}-5}+\sqrt{5}\right.}{64}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^2 + x^3),x)
```

```
[Out] log(x + 1)/5 - log(1 - (x*(2^(1/2)*(- 5^(1/2) - 5)^(1/2) - 5^(1/2) + 1)^3)/
64)*((2^(1/2)*(- 5^(1/2) - 5)^(1/2))/20 - 5^(1/2)/20 + 1/20) + log((x*(2^(1
/2)*(- 5^(1/2) - 5)^(1/2) + 5^(1/2) - 1)^3)/64 + 1)*((2^(1/2)*(- 5^(1/2) -
5)^(1/2))/20 + 5^(1/2)/20 - 1/20) - log(1 - (x*(5^(1/2) + 2^(1/2)*(5^(1/2)
- 5)^(1/2) + 1)^3)/64)*(5^(1/2)/20 + (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/2
0) - log(1 - (x*(5^(1/2) - 2^(1/2)*(5^(1/2) - 5)^(1/2) + 1)^3)/64)*(5^(1/2)
/20 - (2^(1/2)*(5^(1/2) - 5)^(1/2))/20 + 1/20)
```

sympy [A] time = 1.54, size = 36, normalized size = 0.19

$$\frac{\log(x+1)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(25t^2+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**2+x**3),x)
```

```
[Out] log(x + 1)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda(
_t, _t*log(25*_t**2 + x)))
```

3.348 $\int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} dx$

Optimal. Leaf size=29

$$\frac{\left(a + bx^{12n+p+1}\right)^{13}}{13b(12n + p + 1)}$$

```
[Out] 1/13*(a+b*x^(1+12*n+p))^13/b/(1+12*n+p)
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1584, 261}

$$\frac{\left(a + bx^{12n+p+1}\right)^{13}}{13b(12n + p + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int x^p \left(ax^n + bx^{1+13n+p}\right)^{12} dx &= \int x^{12n+p} \left(a + bx^{1+12n+p}\right)^{12} dx \\ &= \frac{\left(a + bx^{1+12n+p}\right)^{13}}{13b(1 + 12n + p)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{\left(a + bx^{12n+p+1}\right)^{13}}{13b(12n + p + 1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^p*(a*x^n + b*x^(1 + 13*n + p))^12,x]
```

```
[Out] (a + b*x^(1 + 12*n + p))^13/(13*b*(1 + 12*n + p))
```

fricas [B] time = 0.41, size = 297, normalized size = 10.24

$$78\, a^2 b^{10} x^{2\,n} x^{143\,n+11\,p+11} + 286\, a^3 b^9 x^{3\,n} x^{130\,n+10\,p+10} + 715\, a^4 b^8 x^{4\,n} x^{117\,n+9\,p+9} + 1287\, a^5 b^7 x^{5\,n} x^{104\,n+8\,p+8} + 17$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="fricas")
```

```
[Out] 1/13*(78*a^2*b^10*x^(2*n)*x^(143*n + 11*p + 11) + 286*a^3*b^9*x^(3*n)*x^(13
0*n + 10*p + 10) + 715*a^4*b^8*x^(4*n)*x^(117*n + 9*p + 9) + 1287*a^5*b^7*x
^(5*n)*x^(104*n + 8*p + 8) + 1716*a^6*b^6*x^(6*n)*x^(91*n + 7*p + 7) + 1716
*a^7*b^5*x^(7*n)*x^(78*n + 6*p + 6) + 1287*a^8*b^4*x^(8*n)*x^(65*n + 5*p +
5) + 715*a^9*b^3*x^(9*n)*x^(52*n + 4*p + 4) + 286*a^10*b^2*x^(10*n)*x^(39*n
 + 3*p + 3) + 78*a^11*b*x^(11*n)*x^(26*n + 2*p + 2) + 13*a^12*x^(12*n)*x^(1
3*n + p + 1) + 13*a*b^11*x^(156*n + 12*p + 12)*x^n + b^12*x^(169*n + 13*p +
 13))/((12*n + p + 1)*x^(13*n))
```

giac [B] time = 2.47, size = 269, normalized size = 9.28

$$\frac{b^{12}x^{13}x^{156\,n}x^{13\,p} + 13\,ab^{11}x^{12}x^{144\,n}x^{12\,p} + 78\,a^2b^{10}x^{11}x^{132\,n}x^{11\,p} + 286\,a^3b^9x^{10}x^{120\,n}x^{10\,p} + 715\,a^4b^8x^9x^{108\,n}x^{9\,p} + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="giac")
```

```
[Out] 1/13*(b^12*x^13*x^(156*n)*x^(13*p) + 13*a*b^11*x^12*x^(144*n)*x^(12*p) + 78
*a^2*b^10*x^11*x^(132*n)*x^(11*p) + 286*a^3*b^9*x^10*x^(120*n)*x^(10*p) + 7
15*a^4*b^8*x^9*x^(108*n)*x^(9*p) + 1287*a^5*b^7*x^8*x^(96*n)*x^(8*p) + 1716
*a^6*b^6*x^7*x^(84*n)*x^(7*p) + 1716*a^7*b^5*x^6*x^(72*n)*x^(6*p) + 1287*a^
8*b^4*x^5*x^(60*n)*x^(5*p) + 715*a^9*b^3*x^4*x^(48*n)*x^(4*p) + 286*a^10*b^
2*x^3*x^(36*n)*x^(3*p) + 78*a^11*b*x^2*x^(24*n)*x^(2*p) + 13*a^12*x*x^(12*n
)*x^p)/(12*n + p + 1)
```

maple [B] time = 0.21, size = 363, normalized size = 12.52

$$\frac{b^{12}x^{13}x^{156n}x^{13p}}{13+156n+13p}+\frac{a\,b^{11}x^{12}x^{144n}x^{12p}}{12n+p+1}+\frac{6a^2b^{10}x^{11}x^{132n}x^{11p}}{12n+p+1}+\frac{22a^3b^9x^{10}x^{120n}x^{10p}}{12n+p+1}+\frac{55a^4b^8x^9x^{108n}x^{9p}}{12n+p+1}+\frac{99a^5b^7x^8x^{96n}}{12n+p+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(a*x^n+b*x^(1+13*n+p))^12,x)
```

```
[Out] 1/13*b^12*x^13*(x^n)^156/(1+12*n+p)*(x^p)^13+a*b^11*x^12*(x^n)^144/(1+12*n+
p)*(x^p)^12+6*a^2*b^10*x^11*(x^n)^132/(1+12*n+p)*(x^p)^11+22*a^3*b^9*x^10*(
x^n)^120/(1+12*n+p)*(x^p)^10+55*a^4*b^8*x^9*(x^n)^108/(1+12*n+p)*(x^p)^9+99
*a^5*b^7*x^8*(x^n)^96/(1+12*n+p)*(x^p)^8+132*a^6*b^6*x^7*(x^n)^84/(1+12*n+p
)*(x^p)^7+132*a^7*b^5*x^6*(x^n)^72/(1+12*n+p)*(x^p)^6+99*a^8*b^4*x^5*(x^n)^
60/(1+12*n+p)*(x^p)^5+55*a^9*b^3*x^4*(x^n)^48/(1+12*n+p)*(x^p)^4+22*a^10*b^
2*x^3*(x^n)^36/(1+12*n+p)*(x^p)^3+6*a^11*b*x^2*(x^n)^24/(1+12*n+p)*(x^p)^2+
a^12/(1+12*n+p)*x*(x^n)^12*x^p
```

maxima [B] time = 1.48, size = 325, normalized size = 11.21

$$\frac{b^{12}x^{156\,n+13\,p+13}}{13\left(12\,n+p+1\right)}+\frac{ab^{11}x^{144\,n+12\,p+12}}{12\,n+p+1}+\frac{6\,a^2b^{10}x^{132\,n+11\,p+11}}{12\,n+p+1}+\frac{22\,a^3b^9x^{120\,n+10\,p+10}}{12\,n+p+1}+\frac{55\,a^4b^8x^{108\,n+9\,p+9}}{12\,n+p+1}+\frac{99\,a^5b^7x^{96}}{12\,n+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*(a*x^n+b*x^(1+13*n+p))^12,x, algorithm="maxima")
```

```
[Out] 1/13*b^12*x^(156*n + 13*p + 13)/(12*n + p + 1) + a*b^11*x^(144*n + 12*p + 1
2)/(12*n + p + 1) + 6*a^2*b^10*x^(132*n + 11*p + 11)/(12*n + p + 1) + 22*a^
3*b^9*x^(120*n + 10*p + 10)/(12*n + p + 1) + 55*a^4*b^8*x^(108*n + 9*p + 9)
/(12*n + p + 1) + 99*a^5*b^7*x^(96*n + 8*p + 8)/(12*n + p + 1) + 132*a^6*b^
6*x^(84*n + 7*p + 7)/(12*n + p + 1) + 132*a^7*b^5*x^(72*n + 6*p + 6)/(12*n
```

```
+ p + 1) + 99*a^8*b^4*x^(60*n + 5*p + 5)/(12*n + p + 1) + 55*a^9*b^3*x^(48*
n + 4*p + 4)/(12*n + p + 1) + 22*a^10*b^2*x^(36*n + 3*p + 3)/(12*n + p + 1)
 + 6*a^11*b*x^(24*n + 2*p + 2)/(12*n + p + 1) + a^12*x^(12*n + p + 1)/(12*n
 + p + 1)
```

mupad [B] time = 6.78, size = 363, normalized size = 12.52

$$\frac{a^{12}\,x\,x^p\,x^{12\,n}}{12\,n+p+1}+\frac{b^{12}\,x^{156\,n}\,x^{13\,p}\,x^{13}}{156\,n+13\,p+13}+\frac{22\,a^{10}\,b^2\,x^{36\,n}\,x^{3\,p}\,x^3}{12\,n+p+1}+\frac{55\,a^9\,b^3\,x^{48\,n}\,x^{4\,p}\,x^4}{12\,n+p+1}+\frac{99\,a^8\,b^4\,x^{60\,n}\,x^{5\,p}\,x^5}{12\,n+p+1}+\frac{132\,a^7\,b^5\,x^{7}}{12\,n+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*(a*x^n + b*x^(13*n + p + 1))^12,x)
```

```
[Out] (a^12*x*x^p*x^(12*n))/(12*n + p + 1) + (b^12*x^(156*n)*x^(13*p)*x^13)/(156*
n + 13*p + 13) + (22*a^10*b^2*x^(36*n)*x^(3*p)*x^3)/(12*n + p + 1) + (55*a^
9*b^3*x^(48*n)*x^(4*p)*x^4)/(12*n + p + 1) + (99*a^8*b^4*x^(60*n)*x^(5*p)*x
^5)/(12*n + p + 1) + (132*a^7*b^5*x^(72*n)*x^(6*p)*x^6)/(12*n + p + 1) + (1
32*a^6*b^6*x^(84*n)*x^(7*p)*x^7)/(12*n + p + 1) + (99*a^5*b^7*x^(96*n)*x^(8
*p)*x^8)/(12*n + p + 1) + (55*a^4*b^8*x^(108*n)*x^(9*p)*x^9)/(12*n + p + 1)
 + (22*a^3*b^9*x^(120*n)*x^(10*p)*x^10)/(12*n + p + 1) + (6*a^2*b^10*x^(132
*n)*x^(11*p)*x^11)/(12*n + p + 1) + (6*a^11*b*x^(24*n)*x^(2*p)*x^2)/(12*n +
 p + 1) + (a*b^11*x^(144*n)*x^(12*p)*x^12)/(12*n + p + 1)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*(a*x**n+b*x**(1+13*n+p))**12,x)
```

```
[Out] Timed out
```

3.349 $\int x^{12} \left(a+b x^{13}\right)^{12} \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+b x^{13}\right)^{13}}{169 b}$$

```
[Out] 1/169*(b*x^13+a)^13/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+b x^{13}\right)^{13}}{169 b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a + b*x^13)^13/(169*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{12} \left(a+b x^{13}\right)^{12} \, dx=\frac{\left(a+b x^{13}\right)^{13}}{169 b}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12} x^{13}}{13}+\frac{6}{13} a^{11} b x^{26}+\frac{22}{13} a^{10} b^2 x^{39}+\frac{55}{13} a^9 b^3 x^{52}+\frac{99}{13} a^8 b^4 x^{65}+\frac{132}{13} a^7 b^5 x^{78}+\frac{132}{13} a^6 b^6 x^{91}+\frac{99}{13} a^5 b^7 x^{104}+\frac{55}{13} a^4 b^8 x^{117}+\frac{22}{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a + b*x^13)^12,x]
```

```
[Out] (a^12*x^13)/13 + (6*a^11*b*x^26)/13 + (22*a^10*b^2*x^39)/13 + (55*a^9*b^3*x
^52)/13 + (99*a^8*b^4*x^65)/13 + (132*a^7*b^5*x^78)/13 + (132*a^6*b^6*x^91)
/13 + (99*a^5*b^7*x^104)/13 + (55*a^4*b^8*x^117)/13 + (22*a^3*b^9*x^130)/13
 + (6*a^2*b^10*x^143)/13 + (a*b^11*x^156)/13 + (b^12*x^169)/169
```

fricas [B] time = 0.33, size = 134, normalized size = 8.38

$$\frac{1}{169} x^{169} b^{12}+\frac{1}{13} x^{156} b^{11} a+\frac{6}{13} x^{143} b^{10} a^2+\frac{22}{13} x^{130} b^9 a^3+\frac{55}{13} x^{117} b^8 a^4+\frac{99}{13} x^{104} b^7 a^5+\frac{132}{13} x^{91} b^6 a^6+\frac{132}{13} x^{78} b^5 a^7+\frac{99}{13} x^{65} b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="fricas")
```

```
[Out] 1/169*x^169*b^12 + 1/13*x^156*b^11*a + 6/13*x^143*b^10*a^2 + 22/13*x^130*b^
9*a^3 + 55/13*x^117*b^8*a^4 + 99/13*x^104*b^7*a^5 + 132/13*x^91*b^6*a^6 + 1
```

```
32/13*x^78*b^5*a^7 + 99/13*x^65*b^4*a^8 + 55/13*x^52*b^3*a^9 + 22/13*x^39*b
^2*a^10 + 6/13*x^26*b*a^11 + 1/13*x^13*a^12
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{\left(bx^{13} + a\right)^{13}}{169\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="giac")
```

```
[Out] 1/169*(b*x^13 + a)^13/b
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{169}b^{12}x^{169}+\frac{1}{13}a\,b^{11}x^{156}+\frac{6}{13}a^{2}b^{10}x^{143}+\frac{22}{13}a^{3}b^{9}x^{130}+\frac{55}{13}a^{4}b^{8}x^{117}+\frac{99}{13}a^{5}b^{7}x^{104}+\frac{132}{13}a^{6}b^{6}x^{91}+\frac{132}{13}a^{7}b^{5}x^{78}+\frac{99}{13}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^13+a)^12,x)
```

```
[Out] 1/169*b^12*x^169+1/13*a*b^11*x^156+6/13*a^2*b^10*x^143+22/13*a^3*b^9*x^130+
55/13*a^4*b^8*x^117+99/13*a^5*b^7*x^104+132/13*a^6*b^6*x^91+132/13*a^7*b^5*
x^78+99/13*a^8*b^4*x^65+55/13*a^9*b^3*x^52+22/13*a^10*b^2*x^39+6/13*a^11*b*
x^26+1/13*a^12*x^13
```

maxima [A] time = 1.33, size = 14, normalized size = 0.88

$$\frac{\left(bx^{13} + a\right)^{13}}{169\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^13+a)^12,x, algorithm="maxima")
```

```
[Out] 1/169*(b*x^13 + a)^13/b
```

mupad [B] time = 5.33, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^{13} + a\right)^{13}}{169\, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a + b*x^13)^12,x)
```

```
[Out] (a + b*x^13)^13/(169*b)
```

sympy [B] time = 0.11, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{13}}{13}+\frac{6a^{11}bx^{26}}{13}+\frac{22a^{10}b^{2}x^{39}}{13}+\frac{55a^{9}b^{3}x^{52}}{13}+\frac{99a^{8}b^{4}x^{65}}{13}+\frac{132a^{7}b^{5}x^{78}}{13}+\frac{132a^{6}b^{6}x^{91}}{13}+\frac{99a^{5}b^{7}x^{104}}{13}+\frac{55a^{4}b^{8}x^{117}}{13}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**13+a)**12,x)
```

```
[Out] a**12*x**13/13 + 6*a**11*b*x**26/13 + 22*a**10*b**2*x**39/13 + 55*a**9*b**3
*x**52/13 + 99*a**8*b**4*x**65/13 + 132*a**7*b**5*x**78/13 + 132*a**6*b**6*
x**91/13 + 99*a**5*b**7*x**104/13 + 55*a**4*b**8*x**117/13 + 22*a**3*b**9*x
**130/13 + 6*a**2*b**10*x**143/13 + a*b**11*x**156/13 + b**12*x**169/169
```

3.350 $\int x^{12} \left(ax+bx^{26}\right)^{12} \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1584, 261}

$$\frac{\left(a+bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{12}\left(ax+bx^{26}\right)^{12}\,dx &= \int x^{24}\left(a+bx^{25}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{25}\right)^{13}}{325b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6}{25}a^{11}bx^{50}+\frac{22}{25}a^{10}b^2x^{75}+\frac{11}{5}a^9b^3x^{100}+\frac{99}{25}a^8b^4x^{125}+\frac{132}{25}a^7b^5x^{150}+\frac{132}{25}a^6b^6x^{175}+\frac{99}{25}a^5b^7x^{200}+\frac{11}{5}a^4b^8x^{225}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x + b*x^26)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{325}x^{325}b^{12}+\frac{1}{25}x^{300}b^{11}a+\frac{6}{25}x^{275}b^{10}a^2+\frac{22}{25}x^{250}b^9a^3+\frac{11}{5}x^{225}b^8a^4+\frac{99}{25}x^{200}b^7a^5+\frac{132}{25}x^{175}b^6a^6+\frac{132}{25}x^{150}b^5a^7+\frac{99}{25}x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

giac [B] time = 0.16, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}a\,b^{11}x^{300}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{132}{25}a^{7}b^{5}x^{150}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^26+a*x)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*a*b^11*x^300+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

maxima [B] time = 1.34, size = 134, normalized size = 8.38

$$\frac{1}{325}\,b^{12}x^{325}+\frac{1}{25}\,ab^{11}x^{300}+\frac{6}{25}\,a^{2}b^{10}x^{275}+\frac{22}{25}\,a^{3}b^{9}x^{250}+\frac{11}{5}\,a^{4}b^{8}x^{225}+\frac{99}{25}\,a^{5}b^{7}x^{200}+\frac{132}{25}\,a^{6}b^{6}x^{175}+\frac{132}{25}\,a^{7}b^{5}x^{150}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^26+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/325*b^12*x^325 + 1/25*a*b^11*x^300 + 6/25*a^2*b^10*x^275 + 22/25*a^3*b^9*
x^250 + 11/5*a^4*b^8*x^225 + 99/25*a^5*b^7*x^200 + 132/25*a^6*b^6*x^175 + 1
32/25*a^7*b^5*x^150 + 99/25*a^8*b^4*x^125 + 11/5*a^9*b^3*x^100 + 22/25*a^10
*b^2*x^75 + 6/25*a^11*b*x^50 + 1/25*a^12*x^25
```

mupad [B] time = 0.00, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{25}}{25}+\frac{6\,a^{11}\,b\,x^{50}}{25}+\frac{22\,a^{10}\,b^{2}\,x^{75}}{25}+\frac{11\,a^{9}\,b^{3}\,x^{100}}{5}+\frac{99\,a^{8}\,b^{4}\,x^{125}}{25}+\frac{132\,a^{7}\,b^{5}\,x^{150}}{25}+\frac{132\,a^{6}\,b^{6}\,x^{175}}{25}+\frac{99\,a^{5}\,b^{7}\,x^{200}}{25}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a*x + b*x^26)^12,x)
```

```
[Out] (a^12*x^25)/25 + (b^12*x^325)/325 + (6*a^11*b*x^50)/25 + (a*b^11*x^300)/25
+ (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x^100)/5 + (99*a^8*b^4*x^125)/25 + (1
32*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^175)/25 + (99*a^5*b^7*x^200)/25 + (11
*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/25 + (6*a^2*b^10*x^275)/25
```

sympy [B] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^2x^{75}}{25}+\frac{11a^9b^3x^{100}}{5}+\frac{99a^8b^4x^{125}}{25}+\frac{132a^7b^5x^{150}}{25}+\frac{132a^6b^6x^{175}}{25}+\frac{99a^5b^7x^{200}}{25}+\frac{11a^4b^8x^{225}}{5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**26+a*x)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

3.351 $\int x^{12} \left(ax^2+bx^{39}\right)^{12} \, dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1584, 261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{12}\left(ax^2+bx^{39}\right)^{12} dx &= \int x^{36}\left(a+bx^{37}\right)^{12} dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^12*(a*x^2 + b*x^39)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^2+\frac{22}{37}x^{370}b^9a^3+\frac{55}{37}x^{333}b^8a^4+\frac{99}{37}x^{296}b^7a^5+\frac{132}{37}x^{259}b^6a^6+\frac{132}{37}x^{222}b^5a^7+\frac{99}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [B] time = 0.15, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{9}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{99}{37}a^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(b*x^39+a*x^2)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [B] time = 1.35, size = 134, normalized size = 8.38

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}ab^{11}x^{444}+\frac{6}{37}a^2b^{10}x^{407}+\frac{22}{37}a^3b^9x^{370}+\frac{55}{37}a^4b^8x^{333}+\frac{99}{37}a^5b^7x^{296}+\frac{132}{37}a^6b^6x^{259}+\frac{132}{37}a^7b^5x^{222}+\frac{9}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^12*(b*x^39+a*x^2)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

mupad [B] time = 5.22, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+\frac{55}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^12*(a*x^2 + b*x^39)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

sympy [B] time = 0.14, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{3}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**12*(b*x**39+a*x**2)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.352 $\int x^{24} \left(a + bx^{25}\right)^{12} dx$

Optimal. Leaf size=16

$$\frac{\left(a + bx^{25}\right)^{13}}{325b}$$

```
[Out] 1/325*(b*x^25+a)^13/b
```

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a + bx^{25}\right)^{13}}{325b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a + b*x^25)^12,x]
```

```
[Out] (a + b*x^25)^13/(325*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{24} \left(a + bx^{25}\right)^{12} dx = \frac{\left(a + bx^{25}\right)^{13}}{325b}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25} + \frac{6}{25}a^{11}bx^{50} + \frac{22}{25}a^{10}b^2x^{75} + \frac{11}{5}a^9b^3x^{100} + \frac{99}{25}a^8b^4x^{125} + \frac{132}{25}a^7b^5x^{150} + \frac{132}{25}a^6b^6x^{175} + \frac{99}{25}a^5b^7x^{200} + \frac{11}{5}a^4b^8x^{225}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a + b*x^25)^12,x]
```

```
[Out] (a^12*x^25)/25 + (6*a^11*b*x^50)/25 + (22*a^10*b^2*x^75)/25 + (11*a^9*b^3*x
^100)/5 + (99*a^8*b^4*x^125)/25 + (132*a^7*b^5*x^150)/25 + (132*a^6*b^6*x^1
75)/25 + (99*a^5*b^7*x^200)/25 + (11*a^4*b^8*x^225)/5 + (22*a^3*b^9*x^250)/
25 + (6*a^2*b^10*x^275)/25 + (a*b^11*x^300)/25 + (b^12*x^325)/325
```

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{325}x^{325}b^{12} + \frac{1}{25}x^{300}b^{11}a + \frac{6}{25}x^{275}b^{10}a^2 + \frac{22}{25}x^{250}b^9a^3 + \frac{11}{5}x^{225}b^8a^4 + \frac{99}{25}x^{200}b^7a^5 + \frac{132}{25}x^{175}b^6a^6 + \frac{132}{25}x^{150}b^5a^7 + \frac{99}{25}x^{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="fricas")
```

```
[Out] 1/325*x^325*b^12 + 1/25*x^300*b^11*a + 6/25*x^275*b^10*a^2 + 22/25*x^250*b^
9*a^3 + 11/5*x^225*b^8*a^4 + 99/25*x^200*b^7*a^5 + 132/25*x^175*b^6*a^6 + 1
```

```
32/25*x^150*b^5*a^7 + 99/25*x^125*b^4*a^8 + 11/5*x^100*b^3*a^9 + 22/25*x^75
*b^2*a^10 + 6/25*x^50*b*a^11 + 1/25*x^25*a^12
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="giac")
```

```
[Out] 1/325*(b*x^25 + a)^13/b
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{325}b^{12}x^{325}+\frac{1}{25}a\,b^{11}x^{300}+\frac{6}{25}a^{2}b^{10}x^{275}+\frac{22}{25}a^{3}b^{9}x^{250}+\frac{11}{5}a^{4}b^{8}x^{225}+\frac{99}{25}a^{5}b^{7}x^{200}+\frac{132}{25}a^{6}b^{6}x^{175}+\frac{132}{25}a^{7}b^{5}x^{150}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^25+a)^12,x)
```

```
[Out] 1/325*b^12*x^325+1/25*a*b^11*x^300+6/25*a^2*b^10*x^275+22/25*a^3*b^9*x^250+
11/5*a^4*b^8*x^225+99/25*a^5*b^7*x^200+132/25*a^6*b^6*x^175+132/25*a^7*b^5*
x^150+99/25*a^8*b^4*x^125+11/5*a^9*b^3*x^100+22/25*a^10*b^2*x^75+6/25*a^11*
b*x^50+1/25*a^12*x^25
```

maxima [A] time = 1.35, size = 14, normalized size = 0.88

$$\frac{\left(bx^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^25+a)^12,x, algorithm="maxima")
```

```
[Out] 1/325*(b*x^25 + a)^13/b
```

mupad [B] time = 5.20, size = 14, normalized size = 0.88

$$\frac{\left(b\,x^{25}+a\right)^{13}}{325\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(a + b*x^25)^12,x)
```

```
[Out] (a + b*x^25)^13/(325*b)
```

sympy [B] time = 0.11, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{25}}{25}+\frac{6a^{11}bx^{50}}{25}+\frac{22a^{10}b^{2}x^{75}}{25}+\frac{11a^{9}b^{3}x^{100}}{5}+\frac{99a^{8}b^{4}x^{125}}{25}+\frac{132a^{7}b^{5}x^{150}}{25}+\frac{132a^{6}b^{6}x^{175}}{25}+\frac{99a^{5}b^{7}x^{200}}{25}+\frac{11a^{4}b^{8}x^{22}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**25+a)**12,x)
```

```
[Out] a**12*x**25/25 + 6*a**11*b*x**50/25 + 22*a**10*b**2*x**75/25 + 11*a**9*b**3
*x**100/5 + 99*a**8*b**4*x**125/25 + 132*a**7*b**5*x**150/25 + 132*a**6*b**
6*x**175/25 + 99*a**5*b**7*x**200/25 + 11*a**4*b**8*x**225/5 + 22*a**3*b**9
*x**250/25 + 6*a**2*b**10*x**275/25 + a*b**11*x**300/25 + b**12*x**325/325
```

3.353 $\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

```
[Out] 1/481*(b*x^37+a)^13/b
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1584, 261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

```
[In] Int[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a + b*x^37)^13/(481*b)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int x^{24}\left(ax+bx^{38}\right)^{12}\,dx &= \int x^{36}\left(a+bx^{37}\right)^{12}\,dx\\ &= \frac{\left(a+bx^{37}\right)^{13}}{481b}\end{aligned}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8x^{33}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^24*(a*x + b*x^38)^12,x]
```

```
[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481
```

fricas [B] time = 0.35, size = 134, normalized size = 8.38

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^2+\frac{22}{37}x^{370}b^9a^3+\frac{55}{37}x^{333}b^8a^4+\frac{99}{37}x^{296}b^7a^5+\frac{132}{37}x^{259}b^6a^6+\frac{132}{37}x^{222}b^5a^7+\frac{99}{37}x^1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="fricas")
```

```
[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [B] time = 0.21, size = 134, normalized size = 8.38

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="giac")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{9}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(b*x^38+a*x)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [B] time = 1.31, size = 134, normalized size = 8.38

$$\frac{1}{481}\,b^{12}x^{481}+\frac{1}{37}\,ab^{11}x^{444}+\frac{6}{37}\,a^{2}b^{10}x^{407}+\frac{22}{37}\,a^{3}b^{9}x^{370}+\frac{55}{37}\,a^{4}b^{8}x^{333}+\frac{99}{37}\,a^{5}b^{7}x^{296}+\frac{132}{37}\,a^{6}b^{6}x^{259}+\frac{132}{37}\,a^{7}b^{5}x^{222}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^24*(b*x^38+a*x)^12,x, algorithm="maxima")
```

```
[Out] 1/481*b^12*x^481 + 1/37*a*b^11*x^444 + 6/37*a^2*b^10*x^407 + 22/37*a^3*b^9*
x^370 + 55/37*a^4*b^8*x^333 + 99/37*a^5*b^7*x^296 + 132/37*a^6*b^6*x^259 +
132/37*a^7*b^5*x^222 + 99/37*a^8*b^4*x^185 + 55/37*a^9*b^3*x^148 + 22/37*a^
10*b^2*x^111 + 6/37*a^11*b*x^74 + 1/37*a^12*x^37
```

mupad [B] time = 0.00, size = 134, normalized size = 8.38

$$\frac{a^{12}\,x^{37}}{37}+\frac{6\,a^{11}\,b\,x^{74}}{37}+\frac{22\,a^{10}\,b^2\,x^{111}}{37}+\frac{55\,a^9\,b^3\,x^{148}}{37}+\frac{99\,a^8\,b^4\,x^{185}}{37}+\frac{132\,a^7\,b^5\,x^{222}}{37}+\frac{132\,a^6\,b^6\,x^{259}}{37}+\frac{99\,a^5\,b^7\,x^{296}}{37}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^24*(a*x + b*x^38)^12,x)
```

```
[Out] (a^12*x^37)/37 + (b^12*x^481)/481 + (6*a^11*b*x^74)/37 + (a*b^11*x^444)/37
+ (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*x^148)/37 + (99*a^8*b^4*x^185)/37 +
(132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x^259)/37 + (99*a^5*b^7*x^296)/37 + (
55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^370)/37 + (6*a^2*b^10*x^407)/37
```

sympy [B] time = 0.13, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**24*(b*x**38+a*x)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.354 $\int x^{36} \left(a+bx^{37}\right)^{12} dx$

Optimal. Leaf size=16

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

[Out] 1/481*(b*x^37+a)^13/b

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Antiderivative was successfully verified.

[In] Int[x^36*(a + b*x^37)^12,x]

[Out] (a + b*x^37)^13/(481*b)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{36} \left(a+bx^{37}\right)^{12} dx = \frac{\left(a+bx^{37}\right)^{13}}{481b}$$

Mathematica [B] time = 0.01, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6}{37}a^{11}bx^{74}+\frac{22}{37}a^{10}b^2x^{111}+\frac{55}{37}a^9b^3x^{148}+\frac{99}{37}a^8b^4x^{185}+\frac{132}{37}a^7b^5x^{222}+\frac{132}{37}a^6b^6x^{259}+\frac{99}{37}a^5b^7x^{296}+\frac{55}{37}a^4b^8$$

Antiderivative was successfully verified.

[In] Integrate[x^36*(a + b*x^37)^12,x]

[Out] (a^12*x^37)/37 + (6*a^11*b*x^74)/37 + (22*a^10*b^2*x^111)/37 + (55*a^9*b^3*
x^148)/37 + (99*a^8*b^4*x^185)/37 + (132*a^7*b^5*x^222)/37 + (132*a^6*b^6*x
^259)/37 + (99*a^5*b^7*x^296)/37 + (55*a^4*b^8*x^333)/37 + (22*a^3*b^9*x^37
0)/37 + (6*a^2*b^10*x^407)/37 + (a*b^11*x^444)/37 + (b^12*x^481)/481

fricas [B] time = 0.34, size = 134, normalized size = 8.38

$$\frac{1}{481}x^{481}b^{12}+\frac{1}{37}x^{444}b^{11}a+\frac{6}{37}x^{407}b^{10}a^2+\frac{22}{37}x^{370}b^9a^3+\frac{55}{37}x^{333}b^8a^4+\frac{99}{37}x^{296}b^7a^5+\frac{132}{37}x^{259}b^6a^6+\frac{132}{37}x^{222}b^5a^7+\frac{99}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="fricas")

[Out] 1/481*x^481*b^12 + 1/37*x^444*b^11*a + 6/37*x^407*b^10*a^2 + 22/37*x^370*b^
9*a^3 + 55/37*x^333*b^8*a^4 + 99/37*x^296*b^7*a^5 + 132/37*x^259*b^6*a^6 +

```
132/37*x^222*b^5*a^7 + 99/37*x^185*b^4*a^8 + 55/37*x^148*b^3*a^9 + 22/37*x^
111*b^2*a^10 + 6/37*x^74*b*a^11 + 1/37*x^37*a^12
```

giac [A] time = 0.15, size = 14, normalized size = 0.88

$$\frac{(bx^{37}+a)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="giac")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

maple [B] time = 0.04, size = 135, normalized size = 8.44

$$\frac{1}{481}b^{12}x^{481}+\frac{1}{37}a\,b^{11}x^{444}+\frac{6}{37}a^{2}b^{10}x^{407}+\frac{22}{37}a^{3}b^{9}x^{370}+\frac{55}{37}a^{4}b^{8}x^{333}+\frac{99}{37}a^{5}b^{7}x^{296}+\frac{132}{37}a^{6}b^{6}x^{259}+\frac{132}{37}a^{7}b^{5}x^{222}+\frac{99}{37}a^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^36*(b*x^37+a)^12,x)
```

```
[Out] 1/481*b^12*x^481+1/37*a*b^11*x^444+6/37*a^2*b^10*x^407+22/37*a^3*b^9*x^370+
55/37*a^4*b^8*x^333+99/37*a^5*b^7*x^296+132/37*a^6*b^6*x^259+132/37*a^7*b^5
*x^222+99/37*a^8*b^4*x^185+55/37*a^9*b^3*x^148+22/37*a^10*b^2*x^111+6/37*a^
11*b*x^74+1/37*a^12*x^37
```

maxima [A] time = 1.30, size = 14, normalized size = 0.88

$$\frac{(bx^{37}+a)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^36*(b*x^37+a)^12,x, algorithm="maxima")
```

```
[Out] 1/481*(b*x^37 + a)^13/b
```

mupad [B] time = 5.18, size = 14, normalized size = 0.88

$$\frac{(b\,x^{37}+a)^{13}}{481\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^36*(a + b*x^37)^12,x)
```

```
[Out] (a + b*x^37)^13/(481*b)
```

sympy [B] time = 0.12, size = 160, normalized size = 10.00

$$\frac{a^{12}x^{37}}{37}+\frac{6a^{11}bx^{74}}{37}+\frac{22a^{10}b^{2}x^{111}}{37}+\frac{55a^{9}b^{3}x^{148}}{37}+\frac{99a^{8}b^{4}x^{185}}{37}+\frac{132a^{7}b^{5}x^{222}}{37}+\frac{132a^{6}b^{6}x^{259}}{37}+\frac{99a^{5}b^{7}x^{296}}{37}+\frac{55a^{4}b^{8}x^{333}}{37}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**36*(b*x**37+a)**12,x)
```

```
[Out] a**12*x**37/37 + 6*a**11*b*x**74/37 + 22*a**10*b**2*x**111/37 + 55*a**9*b**
3*x**148/37 + 99*a**8*b**4*x**185/37 + 132*a**7*b**5*x**222/37 + 132*a**6*b
**6*x**259/37 + 99*a**5*b**7*x**296/37 + 55*a**4*b**8*x**333/37 + 22*a**3*b
**9*x**370/37 + 6*a**2*b**10*x**407/37 + a*b**11*x**444/37 + b**12*x**481/4
81
```

3.355 $\int \frac{1}{ax+bx^n} dx$

Optimal. Leaf size=23

$$\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

```
[Out] ln(b+a*x^(1-n))/a/(1-n)
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^n)^(-1),x]
```

```
[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx^n}\,dx &= \int \frac{x^{-n}}{b+ax^{1-n}}\,dx\\ &= \frac{\log\left(b+ax^{1-n}\right)}{a(1-n)}\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{\log\left(ax^{1-n}+b\right)}{a(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^n)^(-1),x]
```

```
[Out] Log[b + a*x^(1 - n)]/(a*(1 - n))
```

fricas [A] time = 0.40, size = 27, normalized size = 1.17

$$\frac{n\log(x)-\log\left(ax+bx^n\right)}{an-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="fricas")
```

```
[Out] (n*log(x) - log(a*x + b*x^n))/(a*n - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax + bx^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/(a*x + b*x^n), x)
```

maple [A] time = 0.05, size = 36, normalized size = 1.57

$$\frac{n \ln(x)}{(n-1)\, a} - \frac{\ln\left(ax + b\, \mathrm{e}^{n \ln(x)}\right)}{(n-1)\, a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^n),x)
```

```
[Out] n/a/(n-1)*ln(x)-1/a/(n-1)*ln(a*x+b*exp(n*ln(x)))
```

maxima [A] time = 1.42, size = 37, normalized size = 1.61

$$\frac{n \log(x)}{a(n-1)} - \frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^n),x, algorithm="maxima")
```

```
[Out] n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1))
```

mupad [B] time = 5.26, size = 26, normalized size = 1.13

$$-\frac{\ln\left(b\, x^n + a\, x\right) - n\, \ln(x)}{a\, (n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n + a*x),x)
```

```
[Out] -(log(b*x^n + a*x) - n*log(x))/(a*(n - 1))
```

sympy [A] time = 0.68, size = 53, normalized size = 2.30

$$\begin{cases} \tilde{\infty} \log{\left(x \right)} & \text{for}\: a = 0 \wedge b = 0 \wedge n = 1 \\ - \frac{x}{b \left(n x^{n} - x^{n}\right)} & \text{for}\: a = 0 \\ \frac{\log{\left(x \right)}}{a + b} & \text{for}\: n = 1 \\ \frac{\log{\left(x \right)}}{a} & \text{for}\: b = 0 \\ \frac{n \log{\left(x \right)}}{a n - a} - \frac{\log{\left(\frac{a x}{b} + x^{n} \right)}}{a n - a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**n),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 1)), (-x/(b*(n*x**n - x*
*n)), Eq(a, 0)), (log(x)/(a + b), Eq(n, 1)), (log(x)/a, Eq(b, 0)), (n*log(x
)/(a*n - a) - log(a*x/b + x**n)/(a*n - a), True))
```

3.356 $\int \frac{1}{ax+bx^{1+n}}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}$$

```
[Out] ln(x)/a-ln(a+b*x^n)/a/n
```

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log\left(a+bx^n\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^(1 + n))^(-1),x]
```

```
[Out] Log[x]/a - Log[a + b*x^n]/(a*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{ax+bx^{1+n}}\,dx = \int \frac{1}{x(a+bx^n)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{an} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, x^n\right)}{an}$$
$$= \frac{\log(x)}{a} - \frac{\log(a+bx^n)}{an}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.96

$$\frac{n\log(x) - \log(a+bx^n)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^(1 + n))^(-1),x]
[Out] (n*Log[x] - Log[a + b*x^n])/(a*n)
```

fricas [A] time = 0.41, size = 28, normalized size = 1.22

$$\frac{(n+1)\log(x) - \log\left(ax + bx^{n+1}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="fricas")
[Out] ((n + 1)*log(x) - log(a*x + b*x^(n + 1)))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax+bx^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="giac")
[Out] integrate(1/(a*x + b*x^(n + 1)), x)
```

maple [A] time = 0.05, size = 39, normalized size = 1.70

$$\frac{\ln(x)}{a} + \frac{\ln(x)}{an} - \frac{\ln\left(ax + b\,e^{(n+1)\ln(x)}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(n+1)),x)
[Out] 1/a*ln(x)+1/a/n*ln(x)-1/a/n*ln(a*x+b*exp((n+1)*ln(x)))
```

maxima [A] time = 1.33, size = 27, normalized size = 1.17

$$\frac{\log(x)}{a} - \frac{\log\left(\frac{bx^n+a}{b}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1+n)),x, algorithm="maxima")
```

```
[Out] log(x)/a - log((b*x^n + a)/b)/(a*n)
```

mupad [B] time = 5.23, size = 31, normalized size = 1.35

$$\frac{\ln(x)\ (n+1)}{a\,n}-\frac{\ln\left(x\ \left(a+b\,x^n\right)\right)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(n + 1)),x)
```

```
[Out] (log(x)*(n + 1))/(a*n) - log(x*(a + b*x^n))/(a*n)
```

sympy [A] time = 1.81, size = 41, normalized size = 1.78

$$\begin{cases} \tilde{\infty} \log{\left(x \right)} & \text{for}\: a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log{\left(x \right)}}{a + b} & \text{for}\: n = 0 \\ - \frac{x^{- n}}{b n} & \text{for}\: a = 0 \\ \frac{\log{\left(x \right)}}{a} & \text{for}\: b = 0 \\ \frac{\log{\left(x \right)}}{a} - \frac{\log{\left(\frac{a}{b} + x^{n} \right)}}{a n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**(1+n)),x)
```

```
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq
(n, 0)), (-x**(-n)/(b*n), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/a - log(
a/b + x**n)/(a*n), True))
```

3.357 $\int \frac{1}{ax+bx^{1-n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(ax^n+b)}{an}$$

[Out] ln(b+a*x^n)/a/n

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1593, 260}

$$\frac{\log(ax^n+b)}{an}$$

Antiderivative was successfully verified.

[In] Int[(a*x + b*x^(1 - n))^(-1),x]

[Out] Log[b + a*x^n]/(a*n)

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{ax+bx^{1-n}}\,dx &= \int \frac{x^{-1+n}}{b+ax^n}\,dx\\ &= \frac{\log(b+ax^n)}{an}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(ax^n+b)}{an}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x + b*x^(1 - n))^(-1),x]

[Out] Log[b + a*x^n]/(a*n)

fricas [A] time = 0.42, size = 28, normalized size = 1.87

$$\frac{(n-1)\log(x)+\log\left(ax+bx^{-n+1}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="fricas")

```
[Out] ((n - 1)*log(x) + log(a*x + b*x^(-n + 1)))/(a*n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ax + bx^{-n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="giac")
[Out] integrate(1/(a*x + b*x^(-n + 1)), x)
```

maple [B] time = 0.06, size = 41, normalized size = 2.73

$$\frac{\ln(x)}{a} - \frac{\ln(x)}{an} + \frac{\ln\left(ax + b\,\mathrm{e}^{(-n+1)\ln(x)}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b*x^(-n+1)),x)
[Out] 1/a*ln(x)-1/a/n*ln(x)+1/a/n*ln(a*x+b*exp((-n+1)*ln(x)))
```

maxima [A] time = 1.37, size = 19, normalized size = 1.27

$$\frac{\log\left(\frac{ax^n+b}{a}\right)}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x^(1-n)),x, algorithm="maxima")
[Out] log((a*x^n + b)/a)/(a*n)
```

mupad [B] time = 5.22, size = 34, normalized size = 2.27

$$\frac{\ln\left(a\,x + b\,x^{1-n}\right)}{a\,n} + \frac{\ln(x)\,(n-1)}{a\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^(1 - n)),x)
[Out] log(a*x + b*x^(1 - n))/(a*n) + (log(x)*(n - 1))/(a*n)
```

sympy [A] time = 2.14, size = 39, normalized size = 2.60

$$\begin{cases} \tilde{\infty}\log(x) & \text{for}\ a = 0 \wedge b = 0 \wedge n = 0 \\ \frac{\log(x)}{a+b} & \text{for}\ n = 0 \\ \frac{x^n}{bn} & \text{for}\ a = 0 \\ \frac{\log(x)}{a} & \text{for}\ b = 0 \\ \frac{\log(x)}{a} + \frac{\log\left(\frac{a}{b}+x^{-n}\right)}{an} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b*x**(1-n)),x)
[Out] Piecewise((zoo*log(x), Eq(a, 0) & Eq(b, 0) & Eq(n, 0)), (log(x)/(a + b), Eq
(n, 0)), (x**n/(b*n), Eq(a, 0)), (log(x)/a, Eq(b, 0)), (log(x)/a + log(a/b
+ x**(-n))/(a*n), True))
```

3.358 $\int \frac{1}{2x+3x^{1+n}}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{2} - \frac{\log(3x^n+2)}{2n}$$

```
[Out] 1/2*ln(x)-1/2*ln(2+3*x^n)/n
```

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1593, 266, 36, 29, 31}

$$\frac{\log(x)}{2} - \frac{\log(3x^n+2)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 3*x^(1 + n))^(-1),x]
```

```
[Out] Log[x]/2 - Log[2 + 3*x^n]/(2*n)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{1}{2x+3x^{1+n}}\,dx = \int \frac{1}{x\left(2+3x^n\right)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(2+3x)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^n\right)}{2n} - \frac{3\,\text{Subst}\left(\int \frac{1}{2+3x}\,dx, x, x^n\right)}{2n}$$
$$= \frac{\log(x)}{2} - \frac{\log\left(2+3x^n\right)}{2n}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\frac{n\log(x) - \log\left(3x^n+2\right)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^(1 + n))^(-1),x]
[Out] (n*Log[x] - Log[2 + 3*x^n])/(2*n)
```

fricas [A] time = 0.40, size = 26, normalized size = 1.18

$$\frac{(n+1)\log(x) - \log\left(3\,x^{n+1} + 2\,x\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="fricas")
[Out] 1/2*((n + 1)*log(x) - log(3*x^(n + 1) + 2*x))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3\,x^{n+1} + 2\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="giac")
[Out] integrate(1/(3*x^(n + 1) + 2*x), x)
```

maple [A] time = 0.05, size = 32, normalized size = 1.45

$$\frac{\ln(x)}{2} + \frac{\ln(x)}{2n} - \frac{\ln\left(2x + 3\,e^{(n+1)\ln(x)}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+3*x^(n+1)),x)
[Out] 1/2*ln(x)+1/2/n*ln(x)-1/2/n*ln(2*x+3*exp((n+1)*ln(x)))
```

maxima [A] time = 1.32, size = 16, normalized size = 0.73

$$-\frac{\log\left(x^n + \frac{2}{3}\right)}{2\,n} + \frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1+n)),x, algorithm="maxima")
[Out] -1/2*log(x^n + 2/3)/n + 1/2*log(x)
```

mupad [B] time = 5.23, size = 26, normalized size = 1.18

$$\frac{\ln(x)\,(n+1)}{2\,n} - \frac{\ln\left(\frac{2\,x}{3} + x^{n+1}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + 3*x^(n + 1)),x)
[Out] (log(x)*(n + 1))/(2*n) - log((2*x)/3 + x^(n + 1))/(2*n)
```

sympy [A] time = 1.51, size = 20, normalized size = 0.91

$$\begin{cases} \frac{\log(x)}{2} - \frac{\log\left(x^n + \frac{2}{3}\right)}{2n} & \text{for } n \neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x**(1+n)),x)
[Out] Piecewise((log(x)/2 - log(x**n + 2/3)/(2*n), Ne(n, 0)), (log(x)/5, True))
```

3.359 $\int \frac{1}{2x+3x^{1-n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(2x^n+3)}{2n}$$

```
[Out] 1/2*ln(3+2*x^n)/n
```

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 260}

$$\frac{\log(2x^n+3)}{2n}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3 + 2*x^n]/(2*n)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2x+3x^{1-n}}\,dx &= \int \frac{x^{-1+n}}{3+2x^n}\,dx\\ &= \frac{\log(3+2x^n)}{2n}\end{aligned}$$

Mathematica [A] time = 0.00, size = 15, normalized size = 1.00

$$\frac{\log(2x^n+3)}{2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^(1 - n))^(-1),x]
```

```
[Out] Log[3 + 2*x^n]/(2*n)
```

fricas [A] time = 0.41, size = 26, normalized size = 1.73

$$\frac{(n-1)\log(x)+\log\left(3\,x^{-n+1}+2\,x\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="fricas")
```

```
[Out] 1/2*((n - 1)*log(x) + log(3*x^(-n + 1) + 2*x))/n
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3\,x^{-n+1}+2\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="giac")
[Out] integrate(1/(3*x^(-n + 1) + 2*x), x)
```

maple [B] time = 0.05, size = 34, normalized size = 2.27

$$\frac{\ln(x)}{2}-\frac{\ln(x)}{2n}+\frac{\ln\left(2x+3\,\mathrm{e}^{(-n+1)\ln(x)}\right)}{2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x+3*x^(-n+1)),x)
[Out] 1/2*ln(x)-1/2/n*ln(x)+1/2/n*ln(2*x+3*exp((-n+1)*ln(x)))
```

maxima [A] time = 1.32, size = 11, normalized size = 0.73

$$\frac{\log\left(x^n+\frac{3}{2}\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x^(1-n)),x, algorithm="maxima")
[Out] 1/2*log(x^n + 3/2)/n
```

mupad [B] time = 5.20, size = 28, normalized size = 1.87

$$\frac{\ln\left(\frac{2\,x}{3}+x^{1-n}\right)}{2\,n}+\frac{\ln(x)\,\left(n-1\right)}{2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x + 3*x^(1 - n)),x)
[Out] log((2*x)/3 + x^(1 - n))/(2*n) + (log(x)*(n - 1))/(2*n)
```

sympy [A] time = 1.70, size = 22, normalized size = 1.47

$$\begin{cases} \frac{\log(x)}{2}+\frac{\log\left(\frac{2}{3}+x^{-n}\right)}{2n} & \text{for}\ n \neq 0 \\ \frac{\log(x)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x+3*x**(1-n)),x)
[Out] Piecewise((log(x)/2 + log(2/3 + x**(-n))/(2*n), Ne(n, 0)), (log(x)/5, True)
)
```

3.360 $\int \frac{1}{-\sqrt{x}+x}\,dx$

Optimal. Leaf size=12

$$2\log\left(1-\sqrt{x}\right)$$

```
[Out] 2*ln(1-x^(1/2))
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$2\log\left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[x] + x)^(-1),x]
```

```
[Out] 2*Log[1 - Sqrt[x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\sqrt{x}+x}\,dx &= \int \frac{1}{\left(-1+\sqrt{x}\right)\sqrt{x}}\,dx\\ &= 2\log\left(1-\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$2\log\left(1-\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[x] + x)^(-1),x]
```

```
[Out] 2*Log[1 - Sqrt[x]]
```

fricas [A] time = 0.39, size = 8, normalized size = 0.67

$$2\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*log(sqrt(x) - 1)
```

giac [A] time = 0.15, size = 9, normalized size = 0.75

$$2\,\log\left(\left|\sqrt{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="giac")
[Out] 2*log(abs(sqrt(x) - 1))
```

maple [A] time = 0.05, size = 12, normalized size = 1.00

$$-2\operatorname{arctanh}\left(\sqrt{x}\right)+\ln\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-x^(1/2)),x)
[Out] ln(x-1)-2*arctanh(x^(1/2))
```

maxima [A] time = 1.33, size = 8, normalized size = 0.67

$$2\,\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x^(1/2)),x, algorithm="maxima")
[Out] 2*log(sqrt(x) - 1)
```

mupad [B] time = 0.10, size = 8, normalized size = 0.67

$$2\,\ln\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^(1/2)),x)
[Out] 2*log(x^(1/2) - 1)
```

sympy [A] time = 0.18, size = 8, normalized size = 0.67

$$2\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-x**(1/2)),x)
[Out] 2*log(sqrt(x) - 1)
```

3.361 $\int \frac{1}{-x^{3/5}+x}\,dx$

Optimal. Leaf size=14

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

```
[Out] 5/2*ln(1-x^(2/5))
```

Rubi [A] time = 0.00, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 260}

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(3/5) + x)^(-1),x]
```

```
[Out] (5*Log[1 - x^(2/5)])/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-x^{3/5}+x}\,dx &= \int \frac{1}{\left(-1+x^{2/5}\right)x^{3/5}}\,dx\\ &= \frac{5}{2}\log\left(1-x^{2/5}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$\frac{5}{2}\log\left(1-x^{2/5}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(3/5) + x)^(-1),x]
```

```
[Out] (5*Log[1 - x^(2/5)])/2
```

fricas [A] time = 0.39, size = 8, normalized size = 0.57

$$\frac{5}{2}\,\log\left(x^{\frac{2}{5}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="fricas")
```

```
[Out] 5/2*log(x^(2/5) - 1)
```

giac [A] time = 0.20, size = 18, normalized size = 1.29

$$\frac{5}{2}\,\log\left(x^{\frac{1}{5}}+1\right)+\frac{5}{2}\,\log\left(\left|x^{\frac{1}{5}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="giac")
[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(abs(x^(1/5) - 1))
```

maple [B] time = 0.44, size = 116, normalized size = 8.29

$$2\ln\left(x^{\frac{1}{5}}+1\right)+\frac{\ln\left(x-1\right)}{2}+\frac{\ln\left(x+1\right)}{2}+2\ln\left(x^{\frac{1}{5}}-1\right)-\frac{\ln\left(2x^{\frac{2}{5}}-\sqrt{5}\,x^{\frac{1}{5}}-x^{\frac{1}{5}}+2\right)}{2}-\frac{\ln\left(2x^{\frac{2}{5}}-\sqrt{5}\,x^{\frac{1}{5}}+x^{\frac{1}{5}}+2\right)}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(3/5)+x),x)
[Out] 1/2*ln(x+1)+1/2*ln(x-1)+2*ln(x^(1/5)-1)-1/2*ln(-5^(1/2)*x^(1/5)+2*x^(2/5)+x
^(1/5)+2)-1/2*ln(5^(1/2)*x^(1/5)+2*x^(2/5)+x^(1/5)+2)-1/2*ln(2*x^(2/5)-5^(1
/2)*x^(1/5)-x^(1/5)+2)-1/2*ln(2*x^(2/5)+5^(1/2)*x^(1/5)-x^(1/5)+2)+2*ln(x^(
1/5)+1)
```

maxima [A] time = 1.30, size = 17, normalized size = 1.21

$$\frac{5}{2}\,\log\left(x^{\frac{1}{5}}+1\right)+\frac{5}{2}\,\log\left(x^{\frac{1}{5}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(3/5)+x),x, algorithm="maxima")
[Out] 5/2*log(x^(1/5) + 1) + 5/2*log(x^(1/5) - 1)
```

mupad [B] time = 5.30, size = 8, normalized size = 0.57

$$\frac{5\,\ln\left(x^{2/5}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x - x^(3/5)),x)
[Out] (5*log(x^(2/5) - 1))/2
```

sympy [B] time = 0.37, size = 22, normalized size = 1.57

$$\frac{5\log\left(\sqrt[5]{x}-1\right)}{2}+\frac{5\log\left(\sqrt[5]{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(3/5)+x),x)
[Out] 5*log(x**(1/5) - 1)/2 + 5*log(x**(1/5) + 1)/2
```

3.362 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx$

Optimal. Leaf size=12

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

```
[Out] 3/4*ln(1+x^(4/3))
```

Rubi [A] time = 0.00, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1593, 260}

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x)^(-1),x]
[Out] (3*Log[1 + x^(4/3)])/4
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x}\,dx &= \int \frac{\sqrt[3]{x}}{1+x^{4/3}}\,dx\\ &= \frac{3}{4}\log\left(1+x^{4/3}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 12, normalized size = 1.00

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x)^(-1),x]
[Out] (3*Log[1 + x^(4/3)])/4
```

fricas [A] time = 0.39, size = 8, normalized size = 0.67

$$\frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
[Out] 3/4*log(x^(4/3) + 1)
```

giac [B] time = 0.15, size = 32, normalized size = 2.67

$$\frac{3}{4}\,\log\left(\sqrt{2}\,x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{3}{4}\,\log\left(-\sqrt{2}\,x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
[Out] 3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3)
 + 1)
```

maple [A] time = 0.04, size = 9, normalized size = 0.75

$$\frac{3\ln\left(x^{\frac{4}{3}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+x),x)
[Out] 3/4*ln(1+x^(4/3))
```

maxima [A] time = 2.89, size = 8, normalized size = 0.67

$$\frac{3}{4}\,\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="maxima")
[Out] 3/4*log(x^(4/3) + 1)
```

mupad [B] time = 0.09, size = 8, normalized size = 0.67

$$\frac{3\,\ln\left(x^{4/3}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + 1/x^(1/3)),x)
[Out] (3*log(x^(4/3) + 1))/4
```

sympy [A] time = 0.26, size = 10, normalized size = 0.83

$$\frac{3\log\left(x^{\frac{4}{3}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+x),x)
[Out] 3*log(x**(4/3) + 1)/4
```

3.363 $\int \frac{1}{x+x^{\sqrt{2}}} \, dx$

Optimal. Leaf size=24

$$\log(x) - \left(1+\sqrt{2}\right)\log\left(x^{\sqrt{2}-1}+1\right)$$

```
[Out] ln(x)-ln(1+x^(2^(1/2)-1))*(1+2^(1/2))
```

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {1593, 266, 36, 29, 31}

$$\log(x) - \left(1+\sqrt{2}\right)\log\left(x^{\sqrt{2}-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^Sqrt[2])^(-1),x]
```

```
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x+x^{\sqrt{2}}} \, dx &= \int \frac{1}{x\left(1+x^{-1+\sqrt{2}}\right)} \, dx \\
&= \left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x(1+x)} \, dx, x, x^{-1+\sqrt{2}}\right) \\
&= \left(-1-\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{1+x} \, dx, x, x^{-1+\sqrt{2}}\right) + \left(1+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{x} \, dx, x, x^{-1+\sqrt{2}}\right) \\
&= \log(x) - \left(1+\sqrt{2}\right)\log\left(1+x^{-1+\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$\log(x) - \left(1 + \sqrt{2}\right)\log\left(x^{\sqrt{2}-1} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^Sqrt[2])^(-1),x]
[Out] Log[x] - (1 + Sqrt[2])*Log[1 + x^(-1 + Sqrt[2])]
```

fricas [A] time = 0.42, size = 24, normalized size = 1.00

$$-\left(\sqrt{2} + 1\right)\log\left(x + x^{\left(\sqrt{2}\right)}\right) + \left(\sqrt{2} + 2\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="fricas")
[Out] -(sqrt(2) + 1)*log(x + x^sqrt(2)) + (sqrt(2) + 2)*log(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + x^{\left(\sqrt{2}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="giac")
[Out] integrate(1/(x + x^sqrt(2)), x)
```

maple [A] time = 0.16, size = 39, normalized size = 1.62

$$\sqrt{2}\ \ln(x) + 2\ln(x) - \sqrt{2}\ \ln\left(x + \mathrm{e}^{\sqrt{2}\ \ln(x)}\right) - \ln\left(x + \mathrm{e}^{\sqrt{2}\ \ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+x^(2^(1/2))),x)
[Out] 2^(1/2)*ln(x)+2*ln(x)-ln(x+exp(2^(1/2)*ln(x)))*2^(1/2)-ln(x+exp(2^(1/2)*ln(
x)))
```

maxima [A] time = 2.93, size = 31, normalized size = 1.29

$$\frac{\sqrt{2}\log(x)}{\sqrt{2} - 1} - \frac{\log\left(x + x^{\left(\sqrt{2}\right)}\right)}{\sqrt{2} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x^(2^(1/2))),x, algorithm="maxima")
[Out] sqrt(2)*log(x)/(sqrt(2) - 1) - log(x + x^sqrt(2))/(sqrt(2) - 1)
```

mupad [B] time = 5.25, size = 26, normalized size = 1.08

$$\ln(x)\,\left(\sqrt{2} + 2\right) - \frac{\ln\left(x + x^{\sqrt{2}}\right)}{\sqrt{2} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + x^(2^(1/2))),x)
```

```
[Out] log(x)*(2^(1/2) + 2) - log(x + x^(2^(1/2)))/(2^(1/2) - 1)
```

sympy [**A**] time = 0.44, size = 32, normalized size = 1.33

$$-\frac{2\log(x)}{-2+\sqrt{2}}+\frac{\sqrt{2}\log\left(x+x^{\sqrt{2}}\right)}{-2+\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+x**(2**(1/2))),x)
```

```
[Out] -2*log(x)/(-2 + sqrt(2)) + sqrt(2)*log(x + x**(sqrt(2)))/(-2 + sqrt(2))
```

3.364 $\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

Optimal. Leaf size=75

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

```
[Out] 2*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))*a^(1/2)/(j-n)-2*(a*x^j+b*x
^n)^(1/2)/(j-n)/(x^(1/2*j))
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2028, 2029, 206}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/((j - n)*x^(j/2)) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x^
(j/2))/Sqrt[a*x^j + b*x^n]])/(j - n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx&=-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+a\int\frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx\\&=-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{(2a)\,\mathrm{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}\\&=-\frac{2x^{-j/2}\sqrt{ax^j+bx^n}}{j-n}+\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{j-n}\end{aligned}$$

Mathematica [A] time = 0.23, size = 104, normalized size = 1.39

$$-\frac{2x^{-j/2}\left(-\sqrt{a}\sqrt{b}\,x^{\frac{j+n}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)+ax^j+bx^n\right)}{(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b
]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/((j - n)*x^(j/2)*Sqrt[a*x^j +
b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j+bx^n}\,x^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \sqrt{a\,x^j+b\,x^n}\;x^{-\frac{j}{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j+bx^n}\,x^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*x^(-1/2*j - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^j+b\,x^n}}{x^{\frac{j}{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1),x)
```

```
[Out] int((a*x^j + b*x^n)^(1/2)/x^(j/2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{-\frac{j}{2}-1}\sqrt{ax^j+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral(x**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

3.365 $\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx$

Optimal. Leaf size=99

$$\frac{2\sqrt{a}\,x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

```
[Out] 2*x^(1/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))*a^(1/2)/c/(j-n)/
((c*x)^(1/2*j))-2*(a*x^j+b*x^n)^(1/2)/c/(j-n)/((c*x)^(1/2*j))
```

Rubi [A] time = 0.16, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2031, 2028, 2029, 206}

$$\frac{2\sqrt{a}\,x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}-\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^(j/2)) + (2*Sqrt[a]*x^(j/2)*ArcTa
nh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^(j/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int (cx)^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx = \frac{\left(x^{j/2}(cx)^{-j/2}\right)\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\,dx}{c}$$
$$= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)} + \frac{\left(ax^{j/2}(cx)^{-j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c}$$
$$= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)} + \frac{\left(2ax^{j/2}(cx)^{-j/2}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$
$$= -\frac{2(cx)^{-j/2}\sqrt{ax^j+bx^n}}{c(j-n)} + \frac{2\sqrt{a}\,x^{j/2}(cx)^{-j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}$$

Mathematica [A] time = 0.07, size = 109, normalized size = 1.10

$$-\frac{2(cx)^{-j/2}\left(-\sqrt{a}\,\sqrt{b}\,x^{\frac{j+n}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)+ax^j+bx^n\right)}{c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - j/2)*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (-2*(a*x^j + b*x^n - Sqrt[a]*Sqrt[b]*x^((j + n)/2)*Sqrt[1 + (a*x^(j - n))/b
]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(c*(j - n)*(c*x)^(j/2)*Sqrt[a*
x^j + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j+bx^n}\,(cx)^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

maple [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \sqrt{a\,x^j+b\,x^n}\,(cx)^{-\frac{j}{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1/2*j-1)*(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(-1/2*j-1)*(a*x^j+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j + bx^n}\,(cx)^{-\frac{1}{2}j-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-1/2*j)*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^(-1/2*j - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a\,x^j + b\,x^n}}{(c\,x)^{\frac{j}{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1),x)
[Out] int((a*x^j + b*x^n)^(1/2)/(c*x)^(j/2 + 1), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^{-\frac{j}{2}-1}\sqrt{ax^j + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-1/2*j)*(a*x**j+b*x**n)**(1/2),x)
[Out] Integral((c*x)**(-j/2 - 1)*sqrt(a*x**j + b*x**n), x)
```

3.366 $\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=91

$$\frac{2\sqrt{a}\,\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

```
[Out] 2*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*a^(1/2)*(c*x)^(1/2)/c^3/(3-n
)/x^(1/2)-2*(a*x^3+b*x^n)^(1/2)/c/(3-n)/(c*x)^(3/2)
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{a}\,\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}-\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

```
[Out] (-2*Sqrt[a*x^3 + b*x^n])/(c*(3 - n)*(c*x)^(3/2)) + (2*Sqrt[a]*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(c^3*(3 - n)*Sqrt[x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx = -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{a\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{c^3}$$
$$= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{\left(a\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{c^3\sqrt{x}}$$
$$= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{\left(2a\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}$$
$$= -\frac{2\sqrt{ax^3+bx^n}}{c(3-n)(cx)^{3/2}}+\frac{2\sqrt{a}\,\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^3(3-n)\sqrt{x}}$$

Mathematica [A] time = 0.27, size = 103, normalized size = 1.13

$$\frac{2x\left(-\sqrt{a}\,\sqrt{b}\,x^{\frac{n+3}{2}}\sqrt{\frac{ax^{3-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+ax^3+bx^n\right)}{(n-3)(cx)^{5/2}\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^3 + b*x^n]/(c*x)^(5/2),x]
```

```
[Out] (2*x*(a*x^3 + b*x^n - Sqrt[a]*Sqrt[b]*x^((3 + n)/2)*Sqrt[1 + (a*x^(3 - n))/
b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/((-3 + n)*(c*x)^(5/2)*Sqrt[a*
x^3 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^3+b\,x^n}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

```
[Out] int((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(1/2)/(c*x)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^3 + b*x^n)/(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^n + a\,x^3}}{{(c\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2),x)
[Out] int((b*x^n + a*x^3)^(1/2)/(c*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3 + bx^n}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**n)**(1/2)/(c*x)**(5/2),x)
[Out] Integral(sqrt(a*x**3 + b*x**n)/(c*x)**(5/2), x)
```

3.367 $\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2} dx$

Optimal. Leaf size=71

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}$$

```
[Out] 2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))*a^(1/2)/c^2/(2-n)-2*(a*x^2+b*x^n)^
(1/2)/c^2/(2-n)/x
```

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2028, 2008, 206}

$$\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}-\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] (-2*Sqrt[a*x^2 + b*x^n])/(c^2*(2 - n)*x) + (2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/S
qrt[a*x^2 + b*x^n]])/(c^2*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx = \frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2}$$
$$= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x} + \frac{a\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{c^2}$$
$$= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}$$
$$= -\frac{2\sqrt{ax^2+bx^n}}{c^2(2-n)x} + \frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^2(2-n)}$$

Mathematica [A] time = 0.16, size = 99, normalized size = 1.39

$$\frac{2\left(-\sqrt{a}\,\sqrt{b}\,x^{\frac{n}{2}+1}\sqrt{\frac{ax^{2-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)+ax^2+bx^n\right)}{c^2(n-2)x\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^2 + b*x^n]/(c^2*x^2),x]
```

```
[Out] (2*(a*x^2 + b*x^n - Sqrt[a]*Sqrt[b]*x^(1 + n/2)*Sqrt[1 + (a*x^(2 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(c^2*(-2 + n)*x*Sqrt[a*x^2 + b*x^n]
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2+bx^n}}{c^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^2 + b*x^n)/(c^2*x^2), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\,x^2+b\,x^n}}{c^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

```
[Out] int((a*x^2+b*x^n)^(1/2)/c^2/x^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(1/2)/c^2/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*x^2 + b*x^n)/x^2, x)/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^n+a\,x^2}}{c^2\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^2)^(1/2)/(c^2*x^2),x)
```

```
[Out] int((b*x^n + a*x^2)^(1/2)/(c^2*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x**n)**(1/2)/c**2/x**2,x)
```

```
[Out] Integral(sqrt(a*x**2 + b*x**n)/x**2, x)/c**2
```

3.368 $\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

```
[Out] 2*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/c/(1-n)/(c*x)^
(1/2)-2*(a*x+b*x^n)^(1/2)/c/(1-n)/(c*x)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}-\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[a*x + b*x^n])/(c*(1 - n)*Sqrt[c*x]) + (2*Sqrt[a]*Sqrt[x]*ArcTanh[(
Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(c*(1 - n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{a\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx}{c}\\ &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{\left(a\sqrt{x}\right)\int \frac{1}{\sqrt{x}\,\sqrt{ax+bx^n}}\,dx}{c\sqrt{cx}}\\ &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{\left(2a\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}\\ &= -\frac{2\sqrt{ax+bx^n}}{c(1-n)\sqrt{cx}}+\frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c(1-n)\sqrt{cx}}\end{aligned}$$

Mathematica [A] time = 0.29, size = 100, normalized size = 1.15

$$\frac{x\left(-2\sqrt{a}\,\sqrt{b}\,x^{\frac{n+1}{2}}\sqrt{\frac{ax^{1-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+2ax+2bx^n\right)}{(n-1)(cx)^{3/2}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x + b*x^n]/(c*x)^(3/2),x]
```

```
[Out] (x*(2*a*x + 2*b*x^n - 2*Sqrt[a]*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n)
)/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/((-1 + n)*(c*x)^(3/2)*Sqrt[
a*x + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+bx^n}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax+b\,x^n}}{(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

```
[Out] int((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(1/2)/(c*x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x + b*x^n)/(c*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\, x^n + a\, x}}{{(c\, x)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x)^(1/2)/(c*x)^(3/2),x)
[Out] int((b*x^n + a*x)^(1/2)/(c*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax + bx^n}}{(cx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x**n)**(1/2)/(c*x)**(3/2),x)
[Out] Integral(sqrt(a*x + b*x**n)/(c*x)**(3/2), x)
```

3.369 $\int \frac{\sqrt{a+bx^n}}{cx}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))*a^(1/2)/c/n+2*(a+b*x^n)^(1/2)/c/n
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 266, 50, 63, 208}

$$\frac{2\sqrt{a+bx^n}}{cn}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^n]/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n])/(c*n) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx^n}}{cx}\,dx &= \frac{\int \frac{\sqrt{a+bx^n}}{x}\,dx}{c} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{cn} \\
&= \frac{2\sqrt{a+bx^n}}{cn} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn} \\
&= \frac{2\sqrt{a+bx^n}}{cn} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn} \\
&= \frac{2\sqrt{a+bx^n}}{cn} - \frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 46, normalized size = 0.90

$$\frac{2\sqrt{a+bx^n} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^n]/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

fricas [A] time = 0.42, size = 97, normalized size = 1.90

$$\left[\frac{\sqrt{a}\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)+2\sqrt{bx^n+a}}{cn}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)+\sqrt{bx^n+a}\right)}{cn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*sqrt(b*x^n
 + a))/(c*n), 2*(sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n +
 a))/(c*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n+a}}{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)/(c*x), x)
```

maple [A] time = 0.04, size = 39, normalized size = 0.76

$$\frac{-2\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right)+2\sqrt{b\,x^n+a}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(1/2)/c/x,x)
```

```
[Out] 1/c/n*(2*(b*x^n+a)^(1/2)-2*a^(1/2)*arctanh((b*x^n+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 2.96, size = 58, normalized size = 1.14

$$\frac{\frac{\sqrt{a}\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{n}+\frac{2\sqrt{bx^n+a}}{n}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(1/2)/c/x,x, algorithm="maxima")
```

```
[Out] (sqrt(a)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/n + 2
*sqrt(b*x^n + a)/n)/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+b\,x^n}}{c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(1/2)/(c*x),x)
```

```
[Out] int((a + b*x^n)^(1/2)/(c*x), x)
```

sympy [A] time = 1.78, size = 78, normalized size = 1.53

$$\frac{-\frac{2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{n}+\frac{2ax^{-\frac{n}{2}}}{\sqrt{b}n\sqrt{\frac{ax^{-n}}{b}+1}}+\frac{2\sqrt{b}x^{\frac{n}{2}}}{n\sqrt{\frac{ax^{-n}}{b}+1}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(1/2)/c/x,x)
```

```
[Out] (-2*sqrt(a)*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/n + 2*a*x**(-n/2)/(sqrt(b)*n*s
qrt(a*x**(-n)/b + 1)) + 2*sqrt(b)*x**(n/2)/(n*sqrt(a*x**(-n)/b + 1)))/c
```

3.370 $\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx$

Optimal. Leaf size=84

$$\frac{2\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/(1+n)/(c*x)^(
1/2)+2*(c*x)^(1/2)*(a/x+b*x^n)^(1/2)/c/(1+n)
```

Rubi [A] time = 0.17, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2\sqrt{cx}\,\sqrt{\frac{a}{x}+bx^n}}{c(n+1)}-\frac{2\sqrt{a}\,\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(c*(1 + n)) - (2*Sqrt[a]*Sqrt[x]*ArcTanh[Sq
rt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])]])/((1 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}\,dx = \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} + (ac)\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx$$
$$= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} + \frac{\left(a\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{\sqrt{cx}}$$
$$= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} - \frac{\left(2a\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$
$$= \frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{c(1+n)} - \frac{2\sqrt{a}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

Mathematica [A] time = 0.07, size = 85, normalized size = 1.01

$$\frac{x\sqrt{\frac{a}{x}+bx^n}\left(2\sqrt{a+bx^{n+1}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)\right)}{(n+1)\sqrt{cx}\sqrt{a+bx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x + b*x^n]/Sqrt[c*x],x]
```

```
[Out] (x*Sqrt[a/x + b*x^n]*(2*Sqrt[a + b*x^(1 + n)] - 2*Sqrt[a]*ArcTanh[Sqrt[a + 
b*x^(1 + n)]/Sqrt[a]]))/((1 + n)*Sqrt[c*x]*Sqrt[a + b*x^(1 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n+\frac{a}{x}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,x^n+\frac{a}{x}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^n + \frac{a}{x}}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x^n)^(1/2)/(c*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x)/sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,x^n + \frac{a}{x}}}{\sqrt{c\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a/x)^(1/2)/(c*x)^(1/2),x)
```

```
[Out] int((b*x^n + a/x)^(1/2)/(c*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x} + bx^n}}{\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x+b*x**n)**(1/2)/(c*x)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x + b*x**n)/sqrt(c*x), x)
```

3.371 $\int \sqrt{\frac{a}{x^2}+bx^n}\,dx$

Optimal. Leaf size=61

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))*a^(1/2)/(2+n)+2*x*(a/x^2+b*x^n)^(
1/2)/(2+n)
```

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2007, 2029, 206}

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a/x^2 + b*x^n],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^
2 + b*x^n])])/(2 + n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{a}{x^2}+bx^n}\,dx &= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx\\
&= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{(2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}\\
&= \frac{2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 1.28

$$\frac{x\sqrt{\frac{a}{x^2}+bx^n}\left(2\sqrt{a+bx^{n+2}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)\right)}{(n+2)\sqrt{a+bx^{n+2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a/x^2 + b*x^n],x]
```

```
[Out] (x*Sqrt[a/x^2 + b*x^n]*(2*Sqrt[a + b*x^(2 + n)] - 2*Sqrt[a]*ArcTanh[Sqrt[a
+ b*x^(2 + n)]/Sqrt[a]]))/((2 + n)*Sqrt[a + b*x^(2 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{bx^n+\frac{a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int\sqrt{b\,x^n+\frac{a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a/x^2+b*x^n)^(1/2),x)
```

```
[Out] int((a/x^2+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{bx^n+\frac{a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a/x^2), x)
```

mupad [B] time = 5.17, size = 97, normalized size = 1.59

$$\frac{x\sqrt{b\,x^n+\frac{a}{x^2}}}{\frac{n}{2}+1}+\frac{\sqrt{a}\,x\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,x^{\frac{n}{2}+1}}\right)\sqrt{b\,x^n+\frac{a}{x^2}}\,1\mathrm{i}}{\sqrt{b}\,x^{\frac{n}{2}+1}\left(\frac{n}{2}+1\right)\sqrt{\frac{a}{b\,x^{n+2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a/x^2)^(1/2),x)
```

```
[Out] (x*(b*x^n + a/x^2)^(1/2))/(n/2 + 1) + (a^(1/2)*x*asin((a^(1/2)*1i)/(b^(1/2)
*x^(n/2 + 1)))*(b*x^n + a/x^2)^(1/2)*1i)/(b^(1/2)*x^(n/2 + 1)*(n/2 + 1)*(a/
(b*x^(n + 2)) + 1)^(1/2))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a}{x^2} + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**2 + b*x**n), x)
```

3.372 $\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$

Optimal. Leaf size=85

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}\,c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}$$

```
[Out] -2*c*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*a^(1/2)*x^(1/2)/(3+n)/(c*
x)^(1/2)+2*(c*x)^(3/2)*(a/x^3+b*x^n)^(1/2)/c/(3+n)
```

Rubi [A] time = 0.20, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(n+3)}-\frac{2\sqrt{a}\,c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
```

```
[Out] (2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(c*(3 + n)) - (2*Sqrt[a]*c*Sqrt[x]*ArcT
anh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx &= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} + \left(ac^3\right)\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx \\
&= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} + \frac{\left(ac\sqrt{x}\right)\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{\sqrt{cx}} \\
&= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} - \frac{\left(2ac\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}} \\
&= \frac{2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{c(3+n)} - \frac{2\sqrt{a}\,c\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 85, normalized size = 1.00

$$
\frac{x\sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\left(2\sqrt{a+bx^{n+3}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)\right)}{(n+3)\sqrt{a+bx^{n+3}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*Sqrt[a/x^3 + b*x^n],x]
[Out] (x*Sqrt[c*x]*Sqrt[a/x^3 + b*x^n]*(2*Sqrt[a + b*x^(3 + n)] - 2*Sqrt[a]*ArcTa
nh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/((3 + n)*Sqrt[a + b*x^(3 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{bx^n + \frac{a}{x^3}}\sqrt{cx}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$
\int \sqrt{cx}\,\sqrt{b\,x^n + \frac{a}{x^3}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + \frac{a}{x^3}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*x^n + a/x^3)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x}\,\sqrt{b\,x^n + \frac{a}{x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2),x)
[Out] int((c*x)^(1/2)*(b*x^n + a/x^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\sqrt{\frac{a}{x^3} + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(a/x**3+b*x**n)**(1/2),x)
[Out] Integral(sqrt(c*x)*sqrt(a/x**3 + b*x**n), x)
```

3.373 $\int (cx)^{-1-\frac{3j}{2}} \left(ax^j + bx^n\right)^{3/2} dx$

Optimal. Leaf size=141

$$\frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} - \frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

```
[Out] -2/3*(a*x^j+b*x^n)^(3/2)/c/(j-n)/((c*x)^(3/2*j))+2*a^(3/2)*x^(3/2*j)*arctan
h(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/c/(j-n)/((c*x)^(3/2*j))-2*a*x^j*(a
*x^j+b*x^n)^(1/2)/c/(j-n)/((c*x)^(3/2*j))
```

Rubi [A] time = 0.23, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2031, 2028, 2029, 206}

$$\frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} - \frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*a*x^j*Sqrt[a*x^j + b*x^n])/(c*(j - n)*(c*x)^((3*j)/2)) - (2*(a*x^j + b*
x^n)^(3/2))/(3*c*(j - n)*(c*x)^((3*j)/2)) + (2*a^(3/2)*x^((3*j)/2)*ArcTanh[
(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(c*(j - n)*(c*x)^((3*j)/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int (cx)^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx = \frac{\left(x^{3j/2}(cx)^{-3j/2}\right)\int x^{-1-\frac{3j}{2}} \left(ax^j+bx^n\right)^{3/2} dx}{c}$$
$$= -\frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{\left(ax^{3j/2}(cx)^{-3j/2}\right)\int x^{-1-\frac{j}{2}}\sqrt{ax^j+bx^n}\, dx}{c}$$
$$= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{\left(a^2x^{3j/2}(cx)^{-3j/2}\right)\int \frac{x^-}{\sqrt{ax}}}{c}$$
$$= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{\left(2a^2x^{3j/2}(cx)^{-3j/2}\right)\text{Subs}}{c(}$$
$$= -\frac{2ax^j(cx)^{-3j/2}\sqrt{ax^j+bx^n}}{c(j-n)} - \frac{2(cx)^{-3j/2}\left(ax^j+bx^n\right)^{3/2}}{3c(j-n)} + \frac{2a^{3/2}x^{3j/2}(cx)^{-3j/2}\tanh}{c(j-n)}$$

Mathematica [A] time = 0.26, size = 131, normalized size = 0.93

$$-\frac{2(cx)^{-3j/2}\left(-3a^{3/2}\sqrt{b}\,x^{\frac{1}{2}(3j+n)}\sqrt{\frac{ax^{j-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)+4a^2x^{2j}+5abx^{j+n}+b^2x^{2n}\right)}{3c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 - (3*j)/2)*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(4*a^2*x^(2*j) + b^2*x^(2*n) + 5*a*b*x^(j + n) - 3*a^(3/2)*Sqrt[b]*x^((
3*j + n)/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[
b]]))/(3*c*(j - n)*(c*x)^((3*j)/2)*Sqrt[a*x^j + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^j+bx^n\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \left(a\,x^j+b\,x^n\right)^{\frac{3}{2}} (cx)^{-\frac{3j}{2}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^j + bx^n\right)^{\frac{3}{2}} (cx)^{-\frac{3}{2}j-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1-3/2*j)*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^(-3/2*j - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a\,x^j + b\,x^n\right)^{3/2}}{(c\,x)^{\frac{3\,j}{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1),x)
```

```
[Out] int((a*x^j + b*x^n)^(3/2)/(c*x)^((3*j)/2 + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1-3/2*j)*(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

3.374 $\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}} dx$

Optimal. Leaf size=128

$$\frac{2a^{3/2}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}-\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

```
[Out] -2/3*(a*x^3+b*x^n)^(3/2)/c/(3-n)/(c*x)^(9/2)+2*a^(3/2)*arctanh(x^(3/2)*a^(1
/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/c^6/(3-n)/x^(1/2)-2*a*(a*x^3+b*x^n)^(1
/2)/c^4/(3-n)/(c*x)^(3/2)
```

Rubi [A] time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$\frac{2a^{3/2}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}-\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}}-\frac{2\left(ax^3+bx^n\right)^{3/2}}{3c(3-n)(cx)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (-2*a*Sqrt[a*x^3 + b*x^n])/(c^4*(3 - n)*(c*x)^(3/2)) - (2*(a*x^3 + b*x^n)^(
3/2))/(3*c*(3 - n)*(c*x)^(9/2)) + (2*a^(3/2)*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(
3/2))/Sqrt[a*x^3 + b*x^n]])/(c^6*(3 - n)*Sqrt[x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{(ax^3+bx^n)^{3/2}}{(cx)^{11/2}}\,dx = -\frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{a\int \frac{\sqrt{ax^3+bx^n}}{(cx)^{5/2}}\,dx}{c^3}$$
$$= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{a^2\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{c^6}$$
$$= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{(a^2\sqrt{cx})\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{c^6\sqrt{x}}$$
$$= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{(2a^2\sqrt{cx})\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}$$
$$= -\frac{2a\sqrt{ax^3+bx^n}}{c^4(3-n)(cx)^{3/2}} - \frac{2(ax^3+bx^n)^{3/2}}{3c(3-n)(cx)^{9/2}} + \frac{2a^{3/2}\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{c^6(3-n)\sqrt{x}}$$

Mathematica [A] time = 0.33, size = 126, normalized size = 0.98

$$\frac{2\sqrt{cx}\left(-3a^{3/2}\sqrt{b}\,x^{\frac{n+9}{2}}\sqrt{\frac{ax^{3-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+4a^2x^6+5abx^{n+3}+b^2x^{2n}\right)}{3c^6(n-3)x^5\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2),x]
```

```
[Out] (2*Sqrt[c*x]*(4*a^2*x^6 + b^2*x^(2*n) + 5*a*b*x^(3 + n) - 3*a^(3/2)*Sqrt[b]
*x^((9 + n)/2)*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sq
rt[b]]))/(3*c^6*(-3 + n)*x^5*Sqrt[a*x^3 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^3+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{(a\,x^3+b\,x^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

```
[Out] int((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax^3 + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{11}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^n)^(3/2)/(c*x)^(11/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^3 + b*x^n)^(3/2)/(c*x)^(11/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^n + a\,x^3)^{3/2}}{(c\,x)^{11/2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2),x)
```

```
[Out] int((b*x^n + a*x^3)^(3/2)/(c*x)^(11/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**n)**(3/2)/(c*x)**(11/2),x)
```

```
[Out] Timed out
```

3.375 $\int \frac{\left(ax^2+bx^n\right)^{3/2}}{c^4x^4}\,dx$

Optimal. Leaf size=104

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}-\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}$$

```
[Out] -2/3*(a*x^2+b*x^n)^(3/2)/c^4/(2-n)/x^3+2*a^(3/2)*arctanh(x*a^(1/2)/(a*x^2+b
*x^n)^(1/2))/c^4/(2-n)-2*a*(a*x^2+b*x^n)^(1/2)/c^4/(2-n)/x
```

Rubi [A] time = 0.13, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {12, 2028, 2008, 206}

$$\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}-\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

```
[Out] (-2*a*Sqrt[a*x^2 + b*x^n])/(c^4*(2 - n)*x) - (2*(a*x^2 + b*x^n)^(3/2))/(3*c
^4*(2 - n)*x^3) + (2*a^(3/2)*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(c^4
*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(ax^2+bx^n\right)^{3/2}}{c^4x^4}\,dx &= \frac{\int \frac{\left(ax^2+bx^n\right)^{3/2}}{x^4}\,dx}{c^4}\\
&= -\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}+\frac{a\int \frac{\sqrt{ax^2+bx^n}}{x^2}\,dx}{c^4}\\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}+\frac{a^2\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{c^4}\\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}+\frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}\\
&= -\frac{2a\sqrt{ax^2+bx^n}}{c^4(2-n)x}-\frac{2\left(ax^2+bx^n\right)^{3/2}}{3c^4(2-n)x^3}+\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{c^4(2-n)}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 117, normalized size = 1.12

$$
\frac{2\left(-3a^{3/2}\sqrt{b}\,x^{\frac{n}{2}+3}\sqrt{\frac{ax^{2-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)+4a^2x^4+5abx^{n+2}+b^2x^{2n}\right)}{3c^4(n-2)x^3\sqrt{ax^2+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^2 + b*x^n)^(3/2)/(c^4*x^4),x]
```

```
[Out] (2*(4*a^2*x^4 + b^2*x^(2*n) + 5*a*b*x^(2 + n) - 3*a^(3/2)*Sqrt[b]*x^(3 + n/
2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))/Sqrt[b]]))/(3*c^
4*(-2 + n)*x^3*Sqrt[a*x^2 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{\left(ax^2+bx^n\right)^{\frac{3}{2}}}{c^4x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="giac")
```

```
[Out] integrate((a*x^2 + b*x^n)^(3/2)/(c^4*x^4), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$
\int \frac{\left(a\,x^2+b\,x^n\right)^{\frac{3}{2}}}{c^4x^4}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

```
[Out] int((a*x^2+b*x^n)^(3/2)/c^4/x^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(ax^2+bx^n)^{\frac{3}{2}}}{x^4}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^2+b*x^n)^(3/2)/c^4/x^4,x, algorithm="maxima")
```

```
[Out] integrate((a*x^2 + b*x^n)^(3/2)/x^4, x)/c^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^n+a\,x^2\right)^{3/2}}{c^4\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^2)^(3/2)/(c^4*x^4),x)
```

```
[Out] int((b*x^n + a*x^2)^(3/2)/(c^4*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a\sqrt{ax^2+bx^n}}{x^2}\,dx + \int \frac{bx^n\sqrt{ax^2+bx^n}}{x^4}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**2+b*x**n)**(3/2)/c**4/x**4,x)
```

```
[Out] (Integral(a*sqrt(a*x**2 + b*x**n)/x**2, x) + Integral(b*x**n*sqrt(a*x**2 +
b*x**n)/x**4, x))/c**4
```

3.376 $\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{2a^{3/2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}-\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2\left(ax+bx^n\right)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

```
[Out] -2/3*(a*x+b*x^n)^(3/2)/c/(1-n)/(c*x)^(3/2)+2*a^(3/2)*arctanh(a^(1/2)*x^(1/2
)/(a*x+b*x^n)^(1/2))*x^(1/2)/c^2/(1-n)/(c*x)^(1/2)-2*a*(a*x+b*x^n)^(1/2)/c^
2/(1-n)/(c*x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2028, 2031, 2029, 206}

$$\frac{2a^{3/2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}-\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2\left(ax+bx^n\right)^{3/2}}{3c(1-n)(cx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (-2*a*Sqrt[a*x + b*x^n])/(c^2*(1 - n)*Sqrt[c*x]) - (2*(a*x + b*x^n)^(3/2))/
(3*c*(1 - n)*(c*x)^(3/2)) + (2*a^(3/2)*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sq
rt[a*x + b*x^n]])/(c^2*(1 - n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(ax+bx^n)^{3/2}}{(cx)^{5/2}}\,dx &= -\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{a\int \frac{\sqrt{ax+bx^n}}{(cx)^{3/2}}\,dx}{c}\\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{a^2\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx}{c^2}\\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{\left(a^2\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{c^2\sqrt{cx}}\\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{\left(2a^2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}\\
&= -\frac{2a\sqrt{ax+bx^n}}{c^2(1-n)\sqrt{cx}}-\frac{2(ax+bx^n)^{3/2}}{3c(1-n)(cx)^{3/2}}+\frac{2a^{3/2}\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{c^2(1-n)\sqrt{cx}}
\end{aligned}
$$

Mathematica [A] time = 0.28, size = 120, normalized size = 0.98

$$
\frac{x\left(-6a^{3/2}\sqrt{b}\,x^{\frac{n+3}{2}}\sqrt{\frac{ax^{1-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)+8a^2x^2+10abx^{n+1}+2b^2x^{2n}\right)}{3(n-1)(cx)^{5/2}\sqrt{ax+bx^n}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x + b*x^n)^(3/2)/(c*x)^(5/2),x]
```

```
[Out] (x*(8*a^2*x^2 + 2*b^2*x^(2*n) + 10*a*b*x^(1 + n) - 6*a^(3/2)*Sqrt[b]*x^((3
+ n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]])
)/(3*(-1 + n)*(c*x)^(5/2)*Sqrt[a*x + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{(ax+bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$
\int \frac{(ax+b\,x^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

```
[Out] int((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x^n)^(3/2)/(c*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x + b*x^n)^(3/2)/(c*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,x^n + a\,x)^{3/2}}{(c\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x)^(3/2)/(c*x)^(5/2),x)
```

```
[Out] int((b*x^n + a*x)^(3/2)/(c*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax + bx^n)^{\frac{3}{2}}}{(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b*x**n)**(3/2)/(c*x)**(5/2),x)
```

```
[Out] Integral((a*x + b*x**n)**(3/2)/(c*x)**(5/2), x)
```

3.377 $\int \frac{(a+bx^n)^{3/2}}{cx}\,dx$

Optimal. Leaf size=73

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}+\frac{2a\sqrt{a+bx^n}}{cn}+\frac{2\,(a+bx^n)^{3/2}}{3cn}$$

```
[Out] 2/3*(a+b*x^n)^(3/2)/c/n-2*a^(3/2)*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n+2*a*
(a+b*x^n)^(1/2)/c/n
```

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 266, 50, 63, 208}

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}+\frac{2a\sqrt{a+bx^n}}{cn}+\frac{2\,(a+bx^n)^{3/2}}{3cn}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^(3/2)/(c*x),x]
```

```
[Out] (2*a*Sqrt[a + b*x^n])/(c*n) + (2*(a + b*x^n)^(3/2))/(3*c*n) - (2*a^(3/2)*Ar
cTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(c*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(a+bx^n)^{3/2}}{cx}\,dx = \frac{\int \frac{(a+bx^n)^{3/2}}{x}\,dx}{c}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{a\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{a^2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} + \frac{(2a^2)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn}$$
$$= \frac{2a\sqrt{a+bx^n}}{cn} + \frac{2(a+bx^n)^{3/2}}{3cn} - \frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{cn}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 0.79

$$\frac{2\sqrt{a+bx^n}\,(4a+bx^n) - 6a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{3cn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^(3/2)/(c*x),x]
```

```
[Out] (2*Sqrt[a + b*x^n]*(4*a + b*x^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a
]])/(3*c*n)
```

fricas [A] time = 0.41, size = 120, normalized size = 1.64

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{bx^n-2\,\sqrt{bx^n+a}\,\sqrt{a}+2\,a}{x^n}\right) + 2\,(bx^n+4\,a)\sqrt{bx^n+a}}{3\,cn},\ \frac{2\left(3\,\sqrt{-a}\,a\arctan\left(\frac{\sqrt{bx^n+a}\,\sqrt{-a}}{a}\right) + (bx^n+4\,a)\sqrt{bx^n+a}\right.}{3\,cn}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*a^(3/2)*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n) + 2*(b*x
^n + 4*a)*sqrt(b*x^n + a))/(c*n), 2/3*(3*sqrt(-a)*a*arctan(sqrt(b*x^n + a)*
sqrt(-a)/a) + (b*x^n + 4*a)*sqrt(b*x^n + a))/(c*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n+a)^{\frac{3}{2}}}{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)/(c*x), x)
```

maple [A] time = 0.05, size = 51, normalized size = 0.70

$$\frac{-2a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{n}+a}}{\sqrt{a}}\right)+2\sqrt{b\,x^{n}+a}\;a+\frac{2(b\,x^{n}+a)^{\frac{3}{2}}}{3}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^(3/2)/c/x,x)
```

```
[Out] 1/c/n*(2/3*(b*x^n+a)^(3/2)+2*(b*x^n+a)^(1/2)*a-2*a^(3/2)*arctanh((b*x^n+a)^
(1/2)/a^(1/2)))
```

maxima [A] time = 3.00, size = 73, normalized size = 1.00

$$\frac{\frac{3\,a^{\frac{3}{2}}\log\left(\frac{\sqrt{bx^{n}+a}-\sqrt{a}}{\sqrt{bx^{n}+a}+\sqrt{a}}\right)}{n}+\frac{2\left((bx^{n}+a)^{\frac{3}{2}}+3\,\sqrt{bx^{n}+a}\,a\right)}{n}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^(3/2)/c/x,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^(3/2)*log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))
/n + 2*((b*x^n + a)^(3/2) + 3*sqrt(b*x^n + a)*a)/n)/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,x^{n})^{3/2}}{c\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^(3/2)/(c*x),x)
```

```
[Out] int((a + b*x^n)^(3/2)/(c*x), x)
```

sympy [A] time = 3.08, size = 88, normalized size = 1.21

$$\frac{\frac{8a^{\frac{3}{2}}\sqrt{1+\frac{bx^{n}}{a}}}{3n}+\frac{a^{\frac{3}{2}}\log\left(\frac{bx^{n}}{a}\right)}{n}-\frac{2a^{\frac{3}{2}}\log\left(\sqrt{1+\frac{bx^{n}}{a}}+1\right)}{n}+\frac{2\sqrt{a}\,bx^{n}\sqrt{1+\frac{bx^{n}}{a}}}{3n}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**(3/2)/c/x,x)
```

```
[Out] (8*a**(3/2)*sqrt(1 + b*x**n/a)/(3*n) + a**(3/2)*log(b*x**n/a)/n - 2*a**(3/2
)*log(sqrt(1 + b*x**n/a) + 1)/n + 2*sqrt(a)*b*x**n*sqrt(1 + b*x**n/a)/(3*n)
)/c
```

3.378 $\int \sqrt{cx} \left(\frac{a}{x}+bx^n\right)^{3/2} dx$

Optimal. Leaf size=117

$$-\frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}+\frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{n+1}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

```
[Out] 2/3*(c*x)^(3/2)*(a/x+b*x^n)^(3/2)/c/(1+n)-2*a^(3/2)*c*arctanh(a^(1/2)/x^(1/
2)/(a/x+b*x^n)^(1/2))*x^(1/2)/(1+n)/(c*x)^(1/2)+2*a*(c*x)^(1/2)*(a/x+b*x^n)
^(1/2)/(1+n)
```

Rubi [A] time = 0.23, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$-\frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(n+1)\sqrt{cx}}+\frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{n+1}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (2*a*Sqrt[c*x]*Sqrt[a/x + b*x^n])/(1 + n) + (2*(c*x)^(3/2)*(a/x + b*x^n)^(3
/2))/(3*c*(1 + n)) - (2*a^(3/2)*c*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x
 + b*x^n])])/((1 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \sqrt{cx}\left(\frac{a}{x}+bx^n\right)^{3/2}dx = \frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}+(ac)\int \frac{\sqrt{\frac{a}{x}+bx^n}}{\sqrt{cx}}dx$$
$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}+\left(a^2c^2\right)\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}dx$$
$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}+\frac{\left(a^2c\sqrt{x}\right)\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}dx}{\sqrt{cx}}$$
$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}-\frac{\left(2a^2c\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}dx,x,\frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$
$$= \frac{2a\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}{1+n}+\frac{2(cx)^{3/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}{3c(1+n)}-\frac{2a^{3/2}c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{(1+n)\sqrt{cx}}$$

Mathematica [A] time = 0.08, size = 97, normalized size = 0.83

$$\frac{2\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}\left(\sqrt{a+bx^{n+1}}\left(4a+bx^{n+1}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)\right)}{3(n+1)\sqrt{a+bx^{n+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]*(a/x + b*x^n)^(3/2),x]
```

```
[Out] (2*Sqrt[c*x]*Sqrt[a/x + b*x^n]*(Sqrt[a + b*x^(1 + n)]*(4*a + b*x^(1 + n)) -
 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]]))/(3*(1 + n)*Sqrt[a + b*x
^(1 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}\sqrt{cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \sqrt{cx}\,\left(b\,x^n+\frac{a}{x}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^n+a/x)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)*(b*x^n+a/x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^n + \frac{a}{x}\right)^{\frac{3}{2}} \sqrt{cx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)*(a/x+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a/x)^(3/2)*sqrt(c*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\,x}\,\left(b\,x^n + \frac{a}{x}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)*(b*x^n + a/x)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)*(b*x^n + a/x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cx} \left(\frac{a}{x} + bx^n\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)*(a/x+b*x**n)**(3/2),x)
```

```
[Out] Integral(sqrt(c*x)*(a/x + b*x**n)**(3/2), x)
```

3.379 $\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}\,dx$

Optimal. Leaf size=98

$$-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}+\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}$$

```
[Out] 2/3*c^2*x^3*(a/x^2+b*x^n)^(3/2)/(2+n)-2*a^(3/2)*c^2*arctanh(a^(1/2)/x/(a/x^
2+b*x^n)^(1/2))/(2+n)+2*a*c^2*x*(a/x^2+b*x^n)^(1/2)/(2+n)
```

Rubi [A] time = 0.17, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {12, 2028, 2007, 2029, 206}

$$-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{n+2}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(n+2)}+\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{n+2}$$

Antiderivative was successfully verified.

```
[In] Int[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^2*x*Sqrt[a/x^2 + b*x^n])/(2 + n) + (2*c^2*x^3*(a/x^2 + b*x^n)^(3/2))
/(3*(2 + n)) - (2*a^(3/2)*c^2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(2
+ n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int c^2x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx &= c^2\int x^2\left(\frac{a}{x^2}+bx^n\right)^{3/2}dx\\
&=\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}+\left(ac^2\right)\int\sqrt{\frac{a}{x^2}+bx^n}\,dx\\
&=\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}+\left(a^2c^2\right)\int\frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx\\
&=\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{\left(2a^2c^2\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}\\
&=\frac{2ac^2x\sqrt{\frac{a}{x^2}+bx^n}}{2+n}+\frac{2c^2x^3\left(\frac{a}{x^2}+bx^n\right)^{3/2}}{3(2+n)}-\frac{2a^{3/2}c^2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{2+n}
\end{aligned}
$$

Mathematica [A] time = 0.07, size = 94, normalized size = 0.96

$$
\frac{2c^2x\sqrt{\frac{a}{x^2}+bx^n}\left(\sqrt{a+bx^{n+2}}\left(4a+bx^{n+2}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)\right)}{3(n+2)\sqrt{a+bx^{n+2}}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[c^2*x^2*(a/x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*x*Sqrt[a/x^2 + b*x^n]*(Sqrt[a + b*x^(2 + n)]*(4*a + b*x^(2 + n)) - 3
*a^(3/2)*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]]))/(3*(2 + n)*Sqrt[a + b*x^(
2 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^2x^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a/x^2)^(3/2)*c^2*x^2, x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$
\int\left(b\,x^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^2x^2\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2*(b*x^n+a/x^2)^(3/2),x)
```

```
[Out] int(c^2*x^2*(b*x^n+a/x^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \left(bx^n + \frac{a}{x^2}\right)^{\frac{3}{2}} x^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2*(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] c^2*integrate((b*x^n + a/x^2)^(3/2)*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int c^2 x^2 \left(b x^n + \frac{a}{x^2}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2*(b*x^n + a/x^2)^(3/2),x)
```

```
[Out] int(c^2*x^2*(b*x^n + a/x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \left(\int a \sqrt{\frac{a}{x^2} + bx^n} \, dx + \int bx^2 x^n \sqrt{\frac{a}{x^2} + bx^n} \, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**2*(a/x**2+b*x**n)**(3/2),x)
```

```
[Out] c**2*(Integral(a*sqrt(a/x**2 + b*x**n), x) + Integral(b*x**2*x**n*sqrt(a/x*
*2 + b*x**n), x))
```

3.380 $\int (cx)^{7/2} \left(\frac{a}{x^3}+bx^n\right)^{3/2} dx$

Optimal. Leaf size=122

$$-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}+\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{n+3}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

```
[Out] 2/3*(c*x)^(9/2)*(a/x^3+b*x^n)^(3/2)/c/(3+n)-2*a^(3/2)*c^4*arctanh(a^(1/2)/x
^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/(3+n)/(c*x)^(1/2)+2*a*c^2*(c*x)^(3/2)*(
a/x^3+b*x^n)^(1/2)/(3+n)
```

Rubi [A] time = 0.28, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2028, 2031, 2029, 206}

$$-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(n+3)\sqrt{cx}}+\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{n+3}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(n+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n])/(3 + n) + (2*(c*x)^(9/2)*(a/x^3 +
 b*x^n)^(3/2))/(3*c*(3 + n)) - (2*a^(3/2)*c^4*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3
/2)*Sqrt[a/x^3 + b*x^n])])/((3 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int (cx)^{7/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}dx=\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\left(ac^3\right)\int\sqrt{cx}\sqrt{\frac{a}{x^3}+bx^n}\,dx$$

$$=\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\left(a^2c^6\right)\int\frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}dx$$

$$=\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}+\frac{\left(a^2c^4\sqrt{x}\right)\int\frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}dx}{\sqrt{cx}}$$

$$=\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{\left(2a^2c^4\sqrt{x}\right)\text{Subst}\left(\int\frac{1}{1-ax^2}dx,x,\frac{}{x^{3}}\right.}{(3+n)\sqrt{cx}}$$

$$=\frac{2ac^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}{3+n}+\frac{2(cx)^{9/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}{3c(3+n)}-\frac{2a^{3/2}c^4\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{(3+n)\sqrt{cx}}$$

Mathematica [A] time = 0.09, size = 100, normalized size = 0.82

$$\frac{2c^2(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}\left(\sqrt{a+bx^{n+3}}\left(4a+bx^{n+3}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)\right)}{3(n+3)\sqrt{a+bx^{n+3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)*(a/x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]*(Sqrt[a + b*x^(3 + n)]*(4*a + b*x^(3
 + n)) - 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]]))/(3*(3 + n)*Sqrt
[a + b*x^(3 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int(cx)^{\frac{7}{2}}\left(b\,x^n+\frac{a}{x^3}\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^n+a/x^3)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)*(b*x^n+a/x^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)*(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a/x^3)^(3/2)*(c*x)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int(c\,x)^{7/2}\left(b\,x^n+\frac{a}{x^3}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)*(b*x^n + a/x^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)*(a/x**3+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

3.381 $\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx$

Optimal. Leaf size=100

$$-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}+\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}$$

```
[Out] 2/3*c^5*x^6*(a/x^4+b*x^n)^(3/2)/(4+n)-2*a^(3/2)*c^5*arctanh(a^(1/2)/x^2/(a/
x^4+b*x^n)^(1/2))/(4+n)+2*a*c^5*x^2*(a/x^4+b*x^n)^(1/2)/(4+n)
```

Rubi [A] time = 0.21, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2028, 2029, 206}

$$-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{n+4}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(n+4)}+\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{n+4}$$

Antiderivative was successfully verified.

```
[In] Int[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (2*a*c^5*x^2*Sqrt[a/x^4 + b*x^n])/(4 + n) + (2*c^5*x^6*(a/x^4 + b*x^n)^(3/2
))/(3*(4 + n)) - (2*a^(3/2)*c^5*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/x^4 + b*x^n])])
/(4 + n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2028

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[((c*x)^(m + 1)*(a*x^j + b*x^n)^p)/(c*p*(n - j)), x] + Dist[a/c^j,
 Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, j, m,
 n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] &
& (IntegerQ[j] || GtQ[c, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int c^5x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx=c^5\int x^5\left(\frac{a}{x^4}+bx^n\right)^{3/2}\,dx$$
$$=\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}+\left(ac^5\right)\int x\sqrt{\frac{a}{x^4}+bx^n}\,dx$$
$$=\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}+\left(a^2c^5\right)\int\frac{1}{x^3\sqrt{\frac{a}{x^4}+bx^n}}\,dx$$
$$=\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{\left(2a^2c^5\right)\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{1}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}$$
$$=\frac{2ac^5x^2\sqrt{\frac{a}{x^4}+bx^n}}{4+n}+\frac{2c^5x^6\left(\frac{a}{x^4}+bx^n\right)^{3/2}}{3(4+n)}-\frac{2a^{3/2}c^5\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{4+n}$$

Mathematica [A] time = 0.08, size = 96, normalized size = 0.96

$$\frac{2c^5x^2\sqrt{\frac{a}{x^4}+bx^n}\left(\sqrt{a+bx^{n+4}}\left(4a+bx^{n+4}\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+4}}}{\sqrt{a}}\right)\right)}{3(n+4)\sqrt{a+bx^{n+4}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[c^5*x^5*(a/x^4 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^5*x^2*Sqrt[a/x^4 + b*x^n]*(Sqrt[a + b*x^(4 + n)]*(4*a + b*x^(4 + n)) -
 3*a^(3/2)*ArcTanh[Sqrt[a + b*x^(4 + n)]/Sqrt[a]]))/(3*(4 + n)*Sqrt[a + b*x
^(4 + n)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^5x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a/x^4)^(3/2)*c^5*x^5, x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int\left(b\,x^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^5x^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

```
[Out] int(c^5*x^5*(a/x^4+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{5}\int\left(bx^{n}+\frac{a}{x^{4}}\right)^{\frac{3}{2}}x^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^5*x^5*(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] c^5*integrate((b*x^n + a/x^4)^(3/2)*x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int c^{5}\,x^{5}\,\left(b\,x^{n}+\frac{a}{x^{4}}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^5*x^5*(b*x^n + a/x^4)^(3/2),x)
```

```
[Out] int(c^5*x^5*(b*x^n + a/x^4)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^{5}\left(\int ax\sqrt{\frac{a}{x^{4}}+bx^{n}}\,dx+\int bx^{5}x^{n}\sqrt{\frac{a}{x^{4}}+bx^{n}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**5*x**5*(a/x**4+b*x**n)**(3/2),x)
```

```
[Out] c**5*(Integral(a*x*sqrt(a/x**4 + b*x**n), x) + Integral(b*x**5*x**n*sqrt(a/
x**4 + b*x**n), x))
```

3.382 $\int \sqrt{\frac{a+bx}{x^2}}\,dx$

Optimal. Leaf size=51

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+\frac{b}{x}}}\right)$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b/x)^(1/2))*a^(1/2)+2*x*(a/x^2+b/x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1979, 2007, 2013, 620, 206}

$$2x\sqrt{\frac{a}{x^2}+\frac{b}{x}}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+\frac{b}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] 2*Sqrt[a/x^2 + b/x]*x - 2*Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[a/x^2 + b/x]*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a+bx}{x^2}}\,dx &= \int \sqrt{\frac{a}{x^2}+\frac{b}{x}}\,dx\\ &= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x + a\int \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x^2}\,dx\\ &= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x - a\,\text{Subst}\left(\int \frac{1}{\sqrt{bx+ax^2}}\,dx, x, \frac{1}{x}\right)\\ &= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x - (2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x}\right)\\ &= 2\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x - 2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,x}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 58, normalized size = 1.14

$$\frac{2x\sqrt{\frac{a+bx}{x^2}}\left(\sqrt{a+bx}-\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x)/x^2],x]
```

```
[Out] (2*x*Sqrt[(a + b*x)/x^2]*(Sqrt[a + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqr
t[a]]))/Sqrt[a + b*x]
```

fricas [A] time = 0.41, size = 93, normalized size = 1.82

$$\left[2\,x\sqrt{\frac{bx+a}{x^2}}+\sqrt{a}\,\log\left(\frac{bx-2\,\sqrt{a}\,x\sqrt{\frac{bx+a}{x^2}}+2\,a}{x}\right), 2\,x\sqrt{\frac{bx+a}{x^2}}+2\,\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x\sqrt{\frac{bx+a}{x^2}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*x*sqrt((b*x + a)/x^2) + sqrt(a)*log((b*x - 2*sqrt(a)*x*sqrt((b*x + a)/x^
2) + 2*a)/x), 2*x*sqrt((b*x + a)/x^2) + 2*sqrt(-a)*arctan(sqrt(-a)*x*sqrt((
b*x + a)/x^2)/a)]
```

giac [A] time = 0.16, size = 67, normalized size = 1.31

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\text{sgn}(x)}{\sqrt{-a}}+2\,\sqrt{bx+a}\,\text{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\,\sqrt{a}\right)\text{sgn}(x)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*sqrt(b*x + a)*sgn(x)
 - 2*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

maple [A] time = 0.08, size = 47, normalized size = 0.92

$$\frac{2\sqrt{\frac{bx+a}{x^2}}\left(-\sqrt{a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\right)x}{\sqrt{bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)/x^2)^(1/2),x)
```

```
[Out] 2*((b*x+a)/x^2)^(1/2)*x*(-a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1
/2))/(b*x+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx+a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x + a)/x^2), x)
```

mupad [B] time = 5.18, size = 67, normalized size = 1.31

$$2\,x\,\sqrt{\frac{a}{x^2}+\frac{b}{x}}+\frac{\sqrt{a}\,\sqrt{x}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1\mathrm{i}}{\sqrt{b}\,\sqrt{x}}\right)\,\sqrt{\frac{a}{x^2}+\frac{b}{x}}\,2\mathrm{i}}{\sqrt{b}\,\sqrt{\frac{a}{b\,x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)/x^2)^(1/2),x)
```

```
[Out] 2*x*(a/x^2 + b/x)^(1/2) + (a^(1/2)*x^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(1/
2)))*(a/x^2 + b/x)^(1/2)*2i)/(b^(1/2)*(a/(b*x) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a+bx}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x)/x**2), x)
```

3.383 $\int \sqrt{\frac{a+bx^2}{x^2}}\,dx$

Optimal. Leaf size=42

$$x\sqrt{\frac{a}{x^2}+b}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+b}}\right)$$

```
[Out] -arctanh(a^(1/2)/x/(b+a/x^2)^(1/2))*a^(1/2)+x*(b+a/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1972, 242, 277, 217, 206}

$$x\sqrt{\frac{a}{x^2}+b}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+b}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b + a/x^2]*x - Sqrt[a]*ArcTanh[Sqrt[a]/(Sqrt[b + a/x^2]*x)]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{a+bx^2}{x^2}}\,dx &= \int \sqrt{b+\frac{a}{x^2}}\,dx\\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{b+ax^2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{b+\frac{a}{x^2}}\,x - a\operatorname{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}}\,dx, x, \frac{1}{x}\right)\\
&= \sqrt{b+\frac{a}{x^2}}\,x - a\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{b+\frac{a}{x^2}}\,x}\right)\\
&= \sqrt{b+\frac{a}{x^2}}\,x - \sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{b+\frac{a}{x^2}}\,x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 62, normalized size = 1.48

$$
x\sqrt{\frac{a}{x^2}+b} - \frac{\sqrt{a}\,x\sqrt{\frac{a}{x^2}+b}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a+bx^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b + a/x^2]*x - (Sqrt[a]*Sqrt[b + a/x^2]*x*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]])/Sqrt[a + b*x^2]
```

fricas [A] time = 0.41, size = 108, normalized size = 2.57

$$
\left[x\sqrt{\frac{bx^2+a}{x^2}} + \frac{1}{2}\sqrt{a}\,\log\left(-\frac{bx^2-2\sqrt{a}\,x\sqrt{\frac{bx^2+a}{x^2}}+2a}{x^2}\right), x\sqrt{\frac{bx^2+a}{x^2}} + \sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}\,x\sqrt{\frac{bx^2+a}{x^2}}}{bx^2+a}\right)\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((b*x^2 + a)/x^2) + 1/2*sqrt(a)*log(-(b*x^2 - 2*sqrt(a)*x*sqrt((b*x^
2 + a)/x^2) + 2*a)/x^2), x*sqrt((b*x^2 + a)/x^2) + sqrt(-a)*arctan(sqrt(-a)
*x*sqrt((b*x^2 + a)/x^2)/(b*x^2 + a))]
```

giac [B] time = 0.17, size = 69, normalized size = 1.64

$$
\frac{a\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} + \sqrt{bx^2+a}\,\operatorname{sgn}(x) - \frac{\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\,\sqrt{a}\right)\operatorname{sgn}(x)}{\sqrt{-a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*arctan(sqrt(b*x^2 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + sqrt(b*x^2 + a)*sgn(x)
 - (a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(-a)
```

maple [A] time = 0.08, size = 61, normalized size = 1.45

$$
\frac{\sqrt{\frac{b\,x^2+a}{x^2}}\left(-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{b\,x^2+a}\,\sqrt{a}}{x}\right)+\sqrt{b\,x^2+a}\right)x}{\sqrt{b\,x^2+a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/x^2)^(1/2),x)
```

```
[Out] ((b*x^2+a)/x^2)^(1/2)*x/(b*x^2+a)^(1/2)*((b*x^2+a)^(1/2)-a^(1/2)*ln(2*(a+(b
*x^2+a)^(1/2)*a^(1/2))/x))
```

maxima [A] time = 2.94, size = 53, normalized size = 1.26

$$\sqrt{b + \frac{a}{x^2}}\, x + \frac{1}{2}\, \sqrt{a} \log\left(\frac{\sqrt{b + \frac{a}{x^2}}\, x - \sqrt{a}}{\sqrt{b + \frac{a}{x^2}}\, x + \sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b + a/x^2)*x + 1/2*sqrt(a)*log((sqrt(b + a/x^2)*x - sqrt(a))/(sqrt(b +
 a/x^2)*x + sqrt(a)))
```

mupad [B] time = 5.57, size = 55, normalized size = 1.31

$$x\sqrt{b + \frac{a}{x^2}} + \frac{\sqrt{a}\, \operatorname{asin}\left(\frac{\sqrt{a}\, 1\mathrm{i}}{\sqrt{b}\, x}\right) \sqrt{b + \frac{a}{x^2}}\, 1\mathrm{i}}{\sqrt{b}\, \sqrt{\frac{a}{b\, x^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)/x^2)^(1/2),x)
```

```
[Out] x*(b + a/x^2)^(1/2) + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x))*(b + a/x^2)^(
1/2)*1i)/(b^(1/2)*(a/(b*x^2) + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a + bx^2}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2+a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**2)/x**2), x)
```

3.384 $\int \sqrt{\frac{a+bx^3}{x^2}}\,dx$

Optimal. Leaf size=51

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

```
[Out] -2/3*arctanh(a^(1/2)/x/(a/x^2+b*x)^(1/2))*a^(1/2)+2/3*x*(a/x^2+b*x)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1979, 2007, 2029, 206}

$$\frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x])/3 - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x]
)])/3
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a+bx^3}{x^2}}\,dx &= \int \sqrt{\frac{a}{x^2}+bx}\,dx\\ &= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}+a\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx}}\,dx\\ &= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx}}\right)\\ &= \frac{2}{3}x\sqrt{\frac{a}{x^2}+bx}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx}}\right)\end{aligned}$$

Mathematica [A] time = 0.04, size = 66, normalized size = 1.29

$$\frac{2x\sqrt{\frac{a}{x^2}+bx}\left(\sqrt{a+bx^3}-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)\right)}{3\sqrt{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x]*(Sqrt[a + b*x^3] - Sqrt[a]*ArcTanh[Sqrt[a + b*x^3]/S
qrt[a]]))/(3*Sqrt[a + b*x^3])
```

fricas [A] time = 0.41, size = 104, normalized size = 2.04

$$\left[\frac{2}{3}x\sqrt{\frac{bx^3+a}{x^2}}+\frac{1}{3}\sqrt{a}\log\left(\frac{bx^3-2\sqrt{a}\,x\sqrt{\frac{bx^3+a}{x^2}}+2a}{x^3}\right),\frac{2}{3}x\sqrt{\frac{bx^3+a}{x^2}}+\frac{2}{3}\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\,x\sqrt{\frac{bx^3+a}{x^2}}}{a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*x*sqrt((b*x^3 + a)/x^2) + 1/3*sqrt(a)*log((b*x^3 - 2*sqrt(a)*x*sqrt((b
*x^3 + a)/x^2) + 2*a)/x^3), 2/3*x*sqrt((b*x^3 + a)/x^2) + 2/3*sqrt(-a)*arct
an(sqrt(-a)*x*sqrt((b*x^3 + a)/x^2)/a)]
```

giac [A] time = 0.19, size = 71, normalized size = 1.39

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{3\sqrt{-a}}+\frac{2}{3}\sqrt{bx^3+a}\,\mathrm{sgn}(x)-\frac{2\left(a\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}\right)\mathrm{sgn}(x)}{3\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*a*arctan(sqrt(b*x^3 + a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2/3*sqrt(b*x^3 + a
)*sgn(x) - 2/3*(a*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a))*sgn(x)/sqrt(
-a)
```

maple [A] time = 0.08, size = 55, normalized size = 1.08

$$\frac{2\sqrt{\frac{b\,x^3+a}{x^2}}\left(-\sqrt{a}\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^3+a}}{\sqrt{a}}\right)+\sqrt{b\,x^3+a}\right)x}{3\sqrt{b\,x^3+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3+a)/x^2)^(1/2),x)
```

```
[Out] 2/3*((b*x^3+a)/x^2)^(1/2)*x*(-arctanh((b*x^3+a)^(1/2)/a^(1/2))*a^(1/2)+(b*x
^3+a)^(1/2))/(b*x^3+a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx^3 + a}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^3 + a)/x^2), x)
```

mupad [B] time = 5.34, size = 63, normalized size = 1.24

$$\frac{2\,x\,\sqrt{b\,x+\frac{a}{x^2}}}{3}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}\,1{}\mathrm{i}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x+\frac{a}{x^2}}\,2{}\mathrm{i}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{\frac{a}{b\,x^3}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^3)/x^2)^(1/2),x)
```

```
[Out] (2*x*(b*x + a/x^2)^(1/2))/3 + (a^(1/2)*asin((a^(1/2)*1i)/(b^(1/2)*x^(3/2)))
*(b*x + a/x^2)^(1/2)*2i)/(3*b^(1/2)*x^(1/2)*(a/(b*x^3) + 1)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3+a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.385 $\int \sqrt{\frac{a+bx^n}{x^2}}\,dx$

Optimal. Leaf size=61

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{n-2}}}\right)}{n}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^(-2+n))^(1/2))*a^(1/2)/n+2*x*(a/x^2+b*x^(-2
+n))^(1/2)/n
```

Rubi [A] time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1979, 2007, 2029, 206}

$$\frac{2x\sqrt{\frac{a}{x^2}+bx^{n-2}}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{n-2}}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[a/x^2 + b*x^(-2 + n)])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a]/(x*Sqrt[a/x
^2 + b*x^(-2 + n)])])/n
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}\int\sqrt{\frac{a+bx^n}{x^2}}\,dx &= \int\sqrt{\frac{a}{x^2}+bx^{-2+n}}\,dx\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}+a\int\frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\,dx\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}-\frac{(2a)\,\mathrm{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}\\ &= \frac{2x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.02, size = 70, normalized size = 1.15

$$\frac{x\sqrt{\frac{a+bx^n}{x^2}}\left(2\sqrt{a+bx^n}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)\right)}{n\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + b*x^n)/x^2],x]
```

```
[Out] (x*Sqrt[(a + b*x^n)/x^2]*(2*Sqrt[a + b*x^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*
x^n]/Sqrt[a]]))/(n*Sqrt[a + b*x^n])
```

fricas [A] time = 0.44, size = 112, normalized size = 1.84

$$\left[\frac{2x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{a}\log\left(\frac{bx^n-2\sqrt{a}x\sqrt{\frac{bx^n+a}{x^2}}+2a}{x^n}\right)}{n},\ \frac{2\left(x\sqrt{\frac{bx^n+a}{x^2}}+\sqrt{-a}\arctan\left(\frac{\sqrt{-a}x\sqrt{\frac{bx^n+a}{x^2}}}{a}\right)\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*x*sqrt((b*x^n + a)/x^2) + sqrt(a)*log((b*x^n - 2*sqrt(a)*x*sqrt((b*x^n
+ a)/x^2) + 2*a)/x^n))/n, 2*(x*sqrt((b*x^n + a)/x^2) + sqrt(-a)*arctan(sqrt
(-a)*x*sqrt((b*x^n + a)/x^2)/a))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{bx^n+a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

maple [A] time = 1.87, size = 74, normalized size = 1.21

$$-\frac{2\sqrt{\frac{b\,\mathrm{e}^{n\ln(x)}+a}{x^2}}\,\sqrt{a}\,x\,\mathrm{arctanh}\left(\frac{\sqrt{b\,\mathrm{e}^{n\ln(x)}+a}}{\sqrt{a}}\right)}{\sqrt{b\,\mathrm{e}^{n\ln(x)}+a}\;n}+\frac{2\sqrt{\frac{b\,\mathrm{e}^{n\ln(x)}+a}{x^2}}\,x}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^n+a)/x^2)^(1/2),x)
```

```
[Out] 2/n*((b*exp(n*ln(x))+a)/x^2)^(1/2)*x-2*a^(1/2)/n*arctanh((b*exp(n*ln(x))+a)
^(1/2)/a^(1/2))*((b*exp(n*ln(x))+a)/x^2)^(1/2)/(b*exp(n*ln(x))+a)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx^n + a}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^n + a)/x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{a + b\,x^n}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^n)/x^2)^(1/2),x)
```

```
[Out] int(((a + b*x^n)/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a + bx^n}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+b*x**n)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((a + b*x**n)/x**2), x)
```

3.386 $\int \sqrt{\frac{-a+bx}{x^2}}\,dx$

Optimal. Leaf size=53

$$2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}+2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x}-\frac{a}{x^2}}}\right)$$

```
[Out] 2*arctan(a^(1/2)/x/(-a/x^2+b/x)^(1/2))*a^(1/2)+2*x*(-a/x^2+b/x)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1979, 2007, 2013, 620, 203}

$$2x\sqrt{\frac{b}{x}-\frac{a}{x^2}}+2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{b}{x}-\frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x)/x^2],x]
```

```
[Out] 2*Sqrt[-(a/x^2) + b/x]*x + 2*Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[-(a/x^2) + b/x]*x
)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2013

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist
[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]]
&& EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{-a+bx}{x^2}}\,dx &= \int \sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,dx\\ &= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x - a\int \frac{1}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x^2}\,dx\\ &= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x + a\,\text{Subst}\left(\int \frac{1}{\sqrt{bx-ax^2}}\,dx, x, \frac{1}{x}\right)\\ &= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x + (2a)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x}\right)\\ &= 2\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x + 2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{-\frac{a}{x^2}+\frac{b}{x}}\,x}\right)\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 1.25

$$\frac{2x\sqrt{\frac{bx-a}{x^2}}\left(\sqrt{bx-a}-\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\right)}{\sqrt{bx-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x)/x^2],x]
```

```
[Out] (2*x*Sqrt[(-a + b*x)/x^2]*(Sqrt[-a + b*x] - Sqrt[a]*ArcTan[Sqrt[-a + b*x]/S
qrt[a]]))/Sqrt[-a + b*x]
```

fricas [A] time = 0.41, size = 98, normalized size = 1.85

$$\left[2\,x\sqrt{\frac{bx-a}{x^2}}+\sqrt{-a}\,\log\left(\frac{bx-2\,\sqrt{-a}\,x\sqrt{\frac{bx-a}{x^2}}-2\,a}{x}\right), 2\,x\sqrt{\frac{bx-a}{x^2}}-2\,\sqrt{a}\,\arctan\left(\frac{x\sqrt{\frac{bx-a}{x^2}}}{\sqrt{a}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*x*sqrt((b*x - a)/x^2) + sqrt(-a)*log((b*x - 2*sqrt(-a)*x*sqrt((b*x - a)/
x^2) - 2*a)/x), 2*x*sqrt((b*x - a)/x^2) - 2*sqrt(a)*arctan(x*sqrt((b*x - a)
/x^2)/sqrt(a))]
```

giac [A] time = 0.16, size = 61, normalized size = 1.15

$$-2\,\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)\mathrm{sgn}(x)+2\left(\sqrt{a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\mathrm{sgn}(x)+2\,\sqrt{bx-a}\,\mathrm{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a)*arctan(sqrt(b*x - a)/sqrt(a))*sgn(x) + 2*(sqrt(a)*arctan(sqrt(-a
)/sqrt(a)) - sqrt(-a))*sgn(x) + 2*sqrt(b*x - a)*sgn(x)
```

maple [A] time = 0.08, size = 55, normalized size = 1.04

$$\frac{2\sqrt{\frac{bx-a}{x^2}}\left(-\sqrt{a}\,\arctan\left(\frac{\sqrt{bx-a}}{\sqrt{a}}\right)+\sqrt{bx-a}\right)x}{\sqrt{bx-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x-a)/x^2)^(1/2),x)
```

```
[Out] 2*((b*x-a)/x^2)^(1/2)*x*(-a^(1/2)*arctan((b*x-a)^(1/2)/a^(1/2))+(b*x-a)^(1/
2))/(b*x-a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx-a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x - a)/x^2), x)
```

mupad [B] time = 5.18, size = 67, normalized size = 1.26

$$2\,x\,\sqrt{\frac{b}{x}-\frac{a}{x^2}}+\frac{2\,\sqrt{a}\,\sqrt{x}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,\sqrt{x}}\right)\,\sqrt{\frac{b}{x}-\frac{a}{x^2}}}{\sqrt{b}\,\sqrt{1-\frac{a}{b\,x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x)/x^2)^(1/2),x)
```

```
[Out] 2*x*(b/x - a/x^2)^(1/2) + (2*a^(1/2)*x^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(1/2))
)*(b/x - a/x^2)^(1/2))/(b^(1/2)*(1 - a/(b*x))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{-a+bx}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x-a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x)/x**2), x)
```

3.387 $\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx$

Optimal. Leaf size=43

$$x\sqrt{b-\frac{a}{x^2}}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)$$

```
[Out] arctan(a^(1/2)/x/(b-a/x^2)^(1/2))*a^(1/2)+x*(b-a/x^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1972, 242, 277, 217, 203}

$$x\sqrt{b-\frac{a}{x^2}}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{b-\frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b - a/x^2]*x + Sqrt[a]*ArcTan[Sqrt[a]/(Sqrt[b - a/x^2]*x)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{-a+bx^2}{x^2}}\,dx &= \int \sqrt{b-\frac{a}{x^2}}\,dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{b-ax^2}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{b-\frac{a}{x^2}}\,x + a\,\text{Subst}\left(\int \frac{1}{\sqrt{b-ax^2}}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{b-\frac{a}{x^2}}\,x + a\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{\sqrt{b-\frac{a}{x^2}}\,x}\right) \\
&= \sqrt{b-\frac{a}{x^2}}\,x + \sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{\sqrt{b-\frac{a}{x^2}}\,x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 68, normalized size = 1.58

$$
x\sqrt{b-\frac{a}{x^2}} - \frac{\sqrt{a}\,x\sqrt{b-\frac{a}{x^2}}\,\tan^{-1}\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)}{\sqrt{bx^2-a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^2)/x^2],x]
```

```
[Out] Sqrt[b - a/x^2]*x - (Sqrt[a]*Sqrt[b - a/x^2]*x*ArcTan[Sqrt[-a + b*x^2]/Sqrt
[a]])/Sqrt[-a + b*x^2]
```

fricas [A] time = 0.41, size = 118, normalized size = 2.74

$$
\left[x\sqrt{\frac{bx^2-a}{x^2}} + \frac{1}{2}\sqrt{-a}\,\log\left(-\frac{bx^2-2\sqrt{-a}\,x\sqrt{\frac{bx^2-a}{x^2}}-2a}{x^2}\right), x\sqrt{\frac{bx^2-a}{x^2}} + \sqrt{a}\,\arctan\left(\frac{\sqrt{a}\,x\sqrt{\frac{bx^2-a}{x^2}}}{bx^2-a}\right)\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [x*sqrt((b*x^2 - a)/x^2) + 1/2*sqrt(-a)*log(-(b*x^2 - 2*sqrt(-a)*x*sqrt((b*
x^2 - a)/x^2) - 2*a)/x^2), x*sqrt((b*x^2 - a)/x^2) + sqrt(a)*arctan(sqrt(a)
*x*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a))]
```

giac [A] time = 0.24, size = 63, normalized size = 1.47

$$
-\sqrt{a}\,\arctan\left(\frac{\sqrt{bx^2-a}}{\sqrt{a}}\right)\text{sgn}(x) + \left(\sqrt{a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right) - \sqrt{-a}\right)\text{sgn}(x) + \sqrt{bx^2-a}\,\text{sgn}(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(a)*arctan(sqrt(b*x^2 - a)/sqrt(a))*sgn(x) + (sqrt(a)*arctan(sqrt(-a)/
sqrt(a)) - sqrt(-a))*sgn(x) + sqrt(b*x^2 - a)*sgn(x)
```

maple [B] time = 0.09, size = 81, normalized size = 1.88

$$
\frac{\sqrt{\frac{b\,x^2-a}{x^2}}\,\left(a\ln\left(\frac{-2a+2\sqrt{-a}\,\sqrt{b\,x^2-a}}{x}\right)+\sqrt{-a}\,\sqrt{b\,x^2-a}\right)x}{\sqrt{-a}\,\sqrt{b\,x^2-a}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2-a)/x^2)^(1/2),x)
```

```
[Out] ((b*x^2-a)/x^2)^(1/2)*x*((-a)^(1/2)*(b*x^2-a)^(1/2)+a*ln(2*((-a)^(1/2)*(b*x
^2-a)^(1/2)-a)/x))/(-a)^(1/2)/(b*x^2-a)^(1/2)
```

maxima [A] time = 2.99, size = 34, normalized size = 0.79

$$\sqrt{b-\frac{a}{x^2}}\,x-\sqrt{a}\arctan\left(\frac{\sqrt{b-\frac{a}{x^2}}\,x}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^2-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b - a/x^2)*x - sqrt(a)*arctan(sqrt(b - a/x^2)*x/sqrt(a))
```

mupad [B] time = 5.38, size = 54, normalized size = 1.26

$$x\sqrt{b-\frac{a}{x^2}}+\frac{\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)\sqrt{b-\frac{a}{x^2}}}{\sqrt{b}\,\sqrt{1-\frac{a}{b\,x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x^2)/x^2)^(1/2),x)
```

```
[Out] x*(b - a/x^2)^(1/2) + (a^(1/2)*asin(a^(1/2)/(b^(1/2)*x))*(b - a/x^2)^(1/2))
/(b^(1/2)*(1 - a/(b*x^2))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{-a+bx^2}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**2-a)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x**2)/x**2), x)
```

3.388 $\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}+\frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx-\frac{a}{x^2}}}\right)$$

```
[Out] 2/3*arctan(a^(1/2)/x/(-a/x^2+b*x)^(1/2))*a^(1/2)+2/3*x*(-a/x^2+b*x)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1979, 2007, 2029, 203}

$$\frac{2}{3}x\sqrt{bx-\frac{a}{x^2}}+\frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx-\frac{a}{x^2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x])/3 + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-(a/x^2) +
 b*x])])/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\int \sqrt{\frac{-a+bx^3}{x^2}}\,dx = \int \sqrt{-\frac{a}{x^2}+bx}\,dx$$
$$= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} - a\int \frac{1}{x^2\sqrt{-\frac{a}{x^2}+bx}}\,dx$$
$$= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} + \frac{1}{3}(2a)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2}+bx}}\right)$$
$$= \frac{2}{3}x\sqrt{-\frac{a}{x^2}+bx} + \frac{2}{3}\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx}}\right)$$

Mathematica [A] time = 0.04, size = 73, normalized size = 1.38

$$\frac{2x\sqrt{bx-\frac{a}{x^2}}\left(\sqrt{bx^3-a}-\sqrt{a}\tan^{-1}\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)\right)}{3\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^3)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x]*(Sqrt[-a + b*x^3] - Sqrt[a]*ArcTan[Sqrt[-a + b*x^
3]/Sqrt[a]]))/(3*Sqrt[-a + b*x^3])
```

fricas [A] time = 0.41, size = 109, normalized size = 2.06

$$\left[\frac{2}{3}x\sqrt{\frac{bx^3-a}{x^2}} + \frac{1}{3}\sqrt{-a}\log\left(\frac{bx^3-2\sqrt{-a}\,x\sqrt{\frac{bx^3-a}{x^2}}-2a}{x^3}\right), \frac{2}{3}x\sqrt{\frac{bx^3-a}{x^2}} - \frac{2}{3}\sqrt{a}\arctan\left(\frac{x\sqrt{\frac{bx^3-a}{x^2}}}{\sqrt{a}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [2/3*x*sqrt((b*x^3 - a)/x^2) + 1/3*sqrt(-a)*log((b*x^3 - 2*sqrt(-a)*x*sqrt(
(b*x^3 - a)/x^2) - 2*a)/x^3), 2/3*x*sqrt((b*x^3 - a)/x^2) - 2/3*sqrt(a)*arc
tan(x*sqrt((b*x^3 - a)/x^2)/sqrt(a))]
```

giac [A] time = 0.19, size = 65, normalized size = 1.23

$$-\frac{2}{3}\sqrt{a}\arctan\left(\frac{\sqrt{bx^3-a}}{\sqrt{a}}\right)\text{sgn}(x)+\frac{2}{3}\left(\sqrt{a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{a}}\right)-\sqrt{-a}\right)\text{sgn}(x)+\frac{2}{3}\sqrt{bx^3-a}\,\text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*sqrt(a)*arctan(sqrt(b*x^3 - a)/sqrt(a))*sgn(x) + 2/3*(sqrt(a)*arctan(s
qrt(-a)/sqrt(a)) - sqrt(-a))*sgn(x) + 2/3*sqrt(b*x^3 - a)*sgn(x)
```

maple [A] time = 0.11, size = 73, normalized size = 1.38

$$\frac{2\sqrt{\frac{bx^3-a}{x^2}}\left(a\,\text{arctanh}\left(\frac{\sqrt{bx^3-a}}{\sqrt{-a}}\right)+\sqrt{bx^3-a}\,\sqrt{-a}\right)x}{3\sqrt{bx^3-a}\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^3-a)/x^2)^(1/2),x)
```

```
[Out] 2/3*((b*x^3-a)/x^2)^(1/2)*x*((b*x^3-a)^(1/2)*(-a)^(1/2)+a*arctanh((b*x^3-a)
^(1/2)/(-a)^(1/2)))/(b*x^3-a)^(1/2)/(-a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx^3 - a}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x^3-a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^3 - a)/x^2), x)
```

mupad [B] time = 5.35, size = 63, normalized size = 1.19

$$\frac{2\,x\,\sqrt{b\,x-\frac{a}{x^2}}}{3}+\frac{2\,\sqrt{a}\,\mathrm{asin}\left(\frac{\sqrt{a}}{\sqrt{b}\,x^{3/2}}\right)\,\sqrt{b\,x-\frac{a}{x^2}}}{3\,\sqrt{b}\,\sqrt{x}\,\sqrt{1-\frac{a}{b\,x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x^3)/x^2)^(1/2),x)
```

```
[Out] (2*x*(b*x - a/x^2)^(1/2))/3 + (2*a^(1/2)*asin(a^(1/2)/(b^(1/2)*x^(3/2)))*(b
*x - a/x^2)^(1/2))/(3*b^(1/2)*x^(1/2)*(1 - a/(b*x^3))^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x**3-a)/x**2)**(1/2),x)
```

```
[Out] Timed out
```

3.389 $\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}+\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}\right)}{n}$$

```
[Out] 2*arctan(a^(1/2)/x/(-a/x^2+b*x^(-2+n))^(1/2))*a^(1/2)/n+2*x*(-a/x^2+b*x^(-2
+n))^(1/2)/n
```

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1979, 2007, 2029, 203}

$$\frac{2x\sqrt{bx^{n-2}-\frac{a}{x^2}}}{n}+\frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{bx^{n-2}-\frac{a}{x^2}}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (2*x*Sqrt[-(a/x^2) + b*x^(-2 + n)])/n + (2*Sqrt[a]*ArcTan[Sqrt[a]/(x*Sqrt[-
(a/x^2) + b*x^(-2 + n)])])/n
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2007

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(p*(n - j)), x] + Dist[a, Int[x^j*(a*x^j + b*x^n)^(p - 1), x],
 x] /; FreeQ[{a, b, j, n}, x] && IGtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simpl
ify[j*p + 1], 0]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{-a+bx^n}{x^2}}\,dx &= \int \sqrt{-\frac{a}{x^2}+bx^{-2+n}}\,dx \\
&= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} - a\int \frac{1}{x^2\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\,dx \\
&= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, \frac{1}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n} \\
&= \frac{2x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}{n} + \frac{2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a}}{x\sqrt{-\frac{a}{x^2}+bx^{-2+n}}}\right)}{n}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 78, normalized size = 1.24

$$
\frac{x\sqrt{\frac{bx^n-a}{x^2}}\left(2\sqrt{bx^n-a}-2\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{bx^n-a}}{\sqrt{a}}\right)\right)}{n\sqrt{bx^n-a}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(-a + b*x^n)/x^2],x]
```

```
[Out] (x*Sqrt[(-a + b*x^n)/x^2]*(2*Sqrt[-a + b*x^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*x^n]/Sqrt[a]]))/(n*Sqrt[-a + b*x^n])
```

fricas [A] time = 0.42, size = 118, normalized size = 1.87

$$
\left[\frac{2\,x\sqrt{\frac{bx^n-a}{x^2}}+\sqrt{-a}\,\log\left(\frac{bx^n-2\,\sqrt{-a}\,x\sqrt{\frac{bx^n-a}{x^2}}-2\,a}{x^n}\right)}{n}, \frac{2\left(x\sqrt{\frac{bx^n-a}{x^2}}-\sqrt{a}\,\arctan\left(\frac{x\sqrt{\frac{bx^n-a}{x^2}}}{\sqrt{a}}\right)\right)}{n}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*x*sqrt((b*x^n - a)/x^2) + sqrt(-a)*log((b*x^n - 2*sqrt(-a)*x*sqrt((b*x^n - a)/x^2) - 2*a)/x^n))/n, 2*(x*sqrt((b*x^n - a)/x^2) - sqrt(a)*arctan(x*sqrt((b*x^n - a)/x^2)/sqrt(a)))/n]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \sqrt{\frac{bx^n-a}{x^2}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((b*x^n - a)/x^2), x)
```

maple [A] time = 1.99, size = 105, normalized size = 1.67

$$
-\frac{2\sqrt{\frac{b\,e^{n\ln(x)}-a}{x^2}}\,\sqrt{a}\,x\arctan\left(\frac{\sqrt{b\,e^{n\ln(x)}-a}}{\sqrt{a}}\right)}{\sqrt{b\,e^{n\ln(x)}-a}\,n}-\frac{2\left(-b\,e^{n\ln(x)}+a\right)\sqrt{\frac{b\,e^{n\ln(x)}-a}{x^2}}\,x}{\left(b\,e^{n\ln(x)}-a\right)n}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^n-a)/x^2)^(1/2),x)
```

```
[Out] -2*(a-b*exp(n*ln(x)))/n/(b*exp(n*ln(x))-a)*((b*exp(n*ln(x))-a)/x^2)^(1/2)*x
-2*a^(1/2)/n*arctan((b*exp(n*ln(x))-a)^(1/2)/a^(1/2))*((b*exp(n*ln(x))-a)/x
^2)^(1/2)/(b*exp(n*ln(x))-a)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{bx^n - a}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x^n)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((b*x^n - a)/x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{-\frac{a - b\,x^n}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(a - b*x^n)/x^2)^(1/2),x)
```

```
[Out] int((-(a - b*x^n)/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{-a + bx^n}{x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-a+b*x**n)/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt((-a + b*x**n)/x**2), x)
```

3.390 $\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx$

Optimal. Leaf size=62

$$\frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}\,c(j-n)}$$

```
[Out] 2*(c*x)^(1/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/c/(j-n)/(x^(
1/2*j))/a^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2031, 2029, 206}

$$\frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}\,c(j-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*(c*x)^(j/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(Sqrt[a]*c*(
j - n)*x^(j/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(cx)^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx &= \frac{\left(x^{-j/2}(cx)^{j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{c}\\
&= \frac{\left(2x^{-j/2}(cx)^{j/2}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{c(j-n)}\\
&= \frac{2x^{-j/2}(cx)^{j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{\sqrt{a}\,c(j-n)}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 98, normalized size = 1.58

$$\frac{2\sqrt{b}\,(cx)^{j/2}x^{\frac{n-j}{2}}\sqrt{\frac{ax^{j-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + j/2)/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*Sqrt[b]*x^((-j + n)/2)*(c*x)^(j/2)*Sqrt[1 + (a*x^(j - n))/b]*ArcSinh[(Sq
rt[a]*x^((j - n)/2))/Sqrt[b]])/(Sqrt[a]*c*(j - n)*Sqrt[a*x^j + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

maple [F] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{1}{2}j-1}}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+1/2*j)/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(1/2*j - 1)/sqrt(a*x^j + b*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c\,x)^{\frac{j}{2}-1}}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(j/2 - 1)/(a*x^j + b*x^n)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{j}{2}-1}}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+1/2*j)/(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral((c*x)**(j/2 - 1)/sqrt(a*x**j + b*x**n), x)
```

3.391 $\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$

Optimal. Leaf size=53

$$\frac{2\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}\,(3-n)\sqrt{x}}$$

```
[Out] 2*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/(3-n)/a^(1/2)/x^
(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2031, 2029, 206}

$$\frac{2\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}\,(3-n)\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

```
[Out] (2*Sqrt[c*x]*ArcTanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(Sqrt[a]*(3 -
n)*Sqrt[x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx &= \frac{\sqrt{cx}\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{\sqrt{x}}\\ &= \frac{\left(2\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{(3-n)\sqrt{x}}\\ &= \frac{2\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{\sqrt{a}\,(3-n)\sqrt{x}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 89, normalized size = 1.68

$$-\frac{2\sqrt{b}\sqrt{cx}\,x^{\frac{n-1}{2}}\sqrt{\frac{ax^{3-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,(n-3)\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/Sqrt[a*x^3 + b*x^n],x]
```

```
[Out] (-2*Sqrt[b]*x^((-1 + n)/2)*Sqrt[c*x]*Sqrt[1 + (a*x^(3 - n))/b]*ArcSinh[(Sqr
t[a]*x^(3/2 - n/2))/Sqrt[b]])/(Sqrt[a]*(-3 + n)*Sqrt[a*x^3 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{a\,x^3+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x^3+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x)/sqrt(a*x^3 + b*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{c\,x}}{\sqrt{b\,x^n+a\,x^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2),x)
```

```
[Out] int((c*x)^(1/2)/(b*x^n + a*x^3)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{\sqrt{ax^3 + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(a*x**3+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(c*x)/sqrt(a*x**3 + b*x**n), x)
```

3.392 $\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}\,(2-n)}$$

```
[Out] 2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))/(2-n)/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^2 + b*x^n],x]
```

```
[Out] (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^n]])/(Sqrt[a]*(2 - n))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx &= \frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{\sqrt{a}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.07, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{b}\,x^{n/2}\sqrt{\frac{ax^{2-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,(n-2)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^2 + b*x^n],x]
```

```
[Out] (-2*Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt[a]*x^(1 - n/2))
/Sqrt[b]])/(Sqrt[a]*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,x^2 + b\,x^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x^n)^(1/2),x)
```

```
[Out] int(1/(a*x^2+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x^2 + b*x^n), x)
```

mupad [B] time = 5.39, size = 67, normalized size = 1.81

$$\frac{\sqrt{b}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{b}}\right)\,\sqrt{\frac{a\,x^{2-n}}{b}+1}\,1\mathrm{i}}{\sqrt{a}\,\left(\frac{n}{2}-1\right)\,\sqrt{b\,x^n+a\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^n + a*x^2)^(1/2),x)
```

```
[Out] (b^(1/2)*x^(n/2)*asin((a^(1/2)*x^(1 - n/2)*1i)/b^(1/2))*((a*x^(2 - n))/b +
1)^(1/2)*1i)/(a^(1/2)*(n/2 - 1)*(b*x^n + a*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^2 + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**2 + b*x**n), x)
```

3.393 $\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx$

Optimal. Leaf size=51

$$\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}\,(1-n)\sqrt{cx}}$$

```
[Out] 2*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*x^(1/2)/(1-n)/a^(1/2)/(c*x)^(1
/2)
```

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2031, 2029, 206}

$$\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}\,(1-n)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

```
[Out] (2*Sqrt[x]*ArcTanh[(Sqrt[a]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(Sqrt[a]*(1 - n)*S
qrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx &= \frac{\sqrt{x}\int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{\sqrt{cx}}\\ &= \frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{(1-n)\sqrt{cx}}\\ &= \frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{\sqrt{a}\,(1-n)\sqrt{cx}}\end{aligned}$$

Mathematica [A] time = 0.13, size = 87, normalized size = 1.71

$$-\frac{2\sqrt{b}\,x^{\frac{n+1}{2}}\sqrt{\frac{ax^{1-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,(n-1)\sqrt{cx}\,\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c*x]*Sqrt[a*x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[b]*x^((1 + n)/2)*Sqrt[1 + (a*x^(1 - n))/b]*ArcSinh[(Sqrt[a]*x^(1/2
 - n/2))/Sqrt[b]])/(Sqrt[a]*(-1 + n)*Sqrt[c*x]*Sqrt[a*x + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^n}\,\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

```
[Out] int(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax+bx^n}\,\sqrt{cx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(1/2)/(a*x+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*x + b*x^n)*sqrt(c*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{c\,x}\,\sqrt{b\,x^n+a\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)),x)
```

```
[Out] int(1/((c*x)^(1/2)*(b*x^n + a*x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx}\,\sqrt{ax+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(1/2)/(a*x+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(c*x)*sqrt(a*x + b*x**n)), x)
```

3.394 $\int \frac{1}{cx\sqrt{a+bx^n}}\,dx$

Optimal. Leaf size=31

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/c/n/a^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {12, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c*x*Sqrt[a + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{cx\sqrt{a+bx^n}}\,dx &= \frac{\int \frac{1}{x\sqrt{a+bx^n}}\,dx}{c}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{cn}\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{bcn}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$
-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{\sqrt{a}\,cn}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c*x*Sqrt[a + b*x^n]),x]
[Out] (-2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(Sqrt[a]*c*n)
```

fricas [A] time = 0.42, size = 76, normalized size = 2.45

$$
\left[\frac{\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right)}{\sqrt{a}\,cn}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right)}{acn}\right]
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="fricas")
[Out] [log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x^n)/(sqrt(a)*c*n), 2*sqrt(-a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a)/(a*c*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{\sqrt{bx^n+a}\,cx}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*x^n + a)*c*x), x)
```

maple [A] time = 0.05, size = 26, normalized size = 0.84

$$
-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right)}{\sqrt{a}\;cn}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c/x/(b*x^n+a)^(1/2),x)
```

[Out] -2*arctanh((b*x^n+a)^(1/2)/a^(1/2))/c/n/a^(1/2)

maxima [A] time = 2.96, size = 42, normalized size = 1.35

$$\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{\sqrt{a}\,cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c/x/(a+b*x^n)^(1/2),x, algorithm="maxima")

[Out] log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(sqrt(a)*c*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{c\,x\,\sqrt{a+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x*(a + b*x^n)^(1/2)),x)

[Out] int(1/(c*x*(a + b*x^n)^(1/2)), x)

sympy [A] time = 1.98, size = 27, normalized size = 0.87

$$-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}\,x^{-\frac{n}{2}}}{\sqrt{b}}\right)}{\sqrt{a}\,cn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/c/x/(a+b*x**n)**(1/2),x)

[Out] -2*asinh(sqrt(a)*x**(-n/2)/sqrt(b))/(sqrt(a)*c*n)

3.395 $\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}\,c(n+1)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*x^(1/2)/c/(1+n)/a^(1/2)/(c*x)
^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 2029, 206}

$$-\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\,\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}\,c(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(Sqrt[x]*Sqrt[a/x + b*x^n])])/(Sqrt[a]*c*(1 + n
)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx = \frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{c\sqrt{cx}}$$
$$= -\frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{c(1+n)\sqrt{cx}}$$
$$= -\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{\sqrt{a}\,c(1+n)\sqrt{cx}}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 1.26

$$-\frac{2x\sqrt{a+bx^{n+1}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+1}}}{\sqrt{a}}\right)}{\sqrt{a}\,(n+1)(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(3/2)*Sqrt[a/x + b*x^n]),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^(1 + n)]*ArcTanh[Sqrt[a + b*x^(1 + n)]/Sqrt[a]])/(Sqrt[a
]*(1 + n)*(c*x)^(3/2)*Sqrt[a/x + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x}}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\sqrt{b\,x^n + \frac{a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(3/2)/(b*x^n+a/x)^(1/2),x)
```

```
[Out] int(1/(c*x)^(3/2)/(b*x^n+a/x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x}}\,(cx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(3/2)/(a/x+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^n + a/x)*(c*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{3/2}\,\sqrt{b\,x^n + \frac{a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)),x)
[Out] int(1/((c*x)^(3/2)*(b*x^n + a/x)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{3}{2}}\,\sqrt{\frac{a}{x} + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(3/2)/(a/x+b*x**n)**(1/2),x)
[Out] Integral(1/((c*x)**(3/2)*sqrt(a/x + b*x**n)), x)
```

3.396 $\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx$

Optimal. Leaf size=40

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}\,c^2(n+2)}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))/c^2/(2+n)/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {12, 2029, 206}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}\,c^2(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2 + b*x^n])])/(Sqrt[a]*c^2*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{c^2x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx &= \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}\\
&= -\frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{c^2(2+n)}\\
&= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{\sqrt{a}\,c^2(2+n)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 1.65

$$-\frac{2\sqrt{a+bx^{n+2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+2}}}{\sqrt{a}}\right)}{\sqrt{a}\,c^2(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^2*x^2*Sqrt[a/x^2 + b*x^n]),x]
```

```
[Out] (-2*Sqrt[a + b*x^(2 + n)]*ArcTanh[Sqrt[a + b*x^(2 + n)]/Sqrt[a]])/(Sqrt[a]*
c^2*(2 + n)*x*Sqrt[a/x^2 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}\,c^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^2)*c^2*x^2), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,x^n+\frac{a}{x^2}}\,c^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^2/x^2/(b*x^n+a/x^2)^(1/2),x)
```

```
[Out] int(1/c^2/x^2/(b*x^n+a/x^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{bx^n+\frac{a}{x^2}}\,x^2}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^2/x^2/(a/x^2+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^2)*x^2), x)/c^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{c^2\,x^2\,\sqrt{b\,x^n+\frac{a}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)),x)
```

```
[Out] int(1/(c^2*x^2*(b*x^n + a/x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**2/x**2/(a/x**2+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a/x**2 + b*x**n)), x)/c**2
```

3.397 $\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx$

Optimal. Leaf size=54

$$-\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}\,c^2(n+3)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/c^2/(3+n)/a^(1/2)/(
c*x)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2031, 2029, 206}

$$-\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}\,c^2(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

```
[Out] (-2*Sqrt[x]*ArcTanh[Sqrt[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(Sqrt[a]*c^2*(3
 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx = \frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{c^2\sqrt{cx}}$$

$$= -\frac{\left(2\sqrt{x}\right)\mathrm{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{c^2(3+n)\sqrt{cx}}$$

$$= -\frac{2\sqrt{x}\,\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{\sqrt{a}\,c^2(3+n)\sqrt{cx}}$$

Mathematica [A] time = 0.06, size = 68, normalized size = 1.26

$$-\frac{2x\sqrt{a+bx^{n+3}}\,\tanh^{-1}\left(\frac{\sqrt{a+bx^{n+3}}}{\sqrt{a}}\right)}{\sqrt{a}\,(n+3)(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*Sqrt[a/x^3 + b*x^n]),x]
```

```
[Out] (-2*x*Sqrt[a + b*x^(3 + n)]*ArcTanh[Sqrt[a + b*x^(3 + n)]/Sqrt[a]])/(Sqrt[a
]*(3 + n)*(c*x)^(5/2)*Sqrt[a/x^3 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n+\frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}}\sqrt{b\,x^n+\frac{a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^n+a/x^3)^(1/2),x)
```

```
[Out] int(1/(c*x)^(5/2)/(b*x^n+a/x^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^n + \frac{a}{x^3}}\,(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x^3+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*x^n + a/x^3)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(c\,x)^{5/2}\,\sqrt{b\,x^n + \frac{a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)),x)
[Out] int(1/((c*x)^(5/2)*(b*x^n + a/x^3)^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(a/x**3+b*x**n)**(1/2),x)
[Out] Timed out
```

3.398 $\int \frac{(cx)^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*(c*x)^(3/2*j)*arctanh(x^(1/2*j)*a^(1/2)/(a*x^j+b*x^n)^(1/2))/a^(3/2)/c/(j
-n)/(x^(3/2*j))-2*(c*x)^(3/2*j)/a/c/(j-n)/(x^j)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {2031, 2030, 2029, 206}

$$\frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}-\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(c*x)^((3*j)/2))/(a*c*(j - n)*x^j*Sqrt[a*x^j + b*x^n]) + (2*(c*x)^((3*j
)/2)*ArcTanh[(Sqrt[a]*x^(j/2))/Sqrt[a*x^j + b*x^n]])/(a^(3/2)*c*(j - n)*x^(
(3*j)/2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{(cx)^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx = \frac{\left(x^{-3j/2}(cx)^{3j/2}\right)\int \frac{x^{-1+\frac{3j}{2}}}{\left(ax^j+bx^n\right)^{3/2}}\,dx}{c}$$

$$= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{\left(x^{-3j/2}(cx)^{3j/2}\right)\int \frac{x^{-1+\frac{j}{2}}}{\sqrt{ax^j+bx^n}}\,dx}{ac}$$

$$= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{\left(2x^{-3j/2}(cx)^{3j/2}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{ac(j-n)}$$

$$= -\frac{2x^{-j}(cx)^{3j/2}}{ac(j-n)\sqrt{ax^j+bx^n}} + \frac{2x^{-3j/2}(cx)^{3j/2}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{j/2}}{\sqrt{ax^j+bx^n}}\right)}{a^{3/2}c(j-n)}$$

Mathematica [A] time = 0.20, size = 117, normalized size = 1.09

$$-\frac{2x^{-3j/2}(cx)^{3j/2}\left(\sqrt{a}\,x^{j/2}-\sqrt{b}\,x^{n/2}\sqrt{\frac{ax^{j-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{j-n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}c(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(-1 + (3*j)/2)/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (-2*(c*x)^((3*j)/2)*(Sqrt[a]*x^(j/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(j - n
))/b]*ArcSinh[(Sqrt[a]*x^((j - n)/2))/Sqrt[b]]))/(a^(3/2)*c*(j - n)*x^((3*j
)/2)*Sqrt[a*x^j + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}j-1}}{\left(ax^j+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3j}{2}-1}}{\left(a\,x^j+b\,x^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3}{2}j-1}}{\left(ax^j+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(-1+3/2*j)/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(3/2*j - 1)/(a*x^j + b*x^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{\frac{3j}{2}-1}}{\left(a\,x^j+b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^((3*j)/2 - 1)/(a*x^j + b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{3j}{2}-1}}{\left(ax^j+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(-1+3/2*j)/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((c*x)**(3*j/2 - 1)/(a*x**j + b*x**n)**(3/2), x)
```

3.399 $\int \frac{(cx)^{7/2}}{\left(ax^3+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2c^3\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

```
[Out] 2*c^3*arctanh(x^(3/2)*a^(1/2)/(a*x^3+b*x^n)^(1/2))*(c*x)^(1/2)/a^(3/2)/(3-n
)/x^(1/2)-2*c^2*(c*x)^(3/2)/a/(3-n)/(a*x^3+b*x^n)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2030, 2031, 2029, 206}

$$\frac{2c^3\sqrt{cx}\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}-\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] (-2*c^2*(c*x)^(3/2))/(a*(3 - n)*Sqrt[a*x^3 + b*x^n]) + (2*c^3*Sqrt[c*x]*Arc
Tanh[(Sqrt[a]*x^(3/2))/Sqrt[a*x^3 + b*x^n]])/(a^(3/2)*(3 - n)*Sqrt[x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{(cx)^{7/2}}{\left(ax^3+bx^n\right)^{3/2}}\,dx = -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{c^3\int \frac{\sqrt{cx}}{\sqrt{ax^3+bx^n}}\,dx}{a}$$

$$= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{\left(c^3\sqrt{cx}\right)\int \frac{\sqrt{x}}{\sqrt{ax^3+bx^n}}\,dx}{a\sqrt{x}}$$

$$= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{\left(2c^3\sqrt{cx}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a(3-n)\sqrt{x}}$$

$$= -\frac{2c^2(cx)^{3/2}}{a(3-n)\sqrt{ax^3+bx^n}}+\frac{2c^3\sqrt{cx}\,\tanh^{-1}\left(\frac{\sqrt{a}\,x^{3/2}}{\sqrt{ax^3+bx^n}}\right)}{a^{3/2}(3-n)\sqrt{x}}$$

Mathematica [A] time = 0.21, size = 109, normalized size = 1.16

$$\frac{2c^3\sqrt{cx}\left(\sqrt{a}\,x^{3/2}-\sqrt{b}\,x^{n/2}\sqrt{\frac{ax^{3-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{3}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(n-3)\sqrt{x}\,\sqrt{ax^3+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^3*Sqrt[c*x]*(Sqrt[a]*x^(3/2) - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(3 - n))/
b]*ArcSinh[(Sqrt[a]*x^(3/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-3 + n)*Sqrt[x]*Sqr
t[a*x^3 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(ax^3+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(a\,x^3+b\,x^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^{\frac{7}{2}}}{\left(ax^3+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(7/2)/(a*x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^(7/2)/(a*x^3 + b*x^n)^(3/2), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^{7/2}}{\left(b\,x^n+a\,x^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2),x)
```

```
[Out] int((c*x)^(7/2)/(b*x^n + a*x^3)^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(7/2)/(a*x**3+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

3.400 $\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{3/2}} dx$

Optimal. Leaf size=72

$$\frac{2c^2 \tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)} - \frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}$$

```
[Out] 2*c^2*arctanh(x*a^(1/2)/(a*x^2+b*x^n)^(1/2))/a^(3/2)/(2-n)-2*c^2*x/a/(2-n)/
(a*x^2+b*x^n)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2030, 2008, 206}

$$\frac{2c^2 \tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)} - \frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] (-2*c^2*x)/(a*(2 - n)*Sqrt[a*x^2 + b*x^n]) + (2*c^2*ArcTanh[(Sqrt[a]*x)/Sqr
t[a*x^2 + b*x^n]])/(a^(3/2)*(2 - n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{3/2}}\,dx = c^2\int \frac{x^2}{\left(ax^2+bx^n\right)^{3/2}}\,dx$$
$$= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}} + \frac{c^2\int \frac{1}{\sqrt{ax^2+bx^n}}\,dx}{a}$$
$$= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{x}{\sqrt{ax^2+bx^n}}\right)}{a(2-n)}$$
$$= -\frac{2c^2x}{a(2-n)\sqrt{ax^2+bx^n}} + \frac{2c^2\tanh^{-1}\left(\frac{\sqrt{a}\,x}{\sqrt{ax^2+bx^n}}\right)}{a^{3/2}(2-n)}$$

Mathematica [A] time = 0.15, size = 91, normalized size = 1.26

$$\frac{2c^2\left(\sqrt{a}\,x-\sqrt{b}\,x^{n/2}\sqrt{\frac{ax^{2-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{1-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(n-2)\sqrt{ax^2+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^2*x^2)/(a*x^2 + b*x^n)^(3/2),x]
```

```
[Out] (2*c^2*(Sqrt[a]*x - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(2 - n))/b]*ArcSinh[(Sqrt
[a]*x^(1 - n/2))/Sqrt[b]]))/(a^(3/2)*(-2 + n)*Sqrt[a*x^2 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c^2x^2}{\left(ax^2+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(c^2*x^2/(a*x^2 + b*x^n)^(3/2), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{c^2x^2}{\left(a\,x^2+b\,x^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

```
[Out] int(c^2*x^2/(a*x^2+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \frac{x^2}{\left(ax^2 + bx^n\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c^2*x^2/(a*x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] c^2*integrate(x^2/(a*x^2 + b*x^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c^2\, x^2}{\left(b\, x^n + a\, x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2)/(b*x^n + a*x^2)^(3/2),x)
```

```
[Out] int((c^2*x^2)/(b*x^n + a*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2 \int \frac{x^2}{ax^2\sqrt{ax^2 + bx^n} + bx^n\sqrt{ax^2 + bx^n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(c**2*x**2/(a*x**2+b*x**n)**(3/2),x)
```

```
[Out] c**2*Integral(x**2/(a*x**2*sqrt(a*x**2 + b*x**n) + b*x**n*sqrt(a*x**2 + b*x
**n)), x)
```

3.401 $\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{2c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

```
[Out] 2*c*arctanh(a^(1/2)*x^(1/2)/(a*x+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/(1-n)/(c*x)^
(1/2)-2*(c*x)^(1/2)/a/(1-n)/(a*x+b*x^n)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2030, 2031, 2029, 206}

$$\frac{2c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}-\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] (-2*Sqrt[c*x])/(a*(1 - n)*Sqrt[a*x + b*x^n]) + (2*c*Sqrt[x]*ArcTanh[(Sqrt[a
]*Sqrt[x])/Sqrt[a*x + b*x^n]])/(a^(3/2)*(1 - n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{3/2}}\,dx = -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}} + \frac{c\int \frac{1}{\sqrt{cx}\sqrt{ax+bx^n}}\,dx}{a}$$
$$= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}} + \frac{\left(c\sqrt{x}\right)\int \frac{1}{\sqrt{x}\sqrt{ax+bx^n}}\,dx}{a\sqrt{cx}}$$
$$= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}} + \frac{\left(2c\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a(1-n)\sqrt{cx}}$$
$$= -\frac{2\sqrt{cx}}{a(1-n)\sqrt{ax+bx^n}} + \frac{2c\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{ax+bx^n}}\right)}{a^{3/2}(1-n)\sqrt{cx}}$$

Mathematica [A] time = 0.20, size = 104, normalized size = 1.22

$$\frac{2\sqrt{cx}\left(\sqrt{a}\sqrt{x}-\sqrt{b}\,x^{n/2}\sqrt{\frac{ax^{1-n}}{b}+1}\,\sinh^{-1}\left(\frac{\sqrt{a}\,x^{\frac{1}{2}-\frac{n}{2}}}{\sqrt{b}}\right)\right)}{a^{3/2}(n-1)\sqrt{x}\sqrt{ax+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*x]/(a*x + b*x^n)^(3/2),x]
```

```
[Out] (2*Sqrt[c*x]*(Sqrt[a]*Sqrt[x] - Sqrt[b]*x^(n/2)*Sqrt[1 + (a*x^(1 - n))/b]*A
rcSinh[(Sqrt[a]*x^(1/2 - n/2))/Sqrt[b]]))/(a^(3/2)*(-1 + n)*Sqrt[x]*Sqrt[a*
x + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax+b\,x^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^(1/2)/(a*x+b*x^n)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x)/(a*x + b*x^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x}}{{\left(b\,x^n+a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^(1/2)/(b*x^n + a*x)^(3/2),x)
[Out] int((c*x)^(1/2)/(b*x^n + a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{cx}}{(ax+bx^n)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**(1/2)/(a*x+b*x**n)**(3/2),x)
[Out] Integral(sqrt(c*x)/(a*x + b*x**n)**(3/2), x)
```

3.402 $\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx$

Optimal. Leaf size=54

$$\frac{2}{acn\sqrt{a+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

```
[Out] -2*arctanh((a+b*x^n)^(1/2)/a^(1/2))/a^(3/2)/c/n+2/a/c/n/(a+b*x^n)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {12, 266, 51, 63, 208}

$$\frac{2}{acn\sqrt{a+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c*x*(a + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c*n*Sqrt[a + b*x^n]) - (2*ArcTanh[Sqrt[a + b*x^n]/Sqrt[a]])/(a^(3/2)*c
*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{cx(a+bx^n)^{3/2}}\,dx = \frac{\int \frac{1}{x(a+bx^n)^{3/2}}\,dx}{c}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, x^n\right)}{cn}$$
$$= \frac{2}{acn\sqrt{a+bx^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^n\right)}{acn}$$
$$= \frac{2}{acn\sqrt{a+bx^n}} + \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^n}\right)}{abcn}$$
$$= \frac{2}{acn\sqrt{a+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx^n}}{\sqrt{a}}\right)}{a^{3/2}cn}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.74

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^n}{a}+1\right)}{acn\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c*x*(a + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^n)/a])/(a*c*n*Sqrt[a + b*x^n])
```

fricas [A] time = 0.42, size = 148, normalized size = 2.74

$$\left[\frac{\left(\sqrt{a}\,bx^n + a^{\frac{3}{2}}\right)\log\left(\frac{bx^n-2\sqrt{bx^n+a}\sqrt{a}+2a}{x^n}\right) + 2\sqrt{bx^n+a}\,a}{a^2bcnx^n + a^3cn}, \frac{2\left(\left(\sqrt{-a}\,bx^n + \sqrt{-a}\,a\right)\arctan\left(\frac{\sqrt{bx^n+a}\sqrt{-a}}{a}\right) + \sqrt{bx^n+\cdots}\right.}{a^2bcnx^n + a^3cn}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] [((sqrt(a)*b*x^n + a^(3/2))*log((b*x^n - 2*sqrt(b*x^n + a)*sqrt(a) + 2*a)/x
^n) + 2*sqrt(b*x^n + a)*a)/(a^2*b*c*n*x^n + a^3*c*n), 2*((sqrt(-a)*b*x^n +
sqrt(-a)*a)*arctan(sqrt(b*x^n + a)*sqrt(-a)/a) + sqrt(b*x^n + a)*a)/(a^2*b*
c*n*x^n + a^3*c*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n+a)^{\frac{3}{2}}cx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^(3/2)*c*x), x)
```

maple [A] time = 0.05, size = 42, normalized size = 0.78

$$\frac{-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^n+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{b\,x^n+a}\,a}}{cn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c/x/(b*x^n+a)^(3/2),x)
```

```
[Out] 1/c/n*(-2/a^(3/2)*arctanh((b*x^n+a)^(1/2)/a^(1/2))+2/(b*x^n+a)^(1/2)/a)
```

maxima [A] time = 3.06, size = 61, normalized size = 1.13

$$\frac{\frac{\log\left(\frac{\sqrt{bx^n+a}-\sqrt{a}}{\sqrt{bx^n+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}n} + \frac{2}{\sqrt{bx^n+a}\,an}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] (log((sqrt(b*x^n + a) - sqrt(a))/(sqrt(b*x^n + a) + sqrt(a)))/(a^(3/2)*n) +
 2/(sqrt(b*x^n + a)*a*n))/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{c\,x\,(a+b\,x^n)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x*(a + b*x^n)^(3/2)),x)
```

```
[Out] int(1/(c*x*(a + b*x^n)^(3/2)), x)
```

sympy [B] time = 3.72, size = 185, normalized size = 3.43

$$\frac{\frac{2a^3\sqrt{1+\frac{bx^n}{a}}}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n} + \frac{a^3\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n} + \frac{a^2bx^n\log\left(\frac{bx^n}{a}\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n} - \frac{2a^2bx^n\log\left(\sqrt{1+\frac{bx^n}{a}}+1\right)}{a^{\frac{9}{2}}n+a^{\frac{7}{2}}bnx^n}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c/x/(a+b*x**n)**(3/2),x)
```

```
[Out] (2*a**3*sqrt(1 + b*x**n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**3*log(b*x*
*n/a)/(a**(9/2)*n + a**(7/2)*b*n*x**n) - 2*a**3*log(sqrt(1 + b*x**n/a) + 1)
/(a**(9/2)*n + a**(7/2)*b*n*x**n) + a**2*b*x**n*log(b*x**n/a)/(a**(9/2)*n +
 a**(7/2)*b*n*x**n) - 2*a**2*b*x**n*log(sqrt(1 + b*x**n/a) + 1)/(a**(9/2)*n
 + a**(7/2)*b*n*x**n))/c
```

3.403 $\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(1/2)/(a/x+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/c^2/(1+n)/(c*
x)^(1/2)+2/a/c^2/(1+n)/(c*x)^(1/2)/(a/x+b*x^n)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {2030, 2031, 2029, 206}

$$\frac{2}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(n+1)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^2*(1 + n)*Sqrt[c*x]*Sqrt[a/x + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt[a]/
(Sqrt[x]*Sqrt[a/x + b*x^n]])])/(a^(3/2)*c^2*(1 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{5/2}\left(\frac{a}{x}+bx^n\right)^{3/2}}\,dx = \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} + \frac{\int \frac{1}{(cx)^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{ac}$$

$$= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} + \frac{\sqrt{x}\int \frac{1}{x^{3/2}\sqrt{\frac{a}{x}+bx^n}}\,dx}{ac^2\sqrt{cx}}$$

$$= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} - \frac{\left(2\sqrt{x}\right)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{ac^2(1+n)\sqrt{cx}}$$

$$= \frac{2}{ac^2(1+n)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}} - \frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{x}\sqrt{\frac{a}{x}+bx^n}}\right)}{a^{3/2}c^2(1+n)\sqrt{cx}}$$

Mathematica [C] time = 0.05, size = 55, normalized size = 0.61

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+1}}{a}+1\right)}{ac^2(n+1)\sqrt{cx}\sqrt{\frac{a}{x}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(5/2)*(a/x + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(1 + n))/a])/(a*c^2*(1 + n)*Sqr
t[c*x]*Sqrt[a/x + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n+\frac{a}{x}\right)^{\frac{3}{2}}(cx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

maple [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{5}{2}}\left(b\,x^n+\frac{a}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(5/2)/(b*x^n+a/x)^(3/2),x)
```

```
[Out] int(1/(c*x)^(5/2)/(b*x^n+a/x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n + \frac{a}{x}\right)^{\frac{3}{2}} (cx)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(5/2)/(a/x+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x)^(3/2)*(c*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{5/2}\,\left(b\,x^n + \frac{a}{x}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(5/2)*(b*x^n + a/x)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(5/2)/(a/x+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

3.404 $\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(n+2)}$$

```
[Out] -2*arctanh(a^(1/2)/x/(a/x^2+b*x^n)^(1/2))/a^(3/2)/c^4/(2+n)+2/a/c^4/(2+n)/x
/(a/x^2+b*x^n)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2030, 2029, 206}

$$\frac{2}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^4*(2 + n)*x*Sqrt[a/x^2 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x*Sqrt[a/x^2
+ b*x^n])])/(a^(3/2)*c^4*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{c^4x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{x^4\left(\frac{a}{x^2}+bx^n\right)^{3/2}}\,dx}{c^4}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} + \frac{\int \frac{1}{x^2\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{ac^4}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{ac^4(2+n)}$$

$$= \frac{2}{ac^4(2+n)x\sqrt{\frac{a}{x^2}+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x\sqrt{\frac{a}{x^2}+bx^n}}\right)}{a^{3/2}c^4(2+n)}$$

Mathematica [C] time = 0.04, size = 51, normalized size = 0.71

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+2}}{a}+1\right)}{ac^4(n+2)x\sqrt{\frac{a}{x^2}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^4*x^4*(a/x^2 + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(2 + n))/a])/(a*c^4*(2 + n)*x*S
qrt[a/x^2 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*c^4*x^4), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^n+\frac{a}{x^2}\right)^{\frac{3}{2}}c^4x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^4/x^4/(b*x^n+a/x^2)^(3/2),x)
```

```
[Out] int(1/c^4/x^4/(b*x^n+a/x^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\frac{1}{\left(bx^n+\frac{a}{x^2}\right)^{\frac{3}{2}}x^4}}{c^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^4/x^4/(a/x^2+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x^2)^(3/2)*x^4), x)/c^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{c^4\,x^4\,\left(b\,x^n+\frac{a}{x^2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)),x)
```

```
[Out] int(1/(c^4*x^4*(b*x^n + a/x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax^2\sqrt{\frac{a}{x^2}+bx^n}+bx^4x^n\sqrt{\frac{a}{x^2}+bx^n}}\,dx}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**4/x**4/(a/x**2+b*x**n)**(3/2),x)
```

```
[Out] Integral(1/(a*x**2*sqrt(a/x**2 + b*x**n) + b*x**4*x**n*sqrt(a/x**2 + b*x**n
)), x)/c**4
```

3.405 $\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(n+3)\sqrt{cx}}$$

```
[Out] -2*arctanh(a^(1/2)/x^(3/2)/(a/x^3+b*x^n)^(1/2))*x^(1/2)/a^(3/2)/c^5/(3+n)/(
c*x)^(1/2)+2/a/c^4/(3+n)/(c*x)^(3/2)/(a/x^3+b*x^n)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2030, 2031, 2029, 206}

$$\frac{2}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}-\frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(n+3)\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^4*(3 + n)*(c*x)^(3/2)*Sqrt[a/x^3 + b*x^n]) - (2*Sqrt[x]*ArcTanh[Sqrt
[a]/(x^(3/2)*Sqrt[a/x^3 + b*x^n])])/(a^(3/2)*c^5*(3 + n)*Sqrt[c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rule 2031

```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Dist[(c^IntPart[m]*(c*x)^FracPart[m])/x^FracPart[m], Int[x^m*(a*x^j + b*
x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] && N
eQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

Rubi steps

$$\int \frac{1}{(cx)^{11/2}\left(\frac{a}{x^3}+bx^n\right)^{3/2}}\,dx = \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}} + \frac{\int \frac{1}{(cx)^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{ac^3}$$

$$= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}} + \frac{\sqrt{x}\int \frac{1}{x^{5/2}\sqrt{\frac{a}{x^3}+bx^n}}\,dx}{ac^5\sqrt{cx}}$$

$$= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}} - \frac{\left(2\sqrt{x}\right)\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{ac^5(3+n)\sqrt{cx}}$$

$$= \frac{2}{ac^4(3+n)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}} - \frac{2\sqrt{x}\tanh^{-1}\left(\frac{\sqrt{a}}{x^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}\right)}{a^{3/2}c^5(3+n)\sqrt{cx}}$$

Mathematica [C] time = 0.05, size = 55, normalized size = 0.61

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+3}}{a}+1\right)}{ac^4(n+3)(cx)^{3/2}\sqrt{\frac{a}{x^3}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((c*x)^(11/2)*(a/x^3 + b*x^n)^(3/2)),x]
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(3 + n))/a])/(a*c^4*(3 + n)*(c*
x)^(3/2)*Sqrt[a/x^3 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n+\frac{a}{x^3}\right)^{\frac{3}{2}}(cx)^{\frac{11}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{1}{(cx)^{\frac{11}{2}}\left(b\,x^n+\frac{a}{x^3}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x)^(11/2)/(b*x^n+a/x^3)^(3/2),x)
```

```
[Out] int(1/(c*x)^(11/2)/(b*x^n+a/x^3)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n + \frac{a}{x^3}\right)^{\frac{3}{2}} (cx)^{\frac{11}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)^(11/2)/(a/x^3+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x^3)^(3/2)*(c*x)^(11/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,x)^{11/2}\left(b\,x^n + \frac{a}{x^3}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)),x)
```

```
[Out] int(1/((c*x)^(11/2)*(b*x^n + a/x^3)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x)**(11/2)/(a/x**3+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

3.406 $\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(n+4)}$$

```
[Out] -2*arctanh(a^(1/2)/x^2/(a/x^4+b*x^n)^(1/2))/a^(3/2)/c^7/(4+n)+2/a/c^7/(4+n)
/x^2/(a/x^4+b*x^n)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 2030, 2029, 206}

$$\frac{2}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}-\frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] 2/(a*c^7*(4 + n)*x^2*Sqrt[a/x^4 + b*x^n]) - (2*ArcTanh[Sqrt[a]/(x^2*Sqrt[a/
x^4 + b*x^n])])/(a^(3/2)*c^7*(4 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 2030

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] + Dist[(c^j*(m + n*p + n - j + 1))/(a*(n - j)*(p + 1)), Int[(
c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}, x
] && ILtQ[p + 1/2, 0] && NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0] && (Int
egerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{c^7x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx = \frac{\int \frac{1}{x^7\left(\frac{a}{x^4}+bx^n\right)^{3/2}}\,dx}{c^7}$$

$$= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}} + \frac{\int \frac{1}{x^3\sqrt{\frac{a}{x^4}+bx^n}}\,dx}{ac^7}$$

$$= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}} - \frac{2\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{1}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{ac^7(4+n)}$$

$$= \frac{2}{ac^7(4+n)x^2\sqrt{\frac{a}{x^4}+bx^n}} - \frac{2\tanh^{-1}\left(\frac{\sqrt{a}}{x^2\sqrt{\frac{a}{x^4}+bx^n}}\right)}{a^{3/2}c^7(4+n)}$$

Mathematica [C] time = 0.07, size = 51, normalized size = 0.71

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{bx^{n+4}}{a}+1\right)}{ac^7(n+4)x^2\sqrt{\frac{a}{x^4}+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(c^7*x^7*(a/x^4 + b*x^n)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^(4 + n))/a])/(a*c^7*(4 + n)*x^2
*Sqrt[a/x^4 + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^7x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a/x^4)^(3/2)*c^7*x^7), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^n+\frac{a}{x^4}\right)^{\frac{3}{2}}c^7x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/c^7/x^7/(b*x^n+a/x^4)^(3/2),x)
```

```
[Out] int(1/c^7/x^7/(b*x^n+a/x^4)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(bx^n+\frac{a}{x^4}\right)^{\frac{3}{2}}x^7}\,dx}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c^7/x^7/(a/x^4+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a/x^4)^(3/2)*x^7), x)/c^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{c^7\,x^7\,{\left(b\,x^n+\frac{a}{x^4}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)),x)
```

```
[Out] int(1/(c^7*x^7*(b*x^n + a/x^4)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{ax^3\sqrt{\frac{a}{x^4}+bx^n}+bx^7x^n\sqrt{\frac{a}{x^4}+bx^n}}\,dx}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/c**7/x**7/(a/x**4+b*x**n)**(3/2),x)
```

```
[Out] Integral(1/(a*x**3*sqrt(a/x**4 + b*x**n) + b*x**7*x**n*sqrt(a/x**4 + b*x**n
)), x)/c**7
```

3.407 $\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

```
[Out] 2/3*arctanh(x*b^(1/2)/(a/x+b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x^3)/x],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x + b*x^2]])/(3*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x}+bx^2}}\,dx \\
&= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x}+bx^2}}\right) \\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}+bx^2}}\right)}{3\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.97

$$\frac{2\sqrt{a+bx^3}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{3/2}}{\sqrt{a+bx^3}}\right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a+bx^3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^3)/x],x]
```

```
[Out] (2*Sqrt[a + b*x^3]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a + b*x^3]])/(3*Sqrt[b]*S
qrt[x]*Sqrt[(a + b*x^3)/x])
```

fricas [A] time = 0.54, size = 102, normalized size = 3.19

$$\left[\frac{\log\left(-8\,b^2x^6-8\,abx^3-a^2-4\left(2\,bx^5+ax^2\right)\sqrt{b}\sqrt{\frac{bx^3+a}{x}}\right)}{6\,\sqrt{b}},-\frac{\sqrt{-b}\arctan\left(\frac{2\,\sqrt{-b}\,x^2\sqrt{\frac{bx^3+a}{x}}}{2\,bx^3+a}\right)}{3\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*log(-8*b^2*x^6 - 8*a*b*x^3 - a^2 - 4*(2*b*x^5 + a*x^2)*sqrt(b)*sqrt((b
*x^3 + a)/x))/sqrt(b), -1/3*sqrt(-b)*arctan(2*sqrt(-b)*x^2*sqrt((b*x^3 + a)
/x)/(2*b*x^3 + a))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Limit: Max order reached or unable to make series expansion Error: Bad Argu
ment Value
```

maple [C] time = 0.57, size = 477, normalized size = 14.91

$$-\frac{4\left(b\,x^3+a\right)\left(i\sqrt{3}-1\right)\sqrt{-\frac{\left(i\sqrt{3}-3\right)bx}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)^2\sqrt{\frac{2bx+i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}+\left(-a\,b^2\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+\left(-a\,b^2\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{-2bx+i\sqrt{3}\left(-a\,b^2\right)^{\frac{1}{3}}-\cdots}{\left(i\sqrt{3}-1\right)\left(-bx+\left(-a\cdots\right.\right.}}}{\sqrt{\frac{b\,x^3+a}{x}}\sqrt{\left(b\,x^3+a\right)x}\left(i\sqrt{3}-3\right)\sqrt{\frac{\left(-bx+\left(\cdots\right.\right.}{\cdots}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^3+a)/x)^(1/2),x)
```

```
[Out] -4*(b*x^3+a)*(I*3^(1/2)-1)*(-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/
3))*b*x)^(1/2)*(-b*x+(-a*b^2)^(1/3))^2*((2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a
*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(-a*b^2)^(1/3)))^(1/2)*((-2*b*x+I*3^(1/2)*
```

```
(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3)))^(1/2)/b
^2*(EllipticF((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2
),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3))^(1/2))-Elliptic
Pi((-(I*3^(1/2)-3)/(I*3^(1/2)-1)/(-b*x+(-a*b^2)^(1/3))*b*x)^(1/2),(I*3^(1/2
)-1)/(I*3^(1/2)-3),((I*3^(1/2)+3)*(I*3^(1/2)-1)/(1+I*3^(1/2))/(I*3^(1/2)-3)
)^(1/2)))/((b*x^3+a)/x)^(1/2)/((b*x^3+a)*x)^(1/2)/(I*3^(1/2)-3)/((-b*x+(-a*
b^2)^(1/3))*(2*b*x+I*3^(1/2)*(-a*b^2)^(1/3)+(-a*b^2)^(1/3))*(-2*b*x+I*3^(1/
2)*(-a*b^2)^(1/3)-(-a*b^2)^(1/3))/b^2*x)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{bx^3+a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^3+a)/x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((b*x^3 + a)/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b\,x^3+a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)/x)^(1/2),x)
[Out] int(1/((a + b*x^3)/x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a+bx^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**3+a)/x)**(1/2),x)
[Out] Integral(1/sqrt((a + b*x**3)/x), x)
```

3.408 $\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

[Out] 1/2*arctanh(x*b^(1/2)/(a/x^2+b*x^2)^(1/2))/b^(1/2)

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 2008, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a + b*x^4)/x^2],x]

[Out] ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^2 + b*x^2]]/(2*Sqrt[b])

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^2}+bx^2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}+bx^2}}\right)}{2\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 59, normalized size = 1.84

$$\frac{\sqrt{a+bx^4}\tanh^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a+bx^4}}\right)}{2\sqrt{b}x\sqrt{\frac{a+bx^4}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^4)/x^2],x]
```

```
[Out] (Sqrt[a + b*x^4]*ArcTanh[(Sqrt[b]*x^2)/Sqrt[a + b*x^4]])/(2*Sqrt[b]*x*Sqrt[
(a + b*x^4)/x^2])
```

fricas [A] time = 0.42, size = 80, normalized size = 2.50

$$\left[\frac{\log\left(-2\,bx^4-2\,\sqrt{b}\,x^3\sqrt{\frac{bx^4+a}{x^2}}-a\right)}{4\,\sqrt{b}},-\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x^3\sqrt{\frac{bx^4+a}{x^2}}}{bx^4+a}\right)}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-2*b*x^4 - 2*sqrt(b)*x^3*sqrt((b*x^4 + a)/x^2) - a)/sqrt(b), -1/2*
sqrt(-b)*arctan(sqrt(-b)*x^3*sqrt((b*x^4 + a)/x^2)/(b*x^4 + a))/b]
```

giac [A] time = 0.21, size = 40, normalized size = 1.25

$$\frac{\log\left(|a|\right)\mathrm{sgn}\left(x\right)}{4\,\sqrt{b}}-\frac{\log\left(\left|-\sqrt{b}\,x^2+\sqrt{bx^4+a}\right|\right)}{2\,\sqrt{b}\,\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a))*sgn(x)/sqrt(b) - 1/2*log(abs(-sqrt(b)*x^2 + sqrt(b*x^4 + a)
))/(sqrt(b)*sgn(x))
```

maple [A] time = 0.19, size = 49, normalized size = 1.53

$$\frac{\sqrt{b\,x^4+a}\;\ln\left(\sqrt{b}\;x^2+\sqrt{b\,x^4+a}\right)}{2\sqrt{\frac{b\,x^4+a}{x^2}}\;\sqrt{b}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^4+a)/x^2)^(1/2),x)
```

```
[Out] 1/2/((b*x^4+a)/x^2)^(1/2)/x*(b*x^4+a)^(1/2)*ln(b^(1/2)*x^2+(b*x^4+a)^(1/2))
/b^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{x^5}{\left(bx^4+a\right)^{\frac{3}{2}}}\,dx+\frac{x^2}{2\,\sqrt{bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x^5/(b*x^4 + a)^(3/2), x) + 1/2*x^2/sqrt(b*x^4 + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b\,x^4+a}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^4)/x^2)^(1/2),x)
```

```
[Out] int(1/((a + b*x^4)/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a+bx^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**4+a)/x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a + b*x**4)/x**2), x)
```

3.409 $\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx$

Optimal. Leaf size=32

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

```
[Out] 2/5*arctanh(x*b^(1/2)/(a/x^3+b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a + b*x^5)/x^3],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a/x^3 + b*x^2]])/(5*Sqrt[b])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a+bx^5}{x^3}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^3}+bx^2}}\,dx\\
&= \frac{2}{5}\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)\\
&= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}+bx^2}}\right)}{5\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 63, normalized size = 1.97

$$\frac{2\sqrt{a+bx^5}\tanh^{-1}\left(\frac{\sqrt{b}\,x^{5/2}}{\sqrt{a+bx^5}}\right)}{5\sqrt{b}\,x^{3/2}\sqrt{\frac{a+bx^5}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a + b*x^5)/x^3],x]
```

```
[Out] (2*Sqrt[a + b*x^5]*ArcTanh[(Sqrt[b]*x^(5/2))/Sqrt[a + b*x^5]])/(5*Sqrt[b]*x
^(3/2)*Sqrt[(a + b*x^5)/x^3])
```

fricas [A] time = 0.91, size = 102, normalized size = 3.19

$$\left[\frac{\log\left(-8\,b^2x^{10}-8\,abx^5-a^2-4\left(2\,bx^9+ax^4\right)\sqrt{b}\,\sqrt{\frac{bx^5+a}{x^3}}\right)}{10\,\sqrt{b}}, -\frac{\sqrt{-b}\,\arctan\left(\frac{2\,\sqrt{-b}\,x^4\sqrt{\frac{bx^5+a}{x^3}}}{2\,bx^5+a}\right)}{5\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/10*log(-8*b^2*x^10 - 8*a*b*x^5 - a^2 - 4*(2*b*x^9 + a*x^4)*sqrt(b)*sqrt(
(b*x^5 + a)/x^3))/sqrt(b), -1/5*sqrt(-b)*arctan(2*sqrt(-b)*x^4*sqrt((b*x^5
+ a)/x^3)/(2*b*x^5 + a))/b]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(x)]Limit: Max order reached or unable to make series expan
sion Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{b\,x^5+a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x^5+a)/x^3)^(1/2),x)
```

```
[Out] int(1/((b*x^5+a)/x^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{bx^5+a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt((b*x^5 + a)/x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{b\,x^5+a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^5)/x^3)^(1/2),x)
[Out] int(1/((a + b*x^5)/x^3)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x**5+a)/x**3)**(1/2),x)
[Out] Timed out
```

3.410 $\int \frac{1}{\sqrt{x^{2-n}(a+bx^n)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{b}\,n}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^(2-n))^(1/2))/n/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^{2-n}+bx^2}}\right)}{\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^{2-n}\,(a+bx^n)}}\,dx &= \int \frac{1}{\sqrt{bx^2+ax^{2-n}}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^{2-n}}}\right)}{n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^{2-n}}}\right)}{\sqrt{b}\,n}\end{aligned}$$

Mathematica [B] time = 0.05, size = 76, normalized size = 2.05

$$\frac{2\sqrt{a}\,x^{1-\frac{n}{2}}\sqrt{\frac{bx^n}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}\,n\sqrt{x^{2-n}\,(a+bx^n)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^(2 - n)*(a + b*x^n)],x]
```

```
[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 + (b*x^n)/a]*ArcSinh[(Sqrt[b]*x^(n/2))/Sqrt[a
]])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a + b*x^n)])
```

fricas [A] time = 0.41, size = 102, normalized size = 2.76

$$\left[\frac{\log\left(\frac{2\,bxx^{n}+ax+2\,\sqrt{b}\,x^{n}\sqrt{\frac{bx^{2}x^{n}+ax^{2}}{x^{n}}}}{x}\right)}{\sqrt{b}\,n}, -\frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,\sqrt{\frac{bx^{2}x^{n}+ax^{2}}{x^{n}}}}{bx}\right)}{bn}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] [log((2*b*x*x^n + a*x + 2*sqrt(b)*x^n*sqrt((b*x^2*x^n + a*x^2)/x^n))/x)/(sq
rt(b)*n), -2*sqrt(-b)*arctan(sqrt(-b)*sqrt((b*x^2*x^n + a*x^2)/x^n)/(b*x))/
(b*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(bx^{n}+a)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(b\,x^{n}+a)\,x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2-n)*(b*x^n+a))^(1/2),x)
```

```
[Out] int(1/(x^(2-n)*(b*x^n+a))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(bx^{n}+a)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a+b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^n + a)*x^(-n + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^{2-n}\,\left(a+b\,x^{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2 - n)*(a + b*x^n))^(1/2),x)
```

```
[Out] int(1/(x^(2 - n)*(a + b*x^n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2-n}\left(a+bx^n\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(2-n)*(a+b*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**(2 - n)*(a + b*x**n)), x)
```

3.411 $\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$

Optimal. Leaf size=33

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

```
[Out] 2/3*arctan(x*b^(1/2)/(a/x-b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^3)/x],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x - b*x^2]])/(3*Sqrt[b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x}-bx^2}}\,dx\\ &= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x}-bx^2}}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x}-bx^2}}\right)}{3\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 2.00

$$\frac{2\sqrt{a-bx^3}\tan^{-1}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{a-bx^3}}\right)}{3\sqrt{b}\sqrt{x}\sqrt{\frac{a-bx^3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^3)/x],x]
```

```
[Out] (2*Sqrt[a - b*x^3]*ArcTan[(Sqrt[b]*x^(3/2))/Sqrt[a - b*x^3]])/(3*Sqrt[b]*Sq
rt[x]*Sqrt[(a - b*x^3)/x])
```

fricas [A] time = 0.54, size = 111, normalized size = 3.36

$$\left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^6+8\,abx^3-a^2+4\left(2\,bx^5-ax^2\right)\sqrt{-b}\sqrt{-\frac{bx^3-a}{x}}\right)}{6\,b}, -\frac{\arctan\left(\frac{2\,\sqrt{b}x^2\sqrt{-\frac{bx^3-a}{x}}}{2\,bx^3-a}\right)}{3\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*sqrt(-b)*log(-8*b^2*x^6 + 8*a*b*x^3 - a^2 + 4*(2*b*x^5 - a*x^2)*sqrt(
-b)*sqrt(-(b*x^3 - a)/x))/b, -1/3*arctan(2*sqrt(b)*x^2*sqrt(-(b*x^3 - a)/x)
/(2*b*x^3 - a))/sqrt(b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Undef/Unsigned Inf encountered in limitLimit: Max order reached or unable t
o make series expansion Error: Bad Argument Value
```

maple [C] time = 6.22, size = 471, normalized size = 14.27

$$\frac{4\left(bx^3-a\right)\left(1+i\sqrt{3}\right)\sqrt{-\frac{\left(i\sqrt{3}+3\right)bx}{\left(1+i\sqrt{3}\right)\left(-bx+\left(a\,b^2\right)^{\frac{1}{3}}\right)}}\left(-bx+\left(a\,b^2\right)^{\frac{1}{3}}\right)^2\sqrt{\frac{-2bx+i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}-\left(a\,b^2\right)^{\frac{1}{3}}}{\left(i\sqrt{3}-1\right)\left(-bx+\left(a\,b^2\right)^{\frac{1}{3}}\right)}}\sqrt{\frac{2bx+i\sqrt{3}\left(a\,b^2\right)^{\frac{1}{3}}+\left(a\,b^2\right)^{\frac{1}{3}}}{\left(1+i\sqrt{3}\right)\left(-bx+\left(a\,b^2\right)^{\frac{1}{3}}\right)}}}{\sqrt{-\frac{bx^3-a}{x}}\sqrt{-\left(bx^3-a\right)x}\left(i\sqrt{3}+3\right)\sqrt{-\frac{\left(-bx+\left(a\right.\right.}{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^3+a)/x)^(1/2),x)
```

```
[Out] 4*(b*x^3-a)*(1+I*3^(1/2))*(-(I*3^(1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(a*b^2)^(
1/3)))^(1/2)*(-b*x+(a*b^2)^(1/3))^2*((I*3^(1/2)*(a*b^2)^(1/3)-2*b*x-(a*b^2)
^(1/3))/(I*3^(1/2)-1)/(-b*x+(a*b^2)^(1/3)))^(1/2)*((I*3^(1/2)*(a*b^2)^(1/3)
```

```
+2*b*x+(a*b^2)^(1/3))/(1+I*3^(1/2))/(-b*x+(a*b^2)^(1/3)))^(1/2)/b^2*(Ellipt
icF((-(I*3^(1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(a*b^2)^(1/3)))^(1/2),((I*3^(1/
2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(I*3^(1/2)+3))^(1/2))-EllipticPi((-(I*3^(
1/2)+3)*x*b/(1+I*3^(1/2))/(-b*x+(a*b^2)^(1/3)))^(1/2),(1+I*3^(1/2))/(I*3^(1
/2)+3),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(I*3^(1/2)+3))^(1/2)))/(-
(b*x^3-a)/x)^(1/2)/(-(b*x^3-a)*x)^(1/2)/(I*3^(1/2)+3)/(-1/b^2*x*(-b*x+(a*b^
2)^(1/3))*(I*3^(1/2)*(a*b^2)^(1/3)-2*b*x-(a*b^2)^(1/3))*(I*3^(1/2)*(a*b^2)^
(1/3)+2*b*x+(a*b^2)^(1/3)))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\frac{bx^3-a}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^3+a)/x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-(b*x^3 - a)/x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a-b\,x^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^3)/x)^(1/2),x)
[Out] int(1/((a - b*x^3)/x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a-bx^3}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**3+a)/x)**(1/2),x)
[Out] Integral(1/sqrt((a - b*x**3)/x), x)
```

3.412 $\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$

Optimal. Leaf size=33

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/2*arctan(x*b^(1/2)/(a/x^2-b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 2008, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a/x^2 - b*x^2]]/(2*Sqrt[b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx &= \int \frac{1}{\sqrt{\frac{a}{x^2}-bx^2}}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{\frac{a}{x^2}-bx^2}}\right) \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^2}-bx^2}}\right)}{2\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 1.88

$$\frac{\sqrt{a-bx^4}\tan^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a-bx^4}}\right)}{2\sqrt{b}x\sqrt{\frac{a-bx^4}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^4)/x^2],x]
```

```
[Out] (Sqrt[a - b*x^4]*ArcTan[(Sqrt[b]*x^2)/Sqrt[a - b*x^4]])/(2*Sqrt[b]*x*Sqrt[(
a - b*x^4)/x^2])
```

fricas [A] time = 0.41, size = 88, normalized size = 2.67

$$\left[-\frac{\sqrt{-b}\log\left(2\,bx^4-2\,\sqrt{-b}\,x^3\sqrt{-\frac{bx^4-a}{x^2}}-a\right)}{4\,b},-\frac{\arctan\left(\frac{\sqrt{b}\,x^3\sqrt{-\frac{bx^4-a}{x^2}}}{bx^4-a}\right)}{2\,\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-b)*log(2*b*x^4 - 2*sqrt(-b)*x^3*sqrt(-(b*x^4 - a)/x^2) - a)/b,
-1/2*arctan(sqrt(b)*x^3*sqrt(-(b*x^4 - a)/x^2)/(b*x^4 - a))/sqrt(b)]
```

giac [A] time = 0.19, size = 47, normalized size = 1.42

$$\frac{\log\left(|a|\right)\mathrm{sgn}(x)}{4\,\sqrt{-b}}-\frac{\log\left(\left|-\sqrt{-b}\,x^2+\sqrt{-bx^4+a}\right|\right)}{2\,\sqrt{-b}\,\mathrm{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(a))*sgn(x)/sqrt(-b) - 1/2*log(abs(-sqrt(-b)*x^2 + sqrt(-b*x^4 +
 a)))/(sqrt(-b)*sgn(x))
```

maple [B] time = 0.19, size = 53, normalized size = 1.61

$$\frac{\sqrt{-b\,x^4+a}\,\arctan\left(\frac{\sqrt{b}\,x^2}{\sqrt{-b\,x^4+a}}\right)}{2\sqrt{-\frac{b\,x^4-a}{x^2}}\,\sqrt{b}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^4+a)/x^2)^(1/2),x)
```

```
[Out] 1/2/(-(b*x^4-a)/x^2)^(1/2)/x*(-b*x^4+a)^(1/2)/b^(1/2)*arctan(1/(-b*x^4+a)^(
1/2)*b^(1/2)*x^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{x^5}{\left(bx^4-a\right)\sqrt{-bx^4+a}}\,dx+\frac{x^2}{2\,\sqrt{-bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^4+a)/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x^5/((b*x^4 - a)*sqrt(-b*x^4 + a)), x) + 1/2*x^2/sqrt(-b*x^4 +
a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a-b\,x^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^4)/x^2)^(1/2),x)
```

```
[Out] int(1/((a - b*x^4)/x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{a-bx^4}{x^2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**4+a)/x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt((a - b*x**4)/x**2), x)
```

3.413 $\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx$

Optimal. Leaf size=33

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

```
[Out] 2/5*arctan(x*b^(1/2)/(a/x^3-b*x^2)^(1/2))/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[a/x^3 - b*x^2]])/(5*Sqrt[b])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{a-bx^5}{x^3}}} dx &= \int \frac{1}{\sqrt{\frac{a}{x^3}-bx^2}} dx \\
&= \frac{2}{5}\operatorname{Subst}\left(\int \frac{1}{1+bx^2} dx, x, \frac{x}{\sqrt{\frac{a}{x^3}-bx^2}}\right) \\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{\frac{a}{x^3}-bx^2}}\right)}{5\sqrt{b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 2.00

$$\frac{2\sqrt{a-bx^5}\tan^{-1}\left(\frac{\sqrt{b}x^{5/2}}{\sqrt{a-bx^5}}\right)}{5\sqrt{b}x^{3/2}\sqrt{\frac{a-bx^5}{x^3}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(a - b*x^5)/x^3],x]
```

```
[Out] (2*Sqrt[a - b*x^5]*ArcTan[(Sqrt[b]*x^(5/2))/Sqrt[a - b*x^5]])/(5*Sqrt[b]*x^
(3/2)*Sqrt[(a - b*x^5)/x^3])
```

fricas [A] time = 0.92, size = 111, normalized size = 3.36

$$\left[-\frac{\sqrt{-b}\log\left(-8\,b^2x^{10}+8\,abx^5-a^2+4\left(2\,bx^9-ax^4\right)\sqrt{-b}\sqrt{-\frac{bx^5-a}{x^3}}\right)}{10\,b},-\frac{\arctan\left(\frac{2\sqrt{b}x^4\sqrt{-\frac{bx^5-a}{x^3}}}{2\,bx^5-a}\right)}{5\sqrt{b}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/10*sqrt(-b)*log(-8*b^2*x^10 + 8*a*b*x^5 - a^2 + 4*(2*b*x^9 - a*x^4)*sqr
t(-b)*sqrt(-(b*x^5 - a)/x^3))/b, -1/5*arctan(2*sqrt(b)*x^4*sqrt(-(b*x^5 - a
)/x^3)/(2*b*x^5 - a))/sqrt(b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Warning, integration of
abs or sign assumes constant sign by intervals (correct if the argument is
real):Check [abs(x)]Undef/Unsigned Inf encountered in limitLimit: Max order
 reached or unable to make series expansion Error: Bad Argument Value
```

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{-b\,x^5+a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

```
[Out] int(1/((-b*x^5+a)/x^3)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\frac{bx^5-a}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x^5+a)/x^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(-(b*x^5 - a)/x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\frac{a-b\,x^5}{x^3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a - b*x^5)/x^3)^(1/2),x)
[Out] int(1/((a - b*x^5)/x^3)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-b*x**5+a)/x**3)**(1/2),x)
[Out] Timed out
```

3.414 $\int \frac{1}{\sqrt{x^{2-n}(a-bx^n)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{b}\,n}$$

```
[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^(2-n))^(1/2))/n/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^{2-n}-bx^2}}\right)}{\sqrt{b}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^(2 - n)]])/(Sqrt[b]*n)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^n\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^{2-n}}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^{2-n}}}\right)}{\sqrt{b}\,n}\end{aligned}$$

Mathematica [B] time = 0.06, size = 78, normalized size = 2.05

$$\frac{2\sqrt{a}\,x^{1-\frac{n}{2}}\sqrt{1-\frac{bx^n}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{n/2}}{\sqrt{a}}\right)}{\sqrt{b}\,n\sqrt{x^{2-n}\left(a-bx^n\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^(2 - n)*(a - b*x^n)],x]
```

```
[Out] (2*Sqrt[a]*x^(1 - n/2)*Sqrt[1 - (b*x^n)/a]*ArcSin[(Sqrt[b]*x^(n/2))/Sqrt[a]
])/(Sqrt[b]*n*Sqrt[x^(2 - n)*(a - b*x^n)])
```

fricas [A] time = 0.42, size = 106, normalized size = 2.79

$$\left[-\frac{\sqrt{-b}\log\left(-\frac{2\,bxx^{n}-ax-2\,\sqrt{-b}\,x^{n}\sqrt{-\frac{bx^{2}x^{n}-ax^{2}}{x^{n}}}}{x}\right)}{bn},\,-\frac{2\,\arctan\left(\frac{\sqrt{-\frac{bx^{2}x^{n}-ax^{2}}{x^{n}}}}{\sqrt{b}\,x}\right)}{\sqrt{b}\,n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log(-(2*b*x*x^n - a*x - 2*sqrt(-b)*x^n*sqrt(-(b*x^2*x^n - a*x^2)
/x^n))/x)/(b*n), -2*arctan(sqrt(-(b*x^2*x^n - a*x^2)/x^n)/(sqrt(b)*x))/(sqr
t(b)*n)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(bx^{n}-a)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

maple [F] time = 1.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(-b\,x^{n}+a\right)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(-n+2)*(-b*x^n+a))^(1/2),x)
```

```
[Out] int(1/(x^(-n+2)*(-b*x^n+a))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-(bx^{n}-a)x^{-n+2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(2-n)*(a-b*x^n))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^n - a)*x^(-n + 2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{x^{2-n}\,\left(a-b\,x^{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(2 - n)*(a - b*x^n))^(1/2),x)
```

```
[Out] int(1/(x^(2 - n)*(a - b*x^n))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^{2-n}\left(a-bx^{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(2-n)*(a-b*x**n))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**(2 - n)*(a - b*x**n)), x)
```

3.415 $\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

```
[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

```
[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^n\left(a+bx^{2-n}\right)}}\,dx &= \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.10, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{\frac{bx^{2-n}}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^n*(a + b*x^(2 - n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(bx^{-n+2} + a\right)x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(b\,x^{-n+2} + a\right)x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a+b*x^(-n+2)))^(1/2),x)
```

```
[Out] int(1/(x^n*(a+b*x^(-n+2)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(bx^{-n+2} + a\right)x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a+b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((b*x^(-n + 2) + a)*x^n), x)
```

mupad [B] time = 5.21, size = 67, normalized size = 1.81

$$\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a + b*x^(2 - n)))^(1/2),x)
```

```
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^n \left(a + bx^{2-n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**n*(a+b*x**(2-n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**n*(a + b*x**(2 - n))), x)
```

3.416 $\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^2\left(b+ax^{-2+n}\right)}}\,dx &= \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.04, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{\frac{bx^{2-n}}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(b + a*x^(-2 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

fricas [A] time = 0.41, size = 109, normalized size = 2.95

$$\left[\frac{\sqrt{b}\log\left(\frac{axx^{n-2}+2\,bx-2\,\sqrt{ax^2x^{n-2}+bx^2}\,\sqrt{b}}{xx^{n-2}}\right)}{bn-2\,b}, \frac{2\,\sqrt{-b}\,\arctan\left(\frac{\sqrt{ax^2x^{n-2}+bx^2}\,\sqrt{-b}}{bx}\right)}{bn-2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(b)*log((a*x*x^(n - 2) + 2*b*x - 2*sqrt(a*x^2*x^(n - 2) + b*x^2)*sqrt(
b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(-b)*arctan(sqrt(a*x^2*x^(n - 2) + b*
x^2)*sqrt(-b)/(b*x))/(b*n - 2*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-2}+b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

maple [F] time = 2.10, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a\,x^{n-2}+b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b+a*x^(n-2)))^(1/2),x)
```

```
[Out] int(1/(x^2*(b+a*x^(n-2)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-2}+b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) + b)*x^2), x)
```

mupad [B] time = 5.32, size = 67, normalized size = 1.81

$$\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{\frac{b\,x^{2-n}}{a}+1}\,1\mathrm{i}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n+b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(b + a*x^(n - 2)))^(1/2),x)
```

```
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2 \left(ax^{n-2} + b\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(b+a*x**(-2+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a*x**(n - 2) + b)), x)
```

3.417 $\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx$

Optimal. Leaf size=37

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

[Out] 2*arctanh(x*b^(1/2)/(b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1979, 2008, 206}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n+bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[b*x^2 + a*x^n]])/(Sqrt[b]*(2 - n))

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x\left(bx+ax^{-1+n}\right)}}\,dx &= \int \frac{1}{\sqrt{bx^2+ax^n}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.03, size = 78, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{\frac{bx^{2-n}}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(b*x + a*x^(-1 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 + (b*x^(2 - n))/a]*ArcSinh[(Sqrt[b]*x^(1 - n/2))
/Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[b*x^2 + a*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-1} + bx\right)x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a\, x^{n-1} + bx\right) x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x+a*x^(n-1)))^(1/2),x)
```

```
[Out] int(1/(x*(b*x+a*x^(n-1)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-1} + bx\right)x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) + b*x)*x), x)
```

mupad [B] time = 5.13, size = 67, normalized size = 1.81

$$\frac{\sqrt{a}\, x^{n/2}\, \mathrm{asin}\left(\frac{\sqrt{b}\, x^{1-\frac{n}{2}}\, 1\mathrm{i}}{\sqrt{a}}\right) \sqrt{\frac{b\, x^{2-n}}{a}+1}\, 1\mathrm{i}}{\sqrt{b}\, \left(\frac{n}{2}-1\right) \sqrt{a\, x^n + b\, x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(b*x + a*x^(n - 1)))^(1/2),x)
```

```
[Out] (a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2)*1i)/a^(1/2))*((b*x^(2 - n))/a +
1)^(1/2)*1i)/(b^(1/2)*(n/2 - 1)*(a*x^n + b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\left(ax^{n-1}+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(b*x+a*x**(-1+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a*x**(n - 1) + b*x)), x)
```

3.418 $\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x^n\left(a-bx^{2-n}\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\ &= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}\end{aligned}$$

Mathematica [B] time = 0.11, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^n*(a - b*x^(2 - n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(bx^{-n+2}-a\right)x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(-b\,x^{-n+2}+a\right)x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a-b*x^(-n+2)))^(1/2),x)
```

```
[Out] int(1/(x^n*(a-b*x^(-n+2)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(bx^{-n+2}-a\right)x^{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^n*(a-b*x^(2-n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-(b*x^(-n + 2) - a)*x^n), x)
```

mupad [B] time = 5.17, size = 66, normalized size = 1.74

$$-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\,\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^n*(a - b*x^(2 - n)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^n\left(a - bx^{2-n}\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**n*(a-b*x**(2-n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**n*(a - b*x**(2 - n))), x)
```

3.419 $\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

```
[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x^2\left(-b+ax^{-2+n}\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}
\end{aligned}$$

Mathematica [B] time = 0.02, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x^2*(-b + a*x^(-2 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

fricas [A] time = 0.42, size = 109, normalized size = 2.87

$$\left[-\frac{\sqrt{-b}\log\left(\frac{axx^{n-2}-2\,bx-2\,\sqrt{ax^2x^{n-2}-bx^2}\,\sqrt{-b}}{xx^{n-2}}\right)}{bn-2\,b},\ \frac{2\,\sqrt{b}\,\arctan\left(\frac{\sqrt{ax^2x^{n-2}-bx^2}}{\sqrt{b}\,x}\right)}{bn-2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-b)*log((a*x*x^(n - 2) - 2*b*x - 2*sqrt(a*x^2*x^(n - 2) - b*x^2)*sqr
t(-b))/(x*x^(n - 2)))/(b*n - 2*b), 2*sqrt(b)*arctan(sqrt(a*x^2*x^(n - 2) -
b*x^2)/(sqrt(b)*x))/(b*n - 2*b)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-2}-b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

maple [F] time = 2.15, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a\,x^{n-2}-b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(-b+a*x^(n-2)))^(1/2),x)
```

```
[Out] int(1/(x^2*(-b+a*x^(n-2)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-2}-b\right)x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2*(-b+a*x^(-2+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 2) - b)*x^2), x)
```

mupad [B] time = 5.10, size = 66, normalized size = 1.74

$$-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\,\sqrt{a\,x^n-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2*(b - a*x^(n - 2)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\left(ax^{n-2}-b\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2*(-b+a*x**(-2+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2*(a*x**(n - 2) - b)), x)
```

3.420 $\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

```
[Out] 2*arctan(x*b^(1/2)/(-b*x^2+a*x^n)^(1/2))/(2-n)/b^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1979, 2008, 203}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{ax^n-bx^2}}\right)}{\sqrt{b}\,(2-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

```
[Out] (2*ArcTan[(Sqrt[b]*x)/Sqrt[-(b*x^2) + a*x^n]])/(Sqrt[b]*(2 - n))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2008

```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[2/(2 - n), S
ubst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b, n
}, x] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x\left(-bx+ax^{-1+n}\right)}}\,dx &= \int \frac{1}{\sqrt{-bx^2+ax^n}}\,dx\\
&= \frac{2\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{x}{\sqrt{-bx^2+ax^n}}\right)}{2-n}\\
&= \frac{2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{-bx^2+ax^n}}\right)}{\sqrt{b}\,(2-n)}
\end{aligned}$$

Mathematica [B] time = 0.03, size = 80, normalized size = 2.11

$$-\frac{2\sqrt{a}\,x^{n/2}\sqrt{1-\frac{bx^{2-n}}{a}}\,\sin^{-1}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)}{\sqrt{b}\,(n-2)\sqrt{ax^n-bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x*(-(b*x) + a*x^(-1 + n))],x]
```

```
[Out] (-2*Sqrt[a]*x^(n/2)*Sqrt[1 - (b*x^(2 - n))/a]*ArcSin[(Sqrt[b]*x^(1 - n/2))/
Sqrt[a]])/(Sqrt[b]*(-2 + n)*Sqrt[-(b*x^2) + a*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-1} - bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(a\,x^{n-1} - bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(-b*x+a*x^(n-1)))^(1/2),x)
```

```
[Out] int(1/(x*(-b*x+a*x^(n-1)))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(ax^{n-1} - bx\right)x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x^(-1+n)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((a*x^(n - 1) - b*x)*x), x)
```

mupad [B] time = 5.12, size = 66, normalized size = 1.74

$$-\frac{\sqrt{a}\,x^{n/2}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x^{1-\frac{n}{2}}}{\sqrt{a}}\right)\sqrt{1-\frac{b\,x^{2-n}}{a}}}{\sqrt{b}\,\left(\frac{n}{2}-1\right)\sqrt{a\,x^n-b\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x*(b*x - a*x^(n - 1)))^(1/2),x)
```

```
[Out] -(a^(1/2)*x^(n/2)*asin((b^(1/2)*x^(1 - n/2))/a^(1/2))*(1 - (b*x^(2 - n))/a)
^(1/2))/(b^(1/2)*(n/2 - 1)*(a*x^n - b*x^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x\left(ax^{n-1} - bx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x*(-b*x+a*x**(-1+n)))**(1/2),x)
```

```
[Out] Integral(1/sqrt(x*(a*x**(n - 1) - b*x)), x)
```

3.421 $\int (cx)^m \left(ax^j+bx^n\right)^{3/2}\,dx$

Optimal. Leaf size=107

$$\frac{2bx^{n+1}(cx)^m\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{m+\frac{3n}{2}+1}{j-n};\frac{m+\frac{3n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m+3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

```
[Out] 2*b*x^(1+n)*(c*x)^m*hypergeom([-3/2, (1+m+3/2*n)/(j-n)],[1+(1+m+3/2*n)/(j-n
)],-a*x^(j-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+2*m+3*n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 365, 364}

$$\frac{2bx^{n+1}(cx)^m\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{m+\frac{3n}{2}+1}{j-n};\frac{m+\frac{3n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m+3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + n)*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + m +
 (3*n)/2)/(j - n), 1 + (1 + m + (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/((2
+ 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int (cx)^m \left(ax^j+bx^n\right)^{3/2} dx = \frac{\left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{3n}{2}}\left(b+ax^{j-n}\right)^{3/2} dx}{\sqrt{b+ax^{j-n}}}$$
$$= \frac{\left(bx^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{3n}{2}}\left(1+\frac{ax^{j-n}}{b}\right)^{3/2} dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}$$
$$= \frac{2bx^{1+n}(cx)^m\sqrt{ax^j+bx^n}\ {}_2F_1\left(-\frac{3}{2},\frac{1+m+\frac{3n}{2}}{j-n};1+\frac{1+m+\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2+2m+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

Mathematica [B] time = 0.41, size = 218, normalized size = 2.04

$$\frac{2(cx)^m\left(3a^2(j-n)^2x^{2j+1}\sqrt{\frac{ax^{j-n}}{b}+1}\ {}_2F_1\left(\frac{1}{2},\frac{4j+2m-n+2}{2j-2n};\frac{6j+2m-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)+x^{-m}(4j+2m-n+2)\left(ax^j+bx^n\right)\left(a(-\right.\right.}{(2m+3n+2)(4j+2m-n+2)(2j+2m+n+2)\sqrt{ax^j+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*(c*x)^m*(((2 + 4*j + 2*m - n)*(a*x^j + b*x^n)*(a*(2 - j + 2*m + 4*n)*x^(
1 + j + m) + b*(2 + 2*j + 2*m + n)*x^(1 + m + n)))/x^m + 3*a^2*(j - n)^2*x^
(1 + 2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 4*j + 2*m -
 n)/(2*j - 2*n), (2 + 6*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(
(2 + 4*j + 2*m - n)*(2 + 2*j + 2*m + n)*(2 + 2*m + 3*n)*Sqrt[a*x^j + b*x^n]
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^j+bx^n\right)^{\frac{3}{2}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \left(a\,x^j+b\,x^n\right)^{\frac{3}{2}}(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^m*(a*x^j+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^j + bx^n\right)^{\frac{3}{2}} (cx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2)*(c*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^m \left(a\,x^j + b\,x^n\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a*x^j + b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^m*(a*x^j + b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m \left(ax^j + bx^n\right)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((c*x)**m*(a*x**j + b*x**n)**(3/2), x)
```

3.422 $\int (cx)^m\sqrt{ax^j+bx^n}\,dx$

Optimal. Leaf size=100

$$\frac{2x(cx)^m\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{m+\frac{n}{2}+1}{j-n};\frac{2m+n+2}{2j-2n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m+n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

```
[Out] 2*x*(c*x)^m*hypergeom([-1/2, (1+m+1/2*n)/(j-n)],[1+(2+2*m+n)/(2*j-2*n)],-a*
x^(j-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+2*m+n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 365, 364}

$$\frac{2x(cx)^m\sqrt{ax^j+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{m+\frac{n}{2}+1}{j-n};\frac{2m+n+2}{2j-2n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m+n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (1 + m + n/2)/(j -
 n), 1 + (2 + 2*m + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m + n)*Sqr
t[1 + (a*x^(j - n))/b])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int (cx)^m\sqrt{ax^j+bx^n}\,dx = \frac{\left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{n}{2}}\sqrt{b+ax^{j-n}}\,dx}{\sqrt{b+ax^{j-n}}}$$
$$= \frac{\left(x^{-m-\frac{n}{2}}(cx)^m\sqrt{ax^j+bx^n}\right)\int x^{m+\frac{n}{2}}\sqrt{1+\frac{ax^{j-n}}{b}}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}$$
$$= \frac{2x(cx)^m\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{1}{2},\frac{1+m+\frac{n}{2}}{j-n};1+\frac{2+2m+n}{2j-2n};-\frac{ax^{j-n}}{b}\right)}{(2+2m+n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

Mathematica [A] time = 0.21, size = 156, normalized size = 1.56

$$\frac{2x(cx)^m\left((2j+2m-n+2)\left(ax^j+bx^n\right)-a(j-n)x^j\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{2j+2m-n+2}{2j-2n};\frac{4j+2m-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)\right)}{(2m+n+2)(2j+2m-n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m*Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*((2 + 2*j + 2*m - n)*(a*x^j + b*x^n) - a*(j - n)*x^j*Sqrt[1 +
(a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*j + 2*m - n)/(2*j - 2*n), (2
 + 4*j + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/((2 + 2*j + 2*m - n)
*(2 + 2*m + n)*Sqrt[a*x^j + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j+bx^n}\,(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \sqrt{a\,x^j+b\,x^n}\,\,(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m*(a*x^j+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j + bx^n}\,(cx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^j + b*x^n)*(c*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,x)^m\,\sqrt{a\,x^j+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(a*x^j + b*x^n)^(1/2),x)
[Out] int((c*x)^m*(a*x^j + b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cx)^m\,\sqrt{ax^j + bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(a*x**j+b*x**n)**(1/2),x)
[Out] Integral((c*x)**m*sqrt(a*x**j + b*x**n), x)
```

3.423 $\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$

Optimal. Leaf size=102

$$\frac{2x(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m-\frac{n}{2}+1}{j-n};\frac{m-\frac{n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x*(c*x)^m*hypergeom([1/2, (1+m-1/2*n)/(j-n)],[1+(1+m-1/2*n)/(j-n)],-a*x^(
j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/(2+2*m-n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 365, 364}

$$\frac{2x(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{m-\frac{n}{2}+1}{j-n};\frac{m-\frac{n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (1 + m - n/2)
/(j - n), 1 + (1 + m - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 + 2*m - n)*Sq
rt[a*x^j + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{n}{2}}}{\sqrt{b+ax^{j-n}}}\,dx}{\sqrt{ax^j+bx^n}}$$
$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{n}{2}}}{\sqrt{1+\frac{ax^{j-n}}{b}}}\,dx}{\sqrt{ax^j+bx^n}}$$
$$= \frac{2x(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{1}{2},\frac{1+m-\frac{n}{2}}{j-n};1+\frac{1+m-\frac{n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2+2m-n)\sqrt{ax^j+bx^n}}$$

Mathematica [**A**] time = 0.09, size = 106, normalized size = 1.04

$$\frac{2x(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{2m-n+2}{2j-2n};\frac{2m-n+2}{2j-2n}+1;-\frac{ax^{j-n}}{b}\right)}{(2m-n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 + 2*m - n)
/(2*j - 2*n), 1 + (2 + 2*m - n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + 2*m
 - n)*Sqrt[a*x^j + b*x^n])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

maple [**F**] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((c*x)^m/(a*x^j+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate((c*x)^m/sqrt(a*x^j + b*x^n), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^m}{\sqrt{a\,x^j + b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j + b*x^n)^(1/2),x)
[Out] int((c*x)^m/(a*x^j + b*x^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(1/2),x)
[Out] Integral((c*x)**m/sqrt(a*x**j + b*x**n), x)
```

3.424 $\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2x^{1-n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{m-\frac{3n}{2}+1}{j-n};\frac{m-\frac{3n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b(2m-3n+2)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-n)*(c*x)^m*hypergeom([3/2, (1+m-3/2*n)/(j-n)],[1+(1+m-3/2*n)/(j-n)],
-a*x^(j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/b/(2+2*m-3*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 365, 364}

$$\frac{2x^{1-n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{m-\frac{3n}{2}+1}{j-n};\frac{m-\frac{3n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b(2m-3n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 - n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 +
m - (3*n)/2)/(j - n), 1 + (1 + m - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(
b*(2 + 2*m - 3*n)*Sqrt[a*x^j + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{3/2}}\,dx = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{3n}{2}}}{\left(b+ax^{j-n}\right)^{3/2}}\,dx}{\sqrt{ax^j+bx^n}}$$
$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{3n}{2}}}{\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}}\,dx}{b\sqrt{ax^j+bx^n}}$$
$$= \frac{2x^{1-n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{3}{2},\frac{1+m-\frac{3n}{2}}{j-n};1+\frac{1+m-\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b(2+2m-3n)\sqrt{ax^j+bx^n}}$$

Mathematica [A] time = 0.15, size = 116, normalized size = 1.05

$$\frac{2x^{1-j}(cx)^m\left(\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},\frac{-2j+2m-n+2}{2j-2n};\frac{2m-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)-1\right)}{a(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x^(1 - j)*(c*x)^m*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2,
 (2 - 2*j + 2*m - n)/(2*j - 2*n), (2 + 2*m - 3*n)/(2*j - 2*n), -((a*x^(j -
n))/b)]))/(a*(j - n)*Sqrt[a*x^j + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

maple [F] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(a\,x^j+b\,x^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^m/(a*x^j+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(ax^j + bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^m}{\left(a\,x^j + b\,x^n\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j + b*x^n)^(3/2),x)
```

```
[Out] int((c*x)^m/(a*x^j + b*x^n)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(ax^j + bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((c*x)**m/(a*x**j + b*x**n)**(3/2), x)
```

3.425 $\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx$

Optimal. Leaf size=111

$$\frac{2x^{1-2n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{5}{2},\frac{m-\frac{5n}{2}+1}{j-n};\frac{m-\frac{5n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b^2(2m-5n+2)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-2*n)*(c*x)^m*hypergeom([5/2, (1+m-5/2*n)/(j-n)],[1+(1+m-5/2*n)/(j-n)
],-a*x^(j-n)/b)*(1+a*x^(j-n)/b)^(1/2)/b^2/(2+2*m-5*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2032, 365, 364}

$$\frac{2x^{1-2n}(cx)^m\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{5}{2},\frac{m-\frac{5n}{2}+1}{j-n};\frac{m-\frac{5n}{2}+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b^2(2m-5n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

```
[Out] (2*x^(1 - 2*n)*(c*x)^m*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1
+ m - (5*n)/2)/(j - n), 1 + (1 + m - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])
/(b^2*(2 + 2*m - 5*n)*Sqrt[a*x^j + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{5/2}}\,dx = \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{b+ax^{j-n}}\right)\int \frac{x^{m-\frac{5n}{2}}}{\left(b+ax^{j-n}\right)^{5/2}}\,dx}{\sqrt{ax^j+bx^n}}$$

$$= \frac{\left(x^{-m+\frac{n}{2}}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{m-\frac{5n}{2}}}{\left(1+\frac{ax^{j-n}}{b}\right)^{5/2}}\,dx}{b^2\sqrt{ax^j+bx^n}}$$

$$= \frac{2x^{1-2n}(cx)^m\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{5}{2},\frac{1+m-\frac{5n}{2}}{j-n};1+\frac{1+m-\frac{5n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b^2(2+2m-5n)\sqrt{ax^j+bx^n}}$$

Mathematica [A] time = 0.55, size = 166, normalized size = 1.50

$$\frac{2x^{1-2j}(cx)^m\left(-(2j-2m+3n-2)\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{-4j+2m-n+2}{2j-2n};\frac{-2j+2m-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)-\frac{a(j-n)x^j}{ax^j+bx^n}+2j-2m+3n-2\right.}{3a^2(j-n)^2\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x)^m/(a*x^j + b*x^n)^(5/2),x]
```

```
[Out] (2*x^(1 - 2*j)*(c*x)^m*(-2 + 2*j - 2*m + 3*n - (a*(j - n)*x^j)/(a*x^j + b*x
^n) - (-2 + 2*j - 2*m + 3*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/
2, (2 - 4*j + 2*m - n)/(2*j - 2*n), (2 - 2*j + 2*m - 3*n)/(2*j - 2*n), -((a
*x^(j - n))/b)]))/(3*a^2*(j - n)^2*Sqrt[a*x^j + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(ax^j+bx^n\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{\left(a\,x^j+b\,x^n\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
[Out] int((c*x)^m/(a*x^j+b*x^n)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
[Out] integrate((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\,x)^m}{\left(a\,x^j + b\,x^n\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m/(a*x^j + b*x^n)^(5/2),x)
[Out] int((c*x)^m/(a*x^j + b*x^n)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m}{(ax^j + bx^n)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m/(a*x**j+b*x**n)**(5/2),x)
[Out] Integral((c*x)**m/(a*x**j + b*x**n)**(5/2), x)
```

3.426 $\int \left(ax^j+bx^n\right)^{3/2}\,dx$

Optimal. Leaf size=97

$$\frac{2bx^{n+1}\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{\frac{3n}{2}+1}{j-n};\frac{2j+n+2}{2(j-n)};-\frac{ax^{j-n}}{b}\right)}{(3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

```
[Out] 2*b*x^(1+n)*hypergeom([-3/2, (1+3/2*n)/(j-n)],[1/2*(2+2*j+n)/(j-n)],-a*x^(j
-n)/b)*(a*x^j+b*x^n)^(1/2)/(2+3*n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2011, 365, 364}

$$\frac{2bx^{n+1}\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{\frac{3n}{2}+1}{j-n};\frac{2j+n+2}{2(j-n)};-\frac{ax^{j-n}}{b}\right)}{(3n+2)\sqrt{\frac{ax^{j-n}}{b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*b*x^(1 + n)*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-3/2, (1 + (3*n)/2)/(j
 - n), (2 + 2*j + n)/(2*(j - n)), -((a*x^(j - n))/b)])/((2 + 3*n)*Sqrt[1 +
(a*x^(j - n))/b])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \left(ax^j+bx^n\right)^{3/2}\,dx=\frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{3n/2}\left(b+ax^{j-n}\right)^{3/2}\,dx}{\sqrt{b+ax^{j-n}}}$$
$$=\frac{\left(bx^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{3n/2}\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}$$
$$=\frac{2bx^{1+n}\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{3}{2},\frac{1+\frac{3n}{2}}{j-n};\frac{2+2j+n}{2(j-n)};-\frac{ax^{j-n}}{b}\right)}{(2+3n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

Mathematica [A] time = 0.23, size = 177, normalized size = 1.82

$$\frac{2x\left(3a^2(j-n)^2x^{2j}\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{4j-n+2}{2j-2n};\frac{6j-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)+(4j-n+2)\left(ax^j+bx^n\right)\left(a(-j+4n+2)x^j+b(2j\right.\right.}{(3n+2)(4j-n+2)(2j+n+2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^j + b*x^n)^(3/2),x]
```

```
[Out] (2*x*((2 + 4*j - n)*(a*x^j + b*x^n)*(a*(2 - j + 4*n)*x^j + b*(2 + 2*j + n)*
x^n) + 3*a^2*(j - n)^2*x^(2*j)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[
1/2, (2 + 4*j - n)/(2*j - 2*n), (2 + 6*j - 3*n)/(2*j - 2*n), -((a*x^(j - n)
)/b)]))/((2 + 4*j - n)*(2 + 2*j + n)*(2 + 3*n)*Sqrt[a*x^j + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^j+bx^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2), x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int \left(a\,x^j+b\,x^n\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int((a*x^j+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^j + bx^n\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(3/2), x)
```

mupad [B] time = 5.25, size = 82, normalized size = 0.85

$$\frac{x \left(a\,x^j + b\,x^n\right)^{3/2} {}_2F_1\left(-\frac{3}{2}, \frac{\frac{3\,n}{2}+1}{j-n};\ \frac{\frac{3\,n}{2}+1}{j-n} + 1;\ -\frac{a\,x^{j-n}}{b}\right)}{\left(\frac{3\,n}{2} + 1\right) \left(\frac{a\,x^{j-n}}{b} + 1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(3/2),x)
```

```
[Out] (x*(a*x^j + b*x^n)^(3/2)*hypergeom([-3/2, ((3*n)/2 + 1)/(j - n)], ((3*n)/2
+ 1)/(j - n) + 1, -(a*x^(j - n))/b))/(((3*n)/2 + 1)*((a*x^(j - n))/b + 1)^(
3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^j + bx^n\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(3/2), x)
```

3.427 $\int \sqrt{ax^j + bx^n}\, dx$

Optimal. Leaf size=87

$$\frac{2x\sqrt{ax^j + bx^n}\ {}_2F_1\left(-\frac{1}{2}, \frac{n+2}{2(j-n)}; \frac{n+2}{2j-2n} + 1; -\frac{ax^{j-n}}{b}\right)}{(n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

```
[Out] 2*x*hypergeom([-1/2, 1/2*(2+n)/(j-n)],[1+(2+n)/(2*j-2*n)],-a*x^(j-n)/b)*(a*
x^j+b*x^n)^(1/2)/(2+n)/(1+a*x^(j-n)/b)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2011, 365, 364}

$$\frac{2x\sqrt{ax^j + bx^n}\ {}_2F_1\left(-\frac{1}{2}, \frac{n+2}{2(j-n)}; \frac{n+2}{2j-2n} + 1; -\frac{ax^{j-n}}{b}\right)}{(n+2)\sqrt{\frac{ax^{j-n}}{b} + 1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*Sqrt[a*x^j + b*x^n]*Hypergeometric2F1[-1/2, (2 + n)/(2*(j - n)), 1 + (
2 + n)/(2*j - 2*n), -((a*x^(j - n))/b)])/((2 + n)*Sqrt[1 + (a*x^(j - n))/b]
)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \sqrt{ax^j+bx^n}\,dx = \frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{n/2}\sqrt{b+ax^{j-n}}\,dx}{\sqrt{b+ax^{j-n}}}$$
$$= \frac{\left(x^{-n/2}\sqrt{ax^j+bx^n}\right)\int x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\,dx}{\sqrt{1+\frac{ax^{j-n}}{b}}}$$
$$= \frac{2x\sqrt{ax^j+bx^n}\;{}_2F_1\left(-\frac{1}{2},\frac{2+n}{2(j-n)};1+\frac{2+n}{2j-2n};-\frac{ax^{j-n}}{b}\right)}{(2+n)\sqrt{1+\frac{ax^{j-n}}{b}}}$$

Mathematica [A] time = 0.17, size = 134, normalized size = 1.54

$$\frac{2x\left(a(j-n)x^j\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{1}{2},\frac{2j-n+2}{2j-2n};\frac{4j-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)-(2j-n+2)\left(ax^j+bx^n\right)\right)}{(n+2)(-2j+n-2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*(-((2 + 2*j - n)*(a*x^j + b*x^n)) + a*(j - n)*x^j*Sqrt[1 + (a*x^(j - n
))/b]*Hypergeometric2F1[1/2, (2 + 2*j - n)/(2*j - 2*n), (2 + 4*j - 3*n)/(2*
j - 2*n), -((a*x^(j - n))/b)]))/((2 + n)*(-2 - 2*j + n)*Sqrt[a*x^j + b*x^n]
)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*x^j + b*x^n), x)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \sqrt{a\,x^j+b\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j+b*x^n)^(1/2),x)
```

```
[Out] int((a*x^j+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a*x^j + b*x^n), x)
```

mupad [B] time = 5.23, size = 82, normalized size = 0.94

$$\frac{x\sqrt{a\,x^j+b\,x^n}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{\frac{n}{2}+1}{j-n};\ \frac{\frac{n}{2}+1}{j-n}+1;\ -\frac{a\,x^{j-n}}{b}\right)}{\left(\frac{n}{2}+1\right)\sqrt{\frac{a\,x^{j-n}}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^j + b*x^n)^(1/2),x)
[Out] (x*(a*x^j + b*x^n)^(1/2)*hypergeom([-1/2, (n/2 + 1)/(j - n)], (n/2 + 1)/(j
- n) + 1, -(a*x^(j - n))/b))/((n/2 + 1)*((a*x^(j - n))/b + 1)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax^j + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**j+b*x**n)**(1/2),x)
[Out] Integral(sqrt(a*x**j + b*x**n), x)
```

3.428 $\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$

Optimal. Leaf size=93

$$\frac{2x\sqrt{\frac{ax^{j-n}}{b}+1}\ {}_2F_1\left(\frac{1}{2},\frac{2-n}{2(j-n)};\frac{1-\frac{n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x*hypergeom([1/2, 1/2*(2-n)/(j-n)],[1+1/2*(2-n)/(j-n)],-a*x^(j-n)/b)*(1+a
*x^(j-n)/b)^(1/2)/(2-n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2011, 365, 364}

$$\frac{2x\sqrt{\frac{ax^{j-n}}{b}+1}\ {}_2F_1\left(\frac{1}{2},\frac{2-n}{2(j-n)};\frac{1-\frac{n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*x^j + b*x^n],x]
```

```
[Out] (2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (2 - n)/(2*(j - n)),
1 + (1 - n/2)/(j - n), -((a*x^(j - n))/b)])/((2 - n)*Sqrt[a*x^j + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx = \frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-n/2}}{\sqrt{b+ax^{j-n}}}\,dx}{\sqrt{ax^j+bx^n}}$$
$$= \frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-n/2}}{\sqrt{1+\frac{ax^{j-n}}{b}}}\,dx}{\sqrt{ax^j+bx^n}}$$
$$= \frac{2x\sqrt{1+\frac{ax^{j-n}}{b}}\ {}_2F_1\left(\frac{1}{2},\frac{2-n}{2(j-n)};1+\frac{1-\frac{n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{(2-n)\sqrt{ax^j+bx^n}}$$

Mathematica [A] time = 0.06, size = 88, normalized size = 0.95

$$-\frac{2x\sqrt{\frac{ax^{j-n}}{b}+1}\ {}_2F_1\left(\frac{1}{2},\frac{n-2}{2(n-j)};\frac{n-2}{2(n-j)}+1;-\frac{ax^{j-n}}{b}\right)}{(n-2)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*x^j + b*x^n],x]
[Out] (-2*x*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, (-2 + n)/(2*(-j + n)
), 1 + (-2 + n)/(2*(-j + n)), -((a*x^(j - n))/b)])/((-2 + n)*Sqrt[a*x^j + b
*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^j+bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*x^j + b*x^n), x)
```

maple [F] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\,x^j+b\,x^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j+b*x^n)^(1/2),x)
[Out] int(1/(a*x^j+b*x^n)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(a*x^j + b*x^n), x)
```

mupad [B] time = 5.27, size = 83, normalized size = 0.89

$$-\frac{x\sqrt{\frac{b\,x^{n-j}}{a}+1}\;{}_2F_1\left(\frac{1}{2},\frac{\frac{j}{2}-1}{j-n};\ \frac{\frac{j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{j}{2}-1\right)\sqrt{a\,x^j+b\,x^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j + b*x^n)^(1/2),x)
```

```
[Out] -(x*((b*x^(n - j))/a + 1)^(1/2)*hypergeom([1/2, (j/2 - 1)/(j - n)], (j/2 -
1)/(j - n) + 1, -(b*x^(n - j))/a))/((j/2 - 1)*(a*x^j + b*x^n)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ax^j + bx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**j+b*x**n)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x**j + b*x**n), x)
```

3.429 $\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{2x^{1-n}\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n};\frac{1-\frac{3n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-n)*hypergeom([3/2, (1-3/2*n)/(j-n)],[1+(2-3*n)/(2*j-2*n)],-a*x^(j-n)
/b)*(1+a*x^(j-n)/b)^(1/2)/b/(2-3*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2011, 365, 364}

$$\frac{2x^{1-n}\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n};\frac{1-\frac{3n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^j + b*x^n)^(-3/2),x]
```

```
[Out] (2*x^(1 - n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[3/2, (1 - (3*n)/2)
/(j - n), 1 + (1 - (3*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b*(2 - 3*n)*Sqrt
[a*x^j + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\left(ax^j+bx^n\right)^{3/2}}\,dx = \frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-3n/2}}{\left(b+ax^{j-n}\right)^{3/2}}\,dx}{\sqrt{ax^j+bx^n}}$$
$$= \frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-3n/2}}{\left(1+\frac{ax^{j-n}}{b}\right)^{3/2}}\,dx}{b\sqrt{ax^j+bx^n}}$$
$$= \frac{2x^{1-n}\sqrt{1+\frac{ax^{j-n}}{b}}\,{}_2F_1\left(\frac{3}{2},\frac{1-\frac{3n}{2}}{j-n};1+\frac{1-\frac{3n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b(2-3n)\sqrt{ax^j+bx^n}}$$

Mathematica [A] time = 0.12, size = 104, normalized size = 1.03

$$\frac{2x^{1-j}\left(\sqrt{\frac{ax^{j-n}}{b}+1}\,{}_2F_1\left(\frac{1}{2},-\frac{2j+n-2}{2(j-n)};\frac{2-3n}{2j-2n};-\frac{ax^{j-n}}{b}\right)-1\right)}{a(j-n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^j + b*x^n)^(-3/2),x]
```

```
[Out] (2*x^(1 - j)*(-1 + Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[1/2, -1/2*(-
2 + 2*j + n)/(j - n), (2 - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(a*(j -
n)*Sqrt[a*x^j + b*x^n])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^j+bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(-3/2), x)
```

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^j+b\,x^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j+b*x^n)^(3/2),x)
```

```
[Out] int(1/(a*x^j+b*x^n)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^j + bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(-3/2), x)
```

mupad [B] time = 5.48, size = 83, normalized size = 0.82

$$-\frac{x\left(\frac{b\,x^{n-j}}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{\frac{3\,j}{2}-1}{j-n};\ \frac{\frac{3\,j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{3\,j}{2}-1\right)\,\left(a\,x^j+b\,x^n\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j + b*x^n)^(3/2),x)
```

```
[Out] -(x*((b*x^(n - j))/a + 1)^(3/2)*hypergeom([3/2, ((3*j)/2 - 1)/(j - n)], ((3
*j)/2 - 1)/(j - n) + 1, -(b*x^(n - j))/a))/(((3*j)/2 - 1)*(a*x^j + b*x^n)^(
3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^j + bx^n\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**j+b*x**n)**(3/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(-3/2), x)
```

3.430 $\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx$

Optimal. Leaf size=101

$$\frac{2x^{1-2n}\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n};\frac{1-\frac{5n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

```
[Out] 2*x^(1-2*n)*hypergeom([5/2, (1-5/2*n)/(j-n)],[1+(2-5*n)/(2*j-2*n)],-a*x^(j-
n)/b)*(1+a*x^(j-n)/b)^(1/2)/b^2/(2-5*n)/(a*x^j+b*x^n)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2011, 365, 364}

$$\frac{2x^{1-2n}\sqrt{\frac{ax^{j-n}}{b}+1}\;{}_2F_1\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n};\frac{1-\frac{5n}{2}}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^j + b*x^n)^(-5/2),x]
```

```
[Out] (2*x^(1 - 2*n)*Sqrt[1 + (a*x^(j - n))/b]*Hypergeometric2F1[5/2, (1 - (5*n)/
2)/(j - n), 1 + (1 - (5*n)/2)/(j - n), -((a*x^(j - n))/b)])/(b^2*(2 - 5*n)*
Sqrt[a*x^j + b*x^n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\left(ax^j+bx^n\right)^{5/2}}\,dx = \frac{\left(x^{n/2}\sqrt{b+ax^{j-n}}\right)\int \frac{x^{-5n/2}}{\left(b+ax^{j-n}\right)^{5/2}}\,dx}{\sqrt{ax^j+bx^n}}$$
$$= \frac{\left(x^{n/2}\sqrt{1+\frac{ax^{j-n}}{b}}\right)\int \frac{x^{-5n/2}}{\left(1+\frac{ax^{j-n}}{b}\right)^{5/2}}\,dx}{b^2\sqrt{ax^j+bx^n}}$$
$$= \frac{2x^{1-2n}\sqrt{1+\frac{ax^{j-n}}{b}}\;{}_2F_1\left(\frac{5}{2},\frac{1-\frac{5n}{2}}{j-n};1+\frac{1-\frac{5n}{2}}{j-n};-\frac{ax^{j-n}}{b}\right)}{b^2(2-5n)\sqrt{ax^j+bx^n}}$$

Mathematica [A] time = 0.38, size = 185, normalized size = 1.83

$$\frac{2x^{1-2j}\left(\left(8j^2+2j(7n-6)+3n^2-8n+4\right)\sqrt{\frac{ax^{j-n}}{b}+1}\left(ax^j+bx^n\right){}_2F_1\left(\frac{1}{2},-\frac{4j+n-2}{2(j-n)};\frac{-2j-3n+2}{2j-2n};-\frac{ax^{j-n}}{b}\right)-(4j+n-\right.}{3a^2(-4j-n+2)(j-n)^2\left(ax^j+bx^n\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^j + b*x^n)^(-5/2),x]
```

```
[Out] (2*x^(1 - 2*j)*(-((-2 + 4*j + n)*(a*(-2 + j + 4*n)*x^j + b*(-2 + 2*j + 3*n)
*x^n)) + (4 + 8*j^2 - 8*n + 3*n^2 + 2*j*(-6 + 7*n))*Sqrt[1 + (a*x^(j - n))/
b]*(a*x^j + b*x^n)*Hypergeometric2F1[1/2, -1/2*(-2 + 4*j + n)/(j - n), (2 -
 2*j - 3*n)/(2*j - 2*n), -((a*x^(j - n))/b)]))/(3*a^2*(2 - 4*j - n)*(j - n)
^2*(a*x^j + b*x^n)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^j+bx^n\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^(-5/2), x)
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\,x^j+b\,x^n\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j+b*x^n)^(5/2),x)
```

```
[Out] int(1/(a*x^j+b*x^n)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^j+bx^n\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^j+b*x^n)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^(-5/2), x)
```

mupad [B] time = 5.60, size = 83, normalized size = 0.82

$$-\frac{x\left(\frac{b\,x^{n-j}}{a}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{\frac{5\,j}{2}-1}{j-n};\ \frac{\frac{5\,j}{2}-1}{j-n}+1;\ -\frac{b\,x^{n-j}}{a}\right)}{\left(\frac{5\,j}{2}-1\right)\left(a\,x^j+b\,x^n\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^j + b*x^n)^(5/2),x)
```

```
[Out] -(x*((b*x^(n - j))/a + 1)^(5/2)*hypergeom([5/2, ((5*j)/2 - 1)/(j - n)], ((5
*j)/2 - 1)/(j - n) + 1, -(b*x^(n - j))/a))/(((5*j)/2 - 1)*(a*x^j + b*x^n)^(
5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ax^j+bx^n\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**j+b*x**n)**(5/2),x)
```

```
[Out] Integral((a*x**j + b*x**n)**(-5/2), x)
```

3.431 $\int \sqrt{\frac{1+x}{x^5}}\,dx$

Optimal. Leaf size=18

$$-\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2} x^6$$

```
[Out] -2/3*(1/x^5+1/x^4)^(3/2)*x^6
```

Rubi [A] time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1979, 2000}

$$-\frac{2}{3}\left(\frac{1}{x^4}+\frac{1}{x^5}\right)^{3/2} x^6$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/x^5],x]
```

```
[Out] (-2*(x^(-5) + x^(-4))^(3/2)*x^6)/3
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{1+x}{x^5}}\,dx &= \int \sqrt{\frac{1}{x^5}+\frac{1}{x^4}}\,dx\\ &= -\frac{2}{3}\left(\frac{1}{x^5}+\frac{1}{x^4}\right)^{3/2} x^6\end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.06

$$-\frac{2}{3}x(x+1)\sqrt{\frac{x+1}{x^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/x^5],x]
```

```
[Out] (-2*x*(1 + x)*Sqrt[(1 + x)/x^5])/3
```

fricas [A] time = 0.39, size = 16, normalized size = 0.89

$$-\frac{2}{3}\left(x^2+x\right)\sqrt{\frac{x+1}{x^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="fricas")
[Out] -2/3*(x^2 + x)*sqrt((x + 1)/x^5)
```

giac [B] time = 0.19, size = 50, normalized size = 2.78

$$\frac{2\left(3\left(x-\sqrt{x^2+x}\right)^2\operatorname{sgn}(x)+3\left(x-\sqrt{x^2+x}\right)\operatorname{sgn}(x)+\operatorname{sgn}(x)\right)}{3\left(x-\sqrt{x^2+x}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="giac")
[Out] 2/3*(3*(x - sqrt(x^2 + x))^2*sgn(x) + 3*(x - sqrt(x^2 + x))*sgn(x) + sgn(x)
)/(x - sqrt(x^2 + x))^3
```

maple [A] time = 0.05, size = 16, normalized size = 0.89

$$-\frac{2\,(x+1)\sqrt{\frac{x+1}{x^5}}\;x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x+1)/x^5)^(1/2),x)
[Out] -2/3*x*(x+1)*((x+1)/x^5)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x+1}{x^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x^5)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt((x + 1)/x^5), x)
```

mupad [B] time = 5.26, size = 15, normalized size = 0.83

$$-\frac{2\,x\sqrt{\frac{x+1}{x^5}}\;(x+1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)/x^5)^(1/2),x)
[Out] -(2*x*((x + 1)/x^5)^(1/2)*(x + 1))/3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\frac{x+1}{x^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/x**5)**(1/2),x)
[Out] Integral(sqrt((x + 1)/x**5), x)
```

3.432 $\int \sqrt{x+x^{5/2}}\,dx$

Optimal. Leaf size=20

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

```
[Out] 4/9*(x+x^(5/2))^(3/2)/x^(3/2)
```

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2000}

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^(5/2)],x]
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \sqrt{x+x^{5/2}}\,dx=\frac{4\left(x+x^{5/2}\right)^{3/2}}{9x^{3/2}}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$\frac{4\left(x^{5/2}+x\right)^{3/2}}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^(5/2)],x]
[Out] (4*(x + x^(5/2))^(3/2))/(9*x^(3/2))
```

fricas [A] time = 0.44, size = 19, normalized size = 0.95

$$\frac{4\sqrt{x^{\frac{5}{2}}+x}\left(x^2+\sqrt{x}\right)}{9\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="fricas")
[Out] 4/9*sqrt(x^(5/2) + x)*(x^2 + sqrt(x))/x
```

giac [A] time = 0.17, size = 11, normalized size = 0.55

$$\frac{4}{9}\left(x^{\frac{3}{2}}+1\right)^{\frac{3}{2}}-\frac{4}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/9*(x^(3/2) + 1)^(3/2) - 4/9
```

maple [A] time = 0.05, size = 18, normalized size = 0.90

$$\frac{4\sqrt{x^{\frac{5}{2}}+x}\,\left(x^{\frac{3}{2}}+1\right)}{9\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+x^(5/2))^(1/2),x)
```

```
[Out] 4/9*(x+x^(5/2))^(1/2)/x^(1/2)*(x^(3/2)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{\frac{5}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x^(5/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^(5/2) + x), x)
```

mupad [B] time = 5.32, size = 27, normalized size = 1.35

$$\frac{2\,x\,\sqrt{x+x^{5/2}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},1;\ 2;\ -x^{3/2}\right)}{3\,\sqrt{x^{3/2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + x^(5/2))^(1/2),x)
```

```
[Out] (2*x*(x + x^(5/2))^(1/2)*hypergeom([-1/2, 1], 2, -x^(3/2)))/(3*(x^(3/2) + 1
)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{\frac{5}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+x**(5/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x**(5/2) + x), x)
```

3.433 $\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx$

Optimal. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

[Out] 2*arctan(x^(1/2))

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 63, 203}

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x] + x^(3/2))^(-1),x]

[Out] 2*ArcTan[Sqrt[x]]

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}+x^{3/2}}\,dx &= \int \frac{1}{\sqrt{x}\,(1+x)}\,dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)\\ &= 2\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x] + x^(3/2))^(-1),x]

[Out] 2*ArcTan[Sqrt[x]]

fricas [A] time = 0.38, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="fricas")
[Out] 2*arctan(sqrt(x))
```

giac [A] time = 0.15, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="giac")
[Out] 2*arctan(sqrt(x))
```

maple [A] time = 0.04, size = 7, normalized size = 0.88

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(3/2)+x^(1/2)),x)
[Out] 2*arctan(x^(1/2))
```

maxima [A] time = 3.14, size = 6, normalized size = 0.75

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(3/2)+x^(1/2)),x, algorithm="maxima")
[Out] 2*arctan(sqrt(x))
```

mupad [B] time = 5.24, size = 6, normalized size = 0.75

$$2 \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2) + x^(3/2)),x)
[Out] 2*atan(x^(1/2))
```

sympy [A] time = 0.21, size = 7, normalized size = 0.88

$$2 \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(3/2)+x**(1/2)),x)
[Out] 2*atan(sqrt(x))
```

3.434 $\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$

Optimal. Leaf size=25

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

```
[Out] 2/9*(x^2*(b*x^3+a))^(3/2)/b/x^3
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[x^2*(a + b*x^3)],x]
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx=\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[x^2*(a + b*x^3)],x]
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

fricas [A] time = 0.39, size = 28, normalized size = 1.12

$$\frac{2\,\sqrt{bx^5+ax^2}\left(bx^3+a\right)}{9\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

giac [**A**] time = 0.17, size = 27, normalized size = 1.08

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\operatorname{sgn}(x)}{9\,b}-\frac{2\,a^{\frac{3}{2}}\operatorname{sgn}(x)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

maple [**A**] time = 0.05, size = 29, normalized size = 1.16

$$\frac{2\left(b\,x^3+a\right)\sqrt{\left(b\,x^3+a\right)x^2}}{9bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2*(b*x^3+a))^(1/2),x)
[Out] 2/9*(b*x^3+a)*(x^2*(b*x^3+a))^(1/2)/b/x
```

maxima [**A**] time = 1.46, size = 14, normalized size = 0.56

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2*(b*x^3+a))^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

mupad [**B**] time = 5.23, size = 22, normalized size = 0.88

$$\frac{2\left(b\,x^3+a\right)^{3/2}\sqrt{x^2}}{9\,b\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2*(a + b*x^3))^(1/2),x)
[Out] (2*(a + b*x^3)^(3/2)*(x^2)^(1/2))/(9*b*x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2*(b*x**3+a))**(1/2),x)
[Out] Timed out
```

3.435 $\int x\sqrt{ax^2+bx^5}\,dx$

Optimal. Leaf size=25

$$\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

```
[Out] 2/9*(b*x^5+a*x^2)^(3/2)/b/x^3
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {1588}

$$\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(a*x^2 + b*x^5)^(3/2))/(9*b*x^3)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\sqrt{ax^2+bx^5}\,dx=\frac{2\left(ax^2+bx^5\right)^{3/2}}{9bx^3}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{2\left(x^2\left(a+bx^3\right)\right)^{3/2}}{9bx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a*x^2 + b*x^5],x]
```

```
[Out] (2*(x^2*(a + b*x^3))^(3/2))/(9*b*x^3)
```

fricas [A] time = 0.39, size = 28, normalized size = 1.12

$$\frac{2\sqrt{bx^5+ax^2}\left(bx^3+a\right)}{9\,bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*sqrt(b*x^5 + a*x^2)*(b*x^3 + a)/(b*x)
```

giac [A] time = 0.16, size = 27, normalized size = 1.08

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}\operatorname{sgn}(x)}{9\,b}-\frac{2\,a^{\frac{3}{2}}\operatorname{sgn}(x)}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)*sgn(x)/b - 2/9*a^(3/2)*sgn(x)/b
```

maple [A] time = 0.04, size = 29, normalized size = 1.16

$$\frac{2\left(b\,x^3+a\right)\sqrt{b\,x^5+a\,x^2}}{9bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^5+a*x^2)^(1/2),x)
```

```
[Out] 2/9*(b*x^3+a)*(b*x^5+a*x^2)^(1/2)/b/x
```

maxima [A] time = 1.43, size = 14, normalized size = 0.56

$$\frac{2\left(bx^3+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^5+a*x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

mupad [B] time = 5.26, size = 29, normalized size = 1.16

$$\frac{\left(\frac{2\,a}{9\,b}+\frac{2\,x^3}{9}\right)\sqrt{b\,x^5+a\,x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x^2 + b*x^5)^(1/2),x)
```

```
[Out] (((2*a)/(9*b) + (2*x^3)/9)*(a*x^2 + b*x^5)^(1/2))/x
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^2\left(a+bx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**5+a*x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2*(a + b*x**3)), x)
```

3.436 $\int \sqrt{x^4 (a + bx^3)}\, dx$

Optimal. Leaf size=25

$$\frac{2\left(ax^4 + bx^7\right)^{3/2}}{9bx^6}$$

```
[Out] 2/9*(b*x^7+a*x^4)^(3/2)/b/x^6
```

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1979, 2000}

$$\frac{2\left(ax^4 + bx^7\right)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^4*(a + b*x^3)],x]
```

```
[Out] (2*(a*x^4 + b*x^7)^(3/2))/(9*b*x^6)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{x^4\left(a + bx^3\right)}\, dx &= \int \sqrt{ax^4 + bx^7}\, dx \\ &= \frac{2\left(ax^4 + bx^7\right)^{3/2}}{9bx^6} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2\left(x^4\left(a + bx^3\right)\right)^{3/2}}{9bx^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^4*(a + b*x^3)],x]
```

```
[Out] (2*(x^4*(a + b*x^3))^(3/2))/(9*b*x^6)
```

fricas [A] time = 0.37, size = 28, normalized size = 1.12

$$\frac{2\sqrt{bx^7 + ax^4}\left(bx^3 + a\right)}{9\,bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="fricas")
[Out] 2/9*sqrt(b*x^7 + a*x^4)*(b*x^3 + a)/(b*x^2)
```

giac [A] time = 0.19, size = 14, normalized size = 0.56

$$\frac{2\left(bx^{3}+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="giac")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

maple [A] time = 0.05, size = 29, normalized size = 1.16

$$\frac{2\left(b\,x^{3}+a\right)\sqrt{\left(b\,x^{3}+a\right)x^{4}}}{9b\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(b*x^3+a))^(1/2),x)
[Out] 2/9*(b*x^3+a)*(x^4*(b*x^3+a))^(1/2)/b/x^2
```

maxima [A] time = 1.47, size = 14, normalized size = 0.56

$$\frac{2\left(bx^{3}+a\right)^{\frac{3}{2}}}{9\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4*(b*x^3+a))^(1/2),x, algorithm="maxima")
[Out] 2/9*(b*x^3 + a)^(3/2)/b
```

mupad [B] time = 5.21, size = 22, normalized size = 0.88

$$\frac{2\left(b\,x^{3}+a\right)^{3/2}\sqrt{x^{4}}}{9\,b\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^3))^(1/2),x)
[Out] (2*(a + b*x^3)^(3/2)*(x^4)^(1/2))/(9*b*x^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{4}\left(a+bx^{3}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4*(b*x**3+a))**(1/2),x)
[Out] Integral(sqrt(x**4*(a + b*x**3)), x)
```

3.437 $\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx$

Optimal. Leaf size=988

$$-\frac{45\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\sqrt[3]{-\frac{b\left(\sqrt[3]{x}a+bx^{2/3}\right)}{a^2}}\,E\left(\sin^{-1}\right.}{28\sqrt[3]{2}\,b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}$$

```
[Out] -45/28*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a*x^(1/3)+b*x^(2/3))^(1/3)+9/7*(a+b*x^(
1/3))*x^(2/3)/b/(a*x^(1/3)+b*x^(2/3))^(1/3)-45/28*a^2*(a+2*b*x^(1/3))*(-b*(
a*x^(1/3)+b*x^(2/3))/a^2)^(1/3)*2^(2/3)/b^3/(a*x^(1/3)+b*x^(2/3))^(1/3)/(1-
2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))+15/14*3^(3/4)*a^4*(1-
2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2
)^(1/3)*EllipticF((1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/
(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1
+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3)*(-b*(a+b*x^(1/3))*x
^(1/3)/a^2)^(2/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))
^2)^(1/2)*2^(1/6)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(2/3))^(1/3)/((-1+2^(2
/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1
/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)-45/56*3^(1/4)*a^4*(1-2^(2/3)*(-b*(a+b*x^(1
/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2)^(1/3)*EllipticE((1-
2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/(1-2^(2/3)*(-b*(a+b*x
^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*((1+2^(2/3)*(-b*(a+b*x^(
1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(2/3))/(1
-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)*(1/2*6^(1/2
)+1/2*2^(1/2))*2^(2/3)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(2/3))^(1/3)/((-1
+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))
*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 2.25, antiderivative size = 988, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {2011, 341, 50, 61, 622, 619, 235, 304, 219, 1879}

$$-\frac{45\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\sqrt[3]{-\frac{b\left(\sqrt[3]{x}a+bx^{2/3}\right)}{a^2}}\,E\left(\sin^{-1}\right.}{28\sqrt[3]{2}\,b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{a^2}}-\sqrt{3}+1\right)^2}}\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{\sqrt[3]{x}a+bx^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

```
[Out] (-45*a^2*(a + 2*b*x^(1/3))*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3))/(14*
2^(1/3)*b^3*(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/
```

```
3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) - (45*a*(a + b*x^(1/3))*x^(1/3))/(28*b^2
*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (9*(a + b*x^(1/3))*x^(2/3))/(7*b*(a*x^(1/
3) + b*x^(2/3))^(1/3)) - (45*3^(1/4)*Sqrt[2 + Sqrt[3]]*a^4*(1 - 2^(2/3)*(-(
(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^
2]*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticE[ArcSin[(1 + Sqrt[3]
 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(28*2^(1/
3)*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 -
 Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b
*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3)) + (15*3^(3/4)*a^4*(1 - 2^(2/3)*(-(
(b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(
1/3))*x^(1/3))/a^2))^(1/3) + 2*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))
^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^
2]*(-((b*(a*x^(1/3) + b*x^(2/3)))/a^2))^(1/3)*EllipticF[ArcSin[(1 + Sqrt[3]
 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/
3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(7*2^(5/6
)*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 -
Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b*
x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/(a*c + (b*c + a*d)*x + b*d*x^2)^m, Int[(a*c + (b*c
 + a*d)*x + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,
0] && LtQ[-1, m, 0] && LeQ[3, Denominator[m], 4] && AtomQ[b*c + a*d]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 235

```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 304

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, -Dist[(Sqrt[2]*s)/(Sqrt[2 - Sqrt[3]]*r), Int[1/S
qrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x
^3], x], x]] /; FreeQ[{a, b}, x] && NegQ[a]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 1879

```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[((1 + Sqrt[3])*d)/c]], s = Denom[Simplify[((1 + Sqrt[3])*d)/c
]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 - Sqrt[3])*s + r*x)), x] + S
imp[(3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sq
rt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[-((s*(s + r*x)
)/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}\,dx = \frac{\left(\sqrt[3]{a+b\sqrt[3]{x}}\ \sqrt[9]{x}\right)\int \frac{1}{\sqrt[3]{a+b\sqrt[3]{x}}\ \sqrt[9]{x}}\,dx}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= \frac{\left(3\sqrt[3]{a+b\sqrt[3]{x}}\ \sqrt[9]{x}\right)\text{Subst}\left(\int \frac{x^{5/3}}{\sqrt[3]{a+bx}}\,dx, x, \sqrt[3]{x}\right)}{\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(15a\sqrt[3]{a+b\sqrt[3]{x}}\ \sqrt[9]{x}\right)\text{Subst}\left(\int \frac{x^{2/3}}{\sqrt[3]{a+bx}}\,dx, x, \sqrt[3]{x}\right)}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\sqrt[3]{a+b\sqrt[3]{x}}\ \sqrt[9]{x}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\ \sqrt[3]{a+bx}}\,d\right.}{14b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx, x, \sqrt[3]{x}\right)}{14b^2}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(15a^2\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{-\frac{bx}{a}-\frac{b^2x^2}{a^2}}}\right.}{14b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(15a^4\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-\frac{a^2x^2}{b^2}}}\,d\right.}{14\sqrt[3]{2}\,b^4\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{\left(45a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}}{28\sqrt[3]{2}\,b^3\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{\ }}$$

$$= -\frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{7b\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{\left(45a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}\right)\text{Subst}}{28\sqrt[3]{2}\,b^3\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{\ }}$$

$$= -\frac{45a^2\left(a+2b\sqrt[3]{x}\right)\sqrt[3]{-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}}}{14\sqrt[3]{2}\,b^3\left(1-\sqrt{3}-\sqrt[3]{1-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\right)\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} - \frac{45a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{28b^2\sqrt[3]{a\sqrt[3]{x}+bx^{2/3}}} + \frac{9\left(a+b\right.}{7b\sqrt[3]{a\sqrt[3]{\ }}}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.06

$$\frac{9x\sqrt[3]{\frac{b\sqrt[3]{x}}{a}+1}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};-\frac{b\sqrt[3]{x}}{a}\right)}{8\sqrt[3]{\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^(1/3) + b*x^(2/3))^(-1/3),x]
```

```
[Out] (9*(1 + (b*x^(1/3))/a)^(1/3)*x*Hypergeometric2F1[1/3, 8/3, 11/3, -((b*x^(1/
3))/a)])/(8*((a + b*x^(1/3))*x^(1/3))^(1/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{\frac{2}{3}}+a\,x^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

```
[Out] int(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{2}{3}}+ax^{\frac{1}{3}}\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-1/3), x)
```

mupad [B] time = 5.41, size = 42, normalized size = 0.04

$$\frac{9\,x\,{\left(\frac{b\,x^{1/3}}{a}+1\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{8}{3};\ \frac{11}{3};\ -\frac{b\,x^{1/3}}{a}\right)}{8\,{\left(a\,x^{1/3}+b\,x^{2/3}\right)}^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^(1/3) + b*x^(2/3))^(1/3),x)
```

```
[Out] (9*x*((b*x^(1/3))/a + 1)^(1/3)*hypergeom([1/3, 8/3], 11/3, -(b*x^(1/3))/a))
/(8*(a*x^(1/3) + b*x^(2/3))^(1/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a\sqrt[3]{x}+bx^{\frac{2}{3}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**(1/3)+b*x**(2/3))**(1/3),x)
```

```
[Out] Integral((a*x**(1/3) + b*x**(2/3))**(-1/3), x)
```

3.438 $\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx$

Optimal. Leaf size=487

$$\frac{6\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}F}{5b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}-\sqrt{3}+1\right)^2}}\left(a+2b\sqrt[3]{x}\right)\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

```
[Out] -18/5*a*(a+b*x^(1/3))*x^(1/3)/b^2/(a*x^(1/3)+b*x^(2/3))^(2/3)+9/5*(a+b*x^(1
/3))*x^(2/3)/b/(a*x^(1/3)+b*x^(2/3))^(2/3)+6/5*2^(1/3)*3^(3/4)*a^4*(1-2^(2/
3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))*(-b*(a*x^(1/3)+b*x^(2/3))/a^2)^(2/
3)*EllipticF((1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+3^(1/2))/(1-2^
(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2)),2*I-I*3^(1/2))*(1/2*6^(
1/2)-1/2*2^(1/2))*((1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3)+2*2^(1/3
)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(2/3))/(1-2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3
)/a^2)^(1/3)-3^(1/2))^2)^(1/2)/b^3/(a+2*b*x^(1/3))/(a*x^(1/3)+b*x^(2/3))^(2
/3)/((-1+2^(2/3)*(-b*(a+b*x^(1/3))*x^(1/3)/a^2)^(1/3))/(1-2^(2/3)*(-b*(a+b*
x^(1/3))*x^(1/3)/a^2)^(1/3)-3^(1/2))^2)^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {2011, 341, 50, 61, 622, 619, 236, 219}

$$\frac{6\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4\left(1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}F}{5b^3\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{b\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)}{a^2}}-\sqrt{3}+1\right)^2}}\left(a+2b\sqrt[3]{x}\right)\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

```
[Out] (-18*a*(a + b*x^(1/3))*x^(1/3))/(5*b^2*(a*x^(1/3) + b*x^(2/3))^(2/3)) + (9*
(a + b*x^(1/3))*x^(2/3))/(5*b*(a*x^(1/3) + b*x^(2/3))^(2/3)) + (6*2^(1/3)*3
^(3/4)*Sqrt[2 - Sqrt[3]]*a^4*(1 - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^
2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3) + 2
*2^(1/3)*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)
*(-((b*(a + b*x^(1/3))*x^(1/3))/a^2))^(1/3))^2]*(-((b*(a*x^(1/3) + b*x^(2/3
)))/a^2))^(2/3)*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3
))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3))*x^(1/
3))/a^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*b^3*Sqrt[-((1 - 2^(2/3)*(-((b*(a + b
*x^(1/3))*x^(1/3))/a^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((b*(a + b*x^(1/3)
)*x^(1/3))/a^2))^(1/3))^2)]*(a + 2*b*x^(1/3))*(a*x^(1/3) + b*x^(2/3))^(2/3)
)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 61

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Dist[(
(a + b*x)^m*(c + d*x)^m)/(a*c + (b*c + a*d)*x + b*d*x^2)^m, Int[(a*c + (b*c
 + a*d)*x + b*d*x^2)^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d,
0] && LtQ[-1, m, 0] && LeQ[3, Denominator[m], 4] && AtomQ[b*c + a*d]
```

Rule 219

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 - Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 + Sqrt[3])*s
+ r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[-((s*(s + r*x))/((1 - Sqrt[3])*s + r*x)^2)]), x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

Rule 236

```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Dist[(3*Sqrt[b*x^2])/(2*b*x)
, Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b}
, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 622

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(b*x + c*x^2)^p/(-(
(c*(b*x + c*x^2))/b^2))^p, Int[(-((c*x)/b) - (c^2*x^2)/b^2)^p, x], x] /; Fr
eeQ[{b, c}, x] && RationalQ[p] && 3 <= Denominator[p] <= 4
```

Rule 2011

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x
^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Intege
rQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\int \frac{1}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}\,dx=\frac{\left(\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\int \frac{1}{\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}}\,dx}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$=\frac{\left(3\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\text{Subst}\left(\int \frac{x^{4/3}}{(a+bx)^{2/3}}\,dx,x,\sqrt[3]{x}\right)}{\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$=\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}-\frac{\left(12a\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\text{Subst}\left(\int \frac{\sqrt[3]{x}}{(a+bx)^{2/3}}\,dx,x,\sqrt[3]{x}\right)}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}$$

$$=-\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{\left(6a^2\left(a+b\sqrt[3]{x}\right)^{2/3}x^{2/9}\right)\text{Subst}\left(\int \cdots\right.}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)\cdots}$$

$$=-\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{\left(6a^2\right)\text{Subst}\left(\int \frac{1}{\left(ax+bx^2\right)^{2/3}}\,dx,x,\sqrt[3]{x}\cdots\right.}{5b^2}$$

$$=-\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{\left(6a^2\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}\right)\text{Subst}\left(\int \cdots\right.}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\cdots\right.}$$

$$=-\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}-\frac{\left(6\sqrt[3]{2}\,a^4\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\right)}{a^2}\right)^{2/3}\right)\text{Subst}\cdots}{5b^4\left(a\sqrt[3]{x}+\cdots\right.}$$

$$=-\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}-\frac{\left(9\sqrt[3]{2}\,a^4\sqrt{-\frac{\left(a+2b\sqrt[3]{x}\right)^2}{a^2}}\left(-\frac{b\left(a\sqrt[3]{x}+bx^{2/3}\cdots\right)}{a^2}\cdots\right.\right.}{5b^3\left(a+2b\cdots\right.}$$

$$=-\frac{18a\left(a+b\sqrt[3]{x}\right)\sqrt[3]{x}}{5b^2\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{9\left(a+b\sqrt[3]{x}\right)x^{2/3}}{5b\left(a\sqrt[3]{x}+bx^{2/3}\right)^{2/3}}+\frac{6\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,a^4\left(1-\sqrt[3]{1-\frac{(a+\cdots}{\cdots}}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.02, size = 61, normalized size = 0.13

$$\frac{9x\left(\frac{b\sqrt[3]{x}}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{2}{3},\frac{7}{3};\frac{10}{3};-\frac{b\sqrt[3]{x}}{a}\right)}{7\left(\sqrt[3]{x}\left(a+b\sqrt[3]{x}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^(1/3) + b*x^(2/3))^(-2/3),x]
```

```
[Out] (9*(1 + (b*x^(1/3))/a)^(2/3)*x*Hypergeometric2F1[2/3, 7/3, 10/3, -((b*x^(1/
3))/a)])/(7*((a + b*x^(1/3))*x^(1/3))^(2/3))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{2}{3}} + ax^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

maple [F] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\,x^{\frac{2}{3}} + a\,x^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^(2/3)+a*x^(1/3))^(2/3),x)
```

```
[Out] int(1/(b*x^(2/3)+a*x^(1/3))^(2/3),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(bx^{\frac{2}{3}} + ax^{\frac{1}{3}}\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^(1/3)+b*x^(2/3))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(2/3) + a*x^(1/3))^(-2/3), x)
```

mupad [B] time = 5.25, size = 42, normalized size = 0.09

$$\frac{9\,x\,{\left(\frac{b\,x^{1/3}}{a}+1\right)}^{2/3}\,{}_2\mathrm{F}_1\left(\frac{2}{3},\frac{7}{3};\ \frac{10}{3};\ -\frac{b\,x^{1/3}}{a}\right)}{7\,{\left(a\,x^{1/3}+b\,x^{2/3}\right)}^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^(1/3) + b*x^(2/3))^(2/3),x)
```

```
[Out] (9*x*((b*x^(1/3))/a + 1)^(2/3)*hypergeom([2/3, 7/3], 10/3, -(b*x^(1/3))/a))
/(7*(a*x^(1/3) + b*x^(2/3))^(2/3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sqrt[3]{x} + bx^{\frac{2}{3}}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**(1/3)+b*x**(2/3))**(2/3),x)
```

```
[Out] Integral((a*x**(1/3) + b*x**(2/3))**(-2/3), x)
```

3.439 $\int x^m \left(ax^j + bx^n\right)^p \, dx$

Optimal. Leaf size=89

$$\frac{x^{m+1}\left(a+bx^{n-j}\right)\left(ax^j+bx^n\right)^p {}_2F_1\left(1,p+\frac{m+jp+1}{n-j}+1;\frac{m+jp+1}{n-j}+1;-\frac{bx^{n-j}}{a}\right)}{a(jp+m+1)}$$

```
[Out] x^(1+m)*(a*x^j+b*x^n)^p*(a+b*x^(-j+n))*hypergeom([1, 1+p+(j*p+m+1)/(-j+n)],
[1+(j*p+m+1)/(-j+n)],-b*x^(-j+n)/a)/a/(j*p+m+1)
```

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2032, 365, 364}

$$\frac{x^{m+1}\left(\frac{ax^{j-n}}{b}+1\right)^{-p}\left(ax^j+bx^n\right)^p {}_2F_1\left(-p,\frac{m+np+1}{j-n};\frac{m+np+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*(a*x^j + b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n), 1
 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(j
- n))/b)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int x^m\left(ax^j+bx^n\right)^p\,dx &= \left(x^{-np}\left(b+ax^{j-n}\right)^{-p}\left(ax^j+bx^n\right)^p\right)\int x^{m+np}\left(b+ax^{j-n}\right)^p\,dx\\
&= \left(x^{-np}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p\right)\int x^{m+np}\left(1+\frac{ax^{j-n}}{b}\right)^p\,dx\\
&= \frac{x^{1+m}\left(1+\frac{ax^{j-n}}{b}\right)^{-p}\left(ax^j+bx^n\right)^p {}_2F_1\left(-p,\frac{1+m+np}{j-n};1+\frac{1+m+np}{j-n};-\frac{ax^{j-n}}{b}\right)}{1+m+np}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 92, normalized size = 1.03

$$\frac{x^{m+1}\left(\frac{ax^{j-n}}{b}+1\right)^{-p}\left(ax^j+bx^n\right)^p\,{}_2F_1\left(-p,\frac{m+np+1}{j-n};\frac{m+np+1}{j-n}+1;-\frac{ax^{j-n}}{b}\right)}{m+np+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*(a*x^j + b*x^n)^p,x]
```

```
[Out] (x^(1 + m)*(a*x^j + b*x^n)^p*Hypergeometric2F1[-p, (1 + m + n*p)/(j - n), 1
 + (1 + m + n*p)/(j - n), -((a*x^(j - n))/b)])/((1 + m + n*p)*(1 + (a*x^(j
- n))/b)^p)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ax^j+bx^n\right)^p x^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((a*x^j + b*x^n)^p*x^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^j+bx^n\right)^p x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((a*x^j + b*x^n)^p*x^m, x)
```

maple [F] time = 0.74, size = 0, normalized size = 0.00

$$\int x^m\left(a\,x^j+b\,x^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x^j+b*x^n)^p,x)
```

```
[Out] int(x^m*(a*x^j+b*x^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^j+bx^n\right)^p x^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(a*x^j+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((a*x^j + b*x^n)^p*x^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\left(a\,x^j+b\,x^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(a*x^j + b*x^n)^p,x)
```

```
[Out] int(x^m*(a*x^j + b*x^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \left(ax^j + bx^n\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(a*x**j+b*x**n)**p,x)
```

```
[Out] Integral(x**m*(a*x**j + b*x**n)**p, x)
```

3.440 $\int x^{-1-pq} (bx^n + ax^q)^p \, dx$

Optimal. Leaf size=69

$$-\frac{x^{-pq} (a + bx^{n-q}) (ax^q + bx^n)^p \, {}_2F_1\left(1, p+1; p+2; \frac{bx^{n-q}}{a} + 1\right)}{a(p+1)(n-q)}$$

```
[Out] -(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([1, 1+p],[2+p],1+b*x^(n-q)/a)/a/(1
+p)/(n-q)/(x^(p*q))
```

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2032, 266, 65}

$$-\frac{x^{-pq} (a + bx^{n-q}) (ax^q + bx^n)^p \, {}_2F_1\left(1, p+1; p+2; \frac{bx^{n-q}}{a} + 1\right)}{a(p+1)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -(((a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[1, 1 + p, 2 + p, 1
 + (b*x^(n - q))/a])/(a*(1 + p)*(n - q)*x^(p*q)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-pq} (bx^n + ax^q)^p \, dx &= \left(x^{-pq} (a + bx^{n-q})^{-p} (bx^n + ax^q)^p\right) \int \frac{(a + bx^{n-q})^p}{x} \, dx \\
&= \frac{\left(x^{-pq} (a + bx^{n-q})^{-p} (bx^n + ax^q)^p\right) \text{Subst}\left(\int \frac{(a+bx)^p}{x} \, dx, x, x^{n-q}\right)}{n-q} \\
&= -\frac{x^{-pq} (a + bx^{n-q}) (bx^n + ax^q)^p \, {}_2F_1\left(1, 1+p; 2+p; 1 + \frac{bx^{n-q}}{a}\right)}{a(1+p)(n-q)}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 73, normalized size = 1.06

$$\frac{x^{-pq}\left(ax^q+bx^n\right)^p\left(\frac{ax^{q-n}}{b}+1\right)^{-p}{}_2F_1\left(-p,-p;1-p;-\frac{ax^{q-n}}{b}\right)}{p(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - p*q)*(b*x^n + a*x^q)^p,x]
[Out] ((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((a*x^(-n + q))/b)])/(
p*(n - q)*x^(p*q)*(1 + (a*x^(-n + q))/b)^p)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((bx^n+ax^q)^p x^{-pq-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+ax^q)^p x^{-pq-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int x^{-pq-1}\left(a\,x^q+b\,x^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
[Out] int(x^(-p*q-1)*(b*x^n+a*x^q)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+ax^q)^p x^{-pq-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-p*q-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a*x^q)^p*x^(-p*q - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^n+a\,x^q\right)^p}{x^{p\,q+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^q)^p/x^(p*q + 1),x)
```

```
[Out] int((b*x^n + a*x^q)^p/x^(p*q + 1), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-p*q-1)*(b*x**n+a*x**q)**p,x)
```

```
[Out] Timed out
```

3.441 $\int x^{-1-np} (bx^n + ax^q)^p \, dx$

Optimal. Leaf size=66

$$-\frac{x^{-np} (a + bx^{n-q}) (ax^q + bx^n)^p \, {}_2F_1\left(1,1;1-p;-\frac{bx^{n-q}}{a}\right)}{ap(n-q)}$$

```
[Out] -(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([1, 1],[1-p],-b*x^(n-q)/a)/a/p/(n-
q)/(x^(n*p))
```

Rubi [A] time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {2032, 365, 364}

$$-\frac{x^{-np}\left(\frac{bx^{n-q}}{a}+1\right)^{-p} (ax^q + bx^n)^p \, {}_2F_1\left(-p,-p;1-p;-\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])/
(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-np} (bx^n + ax^q)^p \, dx &= \left(x^{-pq} (a + bx^{n-q})^{-p} (bx^n + ax^q)^p\right) \int x^{-1-np+pq} (a + bx^{n-q})^p \, dx \\
&= \left(x^{-pq}\left(1+\frac{bx^{n-q}}{a}\right)^{-p} (bx^n + ax^q)^p\right) \int x^{-1-np+pq}\left(1+\frac{bx^{n-q}}{a}\right)^p dx \\
&= -\frac{x^{-np}\left(1+\frac{bx^{n-q}}{a}\right)^{-p} (bx^n + ax^q)^p \, {}_2F_1\left(-p,-p;1-p;-\frac{bx^{n-q}}{a}\right)}{p(n-q)}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 74, normalized size = 1.12

$$-\frac{x^{-np}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}(ax^q+bx^n)^p\,{}_2F_1\left(-p,-p;1-p;-\frac{bx^{n-q}}{a}\right)}{p(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*p)*(b*x^n + a*x^q)^p,x]
[Out] -(((b*x^n + a*x^q)^p*Hypergeometric2F1[-p, -p, 1 - p, -((b*x^(n - q))/a)])/
(p*(n - q)*x^(n*p)*(1 + (b*x^(n - q))/a)^p))
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n+ax^q)^p x^{-np-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
[Out] integral((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+ax^q)^p x^{-np-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="giac")
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

maple [F] time = 0.76, size = 0, normalized size = 0.00

$$\int x^{-np-1}\left(a\,x^q+b\,x^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-n*p-1)*(a*x^q+b*x^n)^p,x)
[Out] int(x^(-n*p-1)*(a*x^q+b*x^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+ax^q)^p x^{-np-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-n*p-1)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*p - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,x^n+a\,x^q\right)^p}{x^{n\,p+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^q)^p/x^(n*p + 1),x)
```

```
[Out] int((b*x^n + a*x^q)^p/x^(n*p + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-n*p-1)*(b*x**n+a*x**q)**p,x)
```

```
[Out] Timed out
```

3.442 $\int x^{-1-n-(-1+p)q}\left(bx^n+ax^q\right)^p\,dx$

Optimal. Leaf size=69

$$\frac{bx^{-pq}\left(a+bx^{n-q}\right)\left(ax^q+bx^n\right)^p\,{}_2F_1\left(2,p+1;p+2;\frac{bx^{n-q}}{a}+1\right)}{a^2(p+1)(n-q)}$$

```
[Out] b*(a+b*x^(n-q))*(b*x^n+a*x^q)^p*hypergeom([2, 1+p],[2+p],1+b*x^(n-q)/a)/a^2
/(1+p)/(n-q)/(x^(p*q))
```

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2032, 266, 65}

$$\frac{bx^{-pq}\left(a+bx^{n-q}\right)\left(ax^q+bx^n\right)^p\,{}_2F_1\left(2,p+1;p+2;\frac{bx^{n-q}}{a}+1\right)}{a^2(p+1)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (b*(a + b*x^(n - q))*(b*x^n + a*x^q)^p*Hypergeometric2F1[2, 1 + p, 2 + p, 1
 + (b*x^(n - q))/a])/(a^2*(1 + p)*(n - q)*x^(p*q))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}\int x^{-1-n-(-1+p)q}\left(bx^n+ax^q\right)^p\,dx &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-n-(-1+p)q+pq}\left(a+bx^{n-q}\right)^p\,dx\\ &= \frac{\left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\text{Subst}\left(\int\frac{(a+bx)^p}{x^2}\,dx,x,x^{n-q}\right)}{n-q}\\ &= \frac{bx^{-pq}\left(a+bx^{n-q}\right)\left(bx^n+ax^q\right)^p\,{}_2F_1\left(2,1+p;2+p;1+\frac{bx^{n-q}}{a}\right)}{a^2(1+p)(n-q)}\end{aligned}$$

Mathematica [A] time = 0.20, size = 82, normalized size = 1.19

$$\frac{x^{-n-pq+q}\left(ax^q+bx^n\right)^p\left(\frac{ax^{q-n}}{b}+1\right)^{-p}{}_2F_1\left(1-p,-p;2-p;-\frac{ax^{q-n}}{b}\right)}{(p-1)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n - (-1 + p)*q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (x^(-n + q - p*q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((
a*x^(-n + q))/b)])/((-1 + p)*(n - q)*(1 + (a*x^(-n + q))/b)^p)
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(bx^n+ax^q\right)^p x^{-(p-1)q-n-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^n+ax^q\right)^p x^{-(p-1)q-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

maple [F] time = 0.79, size = 0, normalized size = 0.00

$$\int x^{-n-(p-1)q-1}\left(a\,x^q+b\,x^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n-(p-1)*q)*(a*x^q+b*x^n)^p,x)
```

```
[Out] int(x^(-1-n-(p-1)*q)*(a*x^q+b*x^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(bx^n+ax^q\right)^p x^{-(p-1)q-n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n-(-1+p)*q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-(p - 1)*q - n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\,x^n+a\,x^q\right)^p}{x^{n+q\,(p-1)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^q)^p/x^(n + q*(p - 1) + 1),x)
```

```
[Out] int((b*x^n + a*x^q)^p/x^(n + q*(p - 1) + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n-(-1+p)*q)*(b*x**n+a*x**q)**p,x)
```

```
[Out] Timed out
```

3.443 $\int x^{-1-n(-1+p)-q}\left(bx^n+ax^q\right)^p\,dx$

Optimal. Leaf size=84

$$\frac{x^{n(-p)+n-q}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}\left(ax^q+bx^n\right)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

```
[Out] x^(-n*p+n-q)*(b*x^n+a*x^q)^p*hypergeom([-p, 1-p],[2-p],-b*x^(n-q)/a)/(1-p)/
(n-q)/((1+b*x^(n-q)/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2032, 365, 364}

$$\frac{x^{n(-p)+n-q}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}\left(ax^q+bx^n\right)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] (x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -((b
*x^(n - q))/a)])/((1 - p)*(n - q)*(1 + (b*x^(n - q))/a)^p)
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rubi steps

$$\begin{aligned}
\int x^{-1-n(-1+p)-q}\left(bx^n+ax^q\right)^p\,dx &= \left(x^{-pq}\left(a+bx^{n-q}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-n(-1+p)-q+pq}\left(a+bx^{n-q}\right)^p\,dx\\
&= \left(x^{-pq}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}\left(bx^n+ax^q\right)^p\right)\int x^{-1-n(-1+p)-q+pq}\left(1+\frac{bx^{n-q}}{a}\right)^p\,dx\\
&= \frac{x^{n-np-q}\left(1+\frac{bx^{n-q}}{a}\right)^{-p}\left(bx^n+ax^q\right)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(1-p)(n-q)}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 83, normalized size = 0.99

$$-\frac{x^{n(-p)+n-q}\left(\frac{bx^{n-q}}{a}+1\right)^{-p}(ax^q+bx^n)^p\,{}_2F_1\left(1-p,-p;2-p;-\frac{bx^{n-q}}{a}\right)}{(p-1)(n-q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*(-1 + p) - q)*(b*x^n + a*x^q)^p,x]
```

```
[Out] -((x^(n - n*p - q)*(b*x^n + a*x^q)^p*Hypergeometric2F1[1 - p, -p, 2 - p, -(
(b*x^(n - q))/a)])/((-1 + p)*(n - q)*(1 + (b*x^(n - q))/a)^p))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^n+ax^q)^p x^{-np+n-q-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a*x^q)^p*x^(-n*p + n - q - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+ax^q)^p x^{-n(p-1)-q-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int x^{-(p-1)n-q-1}\left(a\,x^q+b\,x^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*(p-1)-q)*(a*x^q+b*x^n)^p,x)
```

```
[Out] int(x^(-1-n*(p-1)-q)*(a*x^q+b*x^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n+ax^q)^p x^{-n(p-1)-q-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*(-1+p)-q)*(b*x^n+a*x^q)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a*x^q)^p*x^(-n*(p - 1) - q - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,x^n+a\,x^q\right)^p}{x^{q+n\,(p-1)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n + a*x^q)^p/x^(q + n*(p - 1) + 1),x)
```

```
[Out] int((b*x^n + a*x^q)^p/x^(q + n*(p - 1) + 1), x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*(-1+p)-q)*(b*x**n+a*x**q)**p,x)
[Out] Timed out
```

3.444 $\int \left(ax^m + bx^{1+m+mp}\right)^p \, dx$

Optimal. Leaf size=44

$$\frac{x^{-m(p+1)} \left(ax^m + bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

```
[Out] (a*x^m+b*x^(m*p+m+1))^(1+p)/b/(1+p)/(m*p+1)/(x^(m*(1+p)))
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2000}

$$\frac{x^{-m(p+1)} \left(ax^m + bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*x^m + b*x^(1 + m + m*p))^p,x]
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\int \left(ax^m + bx^{1+m+mp}\right)^p \, dx = \frac{x^{-m(1+p)} \left(ax^m + bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.98

$$\frac{x^{-m(p+1)} \left(x^m \left(a + bx^{mp+1}\right)\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*x^m + b*x^(1 + m + m*p))^p,x]
[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

fricas [A] time = 0.60, size = 64, normalized size = 1.45

$$\frac{\left(bxx^{mp+m+1} + axx^m\right)\left(bx^{mp+m+1} + ax^m\right)^p}{\left(bmp^2 + (bm+b)p + b\right)x^{mp+m+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="fricas")
[Out] (b*x*x^(m*p + m + 1) + a*x*x^m)*(b*x^(m*p + m + 1) + a*x^m)^p/((b*m*p^2 + (
b*m + b)*p + b)*x^(m*p + m + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{mp+m+1} + ax^{m}\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="giac")
[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \left(a\,x^{m} + b\,x^{mp+m+1}\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m+b*x^(m*p+m+1))^p,x)
[Out] int((a*x^m+b*x^(m*p+m+1))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{mp+m+1} + ax^{m}\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^m+b*x^(m*p+m+1))^p,x, algorithm="maxima")
[Out] integrate((b*x^(m*p + m + 1) + a*x^m)^p, x)
```

mupad [B] time = 5.30, size = 76, normalized size = 1.73

$$\frac{a\left(a\,x^{m} + b\,x^{m+m\,p+1}\right)^{p}\left(\frac{b\,x^{m\,p+1}}{a} - \frac{1}{\left(\frac{b\,x^{m\,p+1}}{a}+1\right)^{p}} + 1\right)}{b\,x^{m\,p}\,\left(m\,p+1\right)\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^m + b*x^(m + m*p + 1))^p,x)
[Out] (a*(a*x^m + b*x^(m + m*p + 1))^p*((b*x^(m*p + 1))/a - 1/((b*x^(m*p + 1))/a
+ 1)^p + 1))/(b*x^(m*p)*(m*p + 1)*(p + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ax^{m} + bx^{mp+m+1}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**m+b*x**(m*p+m+1))**p,x)
[Out] Integral((a*x**m + b*x**(m*p + m + 1))**p, x)
```

3.445 $\int \left(x^m \left(a+bx^{1+mp}\right)\right)^p \, dx$

Optimal. Leaf size=44

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

```
[Out] (a*x^m+b*x^(m*p+m+1))^(1+p)/b/(1+p)/(m*p+1)/(x^(m*(1+p)))
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1979, 2000}

$$\frac{x^{-m(p+1)}\left(ax^m+bx^{mp+m+1}\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^m*(a + b*x^(1 + m*p)))^p,x]
[Out] (a*x^m + b*x^(1 + m + m*p))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x^m\left(a+bx^{1+mp}\right)\right)^p \, dx &= \int \left(ax^m+bx^{1+m+mp}\right)^p \, dx\\ &= \frac{x^{-m(1+p)}\left(ax^m+bx^{1+m+mp}\right)^{1+p}}{b(1+p)(1+mp)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 43, normalized size = 0.98

$$\frac{x^{-m(p+1)}\left(x^m\left(a+bx^{mp+1}\right)\right)^{p+1}}{b(p+1)(mp+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^m*(a + b*x^(1 + m*p)))^p,x]
[Out] (x^m*(a + b*x^(1 + m*p)))^(1 + p)/(b*(1 + p)*(1 + m*p)*x^(m*(1 + p)))
```

fricas [A] time = 0.56, size = 61, normalized size = 1.39

$$\frac{\left(bxx^{mp+1}+ax\right)\left(bx^{mp+1}x^m+ax^m\right)^p}{\left(bmp^2+(bm+b)p+b\right)x^{mp+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + 1) + a*x)*(b*x^(m*p + 1)*x^m + a*x^m)^p/((b*m*p^2 + (b*m + b)
*p + b)*x^(m*p + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(bx^{mp+1} + a \right) x^{m} \right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \left(\left(b\,x^{mp+1} + a \right) x^{m} \right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a+b*x^(m*p+1)))^p,x)
```

```
[Out] int((x^m*(a+b*x^(m*p+1)))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\left(bx^{mp+1} + a \right) x^{m} \right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^m*(a+b*x^(m*p+1)))^p,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^(m*p + 1) + a)*x^m)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(x^m\,\left(a+b\,x^{m\,p+1}\right)\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(a + b*x^(m*p + 1)))^p,x)
```

```
[Out] int((x^m*(a + b*x^(m*p + 1)))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{m} \left(a + bx^{mp+1}\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**m*(a+b*x**(m*p+1)))**p,x)
```

```
[Out] Integral((x**m*(a + b*x**(m*p + 1)))**p, x)
```

3.446 $\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p \, dx$

Optimal. Leaf size=46

$$\frac{x^{-m(p+1)} \left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

```
[Out] (a*x^m+b*x^(m*p+m+n+1))^(1+p)/b/(1+p)/(m*p+n+1)/(x^(m*(1+p)))
```

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1980, 2014}

$$\frac{x^{-m(p+1)} \left(ax^m+bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)))
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int x^n \left(x^m \left(a+bx^{1+n+mp}\right)\right)^p \, dx &= \int x^n \left(ax^m+bx^{1+m+n+mp}\right)^p \, dx\\
&= \frac{x^{-m(1+p)} \left(ax^m+bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 45, normalized size = 0.98

$$\frac{x^{-m(p+1)} \left(x^m \left(a+bx^{mp+n+1}\right)\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(x^m*(a + b*x^(1 + n + m*p)))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)))
```

fricas [A] time = 0.41, size = 76, normalized size = 1.65

$$\frac{\left(bxx^{mp+n+1}x^n+axx^n\right)\left(bx^{mp+n+1}x^m+ax^m\right)^p}{\left(bmp^2+bn+(bm+bn+b)p+b\right)x^{mp+n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + n + 1)*x^n + a*x*x^n)*(b*x^(m*p + n + 1)*x^m + a*x^m)^p/((b*m
*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + n + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(bx^{mp+n+1}+a\right)x^m\right)^p x^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int x^n\left(\left(b\,x^{mp+n+1}+a\right)x^m\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

```
[Out] int(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\left(bx^{mp+n+1}+a\right)x^m\right)^p x^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(x^m*(a+b*x^(m*p+n+1)))^p,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^(m*p + n + 1) + a)*x^m)^p*x^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^n\left(x^m\,\left(a+b\,x^{n+m\,p+1}\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p,x)
```

```
[Out] int(x^n*(x^m*(a + b*x^(n + m*p + 1)))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(x**m*(a+b*x**(m*p+n+1)))**p,x)
```

```
[Out] Timed out
```

3.447 $\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx$

Optimal. Leaf size=46

$$\frac{x^{-m(p+1)} \left(ax^m + bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

```
[Out] (a*x^m+b*x^(m*p+m+n+1))^(1+p)/b/(1+p)/(m*p+n+1)/(x^(m*(1+p)))
```

Rubi [A] time = 0.05, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2014}

$$\frac{x^{-m(p+1)} \left(ax^m + bx^{mp+m+n+1}\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] (a*x^m + b*x^(1 + m + n + m*p))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^n \left(ax^m + bx^{1+m+n+mp}\right)^p \, dx = \frac{x^{-m(1+p)} \left(ax^m + bx^{1+m+n+mp}\right)^{1+p}}{b(1+p)(1+n+mp)}$$

Mathematica [A] time = 0.00, size = 45, normalized size = 0.98

$$\frac{x^{-m(p+1)} \left(x^m \left(a + bx^{mp+n+1}\right)\right)^{p+1}}{b(p+1)(mp+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*(a*x^m + b*x^(1 + m + n + m*p))^p,x]
```

```
[Out] (x^m*(a + b*x^(1 + n + m*p)))^(1 + p)/(b*(1 + p)*(1 + n + m*p)*x^(m*(1 + p)
))
```

fricas [A] time = 0.42, size = 79, normalized size = 1.72

$$\frac{\left(bxx^{mp+m+n+1}x^n + axx^m x^n\right)\left(bx^{mp+m+n+1} + ax^m\right)^p}{\left(bmp^2 + bn + (bm + bn + b)p + b\right)x^{mp+m+n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^(m*p + m + n + 1)*x^n + a*x*x^m*x^n)*(b*x^(m*p + m + n + 1) + a*x^m)
^p/((b*m*p^2 + b*n + (b*m + b*n + b)*p + b)*x^(m*p + m + n + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{mp+m+n+1} + ax^m\right)^p x^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

maple [F] time = 0.88, size = 0, normalized size = 0.00

$$\int x^n \left(a\,x^m + b\,x^{mp+m+n+1}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

```
[Out] int(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(bx^{mp+m+n+1} + ax^m\right)^p x^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*(a*x^m+b*x^(m*p+m+n+1))^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^(m*p + m + n + 1) + a*x^m)^p*x^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x^n \left(a\,x^m + b\,x^{m+n+m\,p+1}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p,x)
```

```
[Out] int(x^n*(a*x^m + b*x^(m + n + m*p + 1))^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*(a*x**m+b*x**(m*p+m+n+1))**p,x)
```

```
[Out] Timed out
```

3.448 $\int \sqrt{x^{2(-1+n)}(a+bx^n)}\,dx$

Optimal. Leaf size=44

$$\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{3n-2}\right)^{3/2}}{3bn}$$

```
[Out] 2/3*x^(3-3*n)*(a/(x^(2-2*n))+b*x^(-2+3*n))^(3/2)/b/n
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{3n-2}\right)^{3/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

```
[Out] (2*x^(3*(1 - n))*(a/x^(2*(1 - n)) + b*x^(-2 + 3*n))^(3/2))/(3*b*n)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x^{2(-1+n)}(a+bx^n)}\,dx &= \int \sqrt{ax^{2(-1+n)}+bx^{2(-1+n)+n}}\,dx\\ &= \frac{2x^{3(1-n)}\left(ax^{-2(1-n)}+bx^{-2+3n}\right)^{3/2}}{3bn}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.82

$$\frac{2x^{3-3n}\left(x^{2n-2}(a+bx^n)\right)^{3/2}}{3bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x^(2*(-1 + n))*(a + b*x^n)],x]
```

```
[Out] (2*x^(3 - 3*n)*(x^(-2 + 2*n)*(a + b*x^n))^(3/2))/(3*b*n)
```

fricas [A] time = 0.40, size = 44, normalized size = 1.00

$$\frac{2\,(bxx^n+ax)\sqrt{\frac{bx^{3\,n}+ax^{2\,n}}{x^2}}}{3\,bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="fricas")

[Out] 2/3*(b*x*x^n + a*x)*sqrt((b*x^(3*n) + a*x^(2*n))/x^2)/(b*n*x^n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(bx^n + a)x^{2n-2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt((b*x^n + a)*x^(2*n - 2)), x)

maple [A] time = 0.19, size = 40, normalized size = 0.91

$$\frac{2\sqrt{\frac{(b\,x^n+a)x^{2n}}{x^2}}\,(b\,x^n+a)\,x\,x^{-n}}{3bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(2*n-2)*(b*x^n+a))^(1/2),x)

[Out] 2/3*(1/x^2*(x^n)^2*(b*x^n+a))^(1/2)*(b*x^n+a)/(x^n)*x/b/n

maxima [A] time = 1.55, size = 17, normalized size = 0.39

$$\frac{2\,(bx^n + a)^{\frac{3}{2}}}{3\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(-2+2*n)*(a+b*x^n))^(1/2),x, algorithm="maxima")

[Out] 2/3*(b*x^n + a)^(3/2)/(b*n)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x^{2\,n-2}\,\left(a+b\,x^n\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(2*n - 2)*(a + b*x^n))^(1/2),x)

[Out] int((x^(2*n - 2)*(a + b*x^n))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**(-2+2*n)*(a+b*x**n))**(1/2),x)

[Out] Timed out

3.449 $\int \sqrt[3]{x^{3(-1+n)}(a+bx^n)}\,dx$

Optimal. Leaf size=44

$$\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{4n-3}\right)^{4/3}}{4bn}$$

[Out] 3/4*x^(4-4*n)*(a/(x^(3-3*n))+b*x^(-3+4*n))^(4/3)/b/n

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{4n-3}\right)^{4/3}}{4bn}$$

Antiderivative was successfully verified.

[In] Int[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]

[Out] (3*x^(4*(1 - n))*(a/x^(3*(1 - n)) + b*x^(-3 + 4*n))^(4/3))/(4*b*n)

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{x^{3(-1+n)}(a+bx^n)}\,dx &= \int \sqrt[3]{ax^{3(-1+n)}+bx^{3(-1+n)+n}}\,dx\\ &= \frac{3x^{4(1-n)}\left(ax^{-3(1-n)}+bx^{-3+4n}\right)^{4/3}}{4bn}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.82

$$\frac{3x^{4-4n}\left(x^{3n-3}(a+bx^n)\right)^{4/3}}{4bn}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(3*(-1 + n))*(a + b*x^n))^(1/3),x]

[Out] (3*x^(4 - 4*n)*(x^(-3 + 3*n)*(a + b*x^n))^(4/3))/(4*b*n)

fricas [A] time = 0.40, size = 44, normalized size = 1.00

$$\frac{3\,(bxx^n+ax)\left(\frac{bx^{4\,n}+ax^{3\,n}}{x^3}\right)^{\frac{1}{3}}}{4\,bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="fricas")
[Out] 3/4*(b*x*x^n + a*x)*((b*x^(4*n) + a*x^(3*n))/x^3)^(1/3)/(b*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^n + a)x^{3n-3}\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="giac")
[Out] integrate(((b*x^n + a)*x^(3*n - 3))^(1/3), x)
```

maple [A] time = 0.08, size = 40, normalized size = 0.91

$$\frac{3\left(\frac{(b\,x^n+a)x^{3n}}{x^3}\right)^{\frac{1}{3}}(b\,x^n+a)\,x\,x^{-n}}{4bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n-3)*(b*x^n+a))^(1/3),x)
[Out] 3/4*(1/x^3*(x^n)^3*(b*x^n+a))^(1/3)*x/(x^n)*(b*x^n+a)/b/n
```

maxima [A] time = 1.49, size = 17, normalized size = 0.39

$$\frac{3\,(bx^n + a)^{\frac{4}{3}}}{4\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-3+3*n)*(a+b*x^n))^(1/3),x, algorithm="maxima")
[Out] 3/4*(b*x^n + a)^(4/3)/(b*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x^{3\,n-3}\,\left(a+b\,x^n\right)\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(3*n - 3)*(a + b*x^n))^(1/3),x)
[Out] int((x^(3*n - 3)*(a + b*x^n))^(1/3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(-3+3*n)*(a+b*x**n))**(1/3),x)
[Out] Timed out
```

3.450 $\int \sqrt[4]{x^{4(-1+n)}(a+bx^n)}\,dx$

Optimal. Leaf size=44

$$\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{5n-4}\right)^{5/4}}{5bn}$$

```
[Out] 4/5*x^(5-5*n)*(a/(x^(4-4*n))+b*x^(-4+5*n))^(5/4)/b/n
```

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1979, 2000}

$$\frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{5n-4}\right)^{5/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

```
[Out] (4*x^(5*(1 - n))*(a/x^(4*(1 - n)) + b*x^(-4 + 5*n))^(5/4))/(5*b*n)
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt[4]{x^{4(-1+n)}(a+bx^n)}\,dx &= \int \sqrt[4]{ax^{4(-1+n)}+bx^{4(-1+n)+n}}\,dx\\ &= \frac{4x^{5(1-n)}\left(ax^{-4(1-n)}+bx^{-4+5n}\right)^{5/4}}{5bn}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.82

$$\frac{4x^{5-5n}\left(x^{4n-4}(a+bx^n)\right)^{5/4}}{5bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(4*(-1 + n))*(a + b*x^n))^(1/4),x]
```

```
[Out] (4*x^(5 - 5*n)*(x^(-4 + 4*n)*(a + b*x^n))^(5/4))/(5*b*n)
```

fricas [A] time = 0.40, size = 44, normalized size = 1.00

$$\frac{4\,(bxx^n+ax)\left(\frac{bx^{5\,n}+ax^{4\,n}}{x^4}\right)^{\frac{1}{4}}}{5\,bnx^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="fricas")
[Out] 4/5*(b*x*x^n + a*x)*((b*x^(5*n) + a*x^(4*n))/x^4)^(1/4)/(b*n*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^n + a)x^{4n-4}\right)^{\frac{1}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="giac")
[Out] integrate(((b*x^n + a)*x^(4*n - 4))^(1/4), x)
```

maple [A] time = 0.07, size = 40, normalized size = 0.91

$$\frac{4\left(\frac{(b\,x^n+a)x^{4n}}{x^4}\right)^{\frac{1}{4}}(b\,x^n + a)\,x\,x^{-n}}{5bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(4*n-4)*(b*x^n+a))^(1/4),x)
[Out] 4/5*(1/x^4*(x^n)^4*(b*x^n+a))^(1/4)*x/(x^n)*(b*x^n+a)/b/n
```

maxima [A] time = 1.58, size = 17, normalized size = 0.39

$$\frac{4\,(bx^n + a)^{\frac{5}{4}}}{5\,bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^(-4+4*n)*(a+b*x^n))^(1/4),x, algorithm="maxima")
[Out] 4/5*(b*x^n + a)^(5/4)/(b*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x^{4\,n-4}\,\left(a + b\,x^n\right)\right)^{1/4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(4*n - 4)*(a + b*x^n))^(1/4),x)
[Out] int((x^(4*n - 4)*(a + b*x^n))^(1/4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(-4+4*n)*(a+b*x**n))**(1/4),x)
[Out] Timed out
```

3.451 $\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}} dx$

Optimal. Leaf size=57

$$\frac{px^{(1-n)(p+1)}\left(ax^{-((1-n)p)}+bx^{n-(1-n)p}\right)^{\frac{1}{p}+1}}{bn(p+1)}$$

```
[Out] p*x^((1-n)*(1+p))*(a/(x^((1-n)*p))+b*x^(n-(1-n)*p))^(1+1/p)/b/n/(1+p)
```

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{px^{(1-n)(p+1)}\left(ax^{-(1-n)p}+bx^{n-(1-n)p}\right)^{\frac{1}{p}+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

```
[Out] (p*x^((1 - n)*(1 + p))*(a/x^((1 - n)*p) + b*x^(n - (1 - n)*p))^(1 + p^(-1))
)/(b*n*(1 + p))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x^{(-1+n)p}(a+bx^n)\right)^{\frac{1}{p}} dx &= \int \left(ax^{(-1+n)p}+bx^{n+(-1+n)p}\right)^{\frac{1}{p}} dx\\ &= \frac{px^{(1-n)(1+p)}\left(ax^{-(1-n)p}+bx^{n-(1-n)p}\right)^{1+\frac{1}{p}}}{bn(1+p)}\end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.82

$$\frac{x^{1-n}(a+bx^n)\left(x^{(n-1)p}(a+bx^n)\right)^{\frac{1}{p}}}{bn\left(\frac{1}{p}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^((-1 + n)*p)*(a + b*x^n))^p^(-1),x]
```

```
[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)*p)*(a + b*x^n))^p^(-1))/(b*n*(1 + p^(-1
)))
```

fricas [A] time = 0.41, size = 47, normalized size = 0.82

$$\frac{\left(bpxx^n + apx\right)\left((bx^n + a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}}{\left(bnp + bn\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="fricas")
```

```
[Out] (b*p*x*x^n + a*p*x)*((b*x^n + a)*x^((n - 1)*p))^(1/p)/((b*n*p + b*n)*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^n + a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="giac")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

maple [F] time = 0.72, size = 0, normalized size = 0.00

$$\int \left((b\,x^n + a)\,x^{(n-1)p}\right)^{\frac{1}{p}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((n-1)*p)*(b*x^n+a))^(1/p),x)
```

```
[Out] int((x^((n-1)*p)*(b*x^n+a))^(1/p),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^n + a)x^{(n-1)p}\right)^{\left(\frac{1}{p}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)*p)*(a+b*x^n))^(1/p),x, algorithm="maxima")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)*p))^(1/p), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(x^{p\,(n-1)}\,\left(a+b\,x^n\right)\right)}^{1/p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(p*(n - 1))*(a + b*x^n))^(1/p),x)
```

```
[Out] int((x^(p*(n - 1))*(a + b*x^n))^(1/p), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{p(n-1)}\left(a + bx^n\right)\right)^{\frac{1}{p}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**((-1+n)*p)*(a+b*x**n))**(1/p),x)
```

```
[Out] Integral((x**(p*(n - 1))*(a + b*x**n))**(1/p), x)
```

3.452 $\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx$

Optimal. Leaf size=61

$$\frac{x^{\frac{(1-n)(p+1)}{p}}\left(ax^{-\frac{1-n}{p}}+bx^{n-\frac{1-n}{p}}\right)^{p+1}}{bn(p+1)}$$

```
[Out] x^((1-n)*(1+p)/p)*(b*x^(n+(-1+n)/p)+a/(x^((1-n)/p)))^(1+p)/b/n/(1+p)
```

Rubi [A] time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1979, 2000}

$$\frac{x^{\frac{(1-n)(p+1)}{p}}\left(ax^{-\frac{1-n}{p}}+bx^{n-\frac{1-n}{p}}\right)^{p+1}}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

```
[Out] (x^(((1 - n)*(1 + p))/p)*(b*x^(n - (1 - n)/p) + a/x^((1 - n)/p))^(1 + p))/(
b*n*(1 + p))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]
```

Rule 2000

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] && !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \left(x^{\frac{-1+n}{p}}(a+bx^n)\right)^p\,dx &= \int \left(bx^{n+\frac{-1+n}{p}}+ax^{\frac{-1+n}{p}}\right)^p\,dx\\ &= \frac{x^{\frac{(1-n)(1+p)}{p}}\left(bx^{n-\frac{1-n}{p}}+ax^{-\frac{1-n}{p}}\right)^{1+p}}{bn(1+p)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.74

$$\frac{x^{1-n}(a+bx^n)\left(x^{\frac{n-1}{p}}(a+bx^n)\right)^p}{bn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^((-1 + n)/p)*(a + b*x^n))^p,x]
```

```
[Out] (x^(1 - n)*(a + b*x^n)*(x^((-1 + n)/p)*(a + b*x^n))^p)/(b*n*(1 + p))
```

fricas [A] time = 0.41, size = 54, normalized size = 0.89

$$\frac{(bxx^n + ax)\left(bx^n x^{\frac{n-1}{p}} + ax^{\frac{n-1}{p}}\right)^p}{\left(bnp + bn\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="fricas")
```

```
[Out] (b*x*x^n + a*x)*(b*x^n*x^((n - 1)/p) + a*x^((n - 1)/p))^p/((b*n*p + b*n)*x^
n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^n + a)x^{\frac{n-1}{p}}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="giac")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

maple [F] time = 0.73, size = 0, normalized size = 0.00

$$\int \left((b\,x^n + a)\,x^{\frac{n-1}{p}}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((n-1)/p)*(b*x^n+a))^p,x)
```

```
[Out] int((x^((n-1)/p)*(b*x^n+a))^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^n + a)x^{\frac{n-1}{p}}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^((-1+n)/p)*(a+b*x^n))^p,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^n + a)*x^((n - 1)/p))^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x^{\frac{n-1}{p}}\,(a + b\,x^n)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^((n - 1)/p)*(a + b*x^n))^p,x)
```

```
[Out] int((x^((n - 1)/p)*(a + b*x^n))^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x^{\frac{n-1}{p}}\,(a + bx^n)\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**((-1+n)/p)*(a+b*x**n))**p,x)
```

```
[Out] Integral((x**((n - 1)/p)*(a + b*x**n))**p, x)
```

3.453 $\int x^{-1+n-p(1+q)}\left(ax^n+bx^p\right)^q\,dx$

Optimal. Leaf size=39

$$\frac{x^{-p(q+1)}\left(ax^n+bx^p\right)^{q+1}}{a(q+1)(n-p)}$$

```
[Out] (a*x^n+b*x^p)^(1+q)/a/(n-p)/(1+q)/(x^(p*(1+q)))
```

Rubi [A] time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {2014}

$$\frac{x^{-p(q+1)}\left(ax^n+bx^p\right)^{q+1}}{a(q+1)(n-p)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

```
[Out] (a*x^n + b*x^p)^(1 + q)/(a*(n - p)*(1 + q)*x^(p*(1 + q)))
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\int x^{-1+n-p(1+q)}\left(ax^n+bx^p\right)^q\,dx=\frac{x^{-p(1+q)}\left(ax^n+bx^p\right)^{1+q}}{a(n-p)(1+q)}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 1.03

$$-\frac{x^{-p(q+1)}\left(ax^n+bx^p\right)^{q+1}}{a(q+1)(p-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n - p*(1 + q))*(a*x^n + b*x^p)^q,x]
```

```
[Out] -((a*x^n + b*x^p)^(1 + q)/(a*(-n + p)*(1 + q)*x^(p*(1 + q))))
```

fricas [A] time = 0.42, size = 76, normalized size = 1.95

$$\frac{\left(axx^{-pq+n-p-1}x^n+bxx^{-pq+n-p-1}x^p\right)\left(ax^n+bx^p\right)^q}{\left(an-ap+\left(an-ap\right)q\right)x^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="fricas")
```

```
[Out] (a*x*x^(-p*q + n - p - 1)*x^n + b*x*x^(-p*q + n - p - 1)*x^p)*(a*x^n + b*x^
p)^q/((a*n - a*p + (a*n - a*p)*q)*x^n)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(ax^n+bx^p\right)^q x^{-p\left(q+1\right)+n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="giac")
[Out] integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

maple [F] time = 1.27, size = 0, normalized size = 0.00

$$\int x^{n-(q+1)p-1} \left(a\,x^{n}+b\,x^{p}\right)^{q}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x)
[Out] int(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ax^{n} + bx^{p})^{q} x^{-p(q+1)+n-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n-p*(1+q))*(a*x^n+b*x^p)^q,x, algorithm="maxima")
[Out] integrate((a*x^n + b*x^p)^q*x^(-p*(q + 1) + n - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int x^{n-p\,(q+1)-1}\,\left(a\,x^n+b\,x^p\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q,x)
[Out] int(x^(n - p*(q + 1) - 1)*(a*x^n + b*x^p)^q, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n-p*(1+q))*(a*x**n+b*x**p)**q,x)
[Out] Timed out
```

3.454 $\int x^{-1-nq-p(1+q)}\left(x^n\left(a+bx^p\right)\right)^q\,dx$

Optimal. Leaf size=40

$$-\frac{x^{-((q+1)(n+p))}\left(ax^n+bx^{n+p}\right)^{q+1}}{ap(q+1)}$$

```
[Out] -(a*x^n+b*x^(n+p))^(1+q)/a/p/(1+q)/(x^((n+p)*(1+q)))
```

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1980, 2014}

$$-\frac{x^{(q+1)(-(n+p))}\left(ax^n+bx^{n+p}\right)^{q+1}}{ap(q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

```
[Out] -((a*x^n + b*x^(n + p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

Rule 1980

```
Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u,
x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] &&  !Generali
zedBinomialMatchQ[u, x]
```

Rule 2014

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)
*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[n,
 j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int x^{-1-nq-p(1+q)}\left(x^n\left(a+bx^p\right)\right)^q\,dx &= \int x^{-1-nq-p(1+q)}\left(ax^n+bx^{n+p}\right)^q\,dx\\ &= -\frac{x^{-(n+p)(1+q)}\left(ax^n+bx^{n+p}\right)^{1+q}}{ap(1+q)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.95

$$-\frac{x^{-((q+1)(n+p))}\left(x^n\left(a+bx^p\right)\right)^{q+1}}{ap(q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 - n*q - p*(1 + q))*(x^n*(a + b*x^p))^q,x]
```

```
[Out] -((x^n*(a + b*x^p))^(1 + q)/(a*p*(1 + q)*x^((n + p)*(1 + q))))
```

fricas [A] time = 0.41, size = 64, normalized size = 1.60

$$-\frac{\left(bxx^{-(n+p)q-p-1}x^p+axx^{-(n+p)q-p-1}\right)\left(bx^nx^p+ax^n\right)^q}{apq+ap}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="fricas")
```

```
[Out] -(b*x*x^(-(n + p)*q - p - 1)*x^p + a*x*x^(-(n + p)*q - p - 1))*(b*x^n*x^p +
 a*x^n)^q/(a*p*q + a*p)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int ((bx^p + a)x^n)^q \, x^{-p(q+1)-nq-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="giac")
```

```
[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int x^{-nq-(q+1)p-1} \left((b\,x^p + a)\,x^n\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1-n*q-(q+1)*p)*(x^n*(a+b*x^p))^q,x)
```

```
[Out] int(x^(-1-n*q-(q+1)*p)*(x^n*(a+b*x^p))^q,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int ((bx^p + a)x^n)^q \, x^{-p(q+1)-nq-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1-n*q-p*(1+q))*(x^n*(a+b*x^p))^q,x, algorithm="maxima")
```

```
[Out] integrate(((b*x^p + a)*x^n)^q*x^(-p*(q + 1) - n*q - 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(x^n\ \left(a+b\,x^p\right)\right)^q}{x^{n\,q+p\,\left(q+1\right)+1}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1),x)
```

```
[Out] int((x^n*(a + b*x^p))^q/x^(n*q + p*(q + 1) + 1), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1-n*q-p*(1+q))*(x**n*(a+b*x**p))**q,x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```